Part II: For the Teacher

Curriculum Areas

Problem 1 – Data Management
Problem 2 – Probability
Problem 3 – Measurement and Number Sense
Problem 4 – Measurement and Number Sense
Problem 5 – Number Sense
Problem 6 – Pattern and Algebra

Hints and Suggestions

Problem 1
Suggestion: Use a ruler to get accurate readings from the graph. The horizontal graph lines are 1cm apart, so a ruler with 1 mm divisions works well for reading off population levels.
Hint 1 – f) How do you think the growth from 2020 - 2040 might be related to the growth from 2040 – 2060?

Problem 2
Hint 1 – b) Could Sally's footwear and coat be the same colour? Her footwear and hat?
Hint 2 – c) How many outfits are possible in total?
Note: In part c), 'random' means that Sally is no more likely to pick one item of clothing than any other item.

Problem 3
Suggestion: 1) Have students trace each route in a different colour.

Problem 4
Hint 1 – How long does it take Bopp to run 10 metres?

Extension: How long does it take Bopp to run 1 metre?

Problem 5
Hint 1 – a) Why is 5 quarters too few?
Hint 2 – a) Could Jacques have 8 quarters?

Suggestions: 1) This could be solved by trial and adjustment. A table something like the following may be helpful in keeping track of trails.

Coin	0.25	0.10	0.05	Totals
No.	7	1	5	13 Coins
$ Value	1.75	0.10	0.25	$2.10
No.				
$ Value				

2) Some students may benefit from using play money or real coins.

Problem 6
Hint 1 – Which students will open (or close) locker 8? Locker 13? Locker 21?
Hint 2 – If the position of a locker door is changed an even number of times, will it be open or closed after all students have finished? What if it has an odd number of changes?
Hint 3 – Will the 5th student switch locker 7? Locker 14? What lockers will the 5th student switch? What about the 7th student?

Suggestions: One of the following aids may help students in exploring this problem.

1) Use a chart such as the one shown. Place a bean (or other small counter) on each closed locker, changing them as required for each student.
Alternatively, use a coin on each square. Heads indicates the locker is open; tails indicates that it is closed.

Chart:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

2) Similar to 1, but use a line of squares.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

(Distribute graph paper for these aids, if desired.)

Solutions and Notes

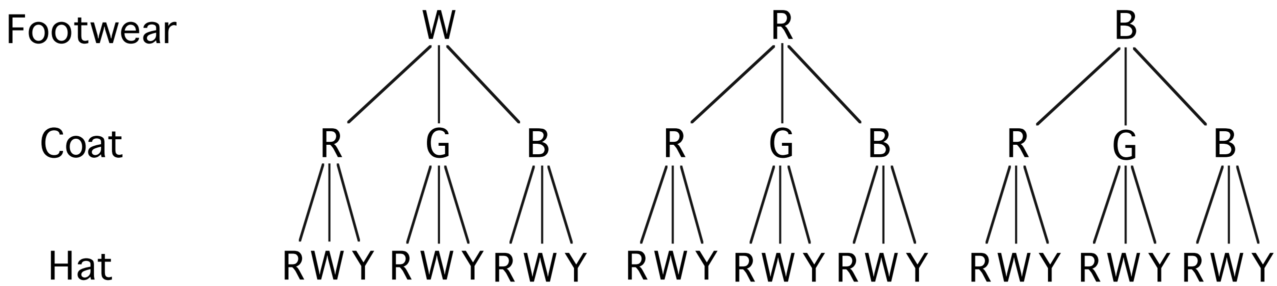
Problem 1

a) The population first exceeded 6 billion people in about the year 2000, say 1998 or 1999.
b) The population in 1985 is just under 5 billion, say 4.8 billion, plus or minus 0.1 billion.
c) The population in 1970 is about 3.7 or 3.8 billion, and in 1990 is about 5.3 billion. Hence the increase is about 1.5 or 1.6 billion.
d) The population in 2020 is predicted to be about 7.5 or 7.6 billion, and in 2040, about 8.8 billion. Thus the predicted increase is approximately 1.2 or 1.3 billion.
e) The graph predicts the population will exceed 9 billion in about 2045 or 2046.
f) Assuming the growth in the 20 year period from 2040 – 2060 is roughly the same as from 2020 – 2040, the predicted population for 2060 is about 8.8 + 1.2 = 10 billion.
g) Assuming the growth from 2050 – 2060 is the same as from 2040 – 2050, the predicted population from 2060 is about 9.2 + (9.2 – 8.8) = 9.2 + 0.4 = 9.6 billion.

Extension: The prediction in g) is more accurate because it uses more recent data than that of f) and extrapolates over a shorter time period.

Problem 2

a) Since red is the only colour common to footwear, coat, and hat, there is only one outfit Sally can wear with all three items the same colour. Suppose we represent the colours by letters as follows: red – R, black – B, white – W, green – G, yellow – Y. We can then construct a tree to record all the possible combinations.



b) Counting the outfits with at least two items of the same colour, we have seven outfits with two red items, three with two white items, and three with two black items, giving us a total of 13 outfits.

c) Since there are 27 outfits in total, and 13 outfits with at least two items of the same colour, the probability is $\frac{13}{27}$.

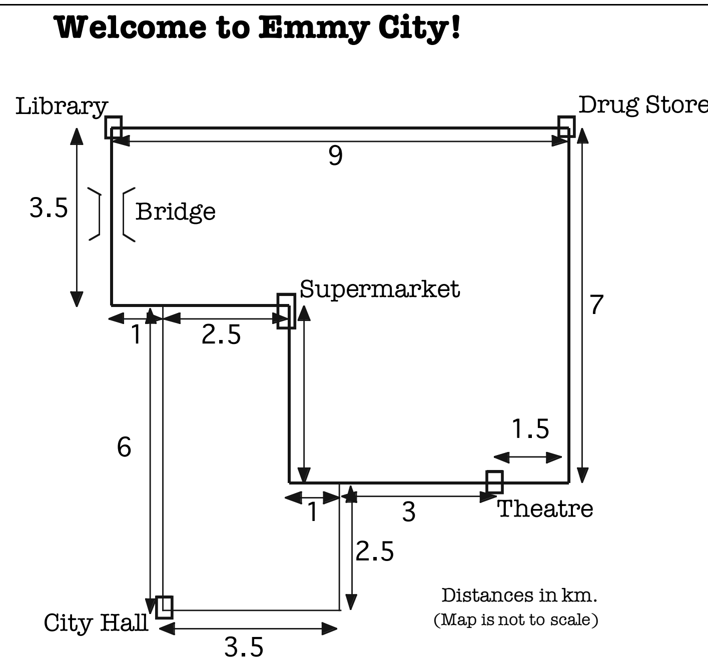
Problem 3

a) The missing distances are shown on the diagram. The shortest route from the library to the theatre is via the bridge and the supermarket, giving
3.5 + 1 + 2.5 + 3.5 + 1 + 3 = 14.5 km.

b) If the bridge is closed, the shortest route is via the drug store, giving
9 + 7 + 1.5 = 17.5 km.

c) From city hall to the supermarket is 8.5 km; from the library to the drug store is 9 km, or 0.5 km further.

d) Yes. The route is city hall to supermarket, to library, to drug store, to theatre, a distance of
10.5 + 7 + 9 + 8.5 = 35 km.



Extensions: 1. No. The supermarket is isolated (not on the perimeter path), so you would either have to retrace the 2.5 km bit, or the 3.5 + 1 km bit.
2. Now you could do it. Start at city hall and go to the theatre, then to the drugstore, then the library, then by the new road to the supermarket, then return to city hall by the left road (2.5 km) and down (6 km), or run these errands in reverse, going first to the Supermarket, then the Library, etc.

Problem 4
Bopp runs 100 m in 11 seconds, so he runs 10 m in 1.1 seconds. Thus, when he runs 110 m, it takes him 11 + 1.1 = 12.1 seconds, allowing Biff to win in 12 seconds.

Extension: To just tie the race, Bopp has to run the total distance in exactly 12 seconds. So we need to know how far he can run in 1 second. Since he runs 100 m in 11 seconds, in 1 second he can run $100 \div 11 = 9\frac{1}{11}$ m. So the answer is c).

Problem 5

a) Trial and adjustment reveals Jacques must have at least 6 quarters (since 5 quarters = $1.25, and 8 dimes = $0.80, giving $2.05 but with no nickels). On the other hand, 7 quarters = $1.75, which makes $2.05 with 6 nickels, but then he'd have no dimes. Thus the only combination with all three types of coins is 6 quarters ($1.50) + 4 dimes ($0.40) + 3 nickels ($0.15) = $2.05. Here are two possible solutions in table form.

No. of 5¢	No. of 10¢	No. of 25¢	Value	Cumulative value	No. of coins
1	1	1	40¢	40¢	3
+2	+2	+2	+80¢	$1.20	9
		+2	+50¢	$1.70	11
		+1	+25¢	$1.95	12
			+10¢	$2.05	13
3	**4**	**6**		**$2.05**	**13**

Notice the student starts with just one of each coin, and gradually adds coins, a few at a time (or even one at a time) constantly checking the cumulative total and the number of coins.

Here's an illustration showing how to delete coins (and their values) if the value goes over $2.05:

No. of 5¢	No. of 10¢	No. of 25¢	Value	Cumulative value	No. of coins
1	1	1	40¢	40¢	3
+2	+2	+2	+80¢	$1.20	9
		+2	+50¢	$1.70	11
		+2	+50¢	$2.20	13
		-1	-25¢	$1.95	12
	+1		+10¢	$2.05	13
3	**4**	**6**		**$2.05**	**13**

b) If Jacques only has 12 coins, then either of these combinations works
   (i) 7 quarters ($1.75) + 1 dime ($0.10) + 4 nickels ($0.20) = $2.05;
   (ii) 6 quarters ($1.50) + 5 dimes ($0.50) + 1 nickel ($0.05) = $2.05.

c) If Jacques exchanges half his quarters in a) for the same number of dimes, he'll exchange 3 quarters ($0.75) for 3 dimes ($0.30), for a net loss of $0.45. So now he has $2.05 - $0.45 = $1.60.

Problem 6
Working through the problem step-by-step reveals that lockers 1, 4, 9, 16 and 25 remain open. These numbers are perfect squares.
There are two key observations which reveal why the square numbered lockers remain open. One is that each student only contacts those lockers of which the student number is a factor, e.g. student 3 opens/closes lockers 3, 6, 9, 12, 15, 18, 21, 24 only; student 7 opens/closes lockers 7, 14, 21 … only, etc. Thus, lockers are only acted on by students whose number is a factor of the locker number.
The second fact is that since the lockers start off closed, they will only be open after the last student if they have had an odd number of contacts, i.e., the locker number must have an odd number of factors. Only perfect squares have an odd number of factors (e.g. 1, 2, 4 for the square number 4, or 1, 3, 9 for the square number 9), so lockers with those numbers are open at the end.

Extension: By the above reasoning, the open lockers would be the square numbers 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.