Emmy Noether - Circle 1 for 2009-2010

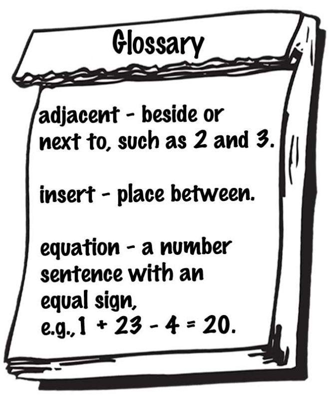
Part I: Problems

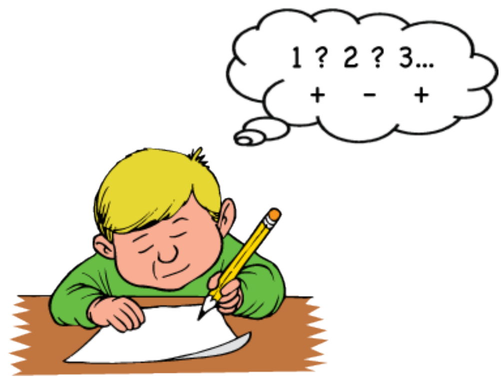
Problem 1

a) Tommy has a problem. He knows the numbers below form an equation if he inserts addition and subtraction signs, and uses two adjacent digits together, in order, to form a two-digit number. Help Tommy make the equation true by inserting addition and subtraction signs, and deciding which two adjacent digits to combine.

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad = \quad 100$$

b) Solve the problem with the digits in reverse order: $9 \quad 8 \quad 7 \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad = \quad 100$.





Extension **:**

1. Solve the problem with the digits in the same order as in part a), but this time use any of the operations addition, subtraction, multiplication, division, powers, and square roots. You may also use two, or three, adjacent digits to form new digits.

Problem 2

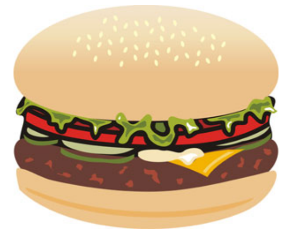
At the school picnic, the teachers are serving hamburgers to the students. Each student gets a single burger, with the choice of any three condiments from the five the teachers provided: lettuce, cheese, tomatoes, pickles, and onions.

a) Ali wants cheese for sure. How many possible combinations of his other two condiments could Ali choose?

b) Tanya wants tomatoes for sure. Does she have fewer, more, or the same number of choices as Ali? Explain your answer.

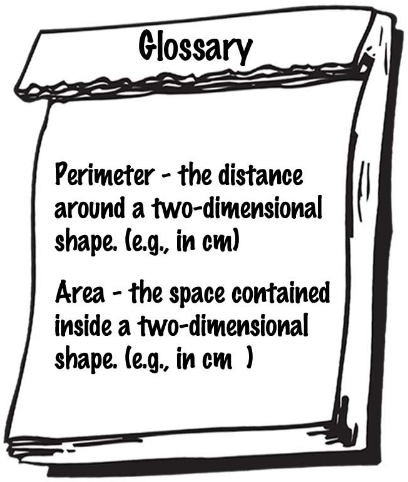
c) Xiao knows he wants either cheese or tomatoes, but not both. How may possible combinations of condiments could Xiao choose?

d) What is the total number of possible combinations of condiments students could choose for their burgers if they have no preferences and use three condiments?



Problem 3

The perimeter of a square is measured in units of length (e.g., mm, cm, m, km, ...), and the area of a square is measured in square units of area (e.g., mm^2, cm^2, m^2, km^2).



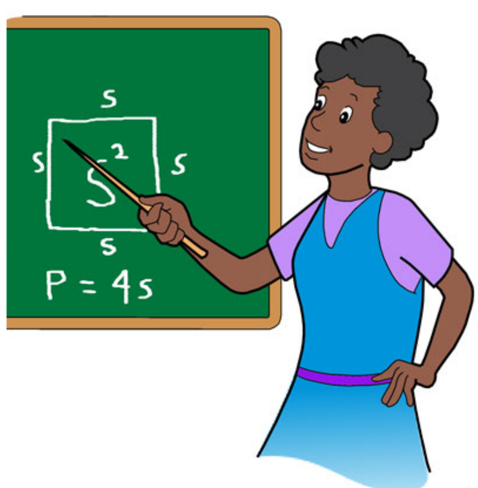
a) Using the grid paper on the following page, draw three squares. The side length of each square should be a whole number.

i) For the first square, the number of units of perimeter is greater than the number of units of area;

ii) For the second square, the number of units of perimeter is less than the number of units of area;

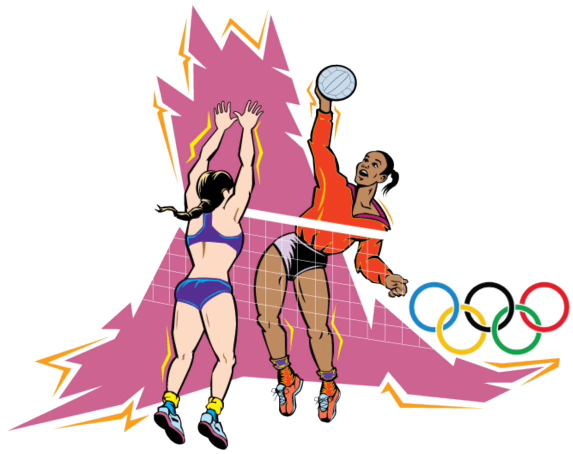
iii) For the third square, the number of units of perimeter is equal to the number of units of area.

b) How many different squares can be drawn for part a)(i)?



Problem 4

OLYMPIC BEACH VOLLEYBALL RULES:



1. A game must be won by at least 2 points.
2. A match has 3 games. To win the match, a team must win 2 of the 3 games.
3. The first two games are played to at least 21 points.
4. Game 3 in the match is played to at least 15 points.

Note: If the game is close (e.g., 21-20) then the winner will have to score more than 21 points (e.g., the final score could be 22-20, or 23-21, or 30-28, etc). Similarly in Game 3, scores may be above 15.

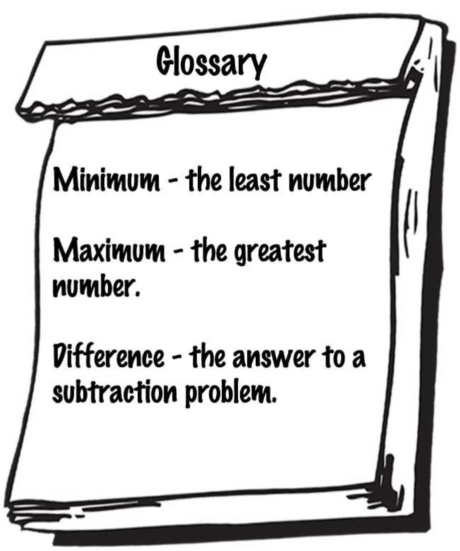
For this problem, you may assume all games were won at either 15 or 21 points.

a) What would be the total difference in points between the two teams in the closest possible match outcome?

b) Assuming a match goes to 3 games, what is the maximum number of points the losing team could score?

c) If one team scores 7 points for every 2 points scored in each game by the other team, how many points will be scored by each team in the match?

d) What is the minimum number of points a team must score to win a match?



	Team A	Team B
Game 1		
Game 2		
Game 3		

Problem 5: Have you got change for a loonie?

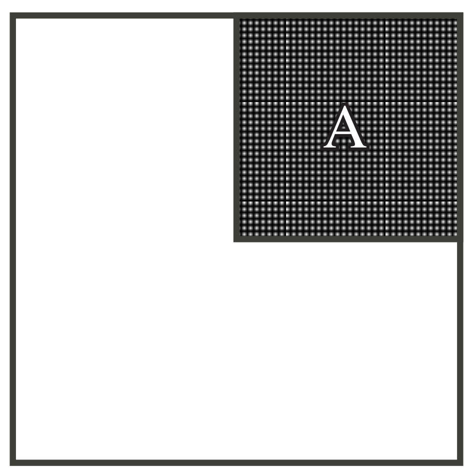
A penny, a nickel, a dime, and a quarter are in a row in front of you. Your goal is to reverse the order of the coins, using ONLY the following moves:

- a coin may be placed on an adjacent coin (on either side) of greater value (e.g., a nickel can be placed on an adjacent dime or quarter, but not on top of a penny);
- a coin may be moved to an adjacent box if it is empty;
- coins may be moved only one at a time (i.e., no moving stacks).

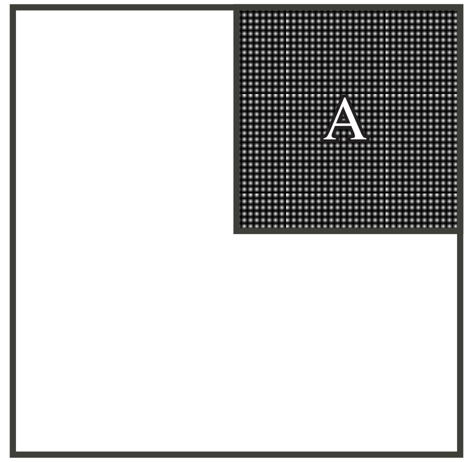
Note: A single coin can move more than one box if, for example, two or more adjacent boxes are empty, or contain coins of greater value. For instance, a legitimate first set of 3 moves would be for the penny to go 3 boxes right, over the nickel and dime onto the quarter.

Problem 6: Inherit this Problem! (For pairs or groups of students)

a) A landowner died and left a large, square piece of land to his wife and six children. His wife received one-quarter of the land (Section A), and his children had to parcel out the remaining three-fourths of land equally. Draw a picture showing how the landowner's children divided the land. Remember, each of the six sections must be the same size and shape.



b) Suppose that there were only four children. Draw a second picture showing how the landowner's children divided the land. Remember, each of the four sections must be the same size and shape.



c) Assume none of the land from b) was fenced. If each side of Section A has length 500 m, how much fencing, in total, would be required to enclose each of the four children's plots you designed in part b)?