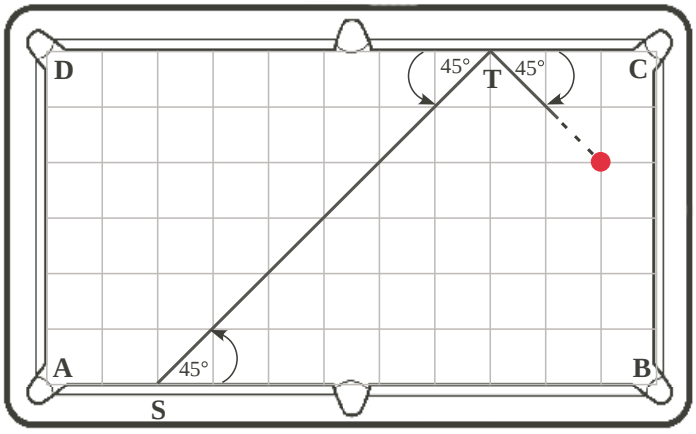
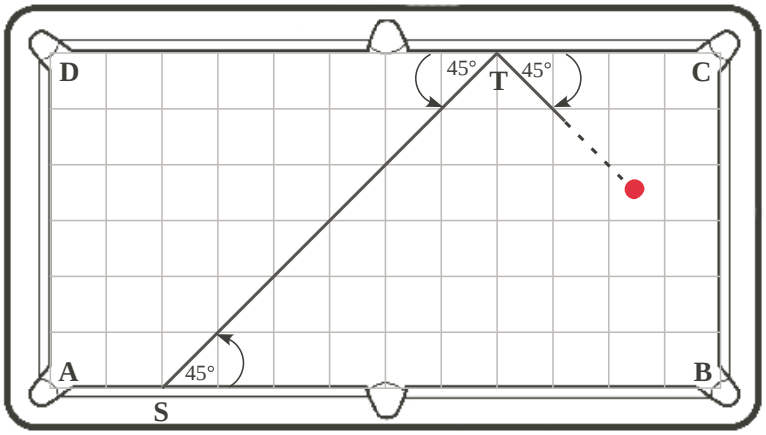
Problem

a) A pool table is 6 units by 11 units, as shown at right, with pockets only at the corners A,B,C,D. Suppose a ball is shot from S at a 45° angle, as shown, and continues to rebound at 45° each time it hits the edge of the table. Will it land in a pocket, eventually? If so, which one?



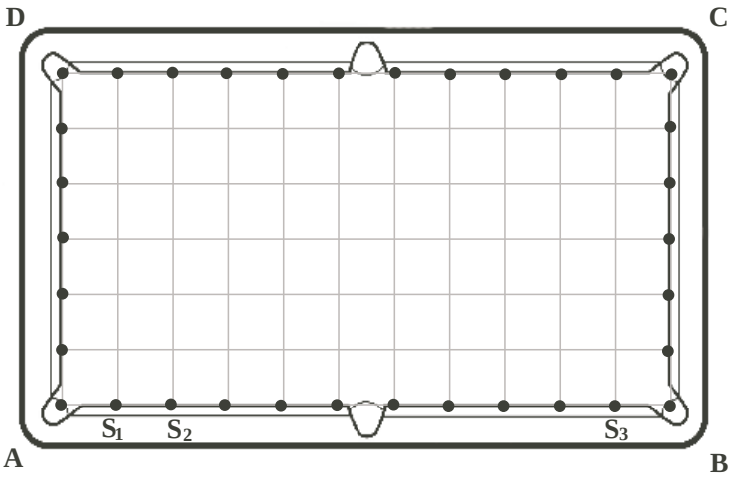
b) Mark on the diagram the position from which you would have to shoot on the table in a) in order to sink the ball in pocket C with just one rebound.

c) Suppose instead that the table is 6 units by 12 units, and S is still 2 units from A. If you shoot from the same initial position S, do you get the same result as in a)? Explain.



Extension:

1. Consider the set of points shown as bold dots on the edges of the 6 unit by 11 unit table from part a). Is there any starting point in the set from which you could shoot the ball with 45° rebounds so that it NEVER lands in a pocket, no matter how many times it rebounds? HINT: Trace the paths (in both directions) from starting points S_1, S_2, S_3. Explain why this reveals the outcome for every path for a ball shot from any point in the set.



Hints

Suggestion: Have students outline several pool tables on graph paper for each question to avoid excessive erasing.

Part a)

Hint 1 - How can the squares on the grid paper help you to draw a 45° angle?

Part b)

Hint 1 - If you shot the ball from corner C at 45° from the table edges, where would it go?

Extension:

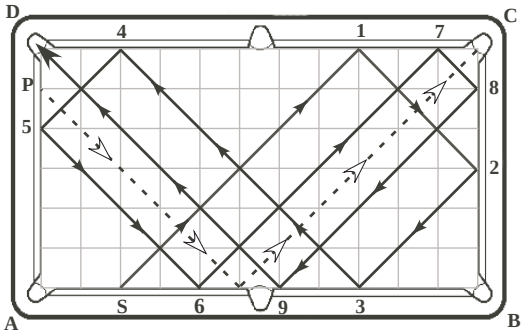
Hint 1 - If a ball lands in pocket C, where was its final rebound? What about if it lands in pocket A? pocket B? pocket D?

Suggestion: Use differenly coloured pencils to experiment with drawing different paths for parts b) and c), and the Extension.

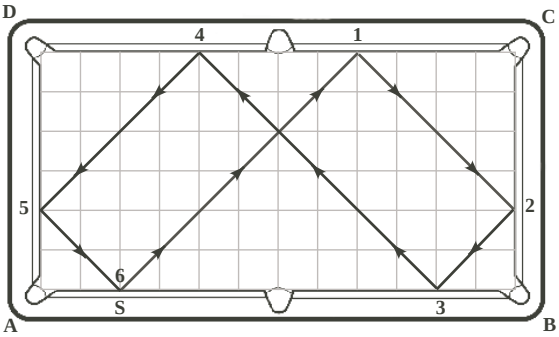
Solution

a) The ball lands in pocket D after 9 rebounds, as shown by the solid path on the diagram.

b) Working backwards from pocket C, we see that we would have to shoot the ball from position P, one unit below pocket D, in order for it to land in C with only one rebound.



c) On this 12 by 6 table, a ball shot from S never lands in a pocket; the sixth rebound occurs at the starting point S, and the ball repeats the same path again and again.



Extension:

1. The diagram at left below shows all the paths taken by a ball shot in either direction from each of the points S_1, S_2, S_3. Note that, as suggested in the HINT, these paths include all possible paths taken by a ball shot from ANY point in the set. That is, eventually, a path from one of these three points hits any other chosen point in the set, after which the two paths would coincide.

   The diagram at right below reveals that a ball which lands in pocket A had its final rebound at the point P_1, moving from right to left; if it lands in pocket B, C, or D, it had its final rebound at P_2 (from left to right), P_3 (from left to right), or P_4 (from right to left), respectively.

   Examining the paths in the left diagram, we see that balls leaving S_1 or S_3 eventually hit either P_2 or P_3, while balls leaving S_2 eventually hit either P_1 or P_4. Since this covers all paths from all points in the set, we see that there is NO point in the set from which a ball can be shot at a 45° angle and NOT land in one of the pockets.