Problem

Arrange the digits 1, 2, 3,..., 9 into three sets of three digits, using each digit only once, so that the sum of the three digits in each set is the same for all three sets.

Extensions:

1. If the three sets can consist of different numbers of digits, but each set must still have the same sum as each of the other two sets, is there more than one way to form the sets? Explain.

2. Try the original problem with a different set of consecutive numbers.
   (e.g., 2, 3, ..., 10, or 5, 6, ..., 14).

Hints

Suggestion: To enable a trial-and-adjustment approach, have students make number cards for the individual digits 1, 2, ..., 9, and arrange them into sets of three with identical sums.

Hint 1 - How could you discover what should be the sum of the three numbers in each set?

Hint 2 - What numbers could NOT be together in the same set (e.g., Could 7 and 8 be together? Could 1 and 2 be together?)

Note to the Teacher: If Hint 1 is not enough, ask "What is the sum of all nine digits?" and then "How does this help to find the sum for each set?"

Suggestion: An alternative approach, once students have discovered that the sum of each set must be 15, is to suggest they simply make a list of all possible combinations of three digits having a sum of 15. Then they must look for three sets with no overlapping digits.

Solution

Since the sum of the digits 1, 2, ..., 9 is 45, and each of the three sets must have the same sum, each set must have sum 15. Thus the 7, 8 and 9 must be in separate sets. A list of the combinations of three digits with sum 15 (with no repeats in any set) give the possibilities $9 + 1 + 5$, $9 + 2 + 4$, $8 + 1 + 6$, $8 + 2 + 5$, $8 + 3 + 4$, $7 + 2 + 6$, $7 + 3 + 5$.

Selecting from this list, the only possible groups of three sets which use all nine digits just once are
(i) $\{9, 1, 5\}$, $\{8 + 3 + 4\}$, $\{7 + 2 + 6\}$,   or   (ii) $\{9, 2, 4\}$, $\{8, 1, 6\}$, $\{7, 3, 5\}$.

Extensions:

1. Here are two solutions (there are others).
   (i) $\{9, 1, 2, 3\}$, $\{7, 8\}$, $\{6, 5, 4\}$ or   (ii) $\{8, 1, 2, 4\}$, $\{9, 6\}$, $\{7, 3, 5\}$

2. Answers will vary, depending on what set of numbers students start with. Have them check one another's work.