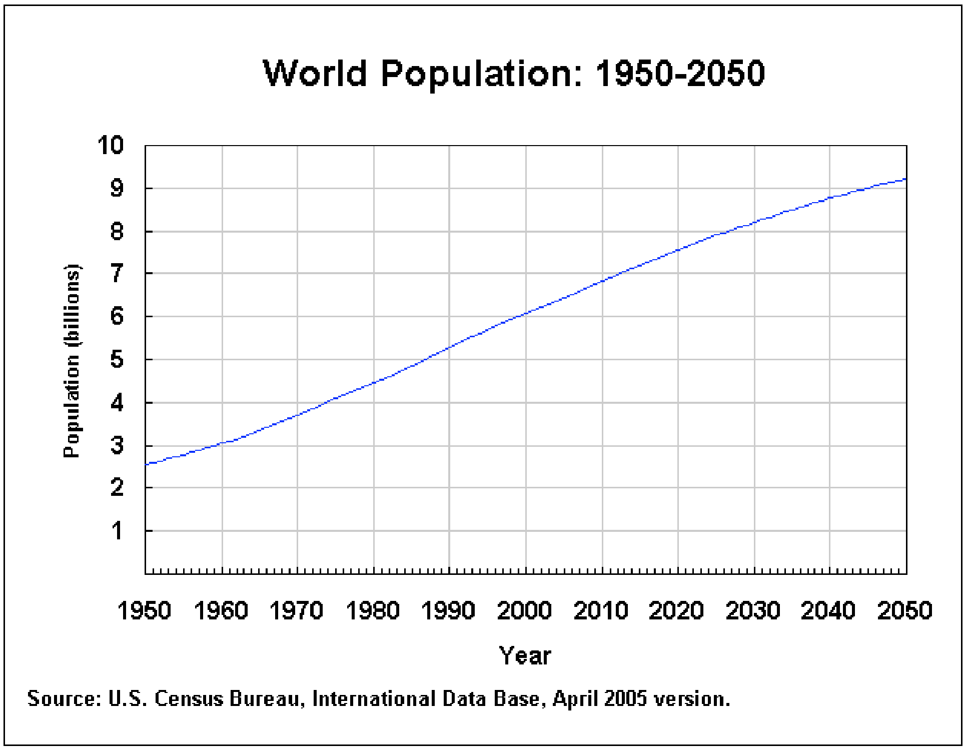
Emmy Noether – Circle 3 for 2005-2006

Part I: Problems

Problem 1:

Here is a graph showing the growth in the world's population from 1950-2000, and projected growth from 2000-2050.

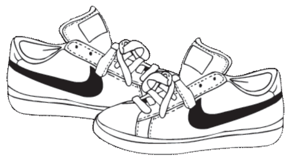


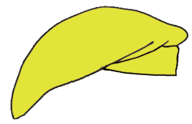
a) Approximately when did the population first exceed 6 billion people?
b) Approximately what was the population in 1985?
c) Approximately what was the increase in population from 1970-1990?
d) Approximately what is the predicted increase from 2020-2040?
e) Approximately when does the graph predict that the population will exceed 9 billion people?
f) Based on the growth from 2020-2040, predict the population in 2060.
g) Based on the growth from 2040-2050, predict the population in 2060.

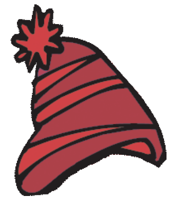
Extension:
Which of your predictions in f) and g) do you think is more accurate? Why?

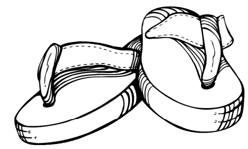
Problem 2:

Sally is getting dressed. For footwear, she can wear white runners, red sandals, or black boots. For a coat, she can choose from a red sweatshirt, a green quilted jacket, or a black leather jacket. For hats, she has a red toque, a white cap, or a yellow beret.





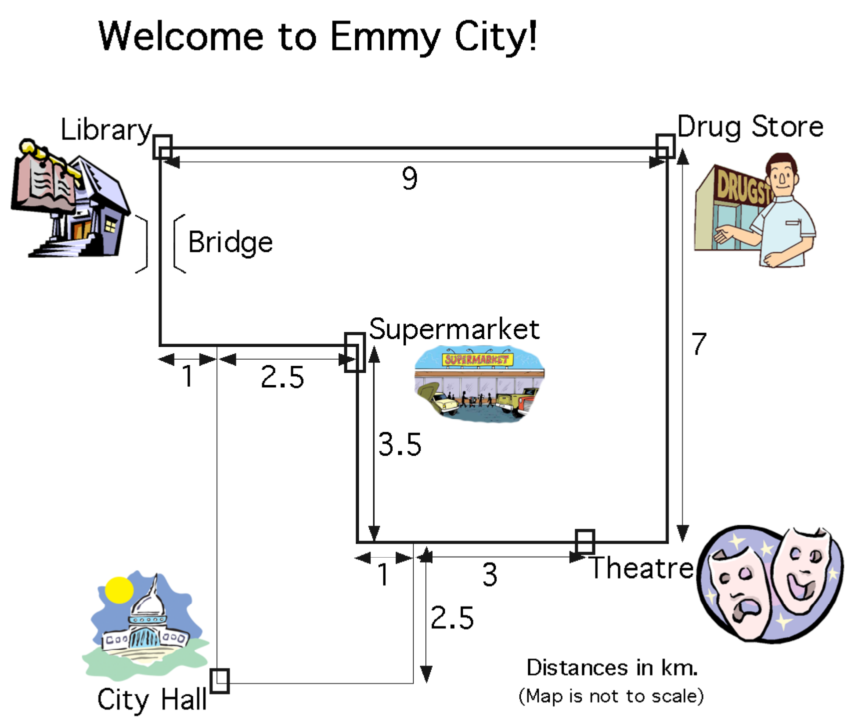




a) How many outfits are possible with all three items of clothing the same colour?
b) How many outfits are there with at least two items of matching colour?
c) If she dressed in the dark, grabbing her clothes at random, what is the probability that her outfit will have a least 2 items of the same colour?

Problem 3:

a) If all angles are right angles, fill in the four missing distances, and then trace the shortest route from the library to the theatre. How far is it?
b) If the bridge is closed, how long is the shortest route for the trip in a)?
c) Which is the further, from city hall to the supermarket, or from the library to the drug store. How much further is it?
d) Is it possible to run errands at the library, supermarket, and drugstore, not necessarily in that order, starting from city hall and finishing at the theatre, without going over the same path twice? If it is possible, how long will the path be, and in what order will the errands be run?



Extensions:
1. Is it possible to run errands starting from city hall and visiting each of the other places marked on the map, then returning to city hall, without retracing any path? Explain.
2. Suppose a new road is built directly (in a straight line) from the library to the supermarket. Repeat Extension 1. Does your answer change?

Problem 4:

Two speedy hares, Biff and Bopp, race each other every day in a 100-metre dash. Bopp always wins in 11 seconds, while Biff takes 12 seconds. One day, Bopp generously decides to start from 10 metres behind the starting line, to give Biff a better chance. What is the outcome of this race?

Extension:
Determine from which of the following distances behind the starting line Bopp should start from in order to tie the race: a) $9\frac{1}{2}$ m, b) $9\frac{1}{12}$ m, c) $9\frac{1}{11}$ m.

Problem 5:

Jacques has 13 coins in his pocket, totalling $2.05. He has only nickels, dimes, and quarters, and he has at least one of each.

a) What is a possible combination of coins Jacques could have? Is this the only possibility? Explain your thinking.
b) What is a possible combination if he only has 12 coins? Is there only one? Explain.
c) From a successful combination of 13 coins you discovered in a), suppose Jacques exchanges half his quarters for the same number of dimes from his coin jar. How much money does he have now?

Problem 6: (Suggested for groups of students)

An Open and Shut Case.
On the first day of school, students in Grade 5 at Emmy Noether Elementary School meet outside and agree on the following prank:

The first student will enter the school and open all 25 of their lockers.
The second student will then enter and close every locker with an even number (2,4,6,…,24).
The third student will then reverse every third locker, i.e. close the locker if it is open, or open it if it is closed.
The fourth student will reverse every fourth locker.
The fifth student will reverse every fifth locker, and so on.

If each of the 25 students enters the building and reverses the lockers in the manner outlined above, which lockers will be open at the end? What is special about these numbers? Why are they still open after the last student?

Extension:
Repeat the problem for 100 lockers. Can you predict which lockers will be open after the 100th student?