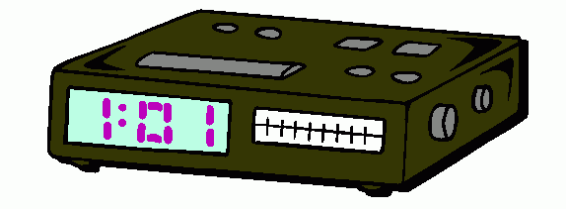
Problem

A 12-hour digital clock displays palindromic numbers many times each day (e.g., 1:01, 1:11, 2:32, ...).

a) What is the least length of time between two consecutive such numbers?

b) What is the greatest length of time between two consecutive such numbers?



Extension:
Is the answer to question b) the same for a 24-hour clock?

Hints

Hint 1 - What is the shortest length of time between two palindromes in the same hour?

Hint 2 - What is the length of time between the last palindrome in one hour and the first palindrome in the next hour (e.g. 1:51 to 2:02)? Is this always the same?

Solution

a) It seems reasonable that the least amount of time between two consecutive palindromes would occur in the same hour, for example 1:01 → 1:11, or 5:25 → 5:35, giving a 10 minutes 'least' time. However, further thought reveals that, while the time between the last palindrome in one hour and the first palindrome in the next is generally 11 minutes (e.g., 7:57 → 8:08), the gap from 9:59 → 10:01 is just 2 minutes, which is the least possible.

b) The greatest length of time between two consecutive palindromes is from 10:01 to 11:11, a gap of 70 minutes.

Extension:
The greatest length of time between two consecutive such numbers using a 24-hour clock is 15:51 to 20:02, a gap of 4 hours and 11 minutes.