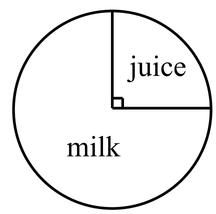
CEMC at Home

Grade 4/5/6 - Monday, May 4, 2020
Contest Day 1

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #6

In the pie chart shown, 80 students chose juice.
How many students chose milk?



(A) 120 **(B)** 160 **(C)** 240

(D) 180 **(E)** 80

2012 Gauss Contest, #15

Yelena chants P, Q, R, S, T, U repeatedly (e.g. $P, Q, R, S, T, U, P, Q, R, \ldots$). Zeno chants $1, 2, 3, 4$ repeatedly (e.g. $1, 2, 3, 4, 1, 2, \ldots$). If Yelena and Zeno begin at the same time and chant at the same rate, which combination will *not* be said?

(A) $T1$ **(B)** $U2$ **(C)** $Q4$ **(D)** $R2$ **(E)** $T3$

More Info:

Check out the CEMC at Home webpage on Monday, May 11 for solutions to the Contest Day 1 problems.

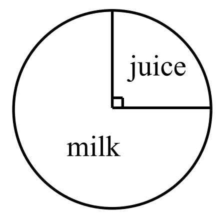
CEMC at Home

Grade 4/5/6 - Monday, May 4, 2020
Contest Day 1 - Solution

Solutions to the two contest problems are provided below, including a video for the second problem.

2020 Gauss Contest, #6

In the pie chart shown, 80 students chose juice.
How many students chose milk?



(A) 120 **(B)** 160 **(C)** 240
(D) 180 **(E)** 80

Solution:

The fraction of the circle which represents students who chose juice is $\frac{1}{4}$.
Therefore, $\frac{1}{4}$ of all students chose juice.
Since the 80 students who chose juice represent $\frac{1}{4}$ of the total number of students, then the total number of students is $4 \times 80 = 320$.
Therefore, $320 - 80 = 240$ students chose milk.

ANSWER: (C)

2012 Gauss Contest, #15

Yelena recites P, Q, R, S, T, U repeatedly (e.g. $P, Q, R, S, T, U, P, Q, R, \ldots$). Zeno recites $1, 2, 3, 4$ repeatedly (e.g. $1, 2, 3, 4, 1, 2, \ldots$). If Yelena and Zeno begin at the same time and recite at the same rate, which combination will *not* be said?

(A) $T1$ **(B)** $U2$ **(C)** $Q4$ **(D)** $R2$ **(E)** $T3$

Solution:

To determine which combination will not be said, we list the letters and numbers chanted by Yelena and Zeno in the table below.

Yelena	P	Q	R	S	T	U	P	Q	R	S	T	U	P	Q	$\cdots$
Zeno	1	2	3	4	1	2	3	4	1	2	3	4	1	2	$\cdots$

Remember that after Yelena chants "U", she will start her pattern all over again beginning from P.
Similarly, after Zeno chants "4", he will start over at 1.
You can see from the table that the pattern for both kids starts to repeat after 12 rounds (it goes back to $P1$, $Q2$, $\ldots$).
To determine which combination will not be said we need only compare the 5 answers with the 12 possibilities given in the table.
The only combination that appears among the 5 answers, but that does not appear in the table, is $R2$.

ANSWER: (D)

Video

Visit the following link for an explanation of the solution to the second contest problem and a discussion of a follow-up problem: https://youtu.be/woHduEnD8M4

CEMC at Home

Grade 4/5/6 - Tuesday, May 5, 2020
Brigitte's Boating Adventure

Brigitte is enjoying rowing her sturdy boat across Blue Lake on a sunny morning. Suddenly, the boat strikes a rock, making a small crack in the hull (the body of the boat).

Water begins to leak into the boat; 3 litres of water leaks in during each minute of time.

But Brigitte's boat is equipped with a small pail and so she can alternate rowing the boat and bailing out the water to keep the boat afloat while she returns to the dock.

(a) If Brigitte does not bail any water out of the boat, how much water would there be in the boat after 1 minute? What about after 30 minutes?

(b) But of course, Brigitte does bail water out of the boat. Using her pail, she removes 1.4 litres during each minute. How many litres of water remain in the bottom of the boat after 1 minute? What about after 30 minutes?

(c) Over each of the next 30 minute periods, the amount that you found in part (b) is added to the water in the bottom of the boat. While alternately bailing and rowing, Brigitte keeps the boat moving toward shore at 2 km per hour.

Alas, despite her efforts, the boat sinks in shallow water just at the end of the dock, at which point it contains a total of 144 litres of water.

Can you use this information to figure out how far away from the end of the dock Brigitte was when the boat struck the rock? Complete the table on the right to help answer this question.

The elapsed time is the time since the boat hit the rock.

Time (hr) Elapsed	**Water (L) in Boat**	**Distance (km) Rowed**
0.5		1 km
1		2 km

Challenge:

Suppose that Brigitte's boat had hit the rock 1 km *further* from the dock than you found in part (c) above. The boat still takes on the same amount of water each minute as before (3 litres) and the boat will still sink once it is holding 144 litres of water.

Brigitte bails faster this time and so she has less time to row. This results in the boat moving at only 1 km per hour. How many litres of water would Brigitte have to bail out of the boat during each minute in order to just reach the dock before the boat sinks?

HINTS:

1. How many minutes will the boat now take to reach the dock?
2. How quickly will the boat need to fill to reach 144 litres of water by this time?

More info: Check out the CEMC at Home webpage on Tuesday, May 12 for a solution to Brigitte's Boating Adventure.

CEMC at Home

Grade 4/5/6 - Tuesday, May 5, 2020
Brigitte's Boating Adventure - Solution

Brigitte is enjoying rowing her sturdy boat across Blue Lake on a sunny morning. Suddenly, the boat strikes a rock, making a small crack in the hull (the body of the boat).

Water begins to leak into the boat; 3 litres of water leaks in during each minute of time. But Brigitte's boat is equipped with a small pail and so she can alternate rowing the boat and bailing out the water to keep the boat afloat while she returns to the dock.

(a) If Brigitte does not bail any water out of the boat, how much water would there be in the boat after 1 minute? What about after 30 minutes?

Solution:

If Brigitte does not bail, after 1 minute there would be 3 litres of water in the boat. So after 30 minutes, there would be $3 \times 30 = 90$ litres.

(b) But of course, Brigitte does bail water out of the boat. Using her pail, she removes 1.4 litres during each minute. How many litres of water remain in the bottom of the boat after 1 minute? What about after 30 minutes?

Solution:

Since 3 litres of water leak into the boat every minute, and Brigitte bails 1.4 litres of water out of the boat each minute, the amount of water in the boat after one minute will be $3 - 1.4 = 1.6$ litres.

After 30 minutes, there will be $30 \times 1.6 = 48$ litres in the boat.

(c) Over each of the next 30 minute periods, the amount that you found in part (b) is added to the water in the bottom of the boat. While alternately bailing and rowing, Brigitte keeps the boat moving toward shore at 2 km per hour. Alas, despite her efforts, the boat sinks in shallow water just at the end of the dock, at which point it contains a total of 144 litres of water. How far away from the end of the dock was Brigitte when the boat struck the rock?

Solution:

Every 30 minutes, the boat gains 48 L of water. Using the given speed of the boat, 2 km per hour (or 1 km per half hour), the completed table reveals that 144 litres of water accumulate in the boat after 1.5 hours.

So Brigitte must have been 3 km from the dock when the boat hit the rock.

Time (hr) Elapsed	**Water (L) in Boat**	**Distance (km) Rowed**
0.5 hr	48 L	1 km
1 hr	96 L	2 km
1.5 hr	144 L	3 km

Challenge:

Suppose that Brigitte's boat had hit the rock 1 km *further* from the dock than you found in part (c) above. The boat still takes on the same amount of water each minute as before (3 litres) and the boat will still sink once it is holding 144 litres of water.

Brigitte bails faster this time and so she has less time to row. This results in the boat moving at only 1 km per hour. How many litres of water would Brigitte have to bail out of the boat during each minute in order to just reach the dock before the boat sinks?

Solution:

The distance to be covered is now $3 + 1 = 4$ km.
Thus, at 1 kilometre per hour, it will take 4 hours, or $4 \times 60 = 240$ minutes to reach the dock.
To accumulate 144 litres of water in 240 minutes, the boat must accumulate $144 \div 240 = 0.6$ litres of water each minute. (0.6 litres each minute, over 240 minutes, would mean $240 \times 0.6 = 144$ litres overall.)

So Brigitte would have to bail $3 - 0.6 = 2.4$ litres of water out of the boat during each minute.

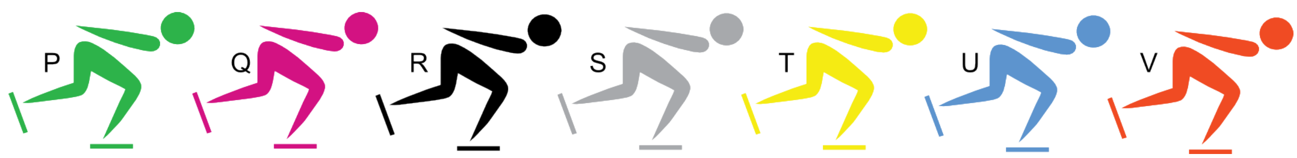
CEMC at Home

Grade 4/5/6 - Wednesday, May 6, 2020
Looping Around

Try the following problems. How are the problems and their solutions similar?

Skaters

Seven people are skating in a line on a very long, frozen canal. They begin as shown below.



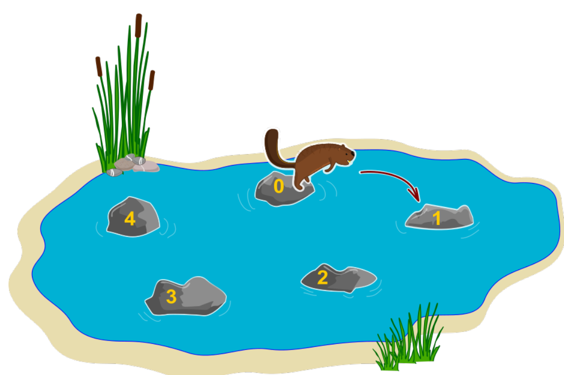
After every minute the person at the front of the line moves to the end of the line. For example, after 1 minute, U will be in front of the line, since V will move behind P.

1. Which skater will be at the front of the line after 3 minutes?
2. Which skater will be at the front of the line after 16 minutes?

Jumping Beaver

An unusual beaver loves to jump. It starts from rock number 0, and jumps clockwise from rock to rock in numerical order. For example, if the beaver jumps 8 times, it ends up on rock number 3:

$$0 \to 1 \to 2 \to 3 \to 4 \to 0 \to 1 \to 2 \to 3$$



1. What rock does the beaver end up on if it jumps 29 times?
2. What rock does the beaver end up on if it jumps 50 times?

More Info:

Check out the CEMC at Home webpage on Wednesday, May 13 for a solution to Looping Around.

These problems appeared on past Beaver Computing Challenges (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

CEMC at Home

Grade 4/5/6 - Wednesday, May 6, 2020
Looping Around - Solution

Skaters

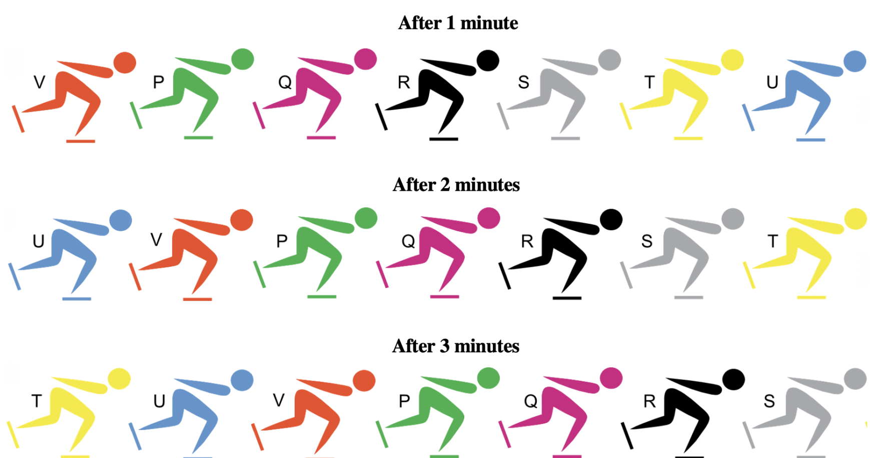
Seven people are skating in a line on a very long, frozen canal. They begin as shown below.

After every minute the person at the front of the line moves to the end of the line. For example, after 1 minute, U will be in front of the line, since V will move behind P.

> 1. Which skater will be at the front of the line after 3 minutes?
> 2. Which skater will be at the front of the line after 16 minutes?

Solution:

1. The positions of the skaters after each of the first 3 minutes are shown below.



As shown in the image above, skater S will be at the front of the line after 3 minutes.

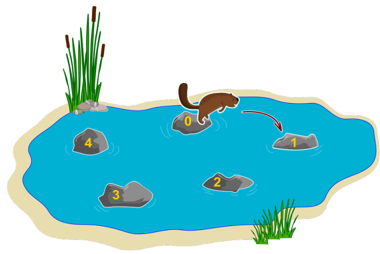
2. We can trace out the positions after each minute.

- at the start, V is in front
- after 1 minute, U is in front
- after 2 minutes, T is in front
- after 3 minutes, S is in front
- after 4 minutes, R is in front
- after 5 minutes, Q is in front
- after 6 minutes, P is in front
- after 7 minutes, V is in front (again)
- after 8 minutes, U is in front (again)
- after 9 minutes, T is in front (again)
- after 10 minutes, S is in front (again)
- after 11 minutes, R is in front (again)
- after 12 minutes, Q is in front (again)
- after 13 minutes, P is in front (again)

Continuing the pattern above, we see that skater T will be in front after 16 minutes.

Jumping Beaver

An unusual beaver loves to jump. It starts from rock number 0, and jumps clockwise from rock to rock in numerical order.



1. What rock does the beaver end up on if it jumps 29 times?
2. What rock does the beaver end up on if it jumps 50 times?

Solution:

We probably do not want to make a list as we did in the previous question because we would have to keep track of many more steps. We will use a different kind of reasoning here.

If the beaver jumps 5 times, then it will end up where it started (at rock number 0). We can think of 5 jumps as a "lap". After another 5 jumps (or another lap) the beaver will again end up at 0. This pattern will continue.

This means the beaver will be at rock number 0 after 5 jumps, 10 jumps, 15 jumps, 20 jumps, 25 jumps, and so on.

1. The beaver ends up at rock number 4 after 29 jumps. Why is this?

   The beaver is at rock number 0 after 25 jumps. The beaver needs to make 4 more jumps to get to a total of 29 jumps. This will take the beaver to rock number 4.

$$0 \to 1 \to 2 \to 3 \to 4$$

2. The beaver ends up at rock number 0 after 50 jumps. Why is this?

   Notice that $50 = 10 \times 5$. This means if the beaver jumps 50 times, then the beaver will complete 10 laps exactly, and end up back at rock number 0.

Notice that the two problems in this activity are similar. In the first problem we are studying a repeating pattern. The skaters that are in front of the line follow the pattern

$$\underbrace{V, U, T, S, R, Q, P}, \underbrace{V, U, T, S, R, Q, P}, \underbrace{V, U, T, S, R, Q, P}, \ldots$$

Since this sequence of seven letters keeps repeating, it is possible for us to figure out which skater will be in front after many minutes without having to actually keep listing the skaters. Can you figure out what skater will be in front after 85 minutes?
In the second problem, the rock numbers of the beaver follow a different repeating pattern:

$$\underbrace{0, 1, 2, 3, 4}, \underbrace{0, 1, 2, 3, 4}, \underbrace{0, 1, 2, 3, 4}, \ldots$$

Using the fact that this sequence of five numbers keeps repeating, can you figure out what rock the beaver will be on after 123 jumps?

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, May 7, 2020
How Many Halves?

Robbie won 300 gumballs and he would like to share the winnings with his friends. He decides to list his friends in order based on how long he has known each of them. He wants to give away the most gumballs to the friend he has known the longest. Then he will give the next friend on the list exactly half as many gumballs as the person he has known the longest. He continues to give away exactly half as many to the next friend on his list, until the pattern cannot continue. (He will not give away half a gumball.)

A) If he gives away 100 gumballs to the first friend on the list, how many friends will receive gumballs? Justify your answer.

B) If the last person he gives gumballs to receives 5 gumballs, what is the largest number of gumballs that the first friend on the list can receive? How many gumballs are given away? Justify your answers.

C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.

More Info:

Check the CEMC at Home webpage on Thursday, May 14 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution emailed to you on Thursday, May 14.

This CEMC at Home resource is the current grade 3/4 problem from Problem of the Week (POTW). This problem was developed for students in grades 3 and 4, but is also appropriate for students in grades 5 and 6. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 5/6 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem A and Solution
How Many Halves?

Problem

Robbie won 300 gumballs and he would like to share the winnings with his friends. He decides to list his friends in order based on how long he has known each of them. He wants to give away the most gumballs to the friend he has known the longest. Then he will give the next friend on the list exactly half as many gumballs as the person he has known the longest. He continues to give away exactly half as many to the next friend on his list, until the pattern cannot continue. (He will not give away half a gumball.)

A) If he gives away 100 gumballs to the first friend on the list, how many friends will receive gumballs? Justify your answer.

B) If the last person he gives gumballs to receives 5 gumballs, what is the largest number of gumballs that the first friend on the list can receive? How many gumballs are given away? Justify your answers.

C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.

Solution

A) If the first friend receives 100 gumballs, then the next friend receives half as many, which is 50 gumballs. Half of 50 is 25 gumballs, which is the amount the next friend on the list receives. Since 25 is an odd number, it cannot be divided in half without ending up with a fraction. So the pattern ends after three friends have received gumballs from Robbie.

B) If we know that the last friend on the list receives 5 gumballs, then as we move up the list, each friend receives twice as many as the previous one. Let's assign the last friend who receives 5 gumballs the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:

Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	5	5
2	10	15
3	20	35
4	40	75
5	80	155
6	160	315

From the information on this table, we can answer the questions.

Since Robbie only has 300 gumballs to give away, if the last person receives 5 gumballs, then the largest amount of gumballs the first person on the list will receive 80 gumballs. There will be a total of 155 gumballs given away.

C) We want to maximize the number of friends to whom Robbie gives gumballs. Using the two solutions above, we notice that the larger the last friend's amount of gumballs the fewer number of friends that can receive gumballs. One way to think about the problem is that we want the last friend to receive the smallest possible amount, which is 1 gumball. So, the second last friend would receive 2 gumballs. As in part (B) let's assign the last friend who receives 1 gumball the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:

Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

From the information on this table, we can answer the questions.

The first person on the list will receive 128 gumballs. A maximum of eight of Robbie's friends can receive gumballs.

Teacher's Notes

In this problem, as we consider one additional friend, the number of gumballs received is reduced by half. For part A), we could write an equation for $g(n)$, the number of gumballs the n^{th} friend gets, as:

$$g(n) = 100 \cdot \left(\frac{1}{2}\right)^{n-1}$$

If we look at the graph of this function, we would see that it is a very steep curve. This is an example of an *exponential function*.

In general, an exponential function is a function of the form:

$$g(n) = m \cdot b^n, \quad \text{where } b > 0 \text{ and } b \neq 1.$$

When $b > 1$, the value of $g(n)$ increases very quickly as n increases. We see exponential growth in real life situations such as compound interest and the spread of viruses.

When $b < 1$ (as in this problem), the value of $g(n)$ decreases very quickly as n increases. We see exponential decay in some chemical reactions and heat transfer.

Exponential functions, exponential growth, and exponential decay are generally seen in high school mathematics and science, and possibly some business studies courses.

CEMC at Home

Grade 4/5/6 - Friday, May 8, 2020
Keeping Your Distance

In this activity, we will play a game with X's and O's on a rectangular grid.

You Will Need:

- Two players
- Some paper
- A pen or pencil

How to Play:

1. Start by drawing a rectangular grid of squares to use as a game board.

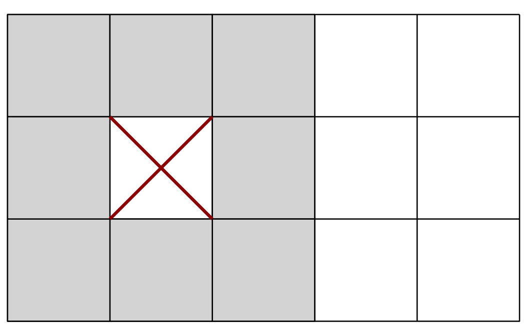
   A 3×5*,* 4×4*, and* 5×5 *game board have been provided for your use on the last page, but you may create game boards of any size.*

2. Players will take turns. Decide which player will go first (Player 1) and which player will go second (Player 2).

3. To begin, Player 1 marks any square in the grid with an X.

4. Next, Player 2 marks any other square in the grid with an O, except that they cannot mark any square that is touching the square with the first X.

   Two squares are touching if they share a side or a vertex. For example, in the game board shown below, Player 2 may not put an O *in any of the squares indicated in grey.*



5. From here, the two players take turns marking the grid with X's (Player 1) and O's (Player 2). Players can never mark a square that touches a square already marked by either player.

6. The first player who is unable to make a move loses, and the other player wins!

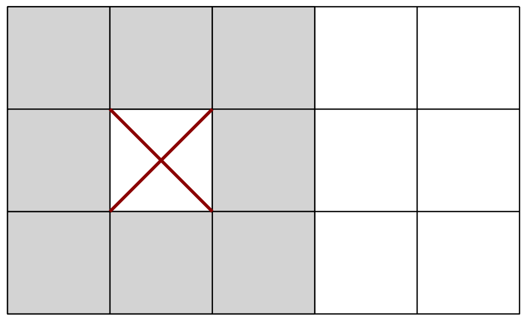
Play this game a number of times.

Try all three of the game boards on the last page, and alternate who goes first and who goes second. Think about the questions on the next page.

Questions:

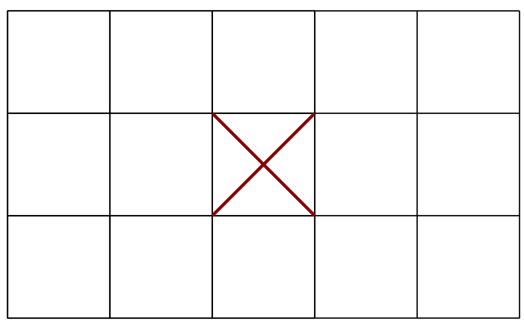
1. Play the game on the 3×5 game board. How many moves are there in the shortest possible game? How many moves are there in the longest possible game?

2. Suppose Player 1 makes the first move shown below.



   How can Player 2 win the game on their first move?

3. Now suppose Player 1 starts the game by claiming the middle square as shown below.



   After making this move, it turns out there is a strategy Player 1 can use to win every time. Can you figure out Player 1's strategy?

4. Can you adjust the strategy you found in 3. so that it works for a 5×5 game board?

5. Play the game on the 4×4 game board. Which player (Player 1 or Player 2) seems to win most often? Can you come up with a strategy that will make sure this player wins the game every time?

6. Play the game on other larger game boards of your choosing. See if you can find strategies for winning each game!

More Info:

Check the CEMC at Home webpage on Friday, May 15 for a discussion of Keeping Your Distance.

Game Boards

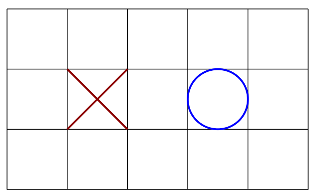
CEMC at Home

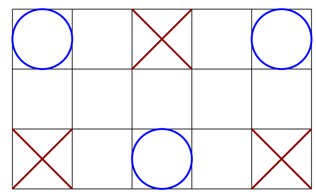
Grade 4/5/6 - Friday, May 8, 2020
Keeping Your Distance - Solution

1. Play the game on the 3×5 game board. How many moves are there in the shortest possible game? How many moves are there in the longest possible game?

   Solution:

   The shortest possible game has 2 moves, and the longest possible game has 6 moves. An example of a finished 2-move game and a finished 6-move game are both given below:

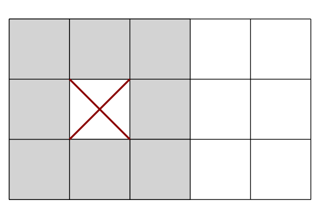
   A game cannot be shorter than 2 moves, because there is no square that Player 1 can mark with an X that will block all of the remaining squares for Player 2.

   To see why a game cannot last longer than 6 moves, think about how many squares could possibly be marked on the game board at the end of a game. On a finished game board, each of the three rows can have at most 3 marked squares. Now, think about the possible number of marked squares in the middle row, and how this affects the other two rows:

   - If there are no marked squares in the middle row, then the top and bottom rows can have up to 3 marked squares each. (This means a maximum of 6 marked squares in total.)
   - If there is 1 marked square in the middle row, then the top and bottom rows can have at most 2 marked squares each. (This means a maximum of 5 marked squares in total.)
   - If there are 2 marked squares in the middle row, then the top and bottom rows can have at most 1 marked square each. (This means a maximum of 4 marked squares in total.)
   - If there are 3 marked squares in the middle row, then there cannot be any other marked squares on the board. (This means a maximum of 3 marked squares in total.)

   This means that a finished game board can have at most 6 marked squares and so a game cannot possibly have more than 6 moves. (A 7th move would mean a 7th marked square which is impossible.)

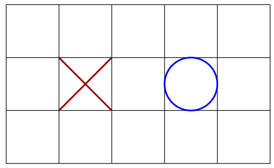
2. Suppose Player 1 makes the first move shown below.

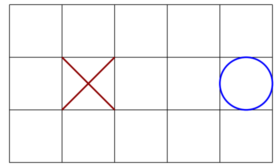
   

   How can Player 2 win the game on their first move?

Solution:

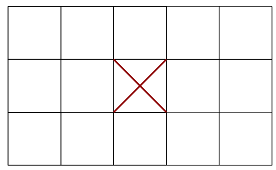
Player 2 actually has two winning moves available. The two moves are shown below.





In either case, every empty square that remains is touching a marked square, and so Player 1 cannot make any legal move on their next turn.

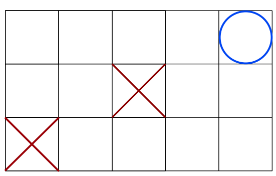
3. Now suppose Player 1 starts the game by claiming the middle square as shown below.

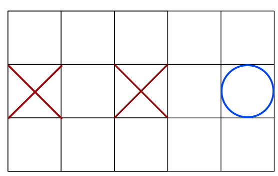


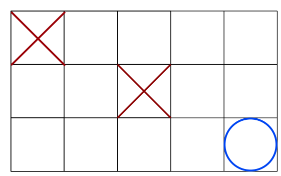
After making this move, it turns out there is a strategy Player 1 can use to win every time. Can you figure out Player 1's strategy?

Solution:

Player 1 can adopt a type of "mirroring strategy", in which they copy everything that Player 2 does, but on the "opposite side of the X" in the centre of the game board. For example, if Player 2 makes any of the three moves shown below (marked with an O) then Player 1 responds by making the corresponding move shown (marked with a second X) on the opposite side.







Player 1's response will be similar if Player 2 chooses to make their first move on the left side of the board.

Because Player 1 is always copying Player 2 by playing on the opposite side of the X in the centre, Player 1 will always be able to make a legal move if Player 2 could. If there's a legal move for Player 2 somewhere on the game board, then there must be a corresponding legal move on the opposite side of the centre X.

Since Player 1 is always one step ahead of Player 2, and will not run out of legal moves until after Player 2 does, Player 1 will win.

4. Can you adjust the strategy you found in 3. so that it works for a 5×5 game board?

Solution:

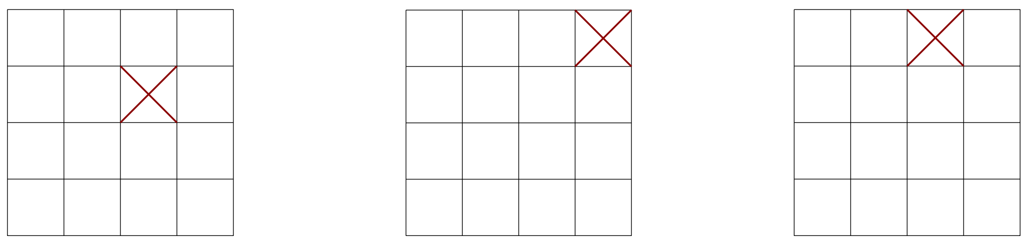
Yes, Player 1's strategy of marking the middle square of the board and then "mirroring" Player 2 will work equally well on a 5×5 game board, for exactly the same reasons as explained above. The key to making this strategy work is that both the rows and the columns on the game board have an odd number of squares. This is necessary in order to make sure there *is* a middle square of the game board (why?). As long as the game board has a middle square, the strategy outlined in in 3. will always work for Player 1.

5. Play the game on the 4×4 game board. Which player (Player 1 or Player 2) seems to win most often? Can you come up with a strategy that will make sure this player wins the game every time?

   Solution:

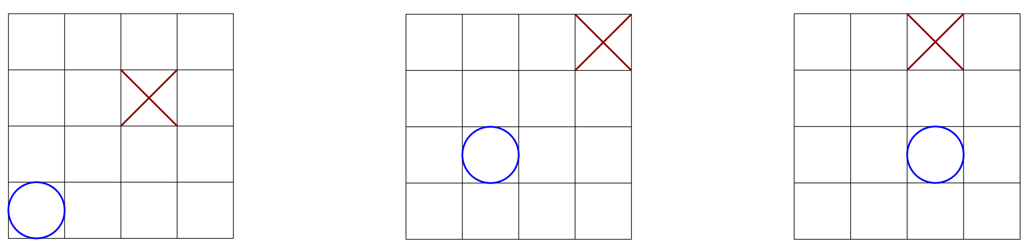
   On the 4×4 game board, the strategy gets more interesting. It turns out that Player 2 can always win, but the strategy is less simple to describe than the 3×5 case.

   Let's suppose that Player 1's first move is one of these three moves:



   Because of the symmetry of the game board, the explanations will be similar if Player 1 chooses a first move other than these three.

   In the three cases above, Player 2 should respond as follows:



   On the first game board, note that Player 1 only has four squares left to play on: the top two squares of the leftmost column, or the rightmost two squares in the bottom row. If Player 1 takes a square in the leftmost column, Player 2 can take either legal square in the bottom row and win. If Player 1 instead goes for one of the squares in the bottom row, then Player 2 can take either legal square in the leftmost column and still win.

   Can you see how to use this explanation above to explain Player 2's strategy on the second game board? Think about reflecting the game board in the diagonal from the upper left corner to the lower right corner. Notice that after this reflection, the marked squares are the same as on the first game board. The strategy here will be similar to the first board.

   On the third game board, Player 1 is only allowed to play in the leftmost column, in which every square is open. But notice that whichever square Player 1 marks, at least one square in that column will remain open for Player 2. Once Player 2 also claims a square in that column, there will be no valid moves left, and so Player 2 wins.

   Think about how to adapt the strategies described above to the other possible game boards after the first two moves.