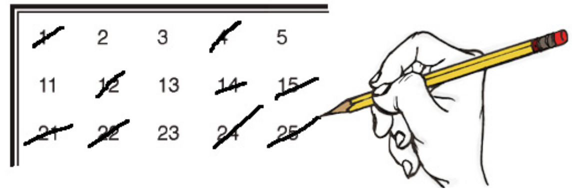
Problem

On the hundred chart below:

- Cross off the number 1.
- Leaving the number 2, cross off all multiples of 2.
- Leaving the number 3, cross off all multiples of 3.
- Leaving the number 5, cross off all multiples of 5.
- Leaving the number 7, cross off all multiples of 7.



To the right of the chart, make a list of all the numbers that are NOT crossed out.
HINT: There should be 24 numbers in your list.

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List of Remaining Numbers

a) Find the factors of each number in your 'List of Remaining Numbers'. What do they have in common? Numbers (other than 1) which have this property are called *prime numbers*.

b) The digit sum of a number is obtained by repeatedly adding digits until a single digit remains. For example, the digit sum of 37 is obtained as follows: $37 \to 3 + 7 = 10 \to 1 + 0 = 1$, giving a digit sum of 1.

   (i) How many of the numbers in your list above have a digit sum of 2?

   (ii) How many of these number have a digit sum of 6?

c) Consider the numbers from 1 to 99.

   (i) Explain why, if one of those numbers has a digit sum of 6, then the sum of its digits must equal to 6 or to 15.

   (ii) Write down all numbers from 1 to 99 that have a digit sum of 6.

*E*xtensions:

1. Determine all the numbers from 101 to 199 that have a digit sum of 6.
2. Show that, if a number from 1 to 99 has a digit sum of 6, then it is divisible by 3.

Hints

Suggestion: The list from the hundred chart, and possibly part a), could be done with the class as a whole, to introduce the more thought-provoking ideas in parts b) and c).

Extension 2:

Hint 1 - What do you know about the sum of the digits of a number that is divisible by 3?

Solution

a),b) The table below shows the list of remaining (prime) numbers and their digit sums.

No.	2	3	5	7	11	13	17	19	23	29	31	37	41	43
Digit Sum	2	3	5	7	2	4	8	1	5	2	4	1	5	7

No.	47	53	59	61	67	71	73	79	83	89	97
Digit Sum	2	8	5	7	4	8	1	7	2	8	7

Note that:

- the factors of each of the remaining (prime) numbers are just 1 and the number itself;
- each of the numbers 2, 11, 29, 47, and 83 has digit sum 2;
- none of the numbers has digit sum 6.

c)(i) If we consider numbers 1 to 9, only number 6 has a digit sum of 6. We now consider the sum of the digits of numbers 10 to 99. For these numbers, the sum of the digits varies from 1 (for number 10) to 18 (for number 99). If this sum is equal to 1, 2, 3, 4, 5, 6, 7, 8 or 9, it is also the digit sum of the number. Therefore if, initially, the sum of the digits of the number is equal to 6, the digit sum of the number is also equal to 6. If the sum of the digits is initially equal to 10, 11, 12, 13, 14, 15, 16, 17 or 18, then only 15 gives a digit sum of 6.b Therefore if a number from 1 to 99 has a digit sum of 6, then the sum of its digits must be equal to 6 or to 15.

(ii) Numbers whose digits have sum 6 are 6, 15, 24, 33, 42, 51 and 60. Numbers whose digits have sum 15 are 69, 78, 87 and 96. Therefore, numbers 6, 15, 24, 33, 42, 51, 60, 69, 78, 87 and 96 have a digit sum of 6.

Extension:

1. Consider the sum of the digits for the numbers 101 to 199, which varies from 2 to 19. As in part c), it must be equal to 6 or to 15 to get a digit sum of 6. To find the numbers whose sum of digits is equal to 6 or to 15, we can work more rapidly if we only look at the units digit and the tens digit whose sum must be equal to 5 or to 14, since the hundreds digit is 1. These numbers are 105, 114, 123, 132, 141, 150, 159, 168, 177, 186 and 195. There are 11 such numbers.
2. We know that a number is divisible by 3 if the sum of its digits is divisible by 3. From part c), we know that numbers whose digit sum is equal to 6 are such that the sum of their digits is equal to 6 or to 15. Since these two sums are divisible by 3, the numbers whose digit sum is equal to 6 must be divisible by 3, and hence will never be prime numbers. Similarly, if the digit sum of a number is 9, then the number is divisible by 9, and hence cannot be prime.