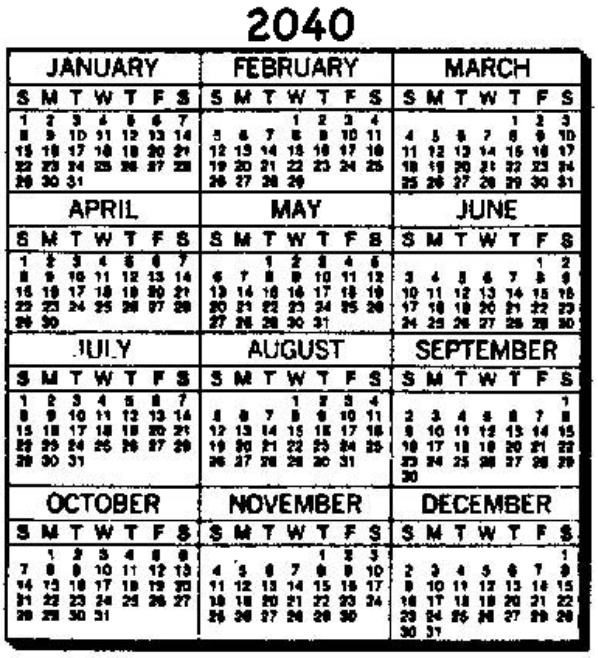
Problem

Suppose a date is 'times-ly' if the product of the month and the day equals the last two digits of the year (e.g., March 31, 1993 is 'times-ly' because $3 \times 31 = 93$).



a) For the calendar year 2040, list all 'times-ly' dates.

b) Repeat part a) for 2085.

c) Predict, without solving, whether there are any 'times-ly' dates in 2006, 2007, 2049 and 2059. Explain your reasoning.

d) Find the first three years in the 21st century with **no** 'times-ly' dates. Explain your reasoning.

Extensions:

1. A date is 'odd' if the day, the month, and the last two digits of the year, in that order, are three sequential odd numbers (e.g., 01/03/05). How many 'odd' dates occur in the 21st century?
2. Does your answer change if you write the month, the day, and then the year? Explain.

Hints

Part a)

Hint 1 - What are the possible month numbers? Day numbers?

Hint 2 - Could March have a 'times-ly' date in the year 2040?

Extension 1:

Hint 1 - What is the greatest odd number that can appear in each slot for month/day/year?

Solution

a) For the calendar year 2040, the day and month of a 'times-ly' year must have a product of 40. Thus we need numbers $d \times m = 40$ such that d (the 'day') is less than 31 and m (the 'month') is less than 12. So the possibilities are:

$$\begin{array}{lll} m = 10, & d = 4 & \text{to give October 4.} \\ m = 8, & d = 5 & \text{to give August 5.} \\ m = 5, & d = 8 & \text{to give May 8.} \\ m = 4, & d = 10 & \text{to give April 10.} \\ m = 2, & d = 20 & \text{to give February 20.} \end{array}$$

Note that $m = 20$, $d = 2$ does not work, since there is no '20th' month.

b) For 2085, we need $m \times d = 85$. Since the only factors of 85 are 1, 5, 17, 85, we see immediately that the *only* 'times-ly' date is May 17, 2085.

c) Since 2006 has factors for '06' of 1×6 or 2×3, which are all less than 12, there are several possibilities (e.g., January 6, June 1,...). For 2007, the only factors are 1 and 7, so January 7 or July 1 work. For 2049, only 7×7 works (July 7), since 49>31. For 2059, there are none, since the only factors are 1×59, and 59>31.

d) The first three years in this century with no 'times-ly' dates are 2037, 2041, 2043. i.e., the first three years in which the last two digits are primes greater than 31. If you consider the year 2000 to be the first year of the 21st century, it could also be included.

Extensions:

1. For an 'odd' date $d/m/y$, we need d, m, and y to be sequential odd numbers with $m \leqq 11$, i.e., the possible choices for the month are 01, 03, 05, 07, 09, 11. So the possibilities are 01/03/05, 03/05/07, 05/07/09, 07/09/11, 09/11/13, i.e., there are five such dates.

2. If we write the month, then the day, then the year, i.e., $m/d/y$, then there are six such dates: 01/03/05,..., 09/11/13, and 11/13/15 since month '11' is the last odd month.