

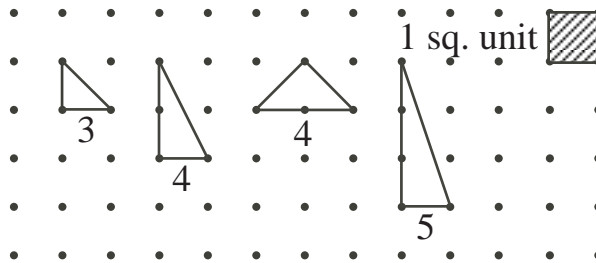
**Problem**

FIGURE IT OUT! (Suggested for groups of two to four students)

Use a geoboard or the dot paper on page 5 to explore the following questions. All vertices are at dots, and NO DOTS can be INSIDE any figure. Use a ruler for accuracy. One square unit of area is the area of the smallest square with four dots at the corners.

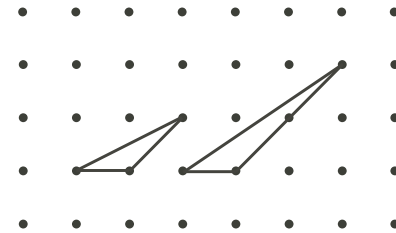
- a) (i) The area of the right-angled triangle with only 3 dots on the boundary is  $\frac{1}{2}$  square unit. What is the area of each right-angled triangle with 4 dots on the boundary? with 5 dots on the boundary? Record these in the chart.

No. of Dots	Area (sq. units)
3	$\frac{1}{2}$
4	
5	
6	
7	

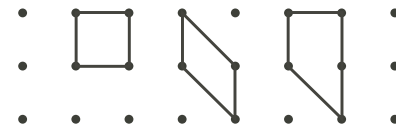


- (ii) Draw a triangle with 6 dots on the boundary and record its area in the chart. Repeat for 7 dots. Use the dot paper on page 5.
- (iii) How much area is added with each increase of one in the number of dots?
- (iv) If a triangle had 20 dots on the boundary, what would be the area of the triangle?

- b) Repeat part of a) for any triangle (not necessarily right-angled).

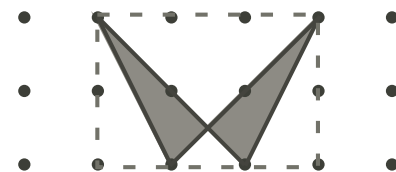


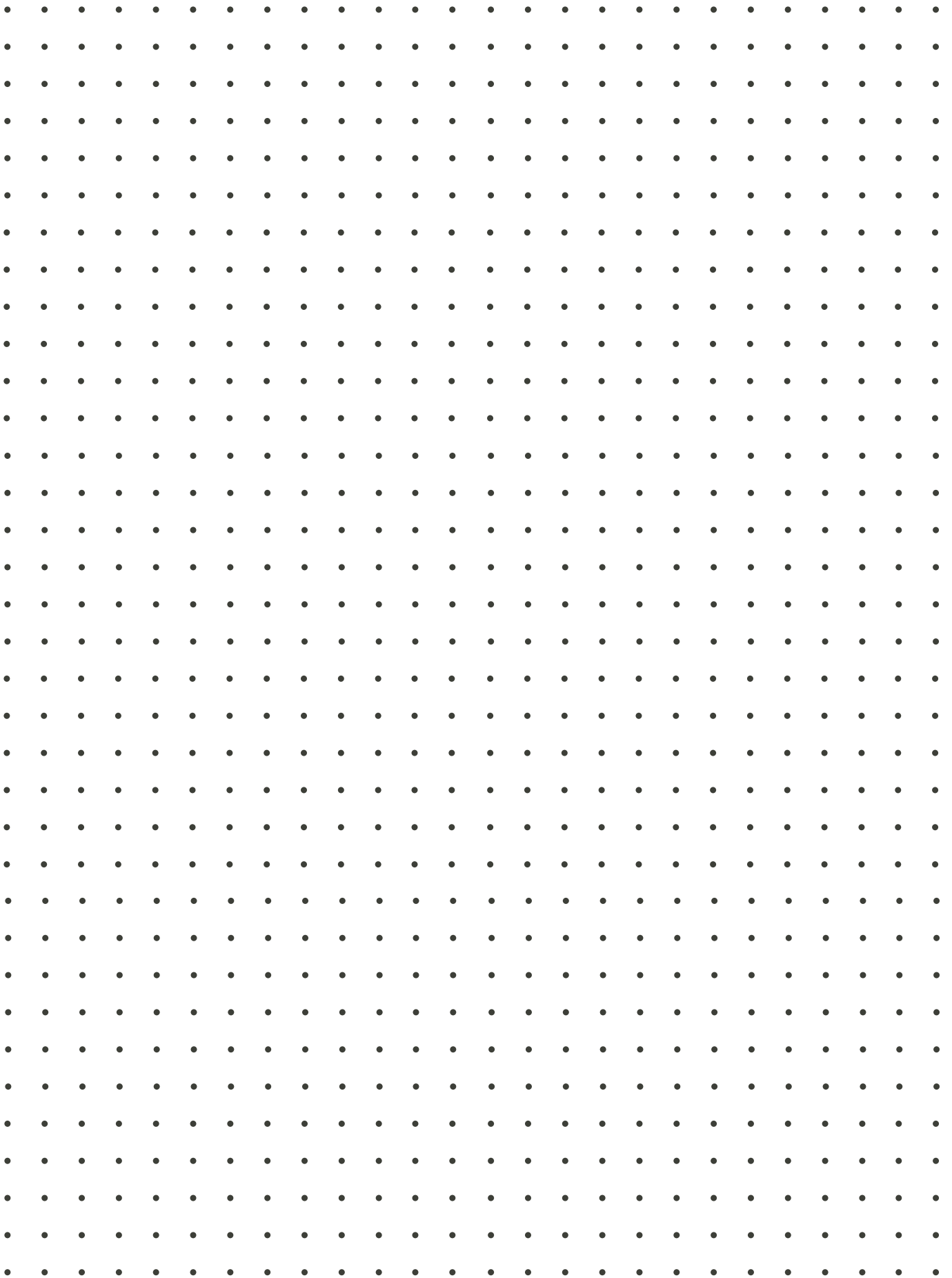
- c) Repeat parts a) and b) for four sided figures, using 4 dots, then 5 dots, then 6 dots, etc. Start with rectangles, then generalize to any quadrilateral.



*Extensions:*

1. Draw at least 8 polygons, each with an area of 2 square units.
2. By finding the unshaded area inside the dashed rectangle, find the shaded area of the W-shaped region.

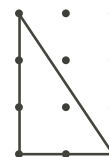




**Hints**

**Part a)**

*Note to the teacher:* We cannot use a base of more than one unit without enclosing a dot, except for the case when the base and height are both 2.



**Hint 1** - What happens if you make the base of the right-angled triangle 2 units long, and the height 3 or more units?

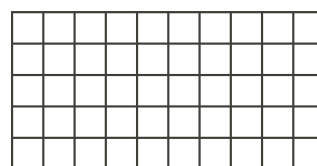
**Part b)**

**Hint 1** - What is the area inside the dashed rectangle, but outside the first triangle?

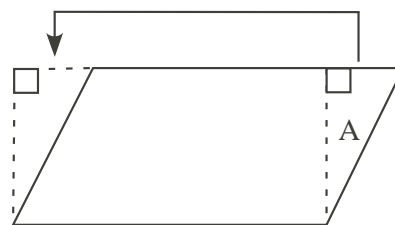


*Suggestion:* If this proves difficult, discuss the area outside the first triangle as “half of a  $2 \times 1$  rectangle (area A) plus half of a  $1 \times 1$  square (area B)”. Thus the triangle has area  $2 - 1 - \frac{1}{2} = \frac{1}{2}$ . Alternatively, use the fact that the triangle has base 1 and height 1, and hence area  $\frac{1}{2}(1 \times 1) = \frac{1}{2}$ . If students are not familiar with the formula  $\frac{1}{2}$  base  $\times$  height, it can be motivated as follows:

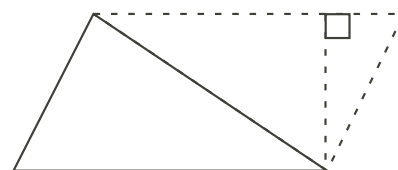
1. For a rectangle, we know that the area equals the product of the number of units in the base length and the number of units in the height, which gives the total number of unit squares (or square units) in the rectangle.
2. For a parallelogram of the same base and height as the rectangle, if we cut off the triangle A on the right and add it to the left side as shown, we obtain a rectangle of exactly the same base and height. Hence the area of a parallelogram also equals its base length times its height.
3. Since any triangle can be pictured as half of a parallelogram, we see that the area of a triangle is  $\frac{1}{2}$  base  $\times$  height.



Area = base  $\times$  height



Area = base  $\times$  height



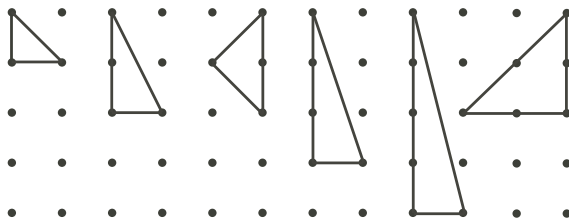
Area =  $\frac{1}{2}$  base  $\times$  height

Note that a diagonal of a parallelogram divides it into two identical triangles, each with area half that of the parallelogram.

### Solution

FIGURE IT OUT!

- a) (i) Each right-angled triangle occupies half of a rectangle or square. Thus the area is easy to calculate.

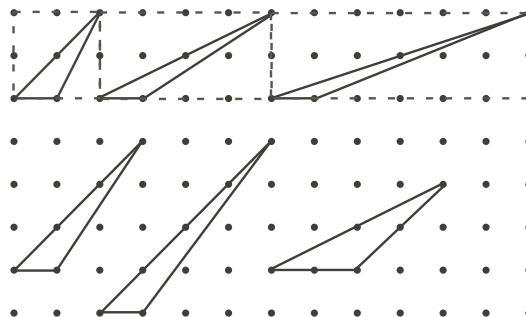


No. of Dots	Area (sq. units)
3	$\frac{1}{2}$
4	1
5	$1\frac{1}{2}$
6	2

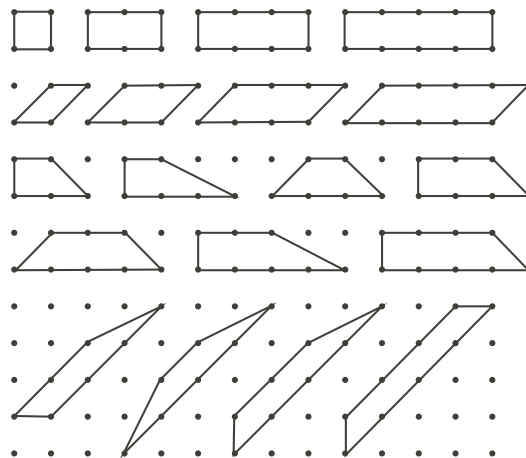
(ii) For each increase of 1 in the number of dots,  $\frac{1}{2}$  square unit of area is added.

(iii) A triangle with 20 dots on the boundary has an area of  $1 + \frac{1}{2}(20 - 4) = 9$  sq. units.

- b) For non-right-angled triangles, we see that all those through 4 dots have base 1 unit and height 2 units and hence area 1 square unit, as in a). It is clear that a similar pattern to a) holds for all triangles of base 1 unit, e.g., 5 dots has height 3 and area  $\frac{3}{2}$ , 6 has height 4 and area 2, etc. A triangle with 6 dots could also have a height 2 and base 2 (hence area 2) as shown at the far right.

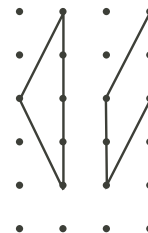


- c) For rectangles, the areas are 1, 2, 3, 4, ... for 4, 6, 8, 10, ... dots, respectively, and the same holds for parallelograms. For trapezoids, those with an even number of dots have the same pattern as rectangles, while those with an odd number fall half-way between. Thus the pattern for the quadrilaterals appears to be: for 4, 5, 6, 7, 8, 9, 10, ... dots, the areas are 1,  $1\frac{1}{2}$ , 2,  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$ , 4, ... etc., the same as for triangles.



Extensions:

- For eight polygons with area 2 square units, we can use the four triangles with 6 dots, the rectangle, parallelogram, and two trapezoids with 6 dots, giving 8 polygons already shown above with area 2. Two others are shown at right.



2. Note that areas  $A_1$  and  $A_2$  are both  $\frac{1}{2}$  square unit: areas  $B_1$ ,  $B_2$  and  $B_3$ , are each 1 square unit, and area  $C_1$  and  $C_2$  are each  $\frac{1}{4}$  square unit. Thus the unshaded area is  $(2 \times \frac{1}{2}) + (3 \times 1) + (2 \times \frac{1}{4}) = 4\frac{1}{2}$  square units, leaving a W-shaped area of  $6 - 4\frac{1}{2} = 1\frac{1}{2}$  square units.

