Part II: For the Teacher

Curriculum Areas

Problem 1 - Number Sense

Problem 2 - Spatial Sense/Visual Logic

Problem 3 - Number Sense

Problem 4 - Geometry

Problem 5 - Probability and Data Management

Problem 6 - Measurement and Data Management

Hints and Suggestions:

Problem 1 a)

Hint 1 - How much change did Axel get?

Hint 2 - Make a chart/table (as shown at right).

Hint 3 - Given that he has at least 2 dimes, how much change consists of other coins?

Dimes	Nickels	Pennies	Total No. of Coins
2	?	?	
2	?	?	
⋮			
3	?	?	
⋮			
4	?	?	

Problem 1 b)

Hint 1 - Would the cashier give as many coins as possible, or the fewest coins possible?

Suggestion: Give the students mock coins to work with.

Problem 2

Suggestion: If students are having trouble visualizing, use the template on the following page, photocopied on cardstock, to make a set of cards with six holes punched in each. Then give them lengths of string to make each of views a) - f), so they can turn them over to seek solutions.

Problem 3

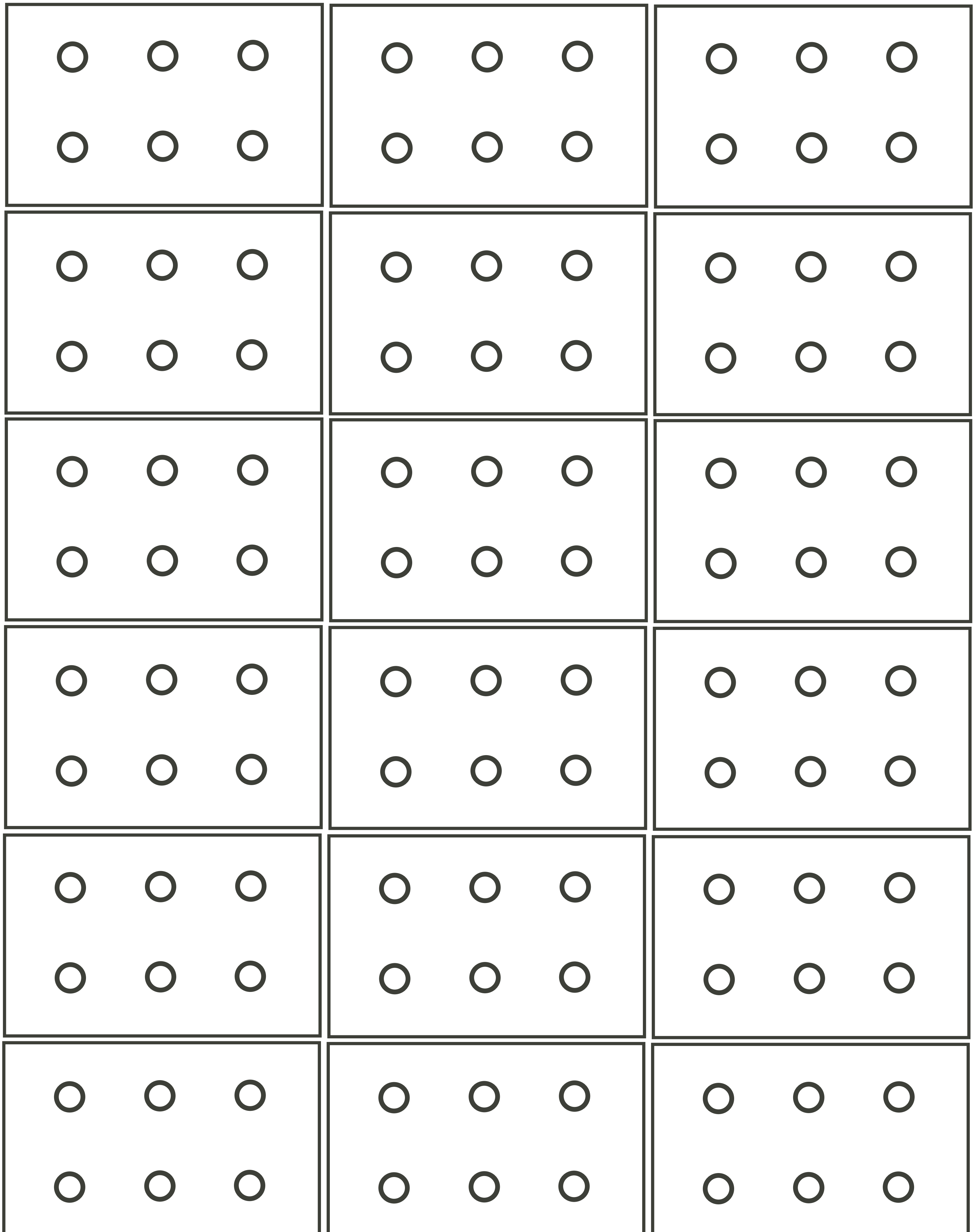
Hint 1 - How many minutes are there in one hour?

Hint 2 - How many hours are there in a day? Minutes in a day?

Hint 3 - How many days are there in one year? Minutes in one year?

Extension:

Hint 1 - How does your age in minutes compare to the given age?



Problem 4

Hint 1 - What skeletons can you make with a square base?

Hint 2 - What skeletons can you make with a triangular base?

Hint 3 - Can you make a skeleton with a pentagonal base using exactly 12 toothpicks?

Hint 4 - Can you make a skeleton with a hexagonal base? Why, or why not?

Suggestion: This activity is best done by supplying students with an ample quantity of toothpicks and mini-marshmallows or licorice bits, and letting them construct the skeletons. Note that it will occupy at least one full math period!

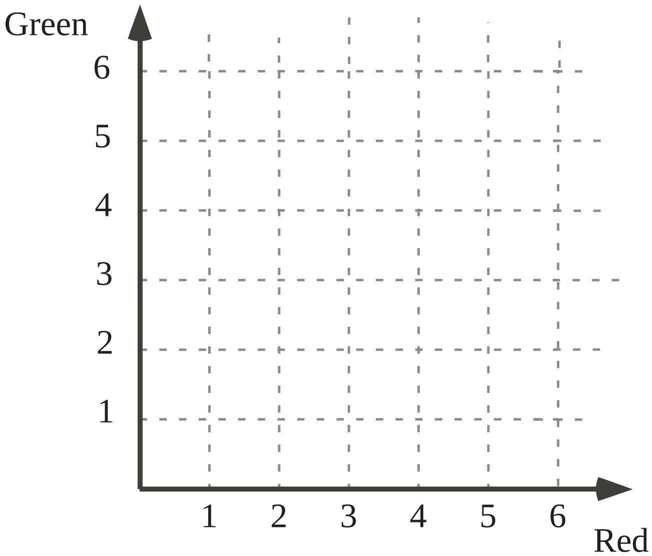
Problem 5

Hint 1 - If you rolled 1 and 4 with the red dice and 5 and 3 with the green dice, what would be your four possible choices of points?

Note to Teacher: A template of six grids is supplied on the following page for use if you wish to have the students play multiple games.

Problem 6

Suggestion: Careful measuring, and a consistent set-up are essential in this problem, with only the string-length varying. Students should be reminded to start each pendulum in motion by drawing it aside about 30° - 45° from the vertical, which will give suitably wide swings. Note that the actual measure of the angle will not noticeably affect the results, within this range.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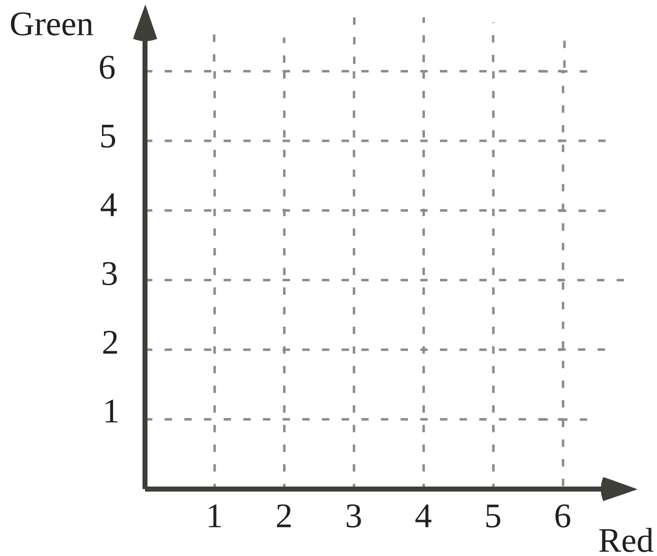
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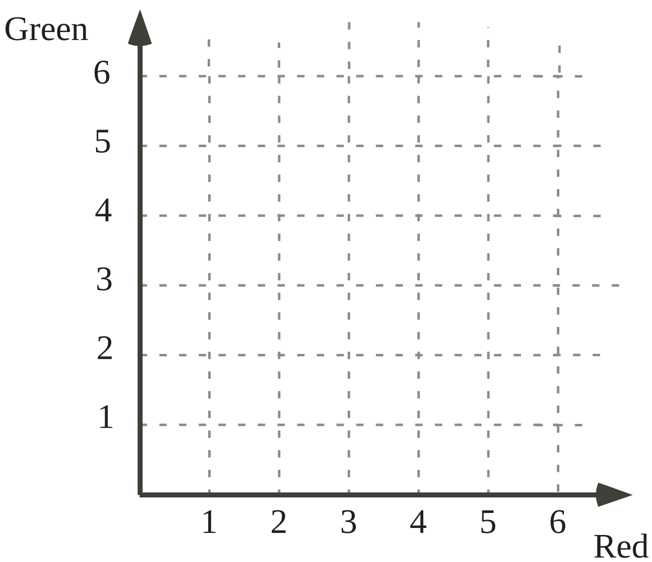
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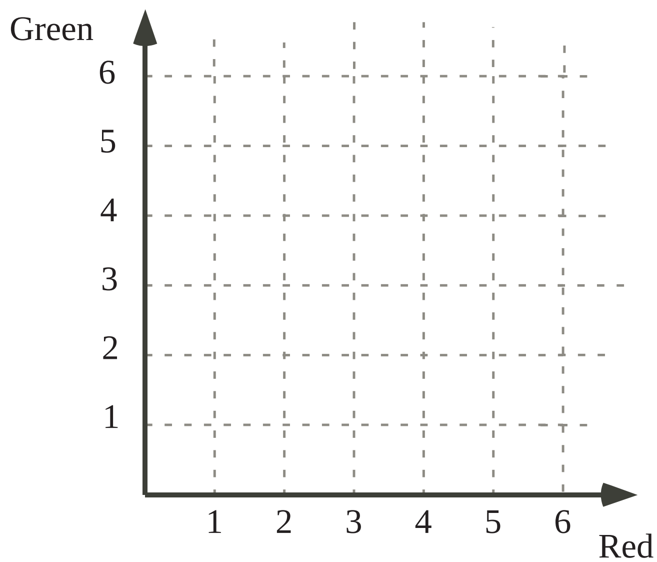
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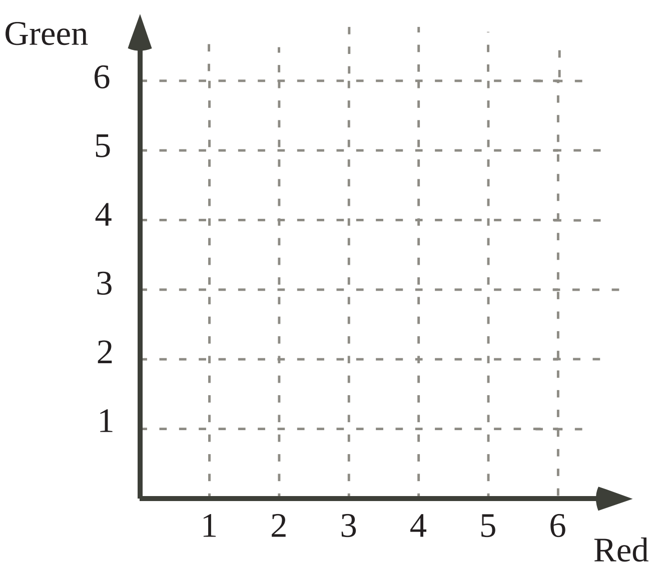
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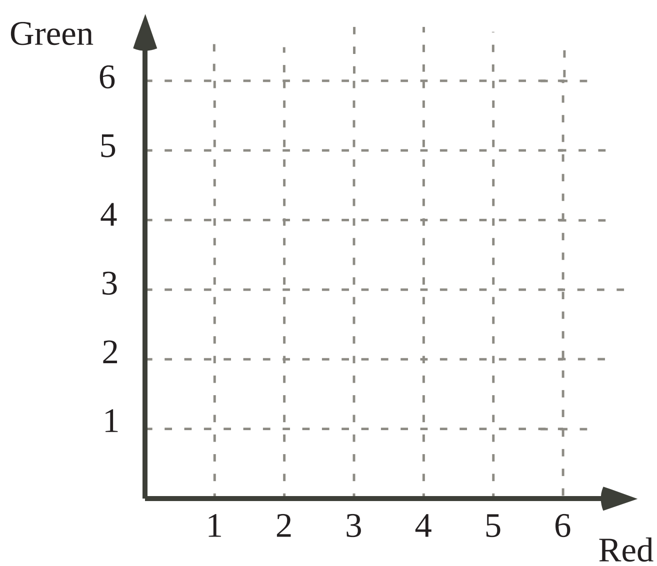
Green
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Red

Solutions

Problem 1

a) Axel should have a total of 41¢ in change. If he has two dimes, he must have 21¢ in nickels and pennies; if he has three dimes, he must have 11¢ in nickels and pennies; if he has four dimes, he can have just one penny. The possible combinations are shown in the table to the right.

Suggestion: You many wish to discuss with students the patterns in the total number of coins (e.g., when one nickel is replaced by five pennies, the number of coins increases by four.)

Dimes	Nickels	Pennies	Total No. of Coins
2	0	21	23
2	1	16	19
2	2	11	15
2	3	6	11
2	4	1	7
3	0	11	14
3	1	6	10
3	2	1	6
4	0	1	5

b) Without assuming he has two dimes, the most likely combination would be one quarter, one dime, one nickel, and one penny, which gives the fewest number of coins.

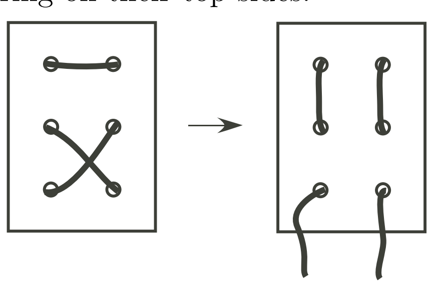
Extensions:

1. If Axel has 10 coins, he could have the correct change, with 3 dimes, 1 nickel, and 6 pennies.
2. If Axel has 18 coins, then he must have incorrect change. Adding to the table in part a) the possibilities for one quarter, and for no quarters (with no dimes, or 1 dime), reveals that no combination of 18 coins has a value of 41¢.

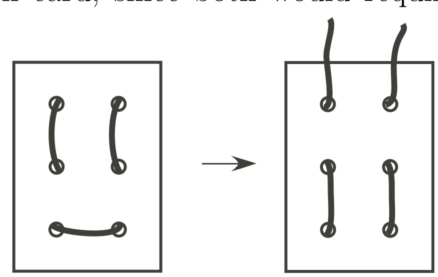
Problem 2

Cards c) and e) could not be the underside of the given card, since both would require 'vertical' segments of string on their top sides.

c)



e)



Problem 3

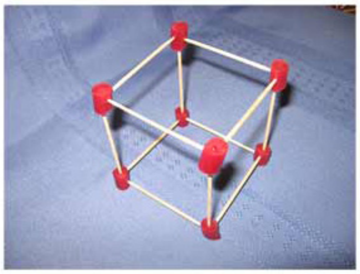
There are $60 \times 24 \times 365 = 525\,600$ minutes in one year. Thus a 10 year old student would be $5\,256\,000$ minutes old. (If you use $365\frac{1}{4}$ days in a year, the figure is $525\,960$ minutes/year; students may also suggest adding two or three days for leap years.) They may also use the estimate of about half a million minutes per year, giving a ten year old's age as about five million minutes.

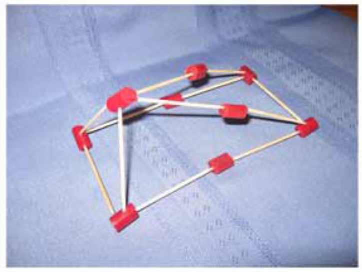
Comment: In making a prediction, students might estimate the number of minutes per year as $60 \times 24 \times 365 \approx 60 \times 25 \times 400 = 600\,000$, or $60 \times 20 \times 400 \approx 480\,000$, or about half a million minutes in one year in either case.

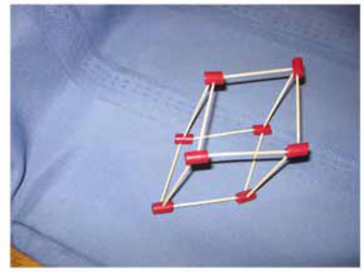
Extension:
The teacher would have to be about 75 years old! Encourage students to estimate, using their own age in minutes. (If they used the estimate of half a million minutes in one year, as above, the teacher would be approximately 80 years old.)

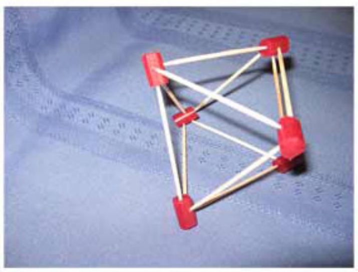
Problem 4

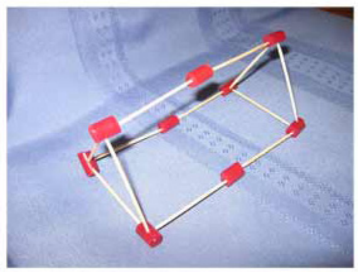
Many polyhedrons are possible. Six actual constructions using licorice vertices are shown in the photos below.


Cube


Rectangular- based pyramid

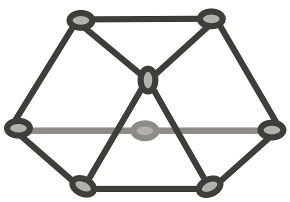

Rhombohedron

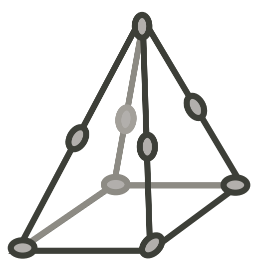

Octahedron

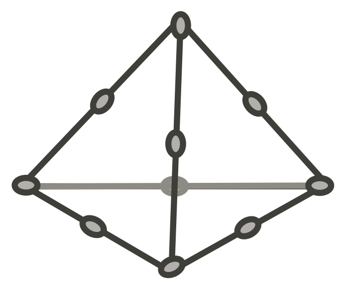

Triangular prism


Triangular-based pyramid

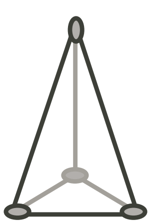
Three others are:


You name it!


Square-based Pyramid


Tetrahedron

Extension:
Again, many are possible. Here are a few; proof is by construction. Some don't have common names; students can have fun inventing names.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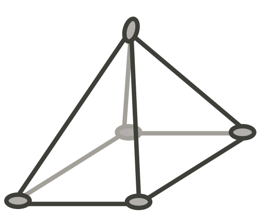
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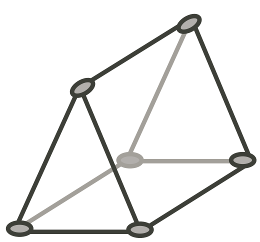
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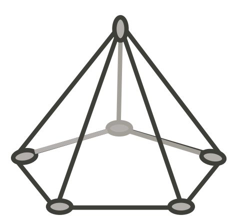
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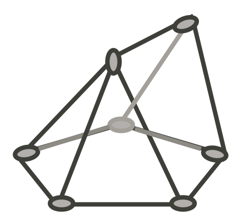
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Problem 5

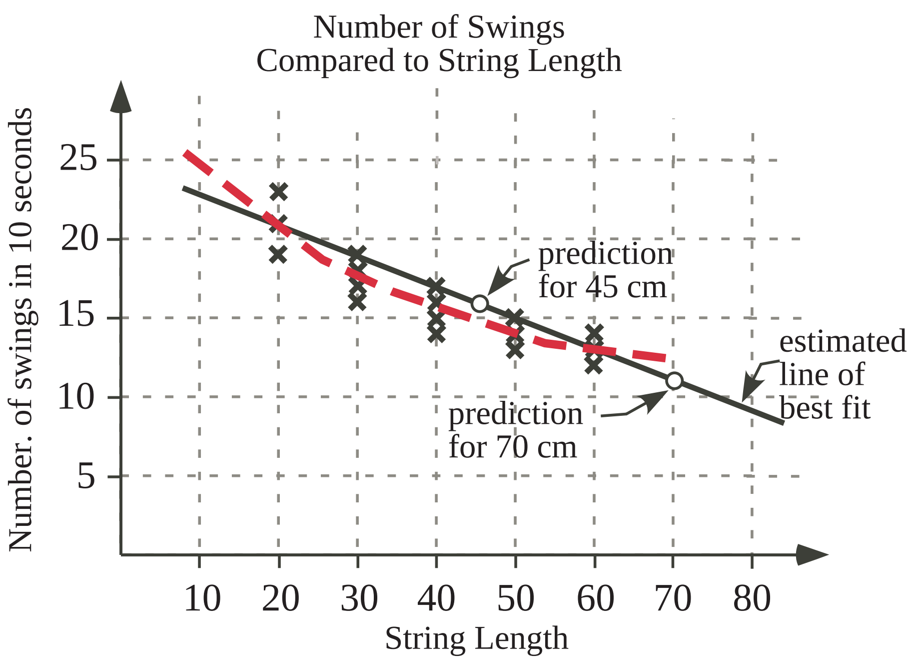
Suggestion: Once a few games have been played, discuss with students the best strategies for winning.

Extension:
Chris can win if she chooses (2, 1) on the first roll, (3, 2) on the second, and (4, 3) on the third.

Problem 6

Suggestion: Average the data from several groups; explain why it is better to use the average of many replications rather than just one set of data. Make a large table and plot on the blackboard or on chart paper. Discuss how to approximate the 'line of best fit' for the data. Here is a sample set of data averaged from five replications of each experiment, and the corresponding plot, with a rough guess 'line of best fit' shown as a solid line.

String Length	Number of Swings in 10 Seconds				
	1 washers	2 washers	3 washers	4 washers	5 washers
60 cm	12	13	13	14	13
50 cm	13	13	15	14	14
40 cm	14	16	17	16	15
30 cm	16	19	18	18	17
20 cm	19	21	23	21	21



a) For these trials, the 'line of the best fit' indicates that a string length of 45 cm would give about 16 swings in 10 seconds. (This process is called interpolation of the graph.)

b) Extending the 'line of best fit' to string length 70 cm predicts about 11 swings in 10 seconds. (This process is called extrapolation of the graph.)

Comments:

1. For this data, if you try for a 'line of best fit' for the four longer string lengths (30-60 cm), it would not quite fit the 20 cm data. This suggests the data actually fits a slightly curved path, such as the dotted line shown.

2. Note that you can also collect data over a longer time. Up to about 30 seconds, this does not appear to affect the slope of the data, ie., in 30 seconds the results appear to be approximately 3 times the data for 10 seconds in our experiments.

Extension: The number of washers (ie., the mass of the pendulum) does not affect the number of swings. Increasing the string length decreases the number of swings, with a drop of about 2 swings in 10 seconds for each increase of 10 cm in string length.