Emmy Noether - Circle 2 for 2007-2008

Part I: Problems

Problem 1

Arrange the digits 1, 2, 3,. . . , 9 into three sets of three digits, using each digit only once, so that the sum of the three digits in each set is the same for all three sets.

Extensions:

1. If the three sets can consist of different numbers of digits, but each set must still have the same sum as each of the other two sets, is there more than one way to form the sets? Explain.

2. Try the original problem with a different set of consecutive numbers.
(e.g., 2, 3, . . . , 10, or 5, 6, . . . , 14).

Problem 2

Farmer Ben has a rectangular vegetable garden that measures 4.5 metres by 9 metres. Unfortunately, the local hungry rabbits are eating all he grows. Being a peaceful man, Ben decides to build a 7.5 metre by 12 metre fence around the garden to protect his harvest.

a) If the garden is centered in the fenced area, how much distance is there between the fence and each edge of the garden?

b) If Ben expands his garden next year to use the whole area inside the fence, how much more planting area will he have, compared to this year?

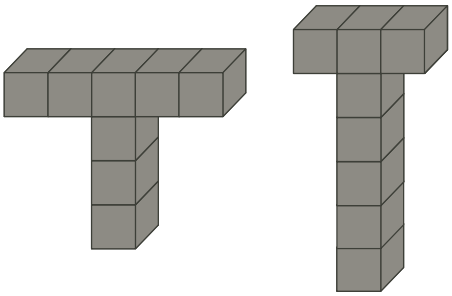
Problem 3

a) Make a list of all the factors of 24. How many different triangles can be formed using any three of these numbers as the side lengths?

b) Write a sentence explaining why not all combinations of any three numbers from your list in a) will form a triangle.

Problem 4

Eight cubes are connected in the shape of a capital "T". Suppose you spray paint all the outside surfaces black.



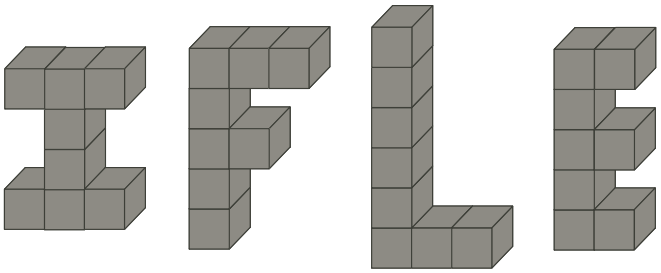
a) How many cube faces would be painted black?

b) Does your answer to a) change if you make the "T" six blocks tall and the three blocks wide?

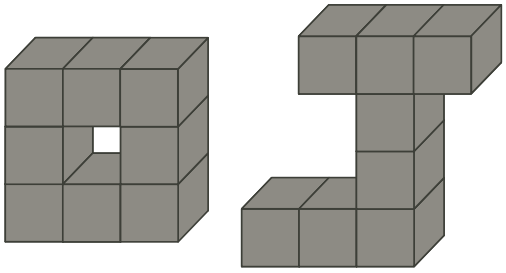
c) If you took apart the "T", describe the different ways the individual cubes would be painted in each of a) and b), WITHOUT turning the cubes.

Extensions:

1. If you use 8 cubes to build the letters shown below, will there be more or fewer cube faces painted black than for the T?

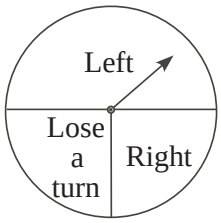


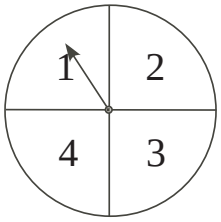
2. Does your answer from 1 hold for the letters at the right?



Problem 5

A board game is played using the two spinners shown at the right, one which tells the number of squares the player can move, and the other tells the direction, or that the player loses a turn (i.e., can make no move at all).





a) Using the number spinner only, what is the probability of spinning a 4? a 2?

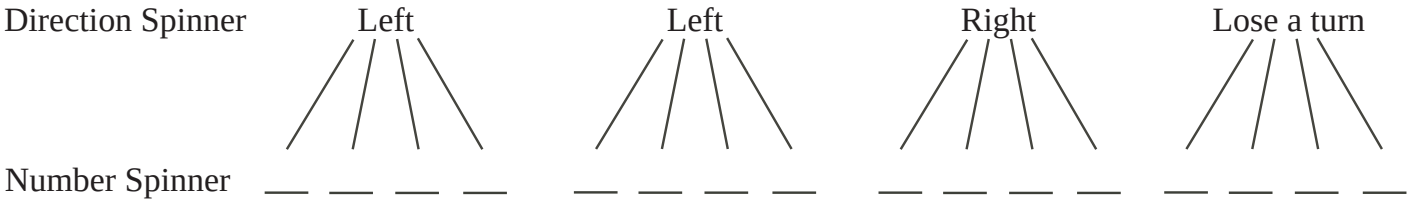
b) Using the direction spinner only, what is the probability of losing a turn?

c) When a player spins both spinners, find the probability of each of the following outcomes:

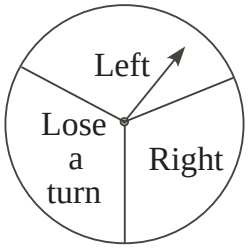
(i) the player moves 2 squares to the left;

(ii) the player moves 1 square to the right.

Complete the tree diagram below to help you decide. Note that there are two 'left' choices because you are twice as likely to spin a 'left' as a 'right' or a 'no turn'.



Extensions:



1. When a player spins both spinners for his turn in the game, is there an outcome which has a probability of $\frac{1}{12}$? Explain.

2. How would the tree diagram change if the direction spinner had three equal subdivision, as shown at right? Would this change your answer to Extension 1. ?

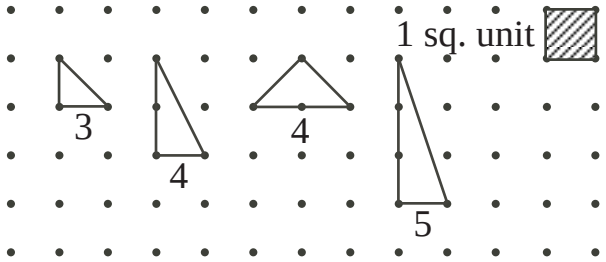
Problem 6

FIGURE IT OUT! (Suggested for groups of two to four students)

Use a geoboard or the dot paper on page 5 to explore the following questions. All vertices are at dots, and NO DOTS can be INSIDE any figure. Use a ruler for accuracy. One square unit of area is the area of the smallest square with four dots at the corners.

No. of Dots	Area (sq. units)
3	$\frac{1}{2}$
4	
5	
6	
7	

a) (i) The area of the right-angled triangle with only 3 dots on the boundary is $\frac{1}{2}$ square unit. What is the area of each right-angled triangle with 4 dots on the boundary? with 5 dots on the boundary? Record these in the chart.

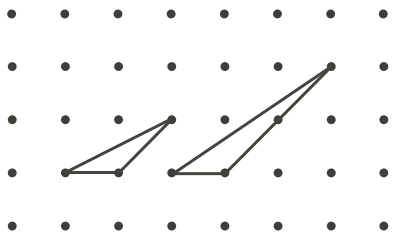


(ii) Draw a triangle with 6 dots on the boundary and record its area in the chart. Repeat for 7 dots. Use the dot paper on page 5.

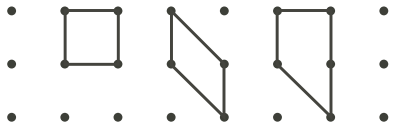
(iii) How much area is added with each increase of one in the number of dots?

(iv) If a triangle had 20 dots on the boundary, what would be the area of the triangle?

b) Repeat part of a) for any triangle (not necessarily right-angled).



c) Repeat parts a) and b) for four sided figures, using 4 dots, then 5 dots, then 6 dots, etc. Start with rectangles, then generalize to any quadrilateral.



Extensions:

1. Draw at least 8 polygons, each with an area of 2 square units.

2. By finding the unshaded area inside the dashed rectangle, find the shaded area of the W-shaped region.