Emmy Noether - Circle 2 for 2008-2009

Part I: Problems

Problem 1

a) I have two digits, I am less than 30, and both 8 and 4 divide me evenly (exactly). The sum of my digits is greater than 6. Who am I?

b) I am greater than 30 but less than 50, and both 7 and 2 divide me evenly. Who am I?

*E*xtensions:

1. I have three digits whose sum is 12. I am less than 200, and evenly divisible by 11. Who am I?
2. Make up your own number clue question. Then find a partner and challenge one another to make sure your clues actually do lead to the number you think they should. Exchange problems with other classmates.

Problem 2

Below are the statistics for Wayne Gretzky's first three years in the NHL, and Sidney Crosby's first two years. (The last column +/- gives goals scored by the player's team while he was on the ice minus goals scored by the opposing team while he was on the ice.)

Wayne Gretzky (The Great One)

Year	Team	Games Played	Goals	Assists	Points	Penalty Minutes	+/-
1979-80	Edmonton	79	51	86	137	21	+15
1980-81	Edmonton	80	55	109	164	28	+41
1981-82	Edmonton	80	92	120	212	26	+81

Sidney Crosby

Year	Team	Games Played	Goals	Assists	Points	Penalty Minutes	+/-
2005-06	Pittsburg	81	39	63	102	21	-1
2006-07	Pittsburg	79	36	84	120	28	+10

a) Calculate Wayne Gretzky's total points in his first three years, and Sidney Crosby's total points in his first two years. How many points would Sid have had to score in 2007-08 to equal Wayne's total for his first three years?

b) In 1981-82, how many points per game did the "Great One" score? How many goals per game? Round your answers to the nearest tenth.

c) For each player, what percentage of their points were goals in their first year in the league? Round your answers to the nearest tenth.

*E*xtension:

1. Who is the better player? Justify your answer using information from the given statistics, your answers above, and any aditional calculations you think are relevant. (You may also wish to look up Crosby's statistics for 2007-08, to see how the two players compare did after three years.)

Problem 3

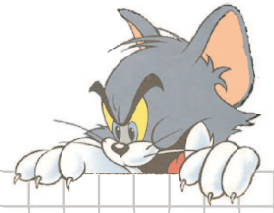
a) A MegaMart parking lot has room for 800 vehicles. One-quarter of the parking spaces are designated for trucks only. On Friday, there were 140 trucks in these spaces, and some cars parked in the lot. If the parking lot was $\frac{5}{8}$ full, how many cars were there in the lot?

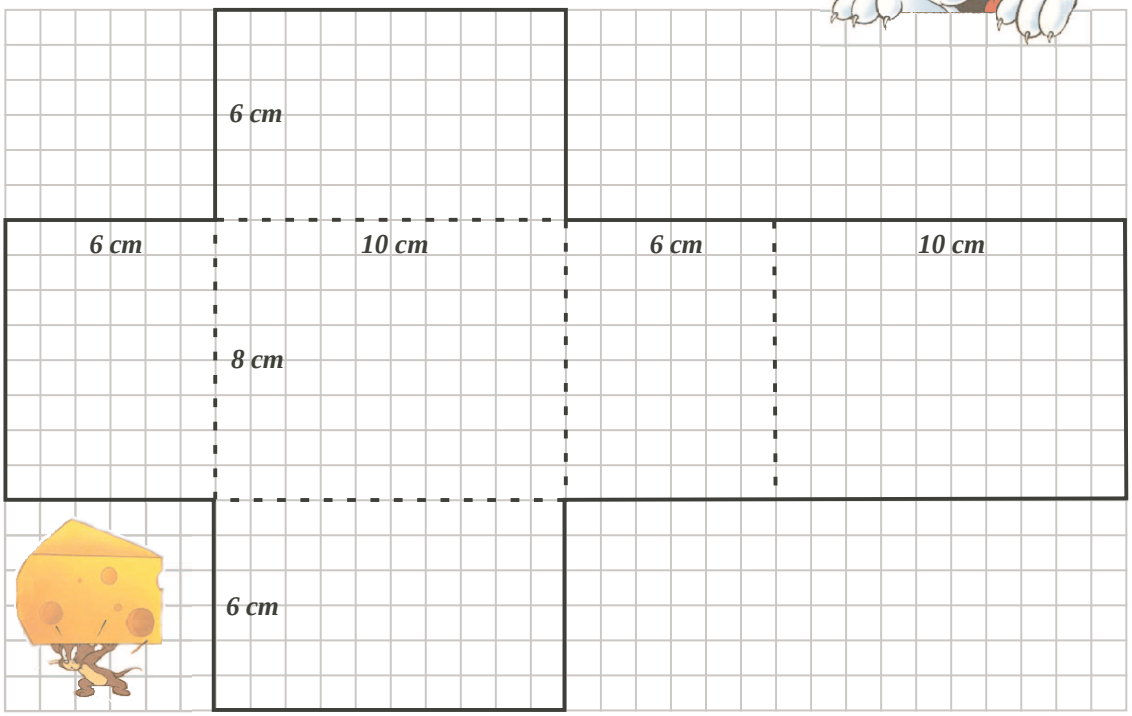
b) Which piece of the given information in part a) is NOT necessary in solving the problem?

c) Of the spaces that are designated for cars, what fraction was empty on Friday?

Problem 4

A rectangular block of cheese covered in wax is 10 cm wide, 6 cm high, and 8 cm deep. (The exterior of the block is covered with wax to keep it fresh.) Use the net given below to construct (mentally or otherwise) a 3-dimensional model of the cheese.





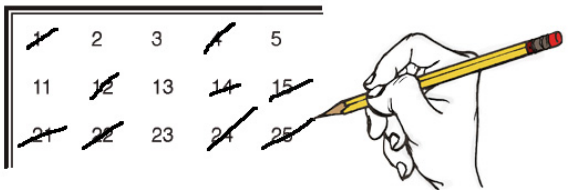
a) How many 2 cm x 2 cm x 2 cm cubes are there in the block?

b) How many of these cubes have no wax on them?

Problem 5

On the hundred chart below:

- Cross off the number 1.
- Leaving the number 2, cross off all multiples of 2.
- Leaving the number 3, cross off all multiples of 3.
- Leaving the number 5, cross off all multiples of 5.
- Leaving the number 7, cross off all multiples of 7.



To the right of the chart, make a list of all the numbers that are NOT crossed out.
HINT: There should be 24 numbers in your list.

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List of Remaining Numbers

a) Find the factors of each number in your 'List of Remaining Numbers'. What do they have in common? Numbers (other than 1) which have this property are called *prime numbers*.

b) The digit sum of a number is obtained by repeatedly adding digits until a single digit remains. For example, the digit sum of 37 is obtained as follows: $37 \to 3+7=10 \to 1+0=1$, giving a digit sum of 1.

(i) How many of the numbers in your list above have a digit sum of 2?

(ii) How many of these number have a digit sum of 6?

c) Consider the numbers from 1 to 99.

(i) Explain why, if one of those numbers has a digit sum of 6, then the sum of its digits must equal to 6 or to 15.

(ii) Write down all numbers from 1 to 99 that have a digit sum of 6.

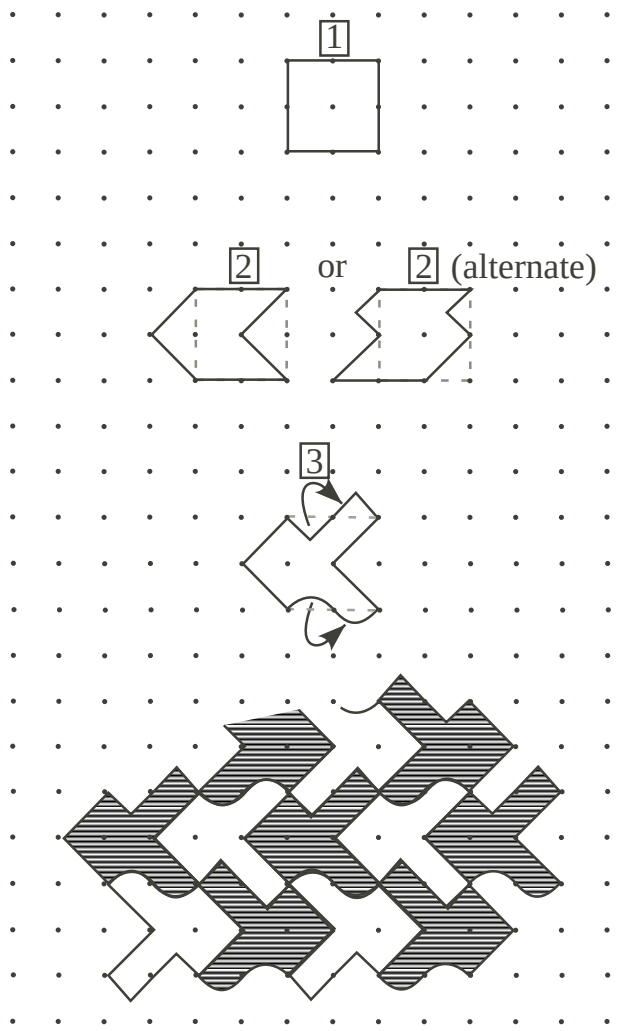
*E*xtensions:

1. Determine all the numbers from 101 to 199 that have a digit sum of 6.
2. Show that, if a number from 1 to 99 has a digit sum of 6, then it is divisible by 3.

Problem 6: Tesselation Creation (Suggested for pairs or groups of students)

Working with your team members, create a tesselation of the plane by following the steps below.

1. Start with a simple shape that tesselates and has at least two pairs of opposite parallel sides (e.g., a square, parallelogram, or hexagon). (In the example shown, this is the square 1 .)
2. Add a shape on one side (say, the left side), and remove it from the opposite side, to form figure 2 . (In the example shown, triangles are added/subtracted.) Figure 2 will also tesselate. (Try it on blank piece of dot paper following this page.)
3. Working with another pair of opposite parallel sides, rotate a piece from half of one side of 2 by a half-turn about the midpoint of that side, and do the same on the opposite side (not necessarily using the same type of piece), to form figure 3 . (In the example, we have rotated a triangle on the left half of the top side of figure 2 , and a semi-circle on the left half of the bottom side.)
4. Finally, use figure 3 to tesselate the plane.



If you have trouble getting started, try doing just step 3, working on the given alternate Shape 2.