## Problem

Pendulum Patterns (suggested for groups of two to four students). Each group will need a 30 cm ruler, a $75-80 \mathrm{~cm}$ length of string, four identical washers, some tape, and a watch or clock that measures seconds.


## Make a Pendulum:

Tie a washer to one end of the string, and tie the other end of the string to one end of a 30 cm ruler taped to a table or desk, as shown, so the pendulum can swing freely.


For the first trial, adjust the string so there is 60 cm between the washer and the ruler. Pull the washer to one side (as shown), keeping the string taut, and release it. Count the number of swings (each time the washer passes from one side to the other under the ruler) in 10 seconds, and record your result in the table below. Repeat the experiment for pendulum string lengths of $50 \mathrm{~cm}, 40$ $\mathrm{cm}, 30 \mathrm{~cm}$ and 20 cm . Then plot your data on the given graph, and use your graph to answer the following questions.
a) How many swings do you think there would be if the pendulum string length were 45 cm ? Explain how you might use your graph to get your answer.
b) How might you use your graph to predict the number of swings for a pendulum string length of 70 cm ? Test your prediction by experimenting with that string length.

Experimental Observations

| String <br> Length | Number of <br> Swings in <br> 10 Seconds |
| :--- | :--- |
| 60 cm |  |
| 50 cm |  |
| 40 cm |  |
| 30 cm |  |
| 20 cm |  |

Number of Swings Compared to String Length


## Extension :

1. a) Tie 2 washers on the end of the string with a length of 60 cm , and again count the number of swings in 10 seconds and record your result in Table 2 below. Repeat for hanging lengths of $50 \mathrm{~cm}, 40 \mathrm{~cm}, 30 \mathrm{~cm}$ and 20 cm .
b) Repeat the experiment in a) with 3 washers, and then 4 washers. Record your results in Table 2.

| String Length | Number of Swings in 10 Seconds |  |  |
| :---: | :---: | :--- | :--- |
|  | 2 washers | 3 washers | 4 washers |
| 60 cm |  |  |  |
| 50 cm |  |  |  |
| 40 cm |  |  |  |
| 30 cm |  |  |  |
| 20 cm |  |  |  |

What do you notice about your results?
How do they compare with your results using only one washer?

Write a short paragraph describing how the number of swings in 10 seconds changes as the string length increases, and as the number of washers changes.

Number of Swings for Different Lengths and Different Weights.

## Hints

Suggestion: Careful measuring, and a consistent set-up are essential in this problem, with only the string-length varying. Students should be reminded to start each pendulum in motion by drawing it aside about $30^{\circ}-45^{\circ}$ from the vertical, which will give suitably wide swings. Note that the actual measure of the angle will not noticeably affect the results, within this range.


## Solution

Suggestion: Average the data from several groups; explain why it is better to use the average of many replications rather than just one set of data. Make a large table and plot on the blackboard or on chart paper. Discuss how to approximate the 'line of best fit' for the data. Here is a sample set of data averaged from five replications of each experiment, and the corresponding plot, with a rough guess 'line of best fit' shown as a solid line.

| String Length | Number of Swings in 10 Seconds |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 washers | 2 washers | 3 washers | 4 washers | 5 washers |
| 60 cm | 12 | 13 | 13 | 14 | 13 |
| 50 cm | 13 | 13 | 15 | 14 | 14 |
| 40 cm | 14 | 16 | 17 | 16 | 15 |
| 30 cm | 16 | 19 | 18 | 18 | 17 |
| 20 cm | 19 | 21 | 23 | 21 | 21 |

Number of Swings

a) For these trials, the 'line of the best fit' indicates that a string length of 45 cm would give about 16 swings in 10 seconds. (This process is called interpolation of the graph.)
b) Extending the 'line of best fit' to string length 70 cm predicts about 11 swings in 10 seconds. (This process is called extrapolation of the graph.)

## Comments:

1. For this data, if you try for a 'line of best fit' for the four longer string lengths ( $30-60 \mathrm{~cm}$ ), it would not quite fit the 20 cm data. This suggests the data actually fits a slightly curved path, such as the dotted line shown.
2. Note that you can also collect data over a longer time. Up to about 30 seconds, this does not appear to affect the slope of the data, ie., in 30 seconds the results appear to be approximately 3 times the data for 10 seconds in our experiments.

Extension: The number of washers (ie., the mass of the pendulum) does not affect the number of swings. Increasing the string length decreases the number of swings, with a drop of about 2 swings in 10 seconds for each increase of 10 cm in string length.

