# Part II: For the Teacher

## Curriculum Areas

**Problem 1 -** Measurement and Number Sense

**Problem 2 -** Number Sense and Problem Solving

**Problem 3 -** Pattern/Algebra

**Problem 4 -** Geometry and Spatial Sense

**Problem 5 -** Measurement and Number Sense

**Problem 6 -** Number Sense and Problem Solving

## Hints and Suggestions:

**Problem 1 c)**

**Hint 1 -** How many minutes are there in one hour? In one 24-hour day?

**Problem 2**

*Suggestions:*

*Extension:*

1. You may wish to review the meanings of the terms listed in the problem.
2. Once students have created a set of clues for the Mystery Grid, have them trade clues with a classmate. Then, using a blank grid, they can try each other's clues to see whether they lead correctly to the given Grid, and whether or not all the clues are needed.

**Problem 3 c)**

**Hint 1 -** Could the three numbers in each of two rows have the same sum? Why/or why not?

**Hint 2 -** Could the three numbers in each of two columns have the same sum? Why/or why not?

**Hint 3 -** Is there a row with the same sum as one of the columns?

**Hint 4 -** How do the sums of the diagonals compare?

**Hint 5 -** Is it necessary to stick to a row, column, or diagonal?

**Problem 4**

**Hint 1 -** What is a line of symmetry?

*Suggestions:*

1. If your students are new to the names of two-dimensional figures, make a list of the 12 names (from the solutions below), in random order, on the blackboard, and have the class match them to the figures.

2. You may wish to suggest that your students cut out the enlarged figures and fold to verify the lines of symmetry.

3. Before students try the extension, you may wish to have a class discussion about what constitutes a *centre point* of a geometric figure.

**Problem 5 a)**

*Suggestion:* Before students begin the problem, you may wish to review the meanings of perimeter and area for a rectangle.

**Hint 1 -** If the pen is to have a perimeter of 16 metres, what must be the sum of the lengths of the two sides?

**Hint 2 -** What is the longest possible side?

**Hint 3 -** How is the area related to the side lengths?

**Problem 5 c)**

**Hint 1 -** About how much space would a dog need to turn around?

**Problem 6 c)**

**Hint 1 -** What number between 1 and 50 should Player 2 guess first in order to eliminate the greatest number of possibilities?

# Solutions

## Problem 1

a) Since 4 gigs = 4000 megabytes (megs, or mb), and one 4-minute song uses 4 mb, we see that 4000 mb holds $4000 \div 4 = 1000$ songs.

b) Since 30 gigs = 30000 mb, the 30-gig player will hold $30000 \div 4 = 7500$ songs.

c) At 4 minutes per song, 7500 songs take $7500 \times 4 = 30000$ minutes. Each day has 24 hours x 60 minutes per hour = 1440 minutes. Thus 7500 songs take $30000 \div 1440 = 20\frac{5}{6}$ days, or 20 days and 20 hours.

## Problem 2

Clues 2 and 3 tell us that the middle number is 9, and column A is 8, 7, 6, in order downward. Clues 1 and 4 tell us column C contains either 2, 1, 4, or 4, 1, 2. Clues 3 and 5 tell us row 3 contains 6, 3, 2, since 1 is already used in the middle of column C, and 4 is not a factor of 6. Thus the remaining number, 5, must go at the top of column B.

| | A | B | C |
|---|---|---|---|
| 1 | 8 | 5 | 4 |
| 2 | 7 | 9 | 1 |
| 3 | 6 | 3 | 2 |

*Extension:*
Many solutions are possible. The important thing is that the set of clues must uniquely determine the given Mystery Grid. Select two or three solutions and work with the class to 'solve' the problem directly with the suggested clues, in order to verify that they work.

## Problem 3

Using the middle number $n = 14$ in the third week of the second calendar gives the excerpt shown at right, which can be used to illustrate the solutions for a), b), c) as follows. In each case, the appropriate numbers or calculations for this excerpt are given in brackets in the solution.

| | | |
|---|---|---|
| 6 | 7 | 8 |
| 13 | 14 | 15 |
| 20 | 21 | 22 |

a) The middle number (14) is 7 greater than the number directly above it, and 7 less than the number directly below it, due to the difference of one week (7 days) between each pair of dates. This would be true of any calendar laid out horizontally in weeks of 7 days.

b) The middle number (14) is one less than the number on its right, and one greater than the number on its left. This would also be true of any calendar laid out horizontally in weeks of 7 days.

c) The key idea in parts (i), (ii), and (iii) is that for a set of three numbers to have an average of $n$ (14), their sum must equal $3n$ (42). Many such sets exist. From the information in parts a) and b), obvious choices which include the middle number $n$ (14) are the middle column (7 + 14 + 21 = 42), and the middle row (13 + 14 + 15 = 42), or the diagonals (6 + 14 + 22 = 42, or 8 + 14 + 20 = 42). In each case, the first number is less than $n$ by the same amount as the last number is greater than $n$.

Other possibilities (NOT containing the middle number $n$) include, for example, upper right + middle left + middle bottom (8 + 13 + 21 = 42), or upper left + middle right + middle bottom (6 + 15 + 21 = 42), or upper middle + middle right + bottom left (7 + 15 + 20 =

42). In these sets, the key idea is to pick three numbers which are each from a different row and a different column than the others, which guarantees that their sum will be $3n$.

For part (iv), where we want two sets with the same sum, but NOT with sum $3n$, we can pick, for example, the upper left + bottom middle + bottom right ($6 + 21 + 22 = 49$) and the upper right + bottom left + bottom middle ($8 + 20 + 21 = 49$). (Other possible pairs exist.)

*Extension:*

1. Observing the relationship between $n$ and the surrounding numbers, we see that the square can be written in the form shown at right. Thus, for example, the averages of the diagonals are $\frac{1}{3}(n-8+n+n+8) = \frac{3n}{n} = n$, or $\frac{1}{3}(n+6+n+n-6) = \frac{3n}{n} = n$. (Proofs for any of the sets found in parts a), b), and c)(i), (ii) above are similar.) On the other hand, for the sets we chose in part c)(iv) above, the general sums give an average of $\frac{1}{3}(n-8+n+7+n+8) = \frac{3n+7}{n} \neq n$, and $\frac{1}{3}(n-6+n+6+n+7) = \frac{3n+7}{n} \neq n$. (Other pairs may have different sums.)

| | | |
|---|---|---|
| $n-8$ | $n-7$ | $n-6$ |
| $n-1$ | $n$ | $n+1$ |
| $n+6$ | $n+7$ | $n+8$ |

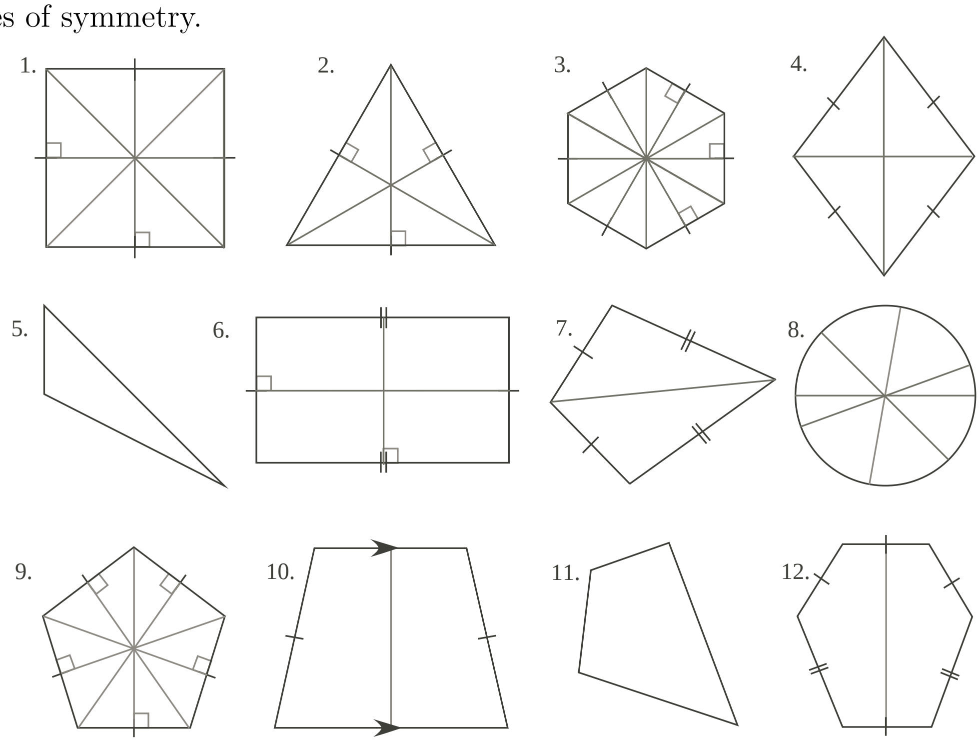
**Problem 4 a)**

Possible names include:

| | | | |
|---|---|---|---|
| 1 - square | 2 - (equilateral) triangle | 3 - (regular) hexagon | 4 - rhombus, diamond |
| 5 - (scalene) triangle | 6 - rectangle | 7 - quadrilateral, kite | 8 - circle |
| 9 - regular pentagon | 10 - trapezoid /quadrilateral | 11 - quadrilateral | 12 - (non-regular) hexagon |

**Problem 4 b), c), d)**

The diagrams below show all lines of symmetry, except for the circle, which has infinitely many lines of symmetry (all diagonals). They reveal that Figures 7, 10, 12 have only one line of symmetry, Figures 4 and 6 have exactly two lines of symmetry, Figure 1 has four, and Figures 3, 8, and 9 have more than 4 lines of symmetry.



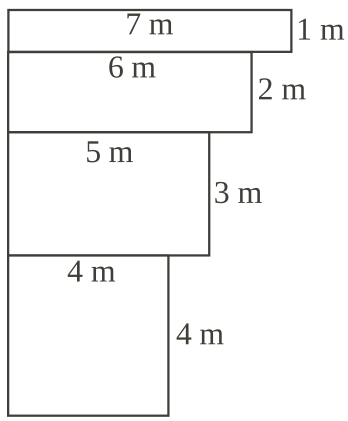
*Extension:*

1. A reasonable definition is to say that a figure has a centre point if it has two or more lines of symmetry meeting in a common point. That point is then the centre point.

**Problem 5**

a) Since a perimeter of 16 metres implies that the lengths of the two sides of the pen must have a sum of 8 metres, possible pen shapes, dimensions, and areas are:

Possible Pens




| Length (m) | Width (m) | Area (sq. m) |
|---|---|---|
| 7 | 1 | 7 |
| 6 | 2 | 12 |
| 5 | 3 | 15 |
| 4 | 4 | 16 |

b) From part a), we see that the greatest possible play area is given by the square 4 m by 4 m pen, with an area of 16 square metres. All the other pens are rectangles, not squares.

c) It seems reasonable that for a large dog to turn around comfortably, a pen of width at least 2 metres would be desirable. Thus the 2 m by 6 m pen would give the longest run possible.

d) If Melanie has 36 metres of fencing, then the sum of the length and width has to be 18 metres. Based on our observations in a) and b), we hypothesize that the pen of maximum area will be the square 9 m by 9 m pen, with are 81 square metres. To verify our 'educated guess', we note that the possible pen dimensions and areas are:
$1 \times 17 = 17$, $2 \times 16 = 32$, $3 \times 15 = 45$, $4 \times 14 = 56$, $5 \times 13 = 65$,
$6 \times 12 = 72$, $7 \times 11 = 77$, $8 \times 10 = 80$, $9 \times 9 = 81$.
These data confirm our hypothesis that 81 square metres is the maximum possible area.

**Problem 6**

The basic idea here is that, to eliminate the greatest number of possibilities at any stage, Player 2 should guess the number in the middle of the remaining range, since this narrows the possibilities by half. For example, if the hidden number is from 1 to 25, and Player 2 first guesses 13, this eliminates either 1-12 (if Player 1 responds "greater") or 14-25 (if Player 1 responds "less"). In the latter case, Player 2 would then guess 6 (or 7), again eliminating half the remaining possibilities, and so on.

The effectiveness of this strategy is more evident when the possible range of the hidden number is larger. If the number is from 1 to 1000, then an initial guess of 500 eliminates either all the numbers 501-1000 (if the response is "less"), or all the numbers 1-499 (if the response is "greater"). In the latter case the next guess would be 750, and the response would eliminate another 250 possible choices for the hidden number.

To illustrate this idea in a concrete way, ask a student to think of a secret last name, but not state it. Then use a telephone directory for your city as follows: go to roughly half-way through it, by thickness, and read the name at the top of the right page. The student then responds "before" or "after" if the secret name is before, or after, the name read. Now you know which half of the phonebook contains the name. Divide that half roughly in two, and repeat the process. Once you have isolated the page, repeat the steps using the set of names on that page, each time reading out the middle name of the remaining set. You may be surprised by how quickly the name is revealed, even in a rather large phonebook.