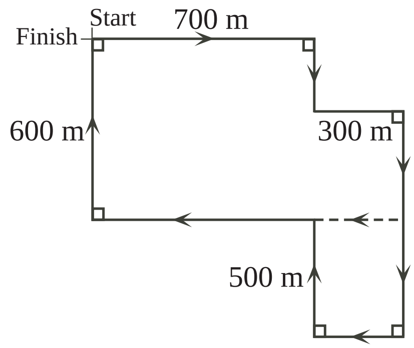
Emmy Noether - Circle 3 for 2006-2007

Part I: Problems

Problem 1

Sabrina is running a cross-country race on the course sketched at right. The race consists of three laps: on the first lap, the runners must complete the entire course (solid line), but on the next two laps, they take the shortcut (dotted line). How far will Sabrina run altogether?



Problem 2

Kamara has \$5.10 worth of stamps. She has an equal number of 50¢, 20¢, 10¢, and 5¢ stamps.

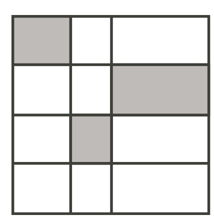
a) How many 50¢ stamps does she have?

b) Kamara has to mail six letters that require 65¢ postage and one larger item that requires \$1.15 postage. Can she mail all seven items without needing more stamps than she has? Explain.

Problem 3

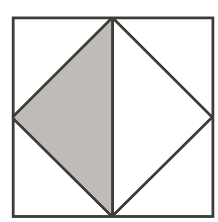
a) The beautiful Princess Morag must solve this problem to escape the evil King Rothbart:

> In the square shown, the horizontal lines are equidistant (equally spaced) from one another. What fraction of the square is shaded?



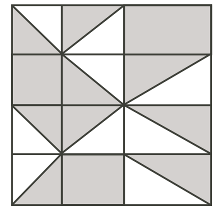
b) King Rothbart goes back on his word and insists Morag solve a further problem to win her freedom:

> The mid-points of the sides of the square are joined as shown. What fraction of the original (larger) square is shaded?



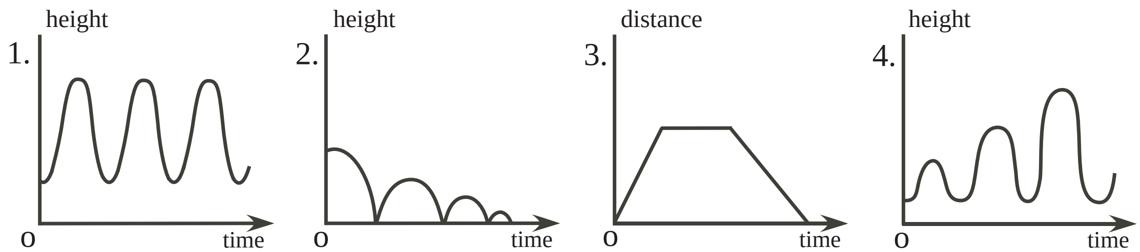
Extension:

The horizontal lines of the square at right are equidistant from one another. What fraction of the square is shaded?



Problem 4

Match each graph to the most suitable story. Indicate what the height or distance is in each case.



a) Sarah runs eagerly to visit a friend, stays for an hour, then walks slowly home.

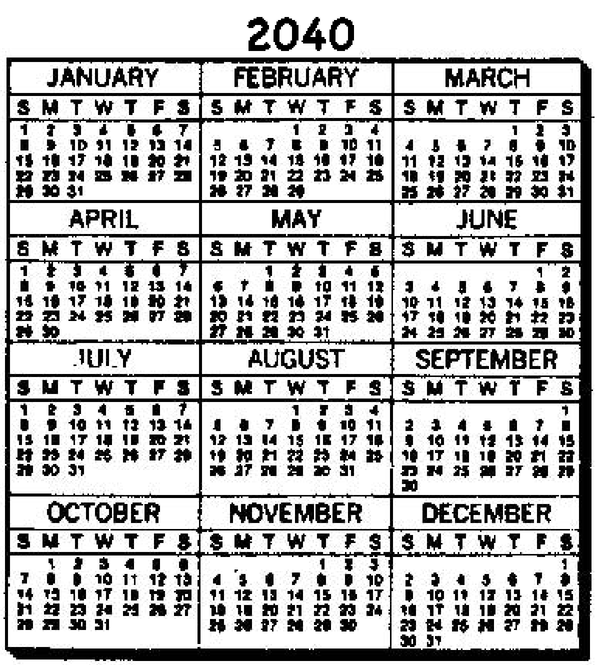
b) Tasha's Grandma has been pushing her on a swing for a while.

c) Wei Li pumps herself up on a swing.

d) A flat stone bounces across the surface of a pond.

Problem 5

Suppose a date is 'times-ly' if the product of the month and the day equals the last two digits of the year (e.g., March 31, 1993 is 'times-ly' because $3 \times 31 = 93$).



a) For the calendar year 2040, list all 'times-ly' dates.

b) Repeat part a) for 2085.

c) Predict, without solving, whether there are any 'times-ly' dates in 2006, 2007, 2049 and 2059. Explain your reasoning.

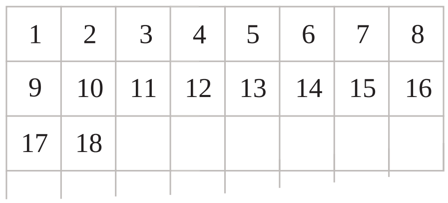
d) Find the first three years in the 21st century with **no** 'times-ly' dates. Explain your reasoning.

Extensions:

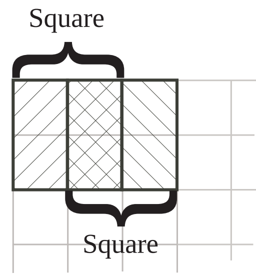
1. A date is 'odd' if the day, the month, and the last two digits of the year, in that order, are three sequential odd numbers (e.g., 01/03/05). How many 'odd' dates occur in the 21st century?
2. Does your answer change if you write the month, the day, and then the year? Explain.

Problem 6

Bart and Lisa got tired of playing checkers because Lisa always won. They remembered one day in math class they counted the number of squares of all sizes on a checkerboard. They counted 64 squares 1 unit by 1 unit, and 49 overlapping squares 2 units by 2 units. They continued until they found the total of 204 squares.

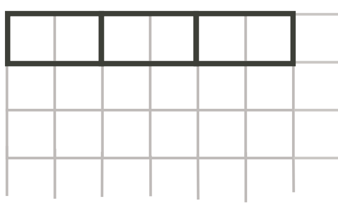


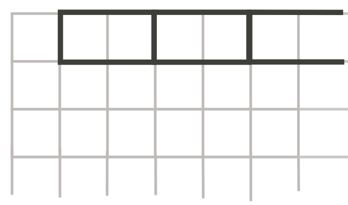
1 x 1 squares



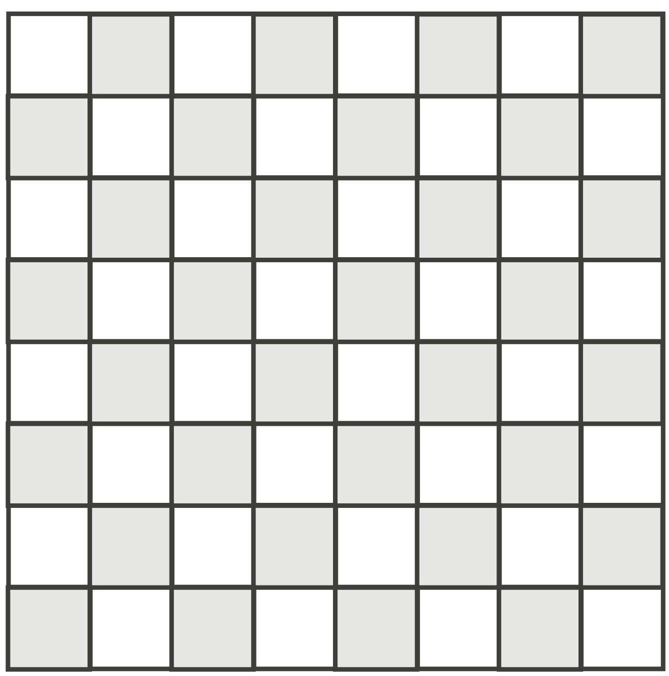
2 x 2 squares

They decided instead, to count the number of rectangles 1 unit by 2 units, such as those shown below. They were careful to count all the overlapping rectangles. If they counted correctly, what was their total?





Here is a checkerboard to work with.



Extension:
Draw diagrams to illustrate how Bart and Lisa counted the total number of squares of all sizes on the 8×8 checkerboard. Is their total correct?