Problem

A 12-hour digital clock displays palindromic numbers many times each day (e.g., 1:01, 1:11, 2:32, \dots).

- a) What is the least length of time between two consecutive such numbers?
- b) What is the greatest length of time between two consecutive such numbers?



Extension:

Is the answer to question b) the same for a 24-hour clock?

Hints

- Hint 1 What is the shortest length of time between two palindromes in the same hour?
- Hint 2 What is the length of time between the last palindrome in one hour and the first palindrome in the next hour (e.g. 1:51 to 2:02)? Is this always the same?

Solution

- a) It seems reasonable that the least amount of time between two consecutive palindromes would occur in the same hour, for example $1:01 \rightarrow 1:11$, or $5:25 \rightarrow 5:35$, giving a 10 minutes 'least' time. However, further thought reveals that, while the time between the last palindrome in one hour and the first palindrome in the next is generally 11 minutes (e.g., $7:57 \rightarrow 8:08$), the gap from $9:59 \rightarrow 10:01$ is just 2 minutes, which is the least possible.
- b) The greatest length of time between two consecutive palindromes is from 10:01 to 11:11, a gap of 70 minutes.

Extension:

The greatest length of time between two consecutive such numbers using a 24-hour clock is 15:51 to 20:02, a gap of 4 hours and 11 minutes.