Part II: For the Teacher

Curriculum Areas

Problem 1 - Measurement and Number Sense

Problem 2 - Measurement, Geometry and Number Sense

Problem 3 - Number Sense and Measurement

Problem 4 - Data Management and Pattern/Algebra

Problem 5 - Geometry and Probability

Problem 6 - Number Sense

Hints and Suggestions:

Problem 1

Hint 1 - How could you draw a diagram of the track and hurdles?

Hint 2 - How many spaces are there between the first hurdle and the last one?

Hint 3 - Remember, the first hurdle is at 13 meters. Where is the second hurdle? The third?

Hint 4 - Remember, the question asks how far the 10th hurdle is from the **finish** line.

Problem 2

Hint 1 - Is it possible to draw a diagram of the box?

Hint 2 - If this box is similar in shape to a cereal box, what shape are the faces?
How do you find the area of these faces?

Hint 3 - What are possible lengths and widths for the top of the box, to make an area of 32 cm^2?
Which of these possibilities are reasonable?

Hint 4 - Remember that the length of one side must match at least one length of the other side and of the top.

Problem 3

Hint 1 - What is the shortest length of time between two palindromes in the same hour?

Hint 2 - What is the length of time between the last palindrome in one hour and the first palindrome in the next hour (e.g. 1:51 to 2:02)? Is this always the same?

Problem 4 a)

Hint 1 - How is 2004 a special year? How would this affect the dates during the year?

Hint 2 - Could the number of days between two Friday the 13ths be 7? 21? 28? 29? 56?
What is the shortest number of days that could occur between two Friday the 13ths?
Does this ever occur in a non-leap year? In a leap year?

Problem 4 b)

Hint 1 - The best chance of having more than two is if January 13th is a Friday. If so, when are the other Friday the 13ths in a non-leap year? A leap year?

Problem 5

Suggestion: Give the students a page of cube nets (see below) to colour in different ways and form into cubes. Have them work in groups if desired. Encourage them to compare one another's cubes, rotating them to see whether their colourings are truly different.

Problem 6

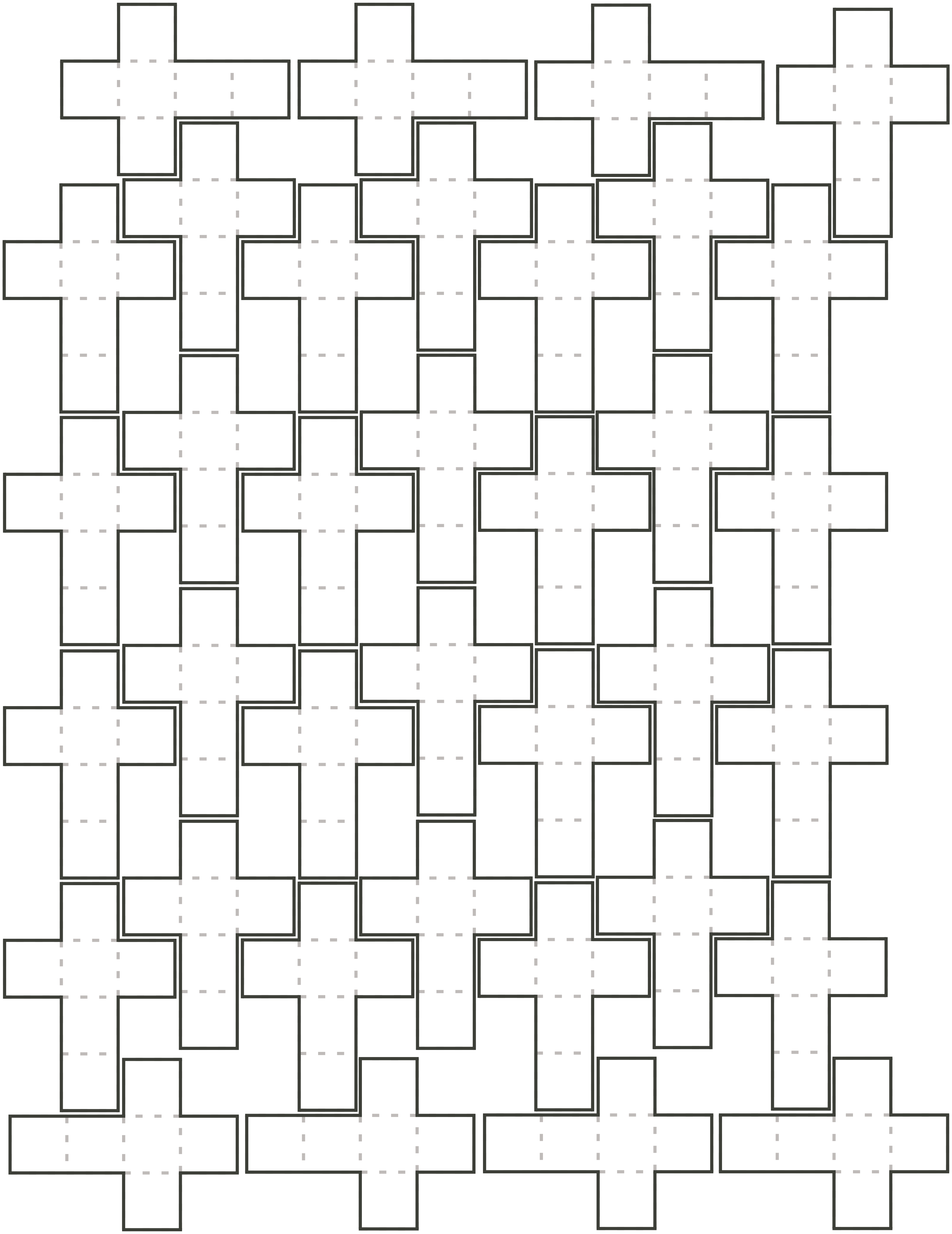
Suggestions:

1. Use one (reasonable) starter set of 5 numbers for the whole class. (See the Solutions for one possible starter set.)

2. Work with finding ways to form the numbers 1-20 first, and then seek more possibilities with the students.

3. Offer bonus points for more than one way to get a number.

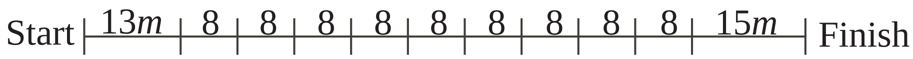
(See solution below for possible ways to play the game.)



Solutions

Problem 1

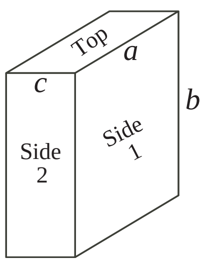
There are 9 spaces between the 10 hurdles; each space has length 8 metres. Thus the distance between the first and last hurdle is $9 \times 8 = 72$ metres. Adding the 13 metres from the starting line to the first hurdle gives 85 metres. Thus the tenth hurdle is $100 - 85 = 15$ metres from the finish line.



Extension:
The distance available for the spaces between the hurdles is 56 metres $(80 - 12 - 12)$. Since there are 7 spaces between the 8 hurdles, consecutive hurdles are $56 \div 7 = 8$ metres apart.

Problem 2

Since each edge length is a whole number, we examine the possible factors of each of the given areas, each area being the product of two lengths. The possibilities are:



Side 1:	96 cm^2	2×48, 3×32, 4×24, 6×16, 8×12
Side 2:	48 cm^2	2×24, 3×16, 4×12, 6×8
Top:	32 cm^2	2×16, 4×8

Now we need to select three lengths a, b, c which appear in pairs among the products of factors, say, a, b for side 1, b, c for side 2, and c, a for the top. Since the top has the fewest possibilities, it is sensible to start with those. If we select 2×16, then side 2 has to be 2×24 (or 3×16), and side 3 has to be 24×16 (or 2×3), neither of which gives 96 cm^2. So the top must be 4×8; then side 2 is 4×12 (or 6×8), and side 3 is 8×12 (or 6×4), of which only $8 \times 12 = 96$. So the dimensions of the box are 4 cm by 8 cm by 12 cm, and its volume is $4 \times 8 \times 12 = 384$ cm^3.

Problem 3

a) It seems reasonable that the least amount of time between two consecutive palindromes would occur in the same hour, for example 1:01 $\rightarrow$ 1:11, or 5:25 $\rightarrow$ 5:35, giving a 10 minutes 'least' time. However, further thought reveals that, while the time between the last palindrome in one hour and the first palindrome in the next is generally 11 minutes (e.g., 7:57 $\rightarrow$ 8:08), the gap from 9:59 $\rightarrow$ 10:01 is just 2 minutes, which is the least possible.

b) The greatest length of time between two consecutive palindromes is from 10:01 to 11:11, a gap of 70 minutes.

Extension:
The greatest length of time between two consecutive such numbers using a 24-hour clock is 15:51 to 20:02, a gap of 4 hours and 11 minutes.

Problem 4

a) Yes, Hakim is right... there can indeed be two Friday the 13ths in a given year. If February has a Friday the 13th (as in the given calendar for 2004), then so will March in the years where February has 28 days.

	S	M	T	W	Th	F	S
Feb.						13	
						20	
						27	
Mar.	1	2	3	4	5	6	
						13	

Another way this can happen is if January 13th is a Friday in a non-leap year. e.g., in the year 2006, both January 13th and October 13th are Fridays.

b) *Solution 1:*

Students who are aware that, in a non-leap year, the days and dates of the first 28 days of March coincide with those of February, may use the given calendar for 2004 to note that, if all the dates labelled 13 are shifted back one day from March 1 onward (so that it becomes a calendar for a non-leap year with January 1 on Thursday), then there will be a Friday the 13th in March, and another in November.

Solution 2:

To determine whether there could be more than two, we can argue as follows. Occurrence of more than one depends on there being an exact number of weeks between two dates labeled 13. Since 28 days form 4 weeks, but most months have 30 or 31 days, we need to look at whether these 'extra' days can form multiples of 7, i.e., can form an exact number of weeks.

Here is the pattern of 'extra' days.

	J	F	M	A	M	J	J	A	S	O	N	D
non-leap year	3	0	3	2	3	2	3	3	2	3	2	3
leap year	3	1	3	2	3	2	3	3	2	3	2	3

So, if we suppose there is a Friday the 13th in January, then we see that in a non-leap year, the extra days from January to September are $3+3+2+3+2+3+3+2 = 21$ days, or 3 weeks, and so, in October, Friday the 13th also occurs, as noted above. On the other hand, in a leap year, we have extra days $3+1+3 = 7$ by the end of March, so April 13th is also a Friday. Then, from April to June, we have $2+3+2 = 7$ so July 13th is also a Friday. Hence we have three Friday the 13ths in such a year (but only 3, since the remaining 'extra' days never give a multiple of 7).

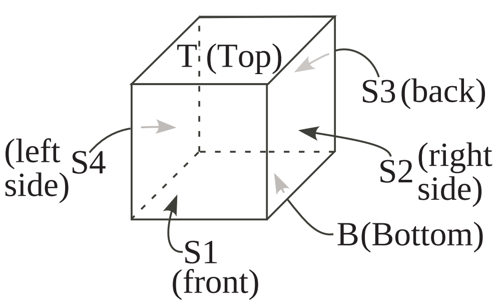
Note: A methodical exploration of the accumulated 'extra' days following a Friday the 13th in any month reveals that there is only one other way that three Friday the 13ths occur in a non-leap year, i.e., February 13, March 13 and November 13.

Extension:
There are only 14 possible calendars for the dates of the year, (i.e., January 1st can occur on any day of the week, giving 7 possible calendars for leap years and 7 for non-leap years). So the simplest way to answer this question is to examine these 14 calendars. It is probably easier to note that, for Friday the 13th to occur, the 1st of the month must be a Sunday, and then see whether each calendar has at least one month where this happens. There is at least one Friday the 13th in each year. They occur as follows:

- If January 1st is a Monday, April 13th is a Friday, (September 13th in a leap year);
- If January 1st is a Tuesday, September 13th is Friday, (June 13th in leap year);
- If January 1st is a Wednesday, June 13th is Friday, (March 13th in leap year);
- If January 1st is a Thursday, February 13th is a Friday in both cases, (plus March 13th, and November 13th in a non-leap year);
- If January 1st is a Friday, August 13th is Friday, (May 13th in leap year);
- If January 1st is a Saturday, May 13th is a Friday, (October 13th in a leap year);
- If January 1st is a Sunday, January 13th is Friday, in both cases, (plus October 13th in a non-leap year, and both April 13th and July 13th in a leap year).

Problem 5

There are 10 'different' cubes Reena can make as follows:

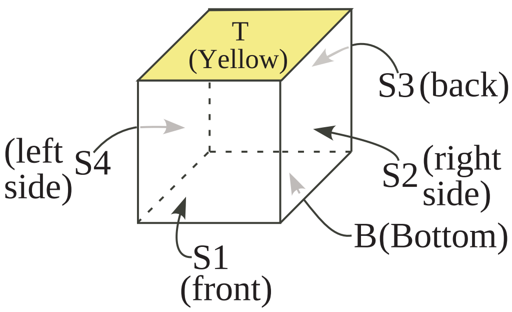


- 2 cubes of solid colours, 1 red and 1 blue;
- 2 cubes with top in one colour and the other 5 faces in the other colour;
- 1 cube with T, S1, S2 in red and B, S3, S4 in blue (i.e., 3 faces which meet in a corner with the same colour, the other 3 the second colour);
- 2 'banded' cubes, with S1, S2, S3, S4 all the same colour, and T and B the second colour;
- 2 cubes with 2 adjacent sides, say S1 and S2, the same colour, and the other 4 sides the second colour;
- 1 cube with a 'partial band', say S1, S2, S3 of one colour and the remaining 'partial band' B, S4, T in the second colour.

Coloured cube nets can also be used to show that there are only 10 such cubes.

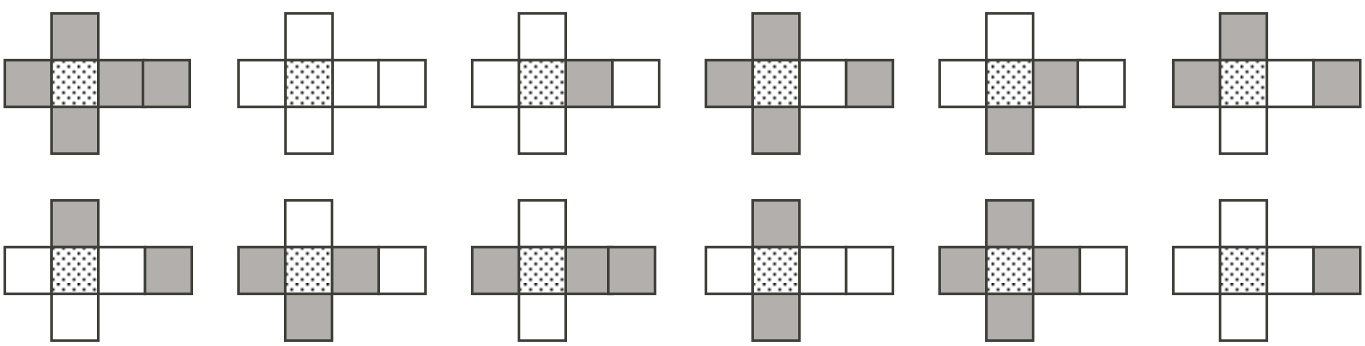
Extension:

1. Allowing for one face to be yellow permits many more cubes. Here are some possibilities:



- 2 with all five remaining sides in one colour (red or blue);
- 2 with 1 adjacent side (say S1) in red (blue), and the other four in blue (red);
- 2 with 2 adjacent sides (S1, S2) in red (blue) and the other three faces in blue (red);
- 2 with 1 adjacent side (S1) and the bottom B in red (blue) and the other three sides in blue (red);
- 2 with opposite sides (S1 and S3) in red (blue), and the remaining faces (S2, B, S4) in blue (red).

Encourage students to come up with more than 10 different cubes. Colouring cube nets is, again, a very good way for students to discover the many possibilities. Here is one possible set of nets for the above ten possibilities plus two more with all four sides S_1, S_2, S_3, S_4 of red (blue) and the reverse.



Problem 6

Two ways to play the game:

1. Set a time limit for the game, suited to your class and the available time. The game could also be used to occupy some students while you are busy with others. Have the teams check each others' work to make sure the operations used are correct.

2. Alternately, play the game as a class. Select a (promising) set of five numbers, and have the whole class work on this. Post the solutions on the bulletin board and have students add to it as they find solutions. Encourage them to check each other's solutions.

Below is a score sheet sample to suggest some possible answers. There are many others.

Note: If you noticed the students slowing down in finding solutions, you may wish to introduce other 'operations'. For example, if you permit a decimal point, then you could write $76 = 56 + (2 \div .1)$, or $85 = 6 \div .1 + 25$ or $16 \div .2 + 5$. Another possibility is to allow squaring (if your starter set contains a 2), so $76 = 5^2 \times (6 \div 2) + 1$ or $(6^2 - 1) \times 2 + 5$.

SAMPLE SCORE SHEET FOR 100 CHALLENGES

$\boxed{1}$ $\boxed{2}$ $\boxed{2}$ $\boxed{5}$ $\boxed{6}$

$2-1=1$	$2\times 1=2$	$5-2=3$	$6-2=4$	$5\times 1=5$
$6\times 1=6$	$5+2=7$	$5+2+1=8$	$5+2+2=9$	$5\times 2=10$
$(5\times 2)+1=11$	$(5\times 2)+2=12$	$(5\times 2)+2+1=13$	$(5+2)\times 2=14$	$5\times(2+1)=15$
$6+5+2+2+1=16$	$15+2=17$	$6\times(2+1)=18$	$15+2+2=19$	$16+2\times 2=20$
$21=21$	$22=22$	$22+1=23$	$25-1=24$	$25=25$
$25+1=26$	$25+2=27$	$25+2+1=28$	$(6\times 5)-1=29$	$6\times 5=30$
$5\times 6+1=31$	$5\times 6+2=32$	$5\times 6+2+1=33$	$5\times 6+2+2=34$	$5\times 6+2+2+1=35$
$6\times(2+1)=36$	$6\times(5+1)+2\div 2=37$	$6\times(5+1)+2=38$	$5\times(6+2)-1=39$	$6\times(5+1)+2+2=40$
$6\times(5+2)-1=41$	$21\times 2=42$	$6\times(5+2)+1=43$	$6\times(5\times 2)+2=44$	$51-6=45$
$(5-1)\times 6\times 2-2=46$	$52-6+1=47$	$6\times(5+2+1)=48$	$25\times 2-1=49$	$25\times 2=50$
$25\times 2+1=51$	$52\times 1=52$	$56-2-1=53$	$56-2=54$	$56-1=55$
$56\times 1=56$	$56+1=57$	$56+2=58$	$56+2+1=59$	$56+2+2=60$
$56+2+2+1=61$	$65-2-1=62$	$65-2=63$	$65-1=64$	$65\times 1=65$
$65+1=66$	$65+2=67$	$65+2+1=68$	$65+2+2=69$	$65+2+2+1=70$
$2\times(2+1)+65=71$	$2\times 5+62=72$	$2\times 5+62+1=73$	$12\times 6+2=74$	$6\times 2\times 2+51=75$
$(16\times 5)-(2\times 2)=76$	$56+21=77$	$56+22=78$	$56+21+2=79$	$2\times 12+56=80$
$(5\times 2)\times(6+2)+1=81$	$16\times 5+2=82$	$26\times(1+2)+5=83$	$12\times(5+2)=84$	$[16+(2\div 2)]\times 5=85$
$65+21=86$	$65+22=87$	$65+21+2=88$	$2\times 12+65=89$	$15\times 6=90$
$15\times 6+(2\div 2)=91$	$122-(5\times 6)=92$	$(26+5)\times(2+1)=93$	$(15\times 6)+2+2=94$	$(21-2)\times 5=95$
$(5-1)\times 6\times 2\times 2=96$	$52\times 2-6-1=97$	$52\times 2-6=98$	$52\times 2-6+1=99$	$(6-1)\times 5\times 2\times 2=100$