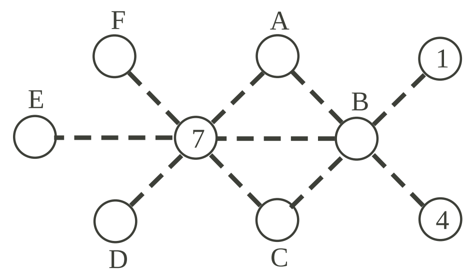
Emmy Noether - Circle 3 for 2008-2009

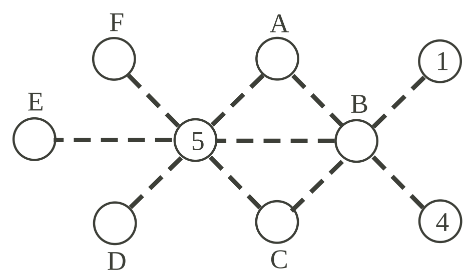
Part I: Problems

Problem 1



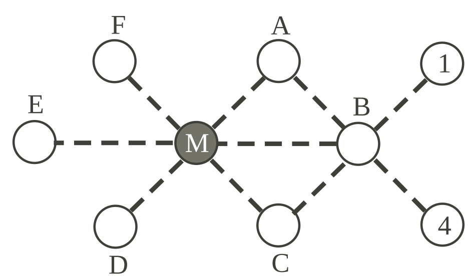
a) In the diagram at right, the sum of the three numbers on each of the five straight lines is equal to 18. Each of the numbers 1,2,3, . . . ,9 is each used once, and only once. Find the numbers which could go in the circles labelled A,B,C,D,E,F.

b) Compare your solution with those of several classmates. Did anyone find a solution different from yours?



c) Try to solve the problem with the 7 replaced by 5, as shown at right. What happens?

Extension:



1. Explore the possible solutions when the number in the shaded circle M is something other than 7 or 5. Hence show that 7 is the only possible choice for which there are solutions to this problem.

Problem 2

a) The skin of an orange has a mass of about $\frac{1}{8}$ of the total mass of the orange. If you buy a 3 kg bag of oranges costing \$ 0.99 per kg, about how much are you paying for the peel? For the fruit itself?

b) A school cafeteria uses 8 bags of oranges per month. One month, the price per bag varies as follows: for week 1, it is \$ 0.99 per kg; week 2, \$ 0.97 per kg; week 3, \$ 1.02 per kg, and week 4, \$ 0.95 per kg. For the cheapest total cost, should the manager have bought all 8 bags the first week, or 2 bags per week over the month?

c) What other costs might be involved that would affect the 'best' choice in part b)? Would your answer change?

Problem 3

In each of the equations (A), (B), (C) below, the bags each contain the same number of loonies, but the number may differ from one equation to another.

(A) 

+ 

= 12 loonies

(B) 

+ 

+ 3 loonies = 7 loonies

(C) 

+ 

+ 

+ 4 loonies = 19 loonies

(i) How many loonies are in each bag in equation (A)? in (B)? in (C)?

(ii) Is it possible to determine the total number of loonies in all seven bags WITHOUT knowing how many are in each bag? If so, show how.

(iii) Match the following story to one of the equations (A), (B), or (C):

Twins Sara and Jesse have each saved the same amount from their allowances. They want to go together to buy a toy spaceship that costs \$7.00, but they need to save \$3.00 more.

(iv) Make up stories that match the other two equations, using different situations from the one in part (iii).

Extension:

1. In the following problems, $\Box$ and $\Diamond$ are two whole positive numbers whose sum is 11, i.e., $\Box + \Diamond = 11$.

    a) What are the possible values of $\Box$ and $\Diamond$?
    b) If it is also true that $\Box - \Diamond = 3$, what could $\Box$ and $\Diamond$ be?
    c) If it is also true that $\Box \times \Diamond = 24$, what could $\Box$ and $\Diamond$ be?
    d) Could $\Box \times \Diamond = 20$? Why or why not?

Problem 4

Xiaomei has four coins in her pocket: a penny, a dime, a nickel, and a quarter. Suppose she reaches in and pulls out one coin, without looking.

a) What is the probability that the coin in her hand is a dime?

b) In the table at right, list the possible sets of coins remaining in Xiaomei's pocket. (One possibility is given.)

c) What is the probability that the total value of the coins remaining in her pocket is greater than 30 cents? less than 30 cents? Explain how you arrive at your answer.

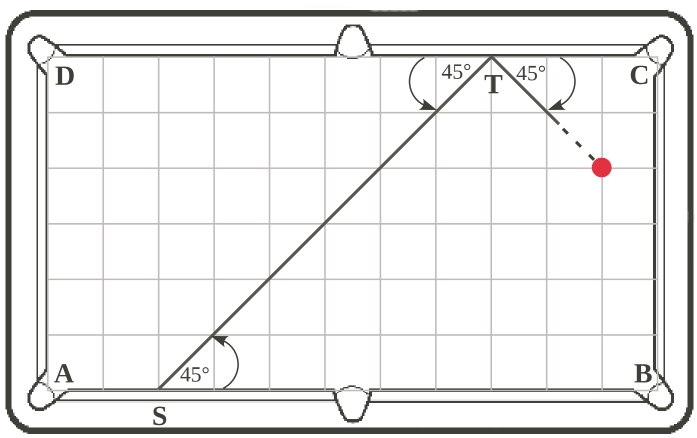
Coin in hand	Remaining coins
5¢	1¢, 10¢, 25¢

Extension:

1. Suppose instead that Xiaomei has two pennies, a nickel, a dime, and a quarter in her pocket.
   (i) Would your answers to a) and c) change? Explain.
   (ii) If she reaches in and pulls out one coin, what is the probability that the change in her pocket totals more than 40 cents?

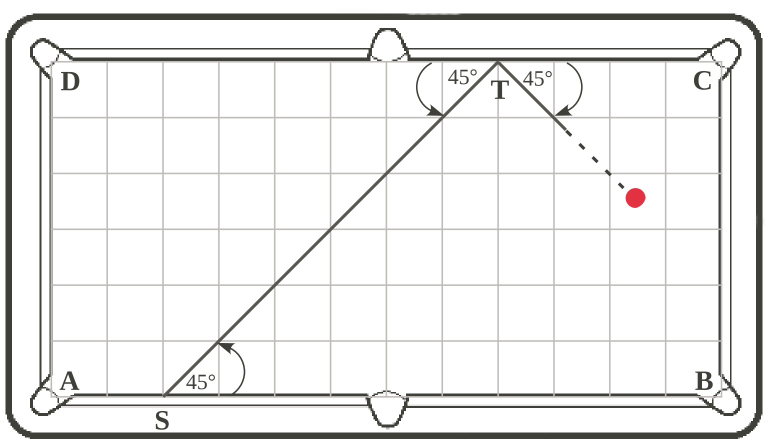
Problem 5

a) A pool table is 6 units by 11 units, as shown at right, with pockets only at the corners A,B,C,D. Suppose a ball is shot from S at a 45° angle, as shown, and continues to rebound at 45° each time it hits the edge of the table. Will it land in a pocket, eventually? If so, which one?



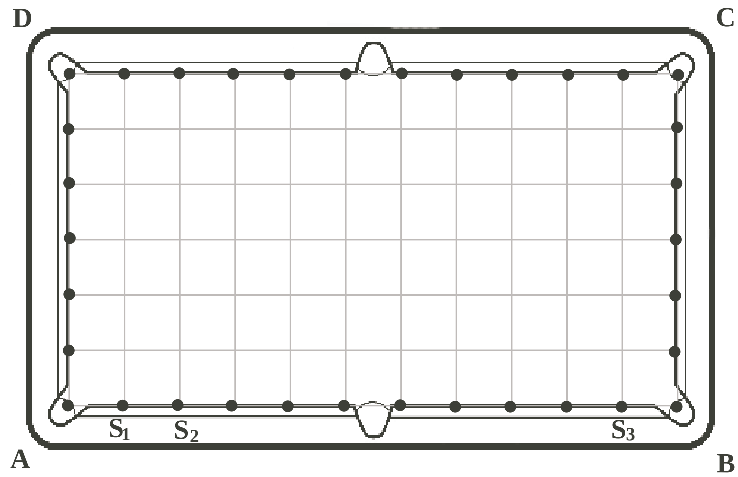
b) Mark on the diagram the position from which you would have to shoot on the table in a) in order to sink the ball in pocket C with just one rebound.

c) Suppose instead that the table is 6 units by 12 units, and S is still 2 units from A. If you shoot from the same initial position S, do you get the same result as in a)? Explain.



Extension:

1. Consider the set of points shown as bold dots on the edges of the 6 unit by 11 unit table from part a). Is there any starting point in the set from which you could shoot the ball with 45° rebounds so that it NEVER lands in a pocket, no matter how many times it rebounds? HINT: Trace the paths (in both directions) from starting points S_1, S_2, S_3. Explain why this reveals the outcome for every path for a ball shot from any point in the set.



Problem 6: The Leaky Tap (Suggested for pairs or groups of students)

For this investigation, you will need a sink with a tap, a watch or clock that measures seconds, a small cup, and a graduated cylinder.

a) Turn on the tap just a little, so that it drips at a slow but steady rate (slow enough that you can count the drips). Once you have achieved this, DO NOT TURN OFF THE TAP until you have completed the following two steps:

   1. Count the number of drips in 20 seconds, and record it in the table below.
   2. Place the cup under the tap and catch the water that drips out for 5 minutes.

b) Transfer the water from the cup to a graduated cylinder to determine the volume of water (in millilitres) that leaked out. Record this volume in #3 in the table.

c) Complete the table at right. (You will have to calculate the entries for items 2, 4, 5, 6, 7.)

d) It has been estimated* that a typical person in the Western world uses about 50 L of water each day for normal household use. How long would your tap have to leak to drip that amount of water? (Recall that 1000 mL = 1 L.)

	Leaky Tap Investigation	
1.	Drips in 20 seconds	drips
2.	Drip rate per minute	drips/min
3.	Volume leaked in 5 minutes	mL
4.	Volume that would leak in 1 hour	mL/hr
5.	Volume that would leak in 1 day	mL/day
6.	Volume that would leak in 1 week	mL/week
7.	Volume that would leak in 1 year	mL/year

e) Compare your results with the other teams in your class. On the web, or in a library, research the average daily water use in the several Third World countries. How does it compare to 50 L?

*The World's Water: The Biennial Report on Freshwater Resources (Island Press, Washington, D.C.)

Extension:

1.(i) Canada has approximately 10×10^6 households. If 1 in 10 of these households has a similar leaky tap to the one in your experiment, how much water is wasted per day? per year?

(ii) If, on average, an inground back yard pool holds 36 000 L, how many such pools could be filled once per year with the water leaked from all these households?