Problem

Aunt Sybil wants to plant herbs to sell at the market. The herb garden is to be a rectangular plot measuring 5 metres by 12 metres. She wants five different herbs, each in a square patch, as follows:

- a 1 m by 1 m square of rosemary;
- a 2 m by 2 m square of sage;
- a 3 m by 3 m square of thyme;
- a 4 m by 4 m square of basil;
- a 5 m by 5 m square of parsley.



Will all of these fit in her plot? Explain your reasoning, and sketch a plan for her garden if possible, using the grid below. How many square metres does Aunt Sybil have unplanted?

Plan for Aunt Sybil's Herb Garden (5 m by 12 m)

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Extensions:

1. a) If Aunt Sybil's plot is only 5 metres by 11 metres, would her five square patches fit? Explain.

Herb Garden (5 m by 11 m)

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b) What if she decides the individual plots do not have to be squares, but must have the same areas as above? Can she then fit them into a 5×11 plot? Explain. Herb Garden (5 m by 11 m)

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Hints

Hint 1 - Which of the five plots should Aunt Sybil fit first into her plan? Why?

Extension 1a):

Hint 1 - If you place the 5×5 square plot at one end, will there be enough space for the 4×4 square and the 3×3 square?

Extension 1 b):

Hint 1 - What is the total area required for all five plots?

Hint 2 - What is the area of the garden?

Solution

Yes, Aunt Sybil can fit all the desired square patches in a 5 metre by 12 metre garden plot. One way to do this is shown at right.



Extension:

- 1 a) If the plot is only 5 metres by 11 metres, then she cannot fit all five square patches, as there is no way to fit both the 3×3 metre and 4×4 metre patches in the 5×6 metre rectangle remaining after the 5×5 metre patch is placed.
- 1 b) If the patches for each herb do not have to be squares, then she needs the following areas:
 - 25 square metres for parsley;
 - 16 square metres for basil;
 - 9 square metres for thyme;
 - 4 square metres for sage;
 - 1 square metre for rose mary.

This gives a total of 55 square metres, which is all the area of a 5×11 metre plot. Thus there are many ways she can fit the desired areas; two are shown below.

Suggestion: Give students the right-hand diagram below, and challenge them to explain why the diagonal patches have the correct areas.



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