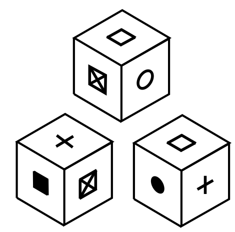
CEMC at Home

Grade 7/8 - Monday, May 4, 2020
Contest Day 1

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #19

Three different views of the same cube are shown. The symbol on the face opposite ● is

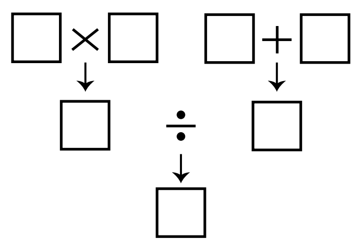


(A) + **(B)** ■ **(C)** ⊠

(D) □ **(E)** ○

2016 Gauss Contest, #20

In the diagram, four different integers from 1 to 9 inclusive are placed in the four boxes in the top row. The integers in the left two boxes are multiplied and the integers in the right two boxes are added and these results are then divided, as shown. The final result is placed in the bottom box. Which of the following integers cannot appear in the bottom box?



(A) 16 **(B)** 24 **(C)** 7

(D) 20 **(E)** 9

More Info:

Check out the CEMC at Home webpage on Monday, May 11 for solutions to the Contest Day 1 problems.

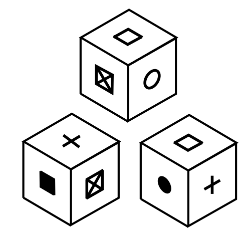
CEMC at Home

Grade 7/8 - Monday, May 4, 2020
Contest Day 1 - Solution

Solutions to the two contest problems are provided below, including a video for the first problem.

2020 Gauss Contest, #19

Three different views of the same cube are shown. The symbol on the face opposite ● is

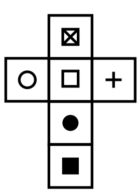


(A) + (B) ■ (C) ⊠

(D) □ (E) ○

Solution:

We begin by recognizing that there are 6 different symbols, and so each face of the cube contains a different symbol.
From left to right, let us number the views of the cube 1, 2 and 3.
Views 1 and 2 each show a face containing the symbol ⊠ .
What symbol is on the face opposite to the face containing ⊠?
In view 1, □ and ○ are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
In view 2, ■ and + are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
There is only one symbol remaining, and so ● must be the symbol that is on the face opposite ⊠, and vice versa.
A net of the cube is shown below.



Answer: (C)

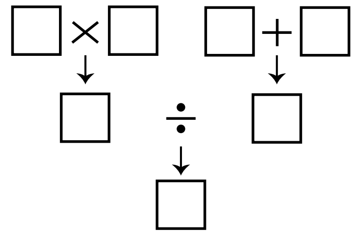
Video

Visit the following link to view another solution to the first contest problem that uses nets:
https://youtu.be/N88l8IXEiHs

See the next page for a solution to the second contest problem.

2016 Gauss Contest, #20

In the diagram, four different integers from 1 to 9 inclusive are placed in the four boxes in the top row. The integers in the left two boxes are multiplied and the integers in the right two boxes are added and these results are then divided, as shown. The final result is placed in the bottom box. Which of the following integers cannot appear in the bottom box?

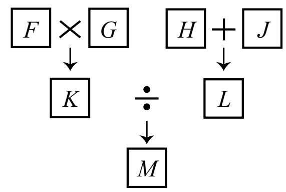


(A) 16 **(B)** 24 **(C)** 7

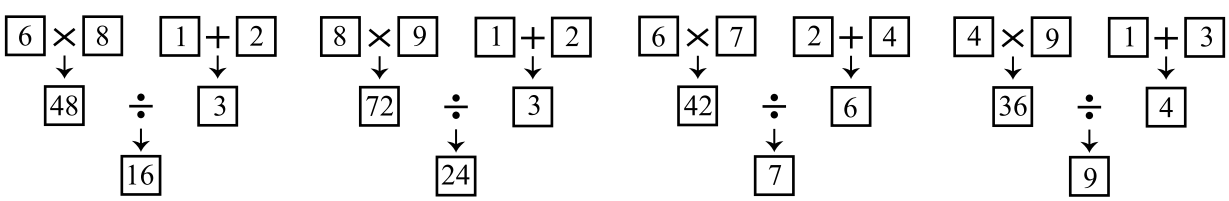
(D) 20 **(E)** 9

Solution:

We begin by naming the boxes as shown to the right.



Of the five answers given, the integer which cannot appear in box M is 20. Why?
Since boxes F and G contain different integers, the maximum value that can appear in box K is $8 \times 9 = 72$.
Since boxes H and J contain different integers, the minimum value that can appear in box L is $1 + 2 = 3$.
Next, we consider the possibilities if 20 is to appear in box M.
If 3 appears in box L (the minimum possible value for this box), then box K must contain 60, since $60 \div 3 = 20$.
However, there are no two integers from 1 to 9 whose product is 60 and so there are no possible integers which could be placed in boxes F and G so that the product in box K is 60.
If any integer greater than or equal to 4 appears in box L, then box K must contain at least $4 \times 20 = 80$.
However, the maximum value that can appear in box K is 72.
Therefore, there are no possible integers from 1 to 9 which can be placed in boxes F, G, H, and J so that 20 appears in box M.
The diagrams below demonstrate how each of the other four answers can appear in box M.



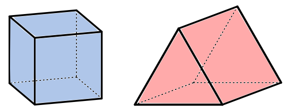
Answer: (D)

CEMC at Home

Grade 7/8 - Tuesday, May 5, 2020

It's in the Bag

Colin has some cubes and some triangular prisms whose rectangular faces are actually squares.



Colin put some number of each of these solids in a bag. In total, the solids in his bag have 21 square faces, 6 triangular faces, and no other types of faces. How many of each solid did Colin put in his bag?

There are several ways to solve this problem. Three different methods are shown below.

Method 1: Make a table

Number of Cubes	Number of Square Faces
1	6
2	12
3	18

Number of Triangular Prisms	Number of Square Faces	Number of Triangular Faces
1	3	2
2	6	4
3	9	6

Notice that if we have 2 cubes and 3 triangular prisms, then we have $12 + 9 = 21$ square faces and 6 triangular faces, as desired. So Colin could have 2 cubes and 3 triangular prisms in his bag. Can you explain why this is the only possibility for the solids in Colin's bag?

Method 2: Use logic

Since cubes do not have triangular faces, the triangular faces must have all come from the triangular prisms. Each triangular prism has 2 triangular faces, so we would need 3 triangular prisms in order to have 6 triangular faces in total. Each triangular prism has 3 square faces, so 3 triangular prisms would have 9 square faces in total. We need 21 square faces. Since $21 - 9 = 12$, that means the remaining 12 square faces must come from cubes. Each cube has 6 square faces, so we would need 2 cubes in order to have 12 square faces in total. This means Colin must have 3 triangular prisms and 2 cubes in his bag.

Method 3: Use variables

Let c represent the number of cubes and t represent the number of triangular prisms in the bag. Since each cube has 0 triangular faces and each triangular prism has 2 triangular faces, the number of triangular faces in the bag must be $2 \times t$. Since there are 6 triangular faces in total, we must have

$$2 \times t = 6$$

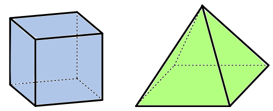
Since each cube has 6 square faces and each triangular prism has 3 square faces, the total number of square faces in the bag must be $6 \times c + 3 \times t$. Since there are 21 square faces in total, we must have

$$6 \times c + 3 \times t = 21$$

Can you use these equations to figure out what the values of t and c must be?

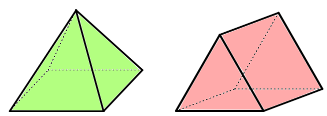
Think about these problem solving methods while you work on the next problems.
Which ones work well for you in these problems, and which ones may not work so well?

Problem 1: Valerie has some cubes and some square-based pyramids.



Valerie put some number of each of these solids in a bag. In total, the solids in her bag have 68 square faces, 56 triangular faces, and no other types of faces. How many of each shape did Valerie put in her bag?

Problem 2: Max has some square-based pyramids and some triangular prisms whose rectangular faces are actually squares.



Max put some number of each of these solids in a bag. In total, the solids in their bag have 13 square faces, some number of triangular faces, and no other types of faces. How many different combinations of solids could Max have put in their bag?

There is no way to know exactly what solids were in Max's bag, but can you find all of the different possibilities? Can you explain how you know you have found them all?

More Info:

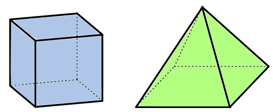
Check out the CEMC at Home webpage on Wednesday, May 6 for a solution to It's in the Bag.

For more practice solving problems using variables, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Tuesday, May 5, 2020
It's in the Bag - Solution

Problem 1: Valerie has some cubes and some square-based pyramids.



Valerie put some number of each of these solids in a bag. In total, the solids in her bag have 68 square faces, 56 triangular faces, and no other types of faces. How many of each shape did Valerie put in her bag?

There are several ways to solve this problem. Three different solutions are shown below.

Method 1: Make a table

Number of Cubes	Number of Square Faces
1	6
2	12
3	18
⋮	⋮
8	48
9	54
10	60
11	66

Number of Pyramids	Number of Square Faces	Number of Triangular Faces
1	1	4
2	2	8
3	3	12
⋮	⋮	⋮
11	11	44
12	12	48
13	13	52
14	14	56

Notice that if we have 9 cubes and 14 pyramids, then we have $54 + 14 = 68$ square faces and 56 triangular faces as needed. So Valerie could have 9 cubes and 14 pyramids in her bag.

Can you use the table to convince yourself that this is the only possibility for the combination of objects in Valerie's bag? We will justify this in the following two solutions.

This method is time consuming because we needed to fill in many rows of the tables before we found a combination that worked. The other methods are more efficient for solving this problem.

Method 2: Use logic

Since cubes do not have triangular faces, the triangular faces must have all come from the pyramids. Each pyramid has 4 triangular faces. Since $56 \div 4 = 14$, there must be 14 pyramids in order to have 56 triangular faces in total.

Each pyramid has 1 square face, so the 14 pyramids have 14 square faces among them. There are 68 square faces in total. Since $68 - 14 = 54$, this means the remaining 54 square faces must come from cubes. Each cube has 6 square faces. Since $54 \div 6 = 9$, there must be 9 cubes in order to have 54 square faces among the cubes.

This means Valerie must have 9 cubes and 14 pyramids in her bag.

Method 3: Use variables

Let c represent the number of cubes and s represent the number of square-based pyramids in Valerie's bag.

Since each cube has 0 triangular faces, and each pyramid has 4 triangular faces, the number of triangular faces in the bag must be $4 \times s$. Since there are 56 triangular faces in total, we must have

$$4 \times s = 56$$

Notice that $4 \times 14 = 56$. The only positive integer that satisfies this equation is $s = 14$ and so we know the number of pyramids must be 14.

Since each cube has 6 square faces and each pyramid has 1 square face, the number of square faces in the bag must be $6 \times c + 1 \times s$. Since there are 68 square faces in total, we must have

$$6 \times c + 1 \times s = 68$$

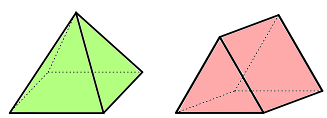
Remember that $s = 14$ so substituting this value we get

$$6 \times c + 1 \times 14 = 68$$

From this we see that $6 \times c$ must equal 54 which means c must equal 9.

Therefore, Valerie must have 9 cubes and 14 pyramids in her bag.

Problem 2: Max has some square-based pyramids and some triangular prisms whose rectangular faces are actually squares.

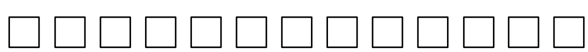


Max put some number of each of these solids in a bag. In total, the solids in their bag have 13 square faces, some number of triangular faces, and no other types of faces. How many different combinations of solids could Max have put in their bag?

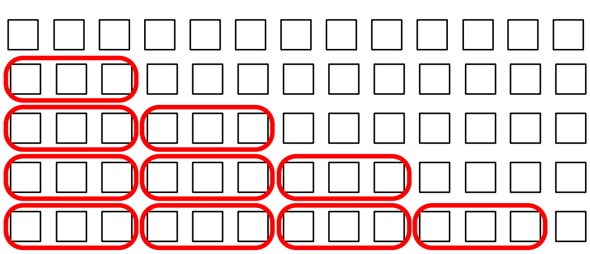
There are several ways to solve this problem. Two different solutions are shown below.

Method 1: Use diagrams

The squares below represent the 13 square faces in Max's bag.



We know each triangular prism has 3 square faces and each pyramid has 1 square face. So we want to figure out how many ways we can put the 13 squares into groups of 3 and 1. The diagram below shows all possible ways to do this, where each group of 3 is circled.



→ 0 triangular prisms and 13 pyramids
→ 1 triangular prism and 10 pyramids
→ 2 triangular prisms and 7 pyramids
→ 3 triangular prisms and 4 pyramids
→ 4 triangular prisms and 1 pyramid

So there are 5 different possible combinations for the solids in Max's bag.

Solution 2: Use variables

Let t represent the number of triangular prisms and s represent the number of square-based pyramids in Max's bag. Each triangular prism has 3 square faces and each pyramid has 1 square face. Since there are 13 square faces in total, we can write the following equation.

$$3 \times t + 1 \times s = 13$$

Notice that there is only one equation and it has two variables. This means we cannot solve this equation in the same way we might solve other equations with only one variable. It also means that the equation may have more than one solution. To find some solutions, we start testing some values.

Suppose there are 0 triangular prisms (and so $t = 0$). Then the equation becomes

$$3 \times 0 + 1 \times s = 13 \quad \text{or} \quad 1 \times s = 13$$

This tells us that in this case we must have $s = 13$.

Suppose there is 1 triangular prism (and so $t = 1$). Then the equation becomes

$$3 \times 1 + 1 \times s = 13 \quad \text{or} \quad 3 + 1 \times s = 13$$

This tells us that in this case we must have $3 + s = 13$ which means $s = 10$.

What are the values of s if there are 2, 3, or 4 triangular prisms (that is, if $t = 2$, $t = 3$, or $t = 4$)? Can there be 5 or more triangular prisms?

It turns out that there are five solutions, and they are shown below:

$$\begin{aligned}
3 \times 0 + 1 \times 13 = 13 &\rightarrow \text{ 0 triangular prisms and 13 pyramids} \\
3 \times 1 + 1 \times 10 = 13 &\rightarrow \text{ 1 triangular prism and 10 pyramids} \\
3 \times 2 + 1 \times 7 = 13 &\rightarrow \text{ 2 triangular prisms and 7 pyramids} \\
3 \times 3 + 1 \times 4 = 13 &\rightarrow \text{ 3 triangular prisms and 4 pyramids} \\
3 \times 4 + 1 \times 1 = 13 &\rightarrow \text{ 4 triangular prisms and 1 pyramid}
\end{aligned}$$

Notice that there cannot be 5 or more triangular prisms is the bag. 5 triangular prisms will contribute $3 \times 5 = 15$ square faces which is more than the total of 13.

So there are 5 different possible combinations for the solids in Max's bag.

Did you use a different approach to solve Problem 2? Is there a way to use a table or other reasoning to solve this problem?

More Info:

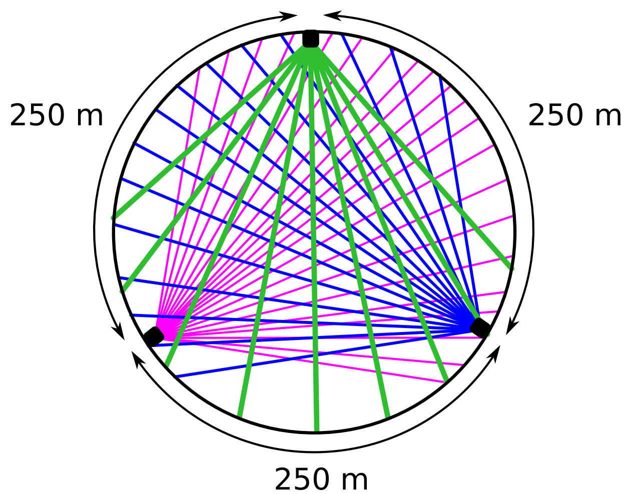
Equations with more than one variable like the one in the solution to Problem 2 are called Diophantine equations. For more practice finding solutions to Diophantine equations, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Wednesday, May 6, 2020

Light Show

William is creating a light show over a circular pond. He has positioned a green light, a pink light, and a blue light so they are equally spaced around the edge of the pond. To get from one light to the next, he must walk 250 metres around the edge of the pond.



Each light has a switch that William can use to turn the light on and off. After positioning the lights, William wants to test each of the different on/off combinations of the three switches to make sure he is happy with how the lights look together.

Problem 1: How many different on/off combinations of the three switches are there?

Need help getting started? Check out the online exploration where you can turn the switches on and off to see all of the different combinations of lights.

Problem 2: William starts at the blue light with all three lights switched off. Explain how William can test four different on/off combinations of the switches (other than "all switches off") by walking a total distance of 750 m around the pond, possibly changing directions during his trip.

Problem 3: William starts at the blue light with all three lights switched off. William wants to test all possible on/off combinations of the switches. What is the shortest possible distance he could walk to complete this task?

Note that William can change directions as many times as he wants during his trip.

More Info:

Check out the CEMC at Home webpage on Thursday, May 7 for a solution to Light Show.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

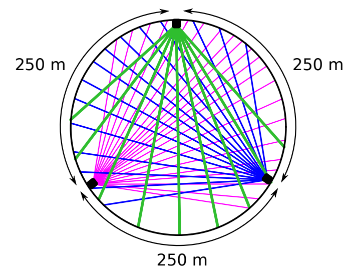
CEMC at Home

Grade 7/8 - Wednesday, May 6, 2020
Light Show - Solution

Summary

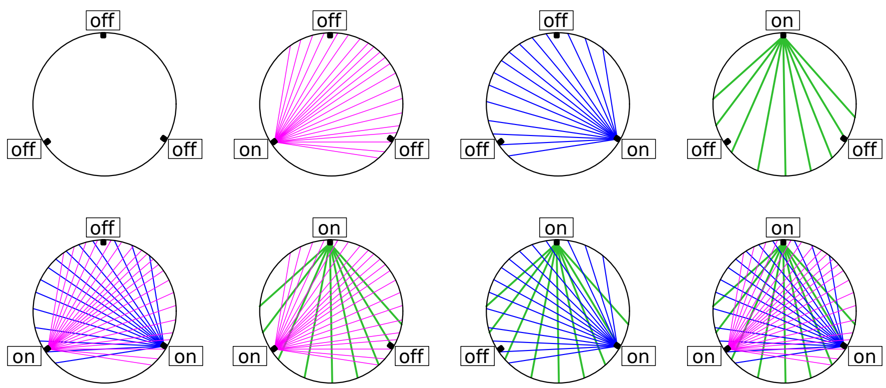
William has positioned a green light, a pink light, and a blue light so they are spaced equally around the pond. To get from one light to another he must walk 250 metres around the edge of the pond.

Each light has a switch that William can use to turn it on and off.



Problem 1: How many different on/off combinations of the three switches are there?

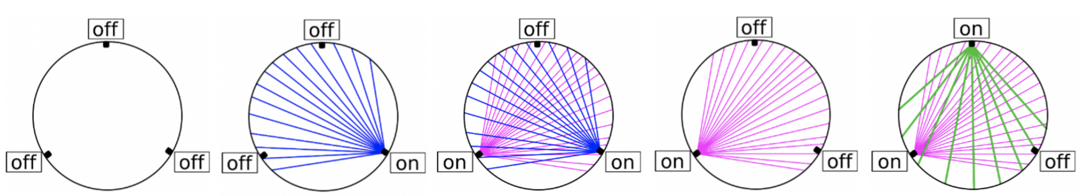
Solution: There are 8 different on/off combinations, as shown below.



Problem 2: William starts at the blue light with all three lights switched off. Explain how William can test four different on/off combinations of the switches (other than "all switches off") by walking a total distance of 750 m around the pond, possibly changing directions during his trip.

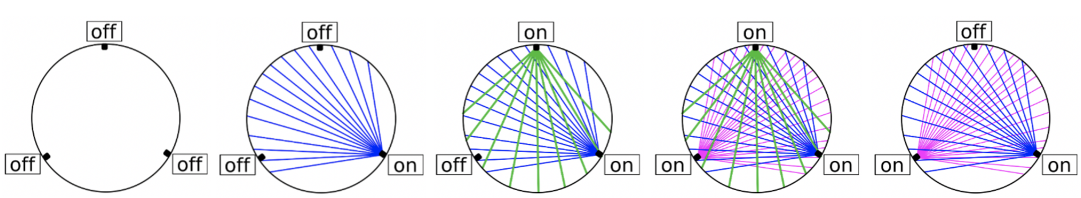
Solution: There are many different ways to achieve this goal.

One option is the following: Start by turning the blue light on. Walk 250 m to the pink light and turn it on. Walk another 250 m *back* to the blue light and turn it off. Walk a final 250 m to the green light and turn it on. The images below show the four different combinations tested on this trip.



Start | After 0 m | After 250 m | After 500 m | After 750 m

Another possible option is illustrated using the images below. (Can you describe the trip here?)



Start | After 0 m | After 250 m | After 500 m | After 750 m

Problem 3: William starts at the blue light with all three lights switched off. William wants to test all possible on/off combinations of the switches. What is the shortest possible distance he could walk to complete this task?

Solution: William can test all eight of the combinations while walking a total of 1500 m. One way to do this is outlined in the table below.

Action	Blue switch	Pink switch	Green switch	Total distance travelled
Start at the blue switch	off	off	off	0 m
Turn the blue switch on	**on**	off	off	0 m
Turn the pink switch on	on	**on**	off	250 m
Turn the blue switch off	**off**	on	off	500 m
Turn the green switch on	off	on	**on**	750 m
Turn the pink switch off	off	**off**	on	1000 m
Turn the blue switch on	**on**	off	on	1250 m
Turn the pink switch on	on	**on**	on	1500 m

We can check that this table contains a row for each of the eight different combinations, and that this trip will take a total of 1500 m.

This means William can complete this task by walking *at most* 1500 m. Since the question is asking for us to find the *shortest possible* distance, we still need to show that it is not possible for William to walk *less than* 1500 m and still test all eight on/off combinations. Let's see why this cannot be done.

At his starting position, William can test at most 2 different combinations: the initial switch combination ("all switches off") and the combination resulting from flipping this switch (the blue switch). So William has not yet moved and has to test 6 more combinations. Since William can only test one new combination each time he walks to a new switch, every 250 m walk he does will result in testing *at most* one new combination. This means that in order to test all 6 remaining combinations, he has to walk at least 6×250 m $= 1500$ m.

This means the shortest possible distance William can walk to complete the task is 1500 m.

Note that it is not important for this question that William starts at the blue light or that the switches all start off. If William starts at any of the three lights looking at any of the eight combinations, the shortest possible distance he can travel to test all eight combinations is 1500 m.

More Info:

The eight combinations of three switches can be generated in seven steps by flipping exactly one switch at each step, and choosing the order carefully. If we write 0 for "off" and 1 for "on", the list of = on/off combinations can be written as a sequence of bit strings (i.e. using only 0s and 1s). The second, third, and fourth columns in the table in the solution to Problem 3 correspond to the following sequence: 000 100 110 010 011 001 101 111. Such sequences are called Gray codes. Each term in a Gray code is unique, even though it differs by only one bit from the adjacent terms. Gray codes have many applications in electronics and computer science. For example, they are used to minimize the number of actions needed during hardware and software testing.

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, May 7, 2020
We Took the Cookies

There is a cookie jar that contains a certain number of cookies. Three friends divide the cookies in the following way. First, Harold takes all of the cookies and places them into three equal piles with none left over. He then keeps one of the piles and puts the other two piles back into the jar. Next, Lucie takes the remaining cookies in the jar and places them into three equal piles with none left over. She then keeps one of the piles and puts the other two piles back into the jar. Finally, Livio takes the remaining cookies in the jar and places them into three equal piles, but there is one cookie left over. He then keeps the leftover cookie and one of the piles and puts the other two piles back into the jar.

If there are now 10 cookies in the jar, how many cookies were originally in the cookie jar?

More Info:

Check the CEMC at Home webpage on Thursday, May 14 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution emailed to you on Thursday, May 14.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
We Took the Cookies

Problem
There is a cookie jar that contains a certain number of cookies. Three friends divide the cookies in the following way. First, Harold takes all of the cookies and places them into three equal piles with none left over. He then keeps one of the piles and puts the other two piles back into the jar. Next, Lucie takes the remaining cookies in the jar and places them into three equal piles with none left over. She then keeps one of the piles and puts the other two piles back into the jar. Finally, Livio takes the remaining cookies in the jar and places them into three equal piles, but there is one cookie left over. He then keeps the leftover cookie and one of the piles and puts the other two piles back into the jar. If there are now 10 cookies in the jar, how many cookies were originally in the cookie jar?

Solution

We will present two different solutions. Solution 1 works backwards through the problem. Solution 2 is an algebraic solution.

Solution 1

The last 10 cookies in the cookie jar are also the remaining two of Livio's three piles. Therefore, each pile he made had $10 \div 2 = 5$ cookies. Therefore, there were $5 \times 3 = 15$ cookies in the three piles he made plus 1 more cookie that he kept, for a total of 16 cookies. Therefore, there were 16 cookies in the cookie jar when Livio started dividing the cookies.

These 16 cookies were two of the three piles that Lucie made. Therefore, each pile that she made had $16 \div 2 = 8$ cookies. Therefore, there were $8 \times 3 = 24$ cookies in the three piles that she made. Therefore, there were 24 cookies in the cookie jar when Lucie started dividing the cookies.

These 24 cookies were two of the three piles that Harold made. Therefore, each pile that he made had $24 \div 2 = 12$ cookies. Therefore, there were $12 \times 3 = 36$ cookies in the three piles that he made. Therefore, there were 36 cookies in the cookie jar when Harold started dividing the cookies.

Therefore, there were originally 36 cookies in the cookie jar.

Solution 2

Let the initial number of cookies in the cookie jar be C.

Harold has $\frac{1}{3}C$ cookies in the pile he keeps. Therefore, $\frac{2}{3}C$ cookies are left for Lucie.

Lucie keeps $\frac{1}{3}$ of $\frac{2}{3}C$ cookies. Therefore, $\frac{2}{3}$ of $\frac{2}{3}C$ cookies are left for Livio. That is, $\frac{2}{3} \times \frac{2}{3}C = \frac{4}{9}C$ cookies are left for Livio.

For Livio, the pile he keeps is $\frac{1}{3}$ of one less than what is left. That is $\frac{1}{3} \times \left(\frac{4}{9}C - 1\right)$, and so the remaining number of cookies that he puts back into the cookie jar is equal to $\frac{2}{3} \times \left(\frac{4}{9}C - 1\right)$.

This is also equal to 10. That is,

$$\frac{2}{3} \times \left(\frac{4}{9}C - 1\right) = 10$$

Dividing both sides by $\frac{2}{3}$,

$$\frac{\frac{2}{3} \times \left(\frac{4}{9}C - 1\right)}{\frac{2}{3}} = \frac{10}{\frac{2}{3}}$$

Since $10 \div \frac{2}{3} = 10 \times \frac{3}{2} = \frac{30}{2} = 15$,

$$\frac{4}{9}C - 1 = 15$$

Therefore,

$$\frac{4}{9}C = 16$$

Dividing both sides by $\frac{4}{9}$,

$$\frac{\frac{4}{9}C}{\frac{4}{9}} = \frac{16}{\frac{4}{9}}$$

Since $16 \div \frac{4}{9} = 16 \times \frac{9}{4} = 36$,

$$C = 36$$

Therefore, there were originally 36 cookies in the cookie jar.

CEMC at Home

Grade 7/8 - Friday, May 8, 2020
Reading Minds

In this activity, you will "read minds" using math!

Activity 1: Follow the steps below.

1. Choose a positive integer.
2. Double it.
3. Add 15.
4. Subtract 1.
5. Divide by 2.
6. Subtract your original number.

Did you get 7? You may think I read your mind, but actually all I did was some math! Let's look at each step more closely. We will use a box to represent the starting number. This way we can show that regardless of what number you start with, you will always end up with the number 7.

Step	**Result**
1. Choose a positive integer.	$\square$
2. Double it.	$\square + \square$
3. Add 15.	$\square + \square + 15$
4. Subtract 1.	$\square + \square + 14$ (which is $\square + 7 + \square + 7$)
5. Divide by 2.	$(\square + 7 + \square + 7) \div 2 = \square + 7$
6. Subtract your original number.	$\square + 7 - \square = 7$

As you can see, regardless of what number you start with, when you follow the steps you will always end up with the number 7.

Activity 2: Explain why following the steps below will always result in the number 3.

1. Choose a positive integer.
2. Multiply it by 4.
3. Add 12.
4. Divide by 4.
5. Subtract your original number.

Activity 3: Follow the steps below a few times with different starting numbers.

1. Choose a positive 3-digit number where all the digits are the same.
2. Divide your 3-digit number by the sum of its digits.

What number do you get each time you follow these steps? Can you explain why this happens?

Activity 4: Make up your own math mind reading trick. Try it out on your family and friends and see if they can figure out your trick!

More Info:

Check out the CEMC at Home webpage on Monday, May 11 for a solution to Reading Minds.

For more practice using variables to solve problems, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Friday, May 8, 2020
Reading Minds - Solution

Activity 2: Explain why following the steps below will always result in the number 3.

1. Choose a positive integer.
2. Multiply it by 4.
3. Add 12.
4. Divide by 4.
5. Subtract your original number.

Solution: We will use a box to represent the starting number. We will show that no matter what number you start with, you will always end up with the number 3 at the end.

Step	**Result**
1. Choose a positive integer.	$\square$
2. Multiply it by 4.	$4 \times \square$ which equals $\square + \square + \square + \square$
3. Add 12.	$\square + \square + \square + \square + 12$ which equals $\square + 3 + \square + 3 + \square + 3 + \square + 3$
4. Divide by 4.	$(\square + 3 + \square + 3 + \square + 3 + \square + 3) \div 4 = \square + 3$
5. Subtract your original number.	$\square + 3 - \square = 3$

The table shows that the result is always 3.

Activity 3: Follow the steps below a few times with different starting numbers.

1. Choose a positive 3-digit number where all the digits are the same.
2. Divide your 3-digit number by the sum of its digits.

What number do you get each time you follow these steps? Can you explain why this happens?

Solution: If you follow these steps, then you will always end up with the number 37.

Since there are only 9 different numbers you could start with in Step 1, you can check for yourself that you get 37 each time. Can you explain why this keeps happening? We check the first few cases in the table below and look for a pattern.

Number chosen	The sum of the digits	Number divided by the sum of its digits
111	3	$\frac{111}{3} = 37$
222	6	$\frac{222}{6} = 37$ since $\frac{222}{6} = \frac{111 \times 2}{3 \times 2} = \frac{111}{3} = 37$
333	9	$\frac{333}{9} = 37$ since $\frac{333}{9} = \frac{111 \times 3}{3 \times 3} = \frac{111}{3} = 37$
444	12	$\frac{444}{12} = 37$ since $\frac{444}{12} = \frac{111 \times 4}{3 \times 4} = \frac{111}{3} = 37$

The table shows that if you start with a number with digits $\square\square\square$ (with the same digit in each box), then the value of the number is $111 \times \square$. This happens because the number $\square\square\square$ has value equal to $100 \times \square + 10 \times \square + 1 \times \square$, which is equal to $(100 + 10 + 1) \times \square$. Also, the sum of the digits in the number is $\square + \square + \square = 3 \times \square$. When you divide $111 \times \square$ by $3 \times \square$ you get the same answer as if you divided 111 by 3, which is 37. This happens no matter what digit is placed in the boxes.