Part II: For the Teacher

Curriculum Areas

Problem 1 - Measurement

Problem 2 - Number Sense

Problem 3 - Geometry

Problem 4 - Data Management

Problem 5 - Number Sense

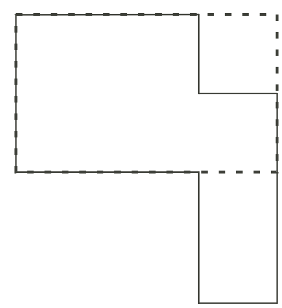
Problem 6 - Pattern and Algebra and Number Sense

Hints and Suggestions:

Problem 1

Hint 1 - What missing distances can you fill in?

Suggestion: For laps 2 and 3, it may help students to visualize the run length as the perimeter of an equivalent rectangle.



Problem 2 a)

Hint 1 - What would be the value of 1 of each stamp? of 2 of each?

Hint 2 - Could there be an odd number of stamps? (Think about the 5¢ stamps...)

Hint 3 - Could Kamara have only two of each stamp? only four? (Make a chart for several different numbers of stamps. Remember Kamara has the same number of each stamp.)

Problem 2 b)

Hint 1 - What combinations of stamps could be used to make the 65¢ required for one letter?

Hint 2 - Might Kamara put more than the required amount of postage on one of the items she wishes to mail?

Problem 3a)

Hint 1 - What fraction of the first column is shaded?

Problem 3b)

Hint 1 - If you draw a horizontal line across the middle of the square, what figures have equal area?

Extension:

Hint 1 - What fraction of column one is shaded? of column 2?

Problem 4

Hint 1 - Which story involves distance travelled?

Hint 2 - As she pumps the swing, which graph could depict Wei Li going higher and higher?

Problem 5a)

Hint 1 - What are the possible month numbers? Day numbers?

Hint 2 - Could March have a 'times-ly' date in the year 2040?

Extension 1:

Hint 1 - What is the greatest odd number that can appear in each slot for month/day/year?

Problem 6

Hint 1 - Make a diagram for the case of a 3×3 square. How many 1×2 rectangles are there? How did you count them? Try a 4×4 square the same way.

Hint 2 - Is a 1×2 rectangle the same as a 2×1 rectangle (i.e., could the rectangle stand "on end"?)

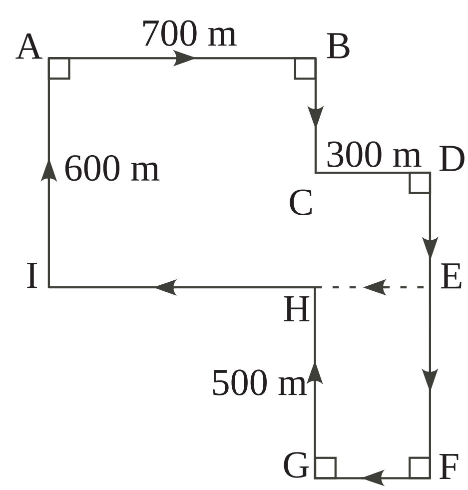
Suggestion: If the class hasn't seen the problem of counting squares of all sizes on a checkerboard, you may wish to challenge them to try the Extension. Encourage students to look for patterns.

Solutions

Problem 1

We can fill in the following distances:

$IH = AB = 700$ metres
$HE = GF = CD = 300$ metres
$EF = HG = 500$ metres

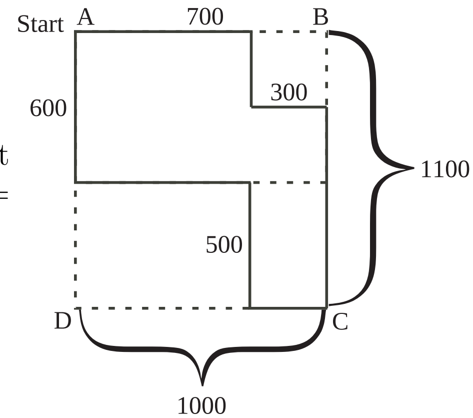


Thus on the first lap, Sabrina runs

$700 + BC + 300 + DE + 500 + 300 + 500 + 700 + 600$
$= 3600 + BC + DE$ metres.
But, $BC + DE = AI = 600$ metres.
Thus Sabrina runs $3600 + 600 = 4200$ metres.

On the second lap, she does not cover the distance $EF + FG + GH = 1300$ metres, but does cover $HE = 300$ metres. Thus she only runs $4200 - 1000 = 3200$ metres.
So all together, Sabrina runs $4200 + 3200 + 3200 = 10600$ metres or 10.6 km.

An alternate solution for the first lap is to note that it is equivalent to the rectangle $ABCD$, which has perimeter $2 \times (1000 + 1100) = 2 \times 2100 = 4200$ metres.



Suggestion: Discuss with the class whether the answer would change if Sabrina started somewhere else. (e.g., at I, or at F).

Problem 2

a) By trial and error, we find that Kamara has 6 of each type of stamp,
since $(6 \times 50¢)+(6 \times 20¢)+(6 \times 10¢)+(6 \times 5¢)= \$3.00 + \$1.20 + \$0.60 + \$0.30 = \$5.10$

Ways the students may reason:

1.

		Values of Stamps					
	no. of each stamp					Total Value	
Value	2	100¢	40¢	20¢	10¢	170¢ or \$1.70	Too low.
Value	5	250¢	100¢	50¢	25¢	425¢ or \$4.25	Still too low, but closer.
Value	6	300¢	120¢	60¢	30¢	510¢ or \$5.10	She has 6 of each.

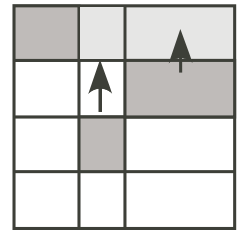
2. One of each stamp will have a value 85¢.
Two of each stamp will have a value \$1.70.
Three of each stamp will have a value \$2.55.
Since \$2.55 is one-half of \$5.10, Kamara must have 6 of each stamp.

A more elegant solution: One of each stamp will have a total value of 85¢ .
$\$5.10 \div 85¢ = 6$. Therefore she has 6 of each stamp.

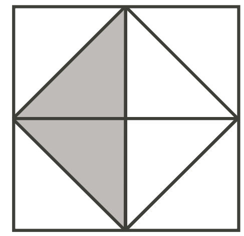
b) Kamara needs $6 \times 65¢ + \$1.15 = \$3.90 + \$1.15 = \$5.05$. Since this is less than \$5.10 worth of stamps she has, some students may assume the answer to the question is 'yes'. To get 65¢ postage for six letters, Kamara could use 6 each of the 50¢, 10¢, and 5¢ stamps. This would leave her with 6 of the 20¢ stamps, or \$1.20. So if she were willing to sacrifice the extra payment, she could just use all of these to mail the \$1.15 item.
However, there is no way to get the *exact* postage on all the items to be mailed, since all six letters require a 5¢ stamp to make 65¢, and the \$1.15 item would also require a 5¢ stamp.

Problem 3

a) Since the horizontal lines are evenly spaced, we see that exactly $\frac{1}{4}$ of each vertical column is shaded. Thus, by shifting all the shaded regions to one row, we see that $\frac{1}{4}$ of the whole square is shaded.

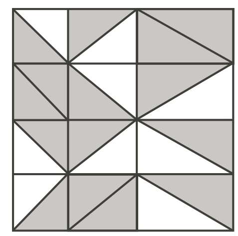


b) By sketching a horizontal line across the middle of the square, we form eight identical triangles, of which two are shaded. Thus $\frac{2}{8}$ or $\frac{1}{4}$ of the square is shaded.



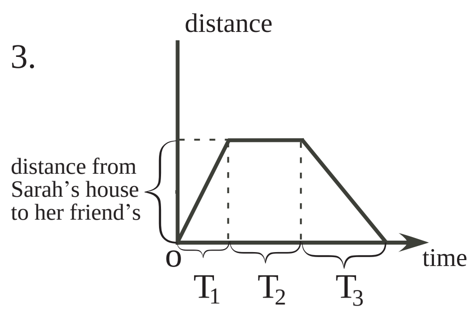
Extension:

Thinking of each column as consisting of eight identical triangles, we see that in each column, five of these triangles are shaded. Thus $\frac{5}{8}$ of each column is shaded, and hence $\frac{5}{8}$ of the whole square is shaded, since the horizontal lines are equidistant from one another.

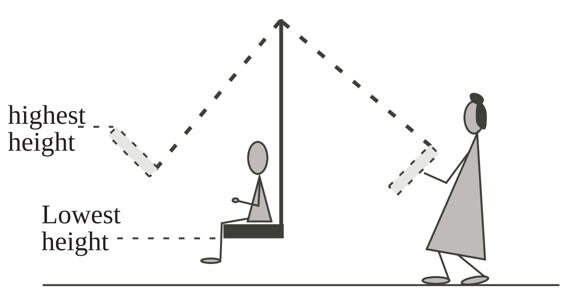


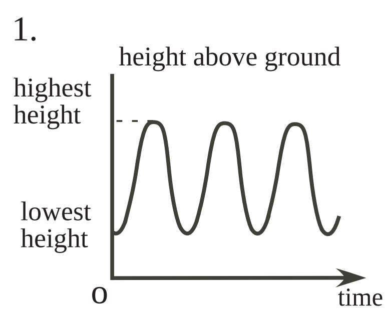
Problem 4

a) Since the only graph for which distance stays the same for a time is graph 3, this seems a logical choice.
Note that the time T_1 is less than the T_3, since Sarah *runs* to her friend's house, but walks slowly home. T_2 is the time she spends at her friend's.



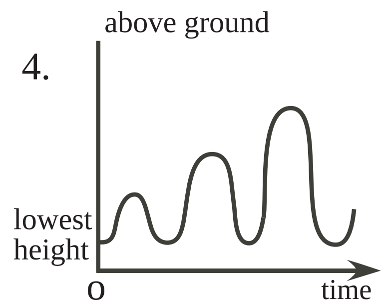
b) As Tasha's Grandma pushes her, the swing moves from lowest to highest height from the ground.



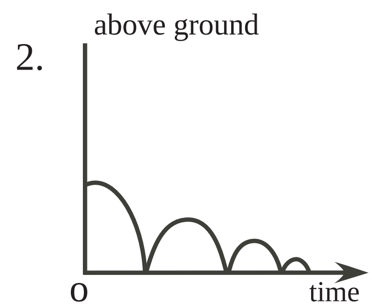


Note: Sometimes a child being pushed on a swing hollers "Higher! Higher!" with each push. Thus graph 4 could also be a valid answer for b).

c) Since Wei Li is pumping herself up on the swing, her height above the ground increases with each cycle of the swing, as depicted in graph 4.



d) As the stone skips across the surface of the pond, it bounces less high with each skip, so its height above the surface is depicted by graph 2.



Problem 5

a) For the calendar year 2040, the day and month of a 'times-ly' year must have a product of 40. Thus we need numbers $d \times m = 40$ such that d (the 'day') is less than 31 and m (the 'month') is less than 12. So the possibilities are:

$m = 10$, $d = 4$ to give October 4.
$m = 8$, $d = 5$ to give August 5.
$m = 5$, $d = 8$ to give May 8.
$m = 4$, $d = 10$ to give April 10.
$m = 2$, $d = 20$ to give February 20.

Note that $m = 20$, $d = 2$ does not work, since there is no '20th' month.

b) For 2085, we need $m \times d = 85$. Since the only factors of 85 are 1, 5, 17, 85, we see immediately that the *only* 'times-ly' date is May 17, 2085.

c) Since 2006 has factors for '06' of 1×6 or 2×3, which are all less than 12, there are several possibilities (e.g., January 6, June 1,...). For 2007, the only factors are 1 and 7, so January 7 or July 1 work. For 2049, only 7×7 works (July 7), since 49>31. For 2059, there are none, since the only factors are 1×59, and 59>31.

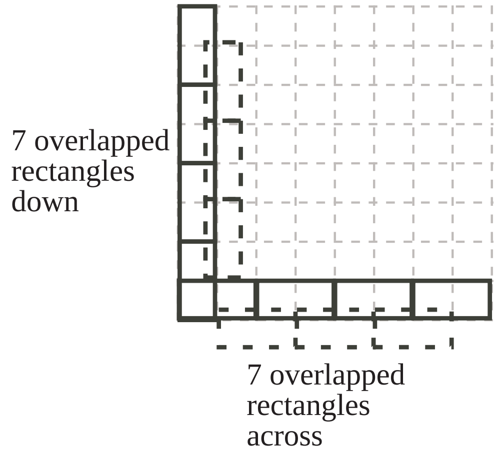
d) The first three years in this century with no 'times-ly' dates are 2037, 2041, 2043. i.e., the first three years in which the last two digits are primes greater than 31. If you consider the year 2000 to be the first year of the 21st century, it could also be included.

Extensions:

1. For an 'odd' date $d/m/y$, we need d, m, and y to be sequential odd numbers with $m \leqq 11$, i.e., the possible choices for the month are 01, 03, 05, 07, 09, 11. So the possibilities are 01/03/05, 03/05/07, 05/07/09, 07/09/11, 09/11/13, i.e., there are five such dates.

2. If we write the month, then the day, then the year, i.e., $m/d/y$, then there are six such dates: 01/03/05,..., 09/11/13, and 11/13/15 since month '11' is the last odd month.

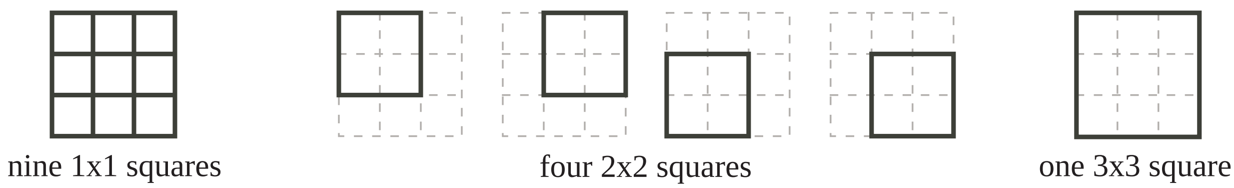
Problem 6



Each column contains 7 overlapping 1 unit $\times$ 2 units vertical rectangles, and similarly for each row there are 7 overlapping horizontal 2 units $\times$ 1 unit rectangles. (Think of shifting the dotted rectangles to the left to see the 7 overlapping vertical rectangles, or up to see the overlapping horizontal rectangles.)
There are 8 rows and 8 columns on the board, so there are 56 horizontal, and 56 vertical rectangles, in total. Thus there are 112 such rectangles altogether.

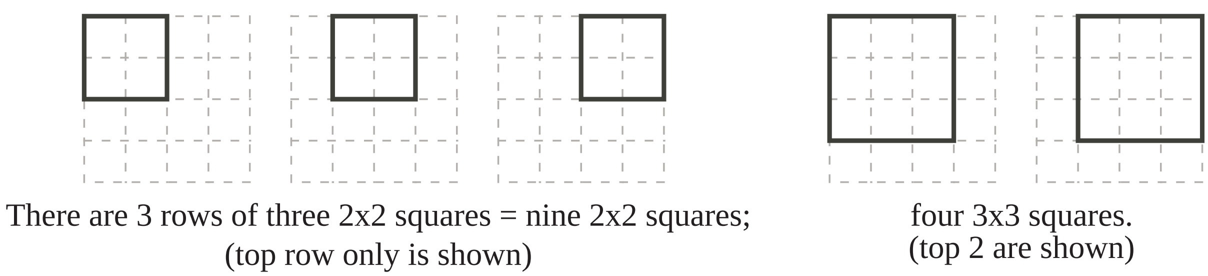
Extension:

a) Consider a 3×3 square as follows:



Thus the 3×3 square contains $1 + 4 + 9 = 14$ squares in total.

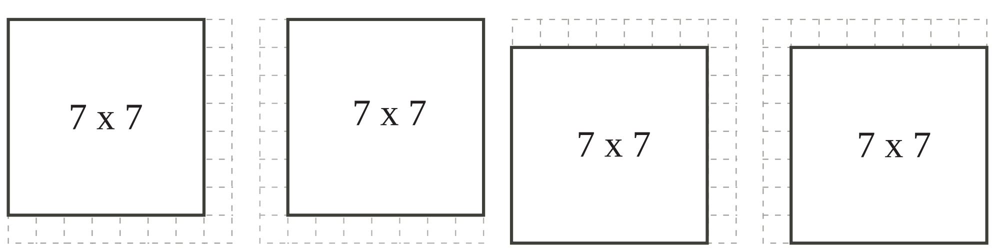
In a 4×4 square, there are sixteen 1×1 squares.



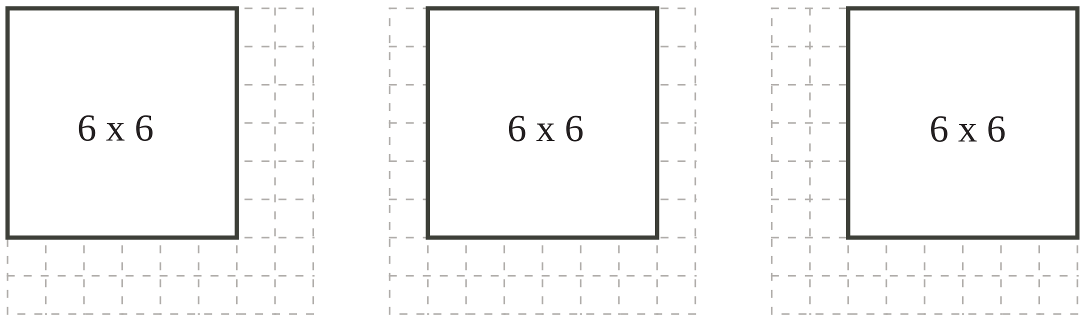
Thus, including the one 4×4 square, there are $1 + 4 + 9 + 16 = 30$ squares in total in the 4×4 square.

b) Examining the pattern in the 3×3 and 4×4 squares in a), we see that there will be one 8×8 square, and four 7×7 squares.

Picture them as:



Then there will be nine 6×6 squares, three of which are shown below.



Note that the pattern is the sum of the squares of all the whole numbers up to n^2, where n is the length of the side of the $n \times n$ square.

In conclusion, the total number of squares for all sizes in an 8×8 checkerboard is $1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 204$ squares. So Bart and Lisa's count was correct.