## Problem

a) Melanie has 16 metres of fencing and wants to build a pen for her Dachshund (weiner) dog, Franz. On the attached grid paper, outline as many rectangles as possible with perimeters of 16 metres, and sides lying on grid lines and having lengths equal to whole numbers. Write down the area of each rectangle.

b) Which pen should Melanie build if she wants Franz to have the greatest possible play area? How is the shape of this pen area different from all the other pens?

c) If Melanie wants her dog to have the longest 'run' (i.e., length) possible, which pen should she build? (Remember that the dog must be able to turn around!)

d) Suppose Melanie has 36 metres of fencing. Based on your results in a) and b), make a hypothesis (an educated guess) about what would be the dimensions of the pen of greatest area that Melanie can make with a perimeter of 36 metres, and sides having lengths equal to whole numbers. Then prove your hypothesis by writing down all the possible dimensions and areas (width x length) in a table. (No diagrams are necessary.)

# Hints

## Part a)

*Suggestion:* Before students begin the problem, you may wish to review the meanings of perimeter and area for a rectangle.

**Hint 1 -** If the pen is to have a perimeter of 16 metres, what must be the sum of the lengths of the two sides?

**Hint 2 -** What is the longest possible side?

**Hint 3 -** How is the area related to the side lengths?

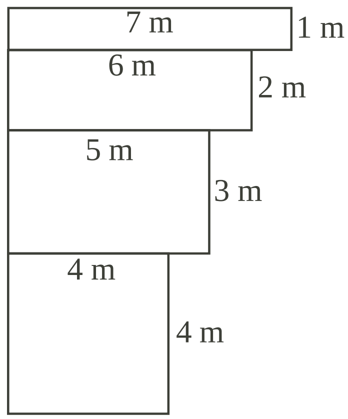
## Part c)

**Hint 1 -** About how much space would a dog need to turn around?

## Solution

a) Since a perimeter of 16 metres implies that the lengths of the two sides of the pen must have a sum of 8 metres, possible pen shapes, dimensions, and areas are:

Possible Pens




| Length (m) | Width (m) | Area (sq. m) |
|---|---|---|
| 7 | 1 | 7 |
| 6 | 2 | 12 |
| 5 | 3 | 15 |
| 4 | 4 | 16 |

b) From part a), we see that the greatest possible play area is given by the square 4 m by 4 m pen, with an area of 16 square metres. All the other pens are rectangles, not squares.

c) It seems reasonable that for a large dog to turn around comfortably, a pen of width at least 2 metres would be desirable. Thus the 2 m by 6 m pen would give the longest run possible.

d) If Melanie has 36 metres of fencing, then the sum of the length and width has to be 18 metres. Based on our observations in a) and b), we hypothesize that the pen of maximum area will be the square 9 m by 9 m pen, with are 81 square metres. To verify our 'educated guess', we note that the possible pen dimensions and areas are:
$1 \times 17 = 17$, $2 \times 16 = 32$, $3 \times 15 = 45$, $4 \times 14 = 56$, $5 \times 13 = 65$,
$6 \times 12 = 72$, $7 \times 11 = 77$, $8 \times 10 = 80$, $9 \times 9 = 81$.
These data confirm our hypothesis that 81 square metres is the maximum possible area.