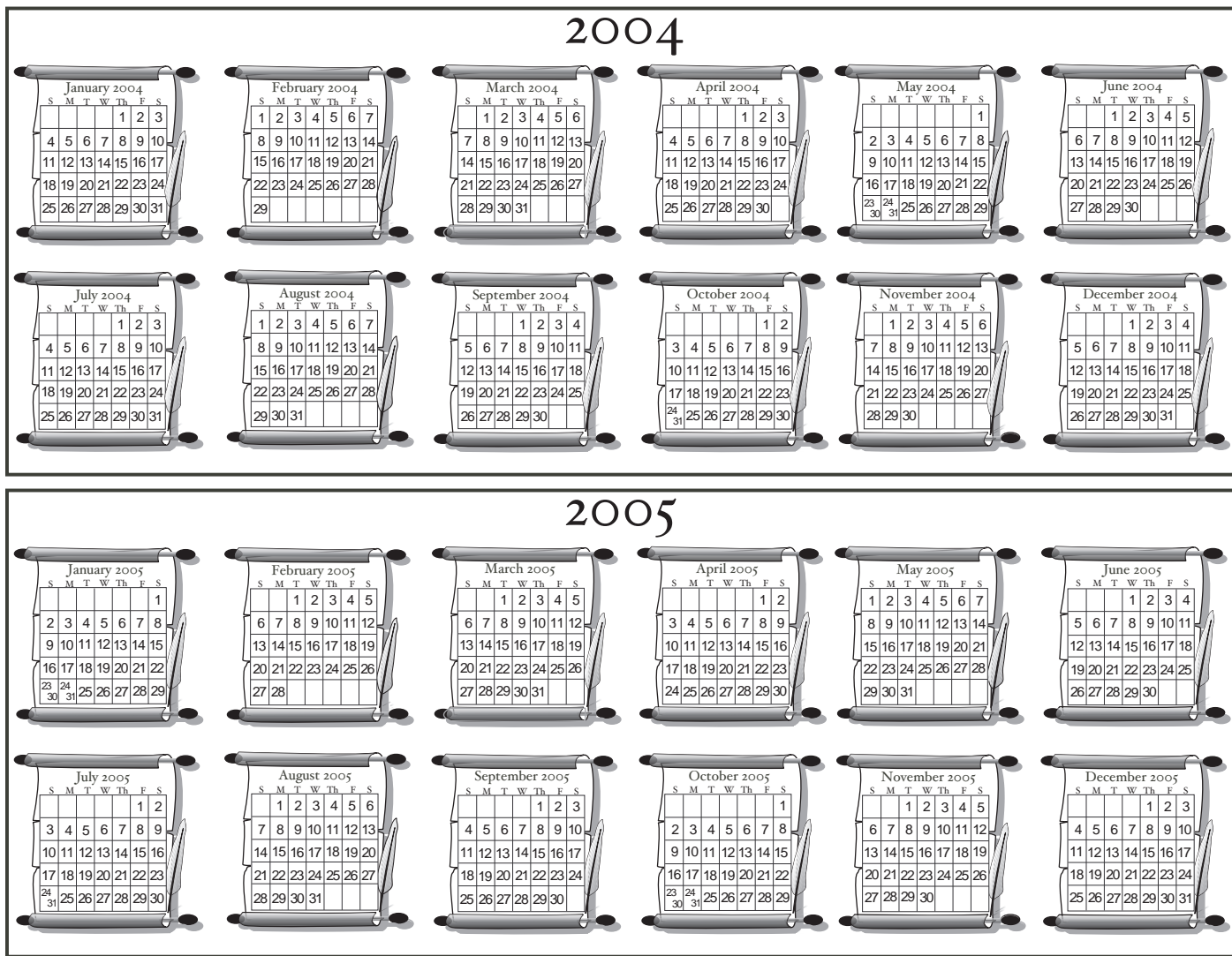


Problem

Freddy felt that Friday the 13th was very bad luck. He was somewhat consoled by his belief that there could be only one Friday the 13th in a calendar year, until his buddy Hakim told him there could be two.

- a) Was Hakim right? Explain how you know.
- b) Could there be more than two Friday the 13ths?



Extension:

Does there have to be at least one Friday the 13th in any given year?



Hints**Part a)**

Hint 1 - How is 2004 a special year? How would this affect the dates during the year?

Hint 2 - Could the number of days between two Friday the 13ths be 7? 21? 28? 29? 56?
What is the shortest number of days that could occur between two Friday the 13ths?
Does this ever occur in a non-leap year? In a leap year?

Part b)

Hint 1 - The best chance of having more than two is if January 13th is a Friday. If so, when are the other Friday the 13ths in a non-leap year? A leap year?

Solution

- a) Yes, Hakim is right... there can indeed be two Friday the 13ths in a given year. If February has a Friday the 13th (as in the given calendar for 2004), then so will March in the years where February has 28 days.

	S	M	T	W	Th	F	S
Feb.						13	
						20	
						27	
Mar.	1	2	3	4	5	6	
						13	

Another way this can happen is if January 13th is a Friday in a non-leap year. e.g., in the year 2006, both January 13th and October 13th are Fridays.

- b) *Solution 1:*

Students who are aware that, in a non-leap year, the days and dates of the first 28 days of March coincide with those of February, may use the given calendar for 2004 to note that, if all the dates labelled 13 are shifted back one day from March 1 onward (so that it becomes a calendar for a non-leap year with January 1 on Thursday), then there will be a Friday the 13th in March, and another in November.

Solution 2:

To determine whether there could be more than two, we can argue as follows. Occurrence of more than one depends on there being an exact number of weeks between two dates labeled 13. Since 28 days form 4 weeks, but most months have 30 or 31 days, we need to look at whether these 'extra' days can form multiples of 7, i.e., can form an exact number of weeks.

Here is the pattern of 'extra' days.

	J	F	M	A	M	J	J	A	S	O	N	D
non-leap year	3	0	3	2	3	2	3	3	2	3	2	3
leap year	3	1	3	2	3	2	3	3	2	3	2	3

So, if we suppose there is a Friday the 13th in January, then we see that in a non-leap year, the extra days from January to September are $3 + 3 + 2 + 3 + 2 + 3 + 3 + 2 = 21$ days, or 3 weeks, and so, in October, Friday the 13th also occurs, as noted above. On the other hand, in a leap year, we have extra days $3 + 1 + 3 = 7$ by the end of March, so April 13th is also a Friday. Then, from April to June, we have $2 + 3 + 2 = 7$ so July 13th is also a Friday. Hence we have three Friday the 13ths in such a year (but only 3, since the remaining 'extra' days never give a multiple of 7).

Note: A methodical exploration of the accumulated 'extra' days following a Friday the 13th in any month reveals that there is only one other way that three Friday the 13ths occur in a non-leap year, i.e., February 13, March 13 and November 13.

Extension:

There are only 14 possible calendars for the dates of the year, (i.e., January 1st can occur on any day of the week, giving 7 possible calendars for leap years and 7 for non-leap years). So the simplest way to answer this question is to examine these 14 calendars. It is probably easier to note that, for Friday the 13th to occur, the 1st of the month must be a Sunday, and then see whether each calendar has at least one month where this happens. There is at least one Friday the 13th in each year. They occur as follows:

- If January 1st is a Monday, April 13th is a Friday, (September 13th in a leap year);
- If January 1st is a Tuesday, September 13th is Friday, (June 13th in leap year);
- If January 1st is a Wednesday, June 13th is Friday, (March 13th in leap year);
- If January 1st is a Thursday, February 13th is a Friday in both cases, (plus March 13th, and November 13th in a non-leap year);
- If January 1st is a Friday, August 13th is Friday, (May 13th in leap year);
- If January 1st is a Saturday, May 13th is a Friday, (October 13th in a leap year);
- If January 1st is a Sunday, January 13th is Friday, in both cases, (plus October 13th in a non-leap year, and both April 13th and July 13th in a leap year).