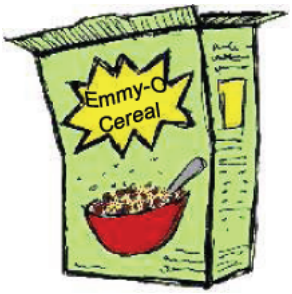
Problem



The area of one side of an Emmy-Os single-serving cereal box is 96 cm^2. The area of another side of the same box is 48 cm^2. The area of the top of the box is 32 cm^2. What is the volume of the box if the length of each edge is a whole number?

Hints

Hint 1 - Is it possible to draw a diagram of the box?

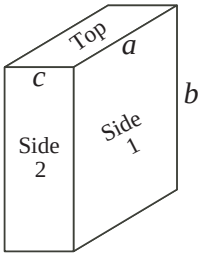
Hint 2 - If this box is similar in shape to a cereal box, what shape are the faces? How do you find the area of these faces?

Hint 3 - What are possible lengths and widths for the top of the box, to make an area of 32 cm^2? Which of these possibilities are reasonable?

Hint 4 - Remember that the length of one side must match at least one length of the other side and of the top.

Solution

Since each edge length is a whole number, we examine the possible factors of each of the given areas, each area being the product of two lengths. The possibilities are:



Side 1:	96 cm^2	2×48, 3×32, 4×24, 6×16, 8×12
Side 2:	48 cm^2	2×24, 3×16, 4×12, 6×8
Top:	32 cm^2	2×16, 4×8

Now we need to select three lengths a, b, c which appear in pairs among the products of factors, say, a, b for side 1, b, c for side 2, and c, a for the top. Since the top has the fewest possibilities, it is sensible to start with those. If we select 2×16, then side 2 has to be 2×24 (or 3×16), and side 3 has to be 24×16 (or 2×3), neither of which gives 96 cm^2. So the top must be 4×8; then side 2 is 4×12 (or 6×8), and side 3 is 8×12 (or 6×4), of which only $8 \times 12 = 96$. So the dimensions of the box are 4 cm by 8 cm by 12 cm, and its volume is $4 \times 8 \times 12 = 384$ cm^3.