Problem

What's My Number? (A game for pairs of students)

Decide who will go first (Player 1).

a) Here are the basic steps in the game:

- Player 1: Pick a number from 1 to 10 and write it on a piece of paper. Then fold the paper in half so the number is hidden from Player 2.
- Player 2: Try to guess the number. Keep track of how many guesses you need to get the correct number.
- Player 1: Respond to each guess only by saying "greater" or "less", meaning the number you picked is greater than, or less than, the guess made by Player 2.

Once Player 2 has made a correct guess, answer the following questions:

(i) How many guesses did it take Player 2 to determine the number?

(ii) Do you think Player 2 could have got the correct number with fewer guesses? Explain.

Switching roles each time, repeat the game several times, recording the number of guesses needed each time to get the correct number.

b) Repeat part a), but this time use a number from 1 to 50.

c) With your partner, discuss possible stategies to minimize the number of guesses needed to determine the unknown number.

d) Now try part b) again, using whatever strategies you devised in part c). Did you use fewer guesses?

Hints

Part c)

Hint 1 - What number between 1 and 50 should Player 2 guess first in order to eliminate the greatest number of possibilities?

Solution

The basic idea here is that, to eliminate the greatest number of possibilities at any stage, Player 2 should guess the number in the middle of the remaining range, since this narrows the possibilities by half. For example, if the hidden number is from 1 to 25, and Player 2 first guesses 13, this eliminates either 1-12 (if Player 1 responds "greater") or 14-25 (if Player 1 responds "less"). In the latter case, Player 2 would then guess 6 (or 7), again eliminating half the remaining possibilities, and so on.

The effectiveness of this strategy is more evident when the possible range of the hidden number is larger. If the number is from 1 to 1000, then an initial guess of 500 eliminates either all the numbers 501-1000 (if the response is "less"), or all the numbers 1-499 (if the response is "greater"). In the latter case the next guess would be 750, and the response would eliminate another 250 possible choices for the hidden number.

To illustrate this idea in a concrete way, ask a student to think of a secret last name, but not state it. Then use a telephone directory for your city as follows: go to roughly half-way through it, by thickness, and read the name at the top of the right page. The student then responds "before" or "after" if the secret name is before, or after, the name read. Now you know which half of the phonebook contains the name. Divide that half roughly in two, and repeat the process. Once you have isolated the page, repeat the steps using the set of names on that page, each time reading out the middle name of the remaining set. You may be surprised by how quickly the name is revealed, even in a rather large phonebook.