Part II: For the Teacher

Curriculum Areas

Problem 1 – Number Sense
Problem 2 – Probability
Problem 3 – Measurement and Geometry (Spatial Sense)
Problem 4 – Number Sense and Pattern Algebra
Problem 5 – Measurement
Problem 6 – Geometry

Hints and Suggestions

Problem 1 a)
Hint 1 – What are the possible days of the week on which you could be born? How many of these are Tuesdays? How do these numbers give the probability of being born on a Tuesday?

Problem 1 b) – f)
Hint 1 – How many days are there in a leap year? in a non-leap year? How often do leap years occur?

Problem 2
Hint 1 – Which clues reveal an exact number that will be part of the solution?
Hint 2 – What are the two possibilities for Jane?
Hint 3 – Is there more than one possibility for Mark?
Hint 4 – How many letters could John have recycled?

Suggestion: A table as shown below may help students organize their thinking by placing the names in the appropriate squares as they work their way through the clues, and shading squares that can't be true.

Letters \ Cans	16	17	22	25	29
101					
102					
105					
136					
138					

Problem 3
Hint 1 – Do two figures which have the same area always have the same shape?

Suggestion: Triangle dot paper may provide an easy way for students to experiment. (See solutions)

Problem 4b)
Hint 1 – How many different digits could be in the 100s place? The units place?
Hint 2 – How many different digits could be in the 10s place?
Hint 3 – List all the 3-digit palindromes with a 100s digit of 1. How many are there?

Extension: Hint 1 – Why isn't 10 a factor
                  Hint 2 – Why isn't the factor even?

Problem 5a)
Hint 1 – What is the perimeter of parking lot B? What is its area?
Hint 2 – If you know the perimeter *p* of a square, how can you find the side length?

Problem 5b)
Hint 1 – What must be the area of each parking lot?
Hint 2 – What are the possible lengths and widths for this area?

Extension: Hint 1 – What must be the perimeter of each parking lot?
                  Hint 2 – What are the length and width of the longest, thinnest such parking lot?
                  Hint 3 – As one dimension changes, what happens to the other dimension?

Problem 6
Suggestion 1: You may wish to tell students from the start that there is no definitive answer of which we are aware. There will always be some waste.
Suggestion 2: Provide cm² graph paper so students can draw and cut out several copies of each template. They may also wish to draw several 12 x 12 cm grids on cm² graph paper for experimenting and recording their answers (or you can make extra copies of the grid given with the problem). Another possibility is to colour the template patterns on a 12 x 12 cm grid.

Extension Suggestion: If using a single cube net seems to give a large amount of waste no matter what arrangement is tried, suggest that students try using more than one net at a time to discover what combination of nets will cover the greatest area of the grid.

Solutions and Notes

Problem 1
Using the ratio of desired outcomes to total possible outcomes for each event gives the following answers:

a) $\dfrac{1}{7}$ b) $\dfrac{1}{365}$ c) $\dfrac{1}{366}$ d) 0   e) $\dfrac{1}{365 \times 3 + 366} = \dfrac{1}{1461}$ f) $\dfrac{12}{365}$

Problem 2
Carbon recycled 16 cans, the least number of cans; Tobias recycled 101 letters, the least number of letters. Jane has 80 more letters than cans, so she could have recycled 102 letters and 22 cans, or 105 letters and 25 cans. But Mark has a total of 124, so he must have done 102 + 22, since that is the only combination that totals 124. So Jane did 105+25. John and Mark total 289 so John's total is 289 – 124 = 165, which means John must have done 136 letters and 29 cans, since that is the only such total. Since Tobias recycled 101 letters and Carbon recycled 16 cans, Tobias must have done 17 cans. The only leftover is 138 letters, which Carbon must have recycled.

Problem 3

a) There is basically one way to create two triangles of equal area. You may wish to discuss whether drawing the dividing line from another vertex gives a 'different' solution.
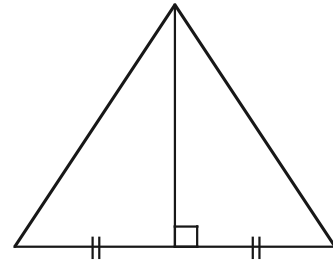
b) There is one solution using congruent triangles with a common vertex at the centre of the triangle, as shown.
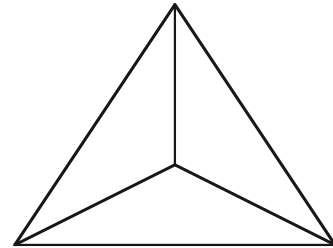
If students know that triangles with the same height and base lengths have equal areas, another solution is found by dividing the base into three equal parts, since all three triangles have a common height.
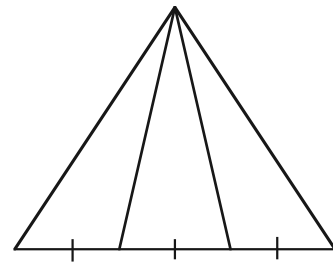
A further solution involves using this idea twice, leaving one triangle from the previous solution, (in this diagram the triangle on the left), and then creating two triangles with equal base lengths and common height h from remaining portion.
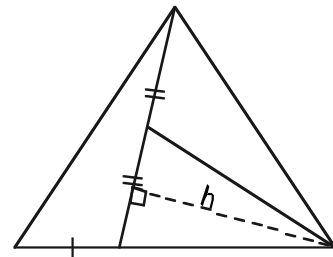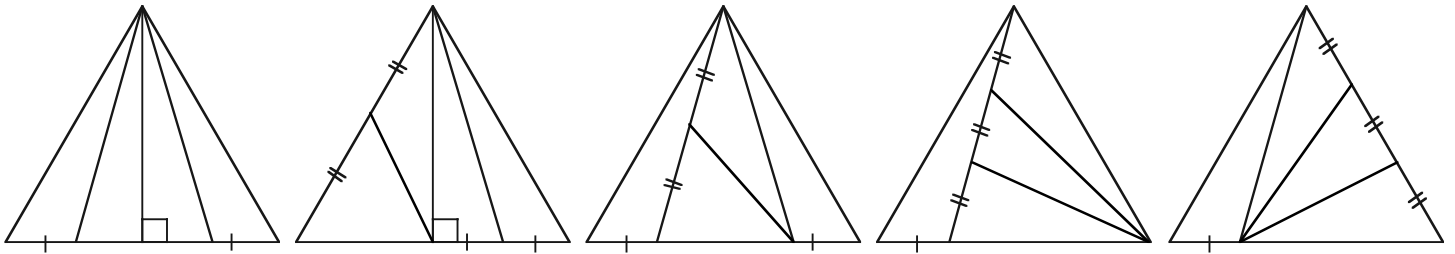
Again, you may wish to discuss whether flips and turns of these solutions are 'different'.

Extension:
Probably the two solutions easiest to see are these:
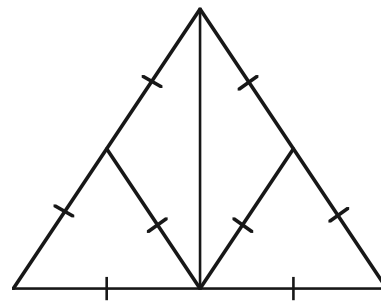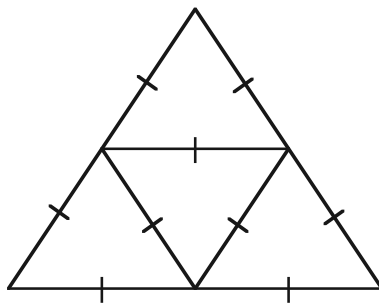
Using equal areas from triangles of equal height and base lengths, many other solutions can be created. For example:

Note that, in the five diagrams above a single ' | ' indicates a line segment with length 1/4 of the side of the equilateral triangle, but 'll' refers to various lengths which are equal in any one diagram, but not necessarily from one diagram to another.

Comment: This problem lends itself to solutions on triangle dot paper. Have students draw triangles with 4-segment sides for part a), 3, 6, or 9 for part b) and 4 or 8 for the extension. This may help them to 'see' the equalities more easily. Here are some samples;

a)

b)
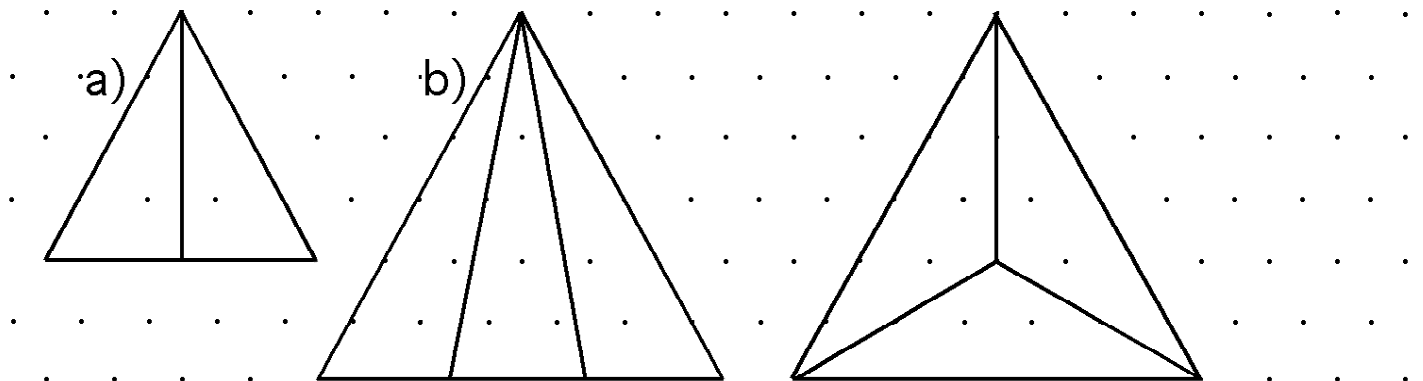
Extension:
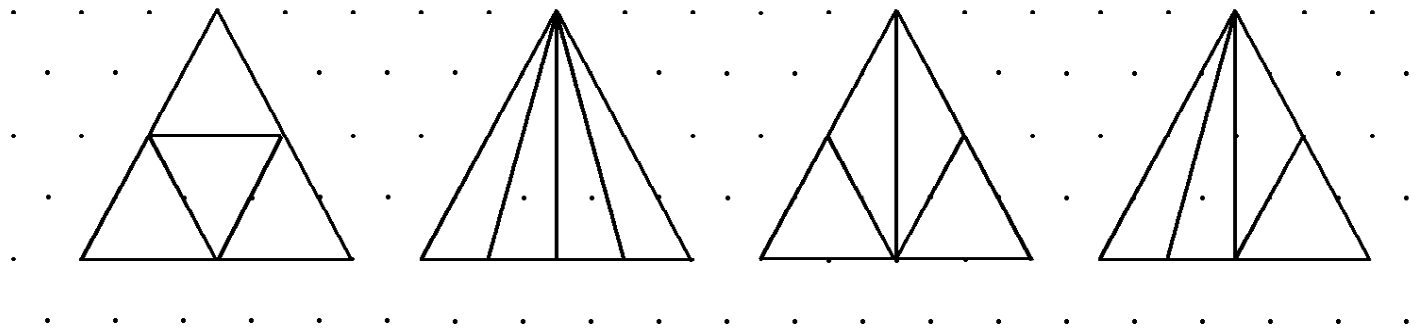
Problem 4
a) The nine 2-digit palindromes are 11, 22, 33, … 99.

b) Since the middle digit can be any one of the ten digits 0, 1, 2, … 9, but the outer digits can only be 1, 2, … 9, there are 9 x 10 = 90, 3-digit palindromes.

c), d) Since the 4-digit palindrome must have a digit pattern *mnnm* with 9 choices for *m* and 10 choices for *n*, there are again 9 x10 = 90 possible 4-digit palindromes. Another way of thinking of this is to take a three digit palindrome *mnm* and just insert a copy of *n*, *mnnm*; hence the number of possibilities does not change.
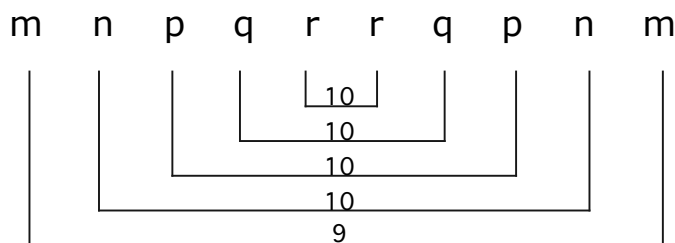
e) The number of 3 or 4 digit palindromes (90) is 10 times the number of 1 or 2 digit palindromes.

Extension 1: Each 4 digit palindrome is divisible by 11. (The reason for this is that the numeric value of *mnnm* is equal to: 1000 x *m* + 100 x *n* + 10 x *n* + *m* = 1001 x *m* + 110 x *n* = 11 x (91 x *m* + 10 x *n*).)

Extension 2: The same principles apply to generating 5-digit palindromes as for 3 or 4-digits. The pattern for 5-digits is *mnpnm*, with 10 choices for *p*; similarly, for 6-digits, the pattern is *mnppnm*. Thus the number of 5 or 6-digit palindromes is 10 times the number of 3 or 4-digit palindromes, which is the same as the relationship revealed above (and in the table) between 3 or 4-digit palindromes and 1 or 2-digit palindromes.

Digits	Number of Palindromes
1	9
2	9
3	90
4	90
5	900
6	900
7	9000
10	90000

For 7-digit palindromes, the pattern is *mnpqpnm* with 10 choices for *q*, hence 10 x 900 possibilities. For 10-digits, the pattern is

with 10 choices for each of the interior digits and 9 for the outer two, giving
9 x 10 x 10 x 10 x 10 = 90 000 choices, or 10 times the number of 7 and 8-digit palindromes.

Problem 5

a) Since the perimeter of lot B is 2 x (40 + 90) = 2 x 130 = 260 metres.

One side must be $\frac{260}{4} = 65$ metres. Thus the area of lot B is 65 x 65 = 4225 square metres.

Hence area of lot B - area of lot A = 4225 - 3600 = 625 square metres.

b) Since the area of a rectangle of length l metres and width m meters is l x w square metres, we seek all whole numbers l and w such that l x w = 3600, and neither l nor w is less than 20. The simplest way to discover the possibilities is to look at combinations of factors of 3600 in a methodical way.

Since $3600 = 2^4 \times 3^2 \times 5^2$, the winning combinations are:

20 x 180, 24 x 150, 25 x 144, 30 x 120, 36 x 100, 40 x 90, 48 x 75, 50 x 72, 60 x 60 and 80 x 45.

You may wish to discuss whether 40 x 90 is the same lot as 90 x 40. If you decide yes, then there are 9 other lots different from the original lot A. If you decide not, there are an additional 9 lots, (i.e., all the products reversed except 60 x 60), giving a total of 18 lots different from A.

Extension: Since the perimeter of a rectangle of length l metres and width w metres is $(l + w) \times 2$ metres, if the perimeter must be the same as parking lot A, then

$(l + w) \times 2 = (90 + 40) \times 2$ metres.

This means $l + w = 130$ metres. If w must be at least 20 metres, then we want all pairs of numbers such that their sum is 130, with one number at least 20. These are 20+110, 21 + 109, 22 + 108, … , 110 + 20, i.e., 91 possible pairs. (This assumes that a parking lot with dimensions 21 by 109 is different from one that is 109 by 21. If you decide they are the same, then there are only 46 different lots.)

Problem 6

The key thing here is that, at the level of Grade 5, 6 mathematics, there is no way to prove that any particular layout for one template, or a variety of them, is the minimal waste solutions, i.e., this is a completely open-ended problem, as is the case in many real world situations. Here are some comments and possible layouts for the three given templates.

Students should note initially that no matter how Net 1 is placed in a corner of the grid, some squares, marked with an "X", will be waste material. This is not true of Nets 2 and 3.

Net 1   Net 1   Net 2   Net 3
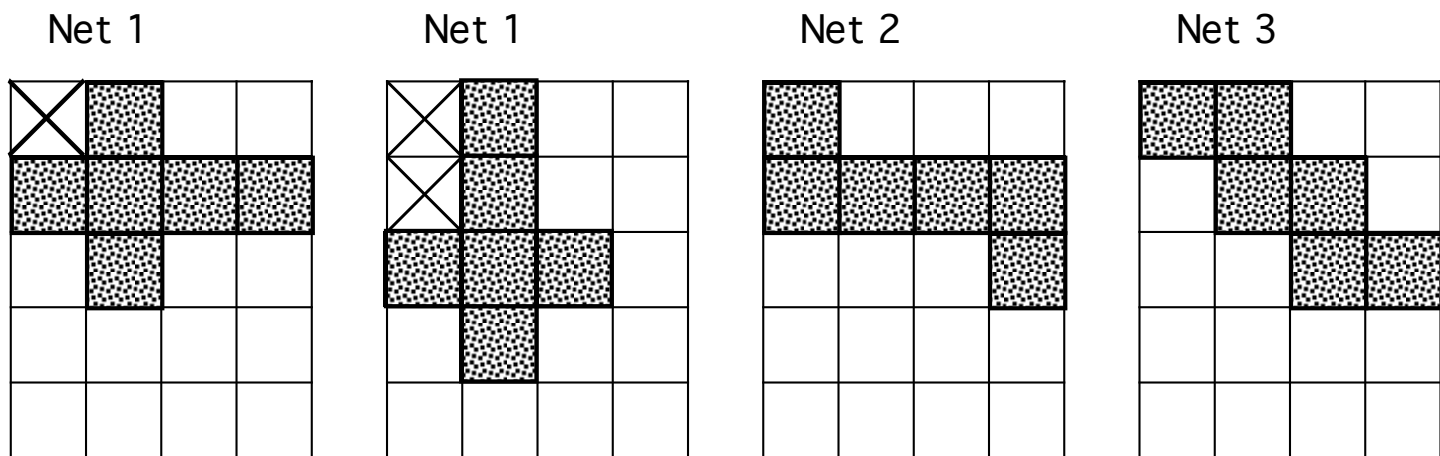
Fitting copies of Net 1 together as closely as possible (Figure 1 below), limits the number of waste squares until the first column is completed, and the 6th copy of the net is placed (Figure 2 below). Continuing to fit nets together closely (Figure 3 below) avoids more waste squares until reaching the top, where the placement of the 11th net necessitates more waste squares.
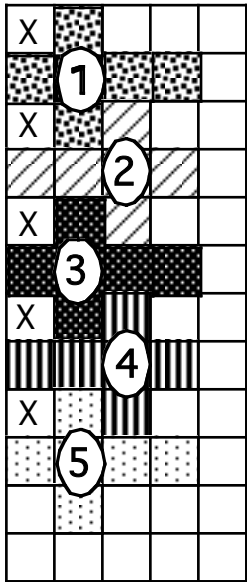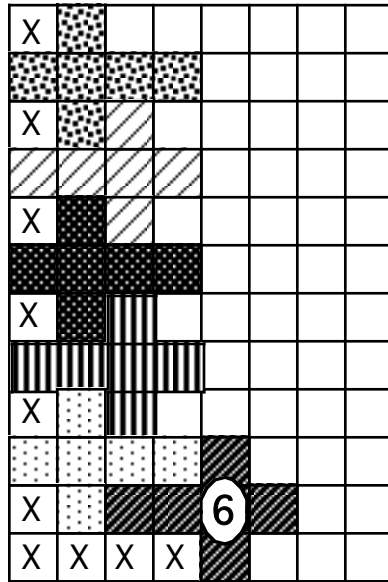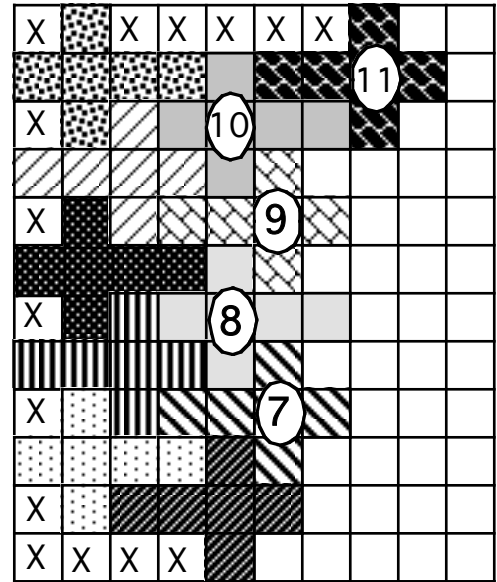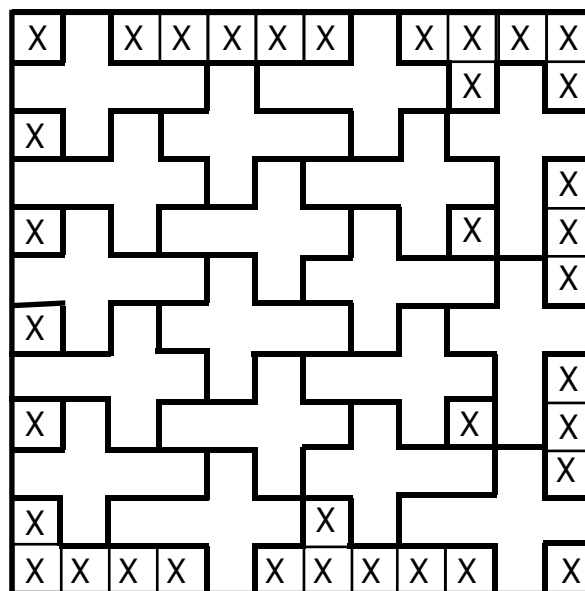
Figure 1

Figure 2

Figure 3

One way the problem may be completed with Net 1 is shown below in Figure 4. This solution contains 36 waste squares. Students could be challenged to find a solution with fewer waste squares.

Figure 4

Although Net 2 can be fitted into a corner so as not to leave any waste squares (Figure 5 below), it is impossible to add a second copy of Net 2 without necessitating waste squares. (Figures 6 and 7)
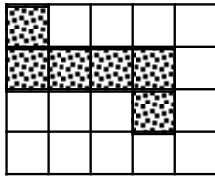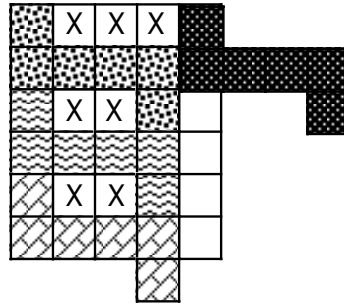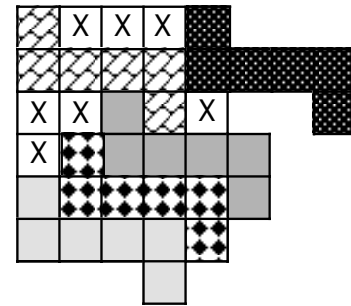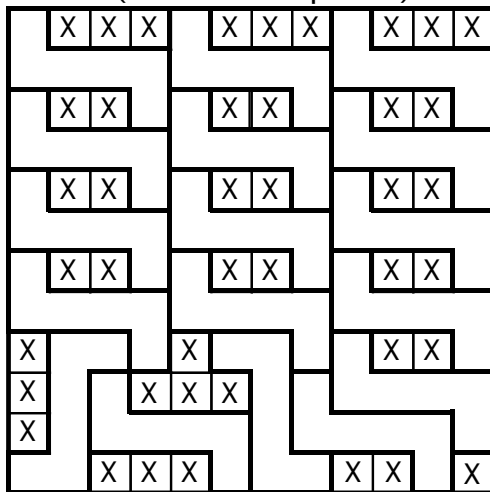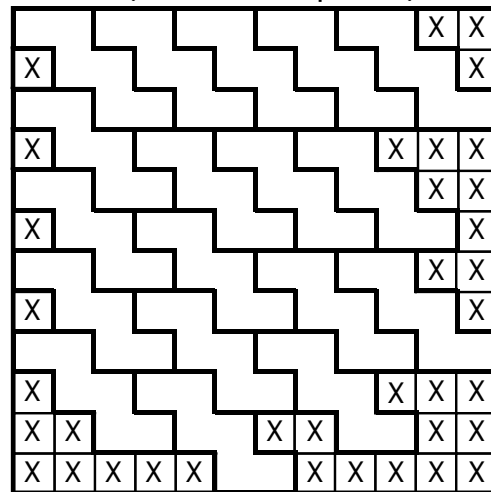
Figure 5	Figure 6	Figure 7

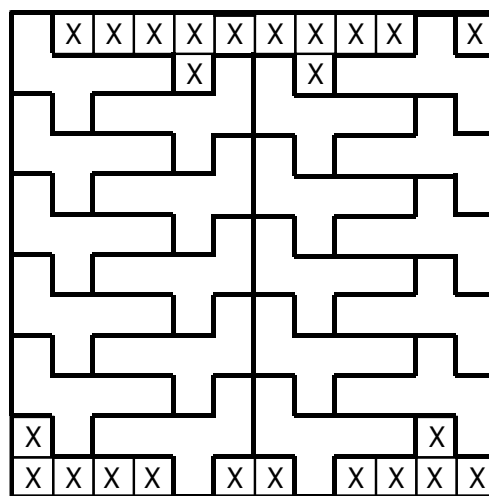The same is true of Net 3. Possible solutions using Nets 2 and 3 are given below.

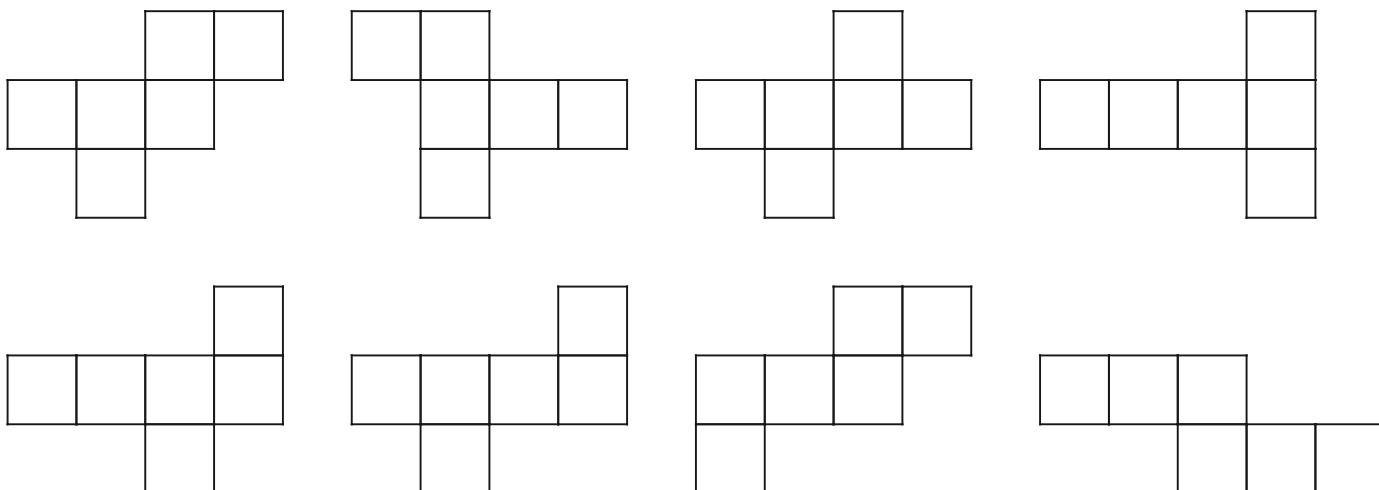| Net 2: (42 waste squares)	Net 3: (36 waste squares)

The diagram below shows a different net that creates only 24 waste squares.

Suggestion: Have student groups compare their solutions with other groups to see which has the least waste. Ask students which net they think will produce the least waste and why.

Extension: Note that the other eight cube nets are as shown below.

If your students do experiment with other nets, ask whether it would be reasonable to use several different nets to avoid waste. Discuss the downside of this, (e.g., if folding is done by machine, this may be impossible; similarly, printing on different nets would require different layouts.)