## ${\bf Problem}$



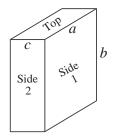
The area of one side of an Emmy-Os single-serving cereal box is  $96 \text{ cm}^2$ . The area of another side of the same box is  $48 \text{ cm}^2$ . The area of the top of the box is  $32 \text{ cm}^2$ . What is the volume of the box if the length of each edge is a whole number?

## Hints

- **Hint 1 -** Is it possible to draw a diagram of the box?
- **Hint 2 -** If this box is similar in shape to a cereal box, what shape are the faces? How do you find the area of these faces?
- **Hint 3 -** What are possible lengths and widths for the top of the box, to make an area of 32 cm<sup>2</sup>? Which of these possibilities are reasonable?
- **Hint 4** Remember that the length of one side must match at least one length of the other side and of the top.

## Solution

Since each edge length is a whole number, we examine the possible factors of each of the given areas, each area being the product of two lengths. The possibilities are:



Side 1:  $96 \text{ cm}^2$   $2 \times 48$ ,  $3 \times 32$ ,  $4 \times 24$ ,  $6 \times 16$ ,  $8 \times 12$ 

Side 2:  $48 \text{ cm}^2$   $2 \times 24$ ,  $3 \times 16$ ,  $4 \times 12$ ,  $6 \times 8$ 

Top:  $32 \text{ cm}^2 \quad 2 \times 16, 4 \times 8$ 

Now we need to select three lengths a, b, c which appear in pairs among the products of factors, say, a, b for side 1, b, c for side 2, and c, a for the top. Since the top has the fewest possibilities, it is sensible to start with those. If we select  $2 \times 16$ , then side 2 has to be  $2 \times 24$  (or  $3 \times 16$ ), and side 3 has to be  $24 \times 16$  (or  $2 \times 3$ ), neither of which gives  $96 \text{ cm}^2$ . So the top must be  $4 \times 8$ ; then side 2 is  $4 \times 12$  (or  $6 \times 8$ ), and side 3 is  $8 \times 12$  (or  $6 \times 4$ ), of which only  $8 \times 12 = 96$ . So the dimensions of the box are 4 cm by 8 cm by 12 cm, and its volume is  $4 \times 8 \times 12 = 384 \text{ cm}^3$ .