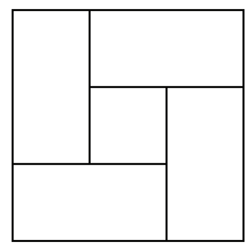
CEMC at Home

Grade 7/8 - Monday, May 25, 2020
Contest Day 4

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2014 Gauss Contest, #18

In the figure shown, the outer square has an area of $9\,\text{cm}^2$, the inner square has an area of $1\,\text{cm}^2$, and the four rectangles are identical. What is the perimeter of one of the four identical rectangles?



(A) 6 cm **(B)** 8 cm **(C)** 10 cm
(D) 9 cm **(E)** 7 cm

2020 Gauss Contest, #22

Celyna bought 300 grams of candy A for \$5.00, and x grams of candy B for \$7.00. She calculated that the average price of all of the candy that she purchased was \$1.50 per 100 grams. What is the value of x?

(A) 525 **(B)** 600 **(C)** 500 **(D)** 450 **(E)** 900

More Info:

Check out the CEMC at Home webpage on Monday, June 1 for solutions to the Contest Day 4 problems.

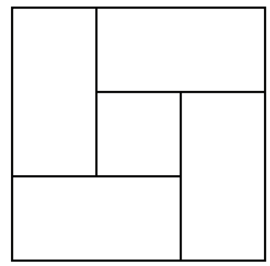
CEMC at Home

Grade 7/8 - Monday, May 25, 2020
Contest Day 4 - Solution

Solutions to the two contest problems are provided below, including a video for the second problem.

2014 Gauss Contest, #18

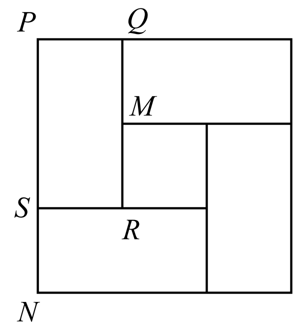
In the figure shown, the outer square has an area of $9\,\text{cm}^2$, the inner square has an area of $1\,\text{cm}^2$, and the four rectangles are identical. What is the perimeter of one of the four identical rectangles?



(A) 6 cm **(B)** 8 cm **(C)** 10 cm

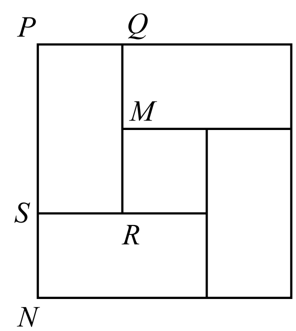
(D) 9 cm **(E)** 7 cm

Solution 1:



The outer square has an area of 9 cm^2, so the sides of this outer square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
The inner square has an area of 1 cm^2, so the sides of this inner square have length 1 cm (since $1 \times 1 = 1$), and thus $MR = 1$ cm.
Since $PN = 3$ cm, then $PS + SN = 3$ cm and so $QR + SN = 3$ cm (since $QR = PS$).
But $QR = QM + MR$, so then $QM + MR + SN = 3$ cm or $QM + 1 + SN = 3$ cm (since $MR = 1$ cm).
From this last equation we get $QM + SN = 2$ cm.
Since each of QM and SN is the width of an identical rectangle, then $QM = SN = 1$ cm.
Using $PS + SN = 3$ cm, we get $PS + 1 = 3$ cm and so $PS = 2$ cm.
Since the rectangles are identical, then $SN = PQ = 1$ cm.
The perimeter of rectangle $PQRS$ is $2 \times (PS + PQ) = 2 \times (2 + 1) = 2 \times 3 = 6$ cm.

Solution 2:



The outer square has an area of 9 cm^2, so the sides of this outer square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
Since $PN = 3$ cm, then $PS + SN = 3$ cm.
Since each of PQ and SN is the width of an identical rectangle, then $PQ = SN$ and so $PS + SN = PS + PQ = 3$ cm.
The perimeter of $PQRS$ is $2 \times (PS + PQ) = 2 \times 3 = 6$ cm.

Answer: (A)

See the next page for a solution to the second contest problem.

2020 Gauss Contest, #22

Celyna bought 300 grams of candy A for \$5.00, and x grams of candy B for \$7.00. She calculated that the average price of all of the candy that she purchased was \$1.50 per 100 grams. What is the value of x?

(A) 525 **(B)** 600 **(C)** 500 **(D)** 450 **(E)** 900

Solution:

Celyna spent \$5.00 on candy A and \$7.00 on candy B, or \$12.00 in total.
The average price of all the candy that she purchased was \$1.50 per 100 grams.
This means that if Celyna bought 100 grams of candy, she would have spent \$1.50.
If she bought 200 grams of candy, she would have spent $2 \times \$1.50 = \$3.00$.
How many grams of candy would Celyna need to buy to spend \$12.00?
Since $8 \times \$1.50 = \$12.00$ (or $\$12.00 \div \$1.50 = 8$), then she would need to buy a total of 800 grams of candy.
Celyna bought 300 grams of candy A, and so she must have purchased $800 - 300 = 500$ grams of candy B. The value of x is 500.

Answer: (C)

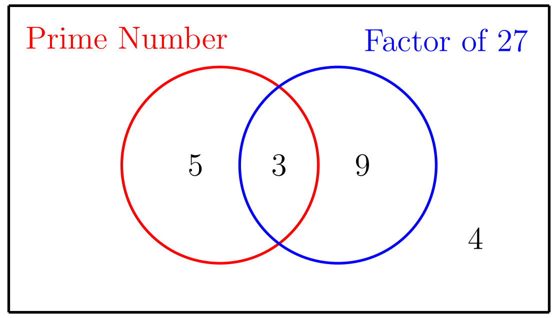
Video

Visit the following link to view three different approaches to solving the second contest problem: https://youtu.be/fWHAtLvCKtA.

CEMC at Home

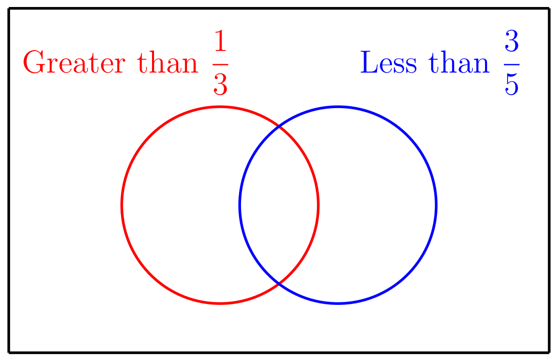
Grade 7/8 - Tuesday, May 26, 2020
Going in Circles

In the Venn diagram shown, the circle on the left contains prime numbers and the circle on the right contains factors of 27. The overlapping area in the middle, contained in both circles, contains prime numbers that are factors of 27. The area outside both circles contains numbers that are neither prime nor factors of 27. We have placed one positive integer in each of the four regions, but they are not the only numbers we could have chosen.



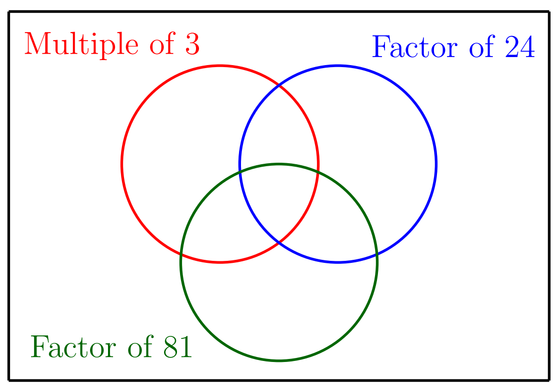
Problem 1

This Venn diagram has four regions. Place a fraction in as many of the regions as you can. Is it possible to find a fraction for each region?



Problem 2

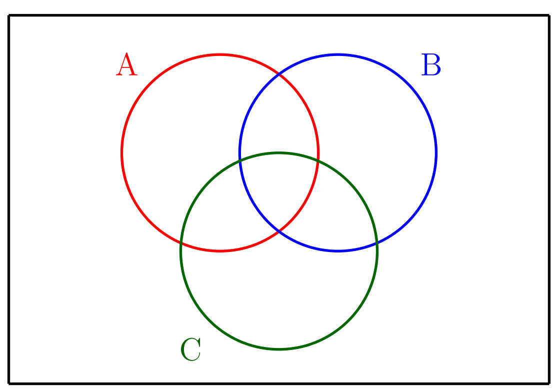
This Venn diagram has eight regions (seven regions "inside" at least one of the circles and one region "outside" all three circles). Place a positive integer in as many of the regions as you can. Is it possible to find a positive integer for each region?



Problem 3

This Venn diagram has eight regions. Place a positive three-digit integer in as many of the regions as you can. Is it possible to find a three-digit integer for each region?

A: 5 is a factor of the sum of the digits
B: The product of the digits is even
C: The mean of the digits is an integer



More Info:

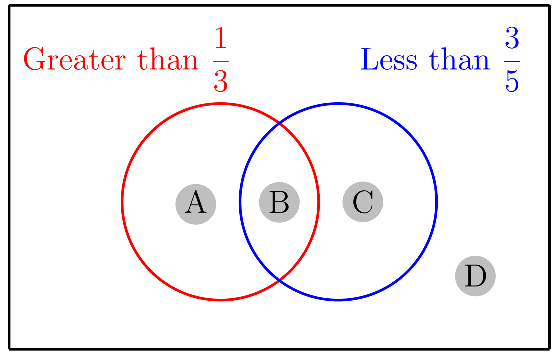
Check the CEMC at Home webpage on Wednesday, May 27 for a solution to Going in Circles.

CEMC at Home

Grade 7/8 - Tuesday, May 26, 2020
Going in Circles - Solution

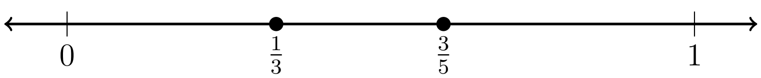
Problem 1

This Venn diagram has four regions. Place a fraction in as many of the regions as you can. Is it possible to find a fraction for each region?



Solution:

We have marked the four regions A, B, C, and D.
We plot the fractions on a number line as a reference:

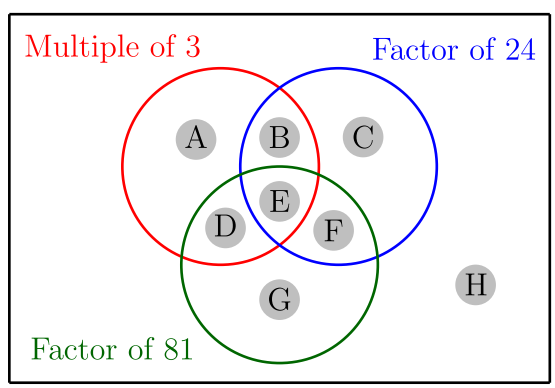


- Any fraction in Region A must be greater than $\frac{1}{3}$ and *not* less than $\frac{3}{5}$.
  This means the fraction must be greater than $\frac{1}{3}$ and greater than or equal to $\frac{3}{5}$.
  Some examples are $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{3}{4}$. (Any fraction greater than or equal to $\frac{3}{5}$ works.)
- Any fraction in Region B must be greater than $\frac{1}{3}$ and less than $\frac{3}{5}$.
  Some examples are $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{4}{7}$. (Any fraction between $\frac{1}{3}$ and $\frac{3}{5}$ works.)
- Any fraction in Region C must be less than $\frac{3}{5}$ and *not* greater than $\frac{1}{3}$.
  This means the fraction must be less than $\frac{3}{5}$ and less than or equal to $\frac{1}{3}$.
  Some examples are $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$. (Any fraction less than or equal to $\frac{1}{3}$ works.)
- Any fraction in Region D must *not* be greater than $\frac{1}{3}$ and *not* be less than $\frac{3}{5}$.
  This means the fraction must be less than or equal to $\frac{1}{3}$ and greater than or equal to $\frac{3}{5}$.
  It is not possible to find such a fraction and so this region must remain empty.

Therefore, we can place a fraction in three of the four regions.
For example, we could place $\frac{4}{5}$ in region A, $\frac{2}{5}$ in region B, $\frac{1}{5}$ in region C, and no fraction in region D.

Problem 2

This Venn diagram has eight regions (seven regions “inside” at least one of the circles and one region “outside” all three circles). Place a positive integer in as many of the regions as you can. Is it possible to find a positive integer for each region?



Solution:

It is helpful if we first write out the factors of 24 and 81.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 81: 1, 3, 9, 27, 81

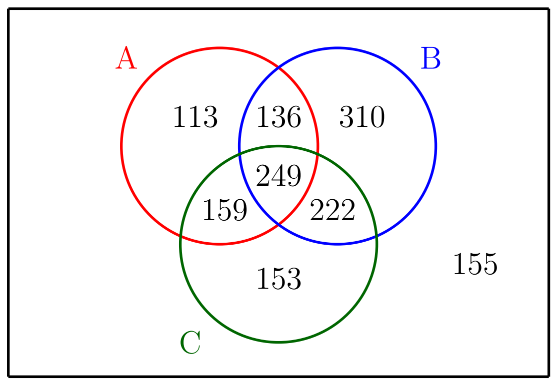
We have marked the eight regions A, B, C, D, E, F, G, and H. We can place a positive integer in each region except for region G.

- Any integer in Region A must be a multiple of 3 but not a factor of 24 and not a factor of 81. Some examples are 18, 21, and 30.
- Any integer in Region B must be a multiple of 3 and a factor of 24 but not a factor of 81. The only options are 6, 12, and 24.
- Any integer in Region C must be a factor of 24 but not a factor of 81 and not a multiple of 3. The only options are 2, 4, and 8.
- Any integer in Region D must be a multiple of 3 and a factor of 81 but not a factor of 24. The only options are 9, 27, and 81.
- Any integer in Region E must be a multiple of 3 and a factor of both 24 and 81. The only option is 3.
- Any integer in Region F must be a factor of both 24 and 81 but not a multiple of 3. The only option is 1.
- Any integer in Region G must be a factor of 81 but not a factor of 24 and not a multiple of 3. It is not possible to find such an integer, so this region must remain empty.
- Any integer in Region H must not be multiple of 3, not be a factor of 24, and not be a factor of 81. Some examples are 5, 7, and 10.

Problem 3

This Venn diagram has eight regions. Place a positive three-digit integer in as many of the regions as you can. Is it possible to find a three-digit integer for each region?

A: 5 is a factor of the sum of the digits
B: The product of the digits is even
C: The mean of the digits is an integer



Solution:

Positive three-digit integers have been placed in the regions in the diagram above. It is possible to place a number in each of the eight regions, and there are other choices you could have made.

There are many ways to go about finding these numbers. You can choose three-digit numbers randomly and then test them to see in which region they belong, hoping to eventually find one for every region, or you can try to reason what digits in each of the regions must look like.

For example, you can note that the product of the digits of an integer is even exactly when the number has at least one even digit. Because of this we know that any number placed within the circle marked B must have at least one even digit, and every number placed outside of this circle must have three odd digits.

We can further note that three odd digits must have an odd sum. So if a number is *outside* circle B but *inside* circle A, then it must have three odd digits that add to an odd multiple of 5. (In fact, they must add to either 5 or 15. Can you see why?) The numbers 113 and 159 both have this property and exactly one of them has the additional property that the mean of its digits is equal to an integer. (The mean of the digits of 113 is $\frac{1+1+3}{3} = \frac{5}{3}$, which is not an integer, and the mean of the digits of 159 is $\frac{1+5+9}{3} = \frac{15}{3} = 5$, which is an integer.)

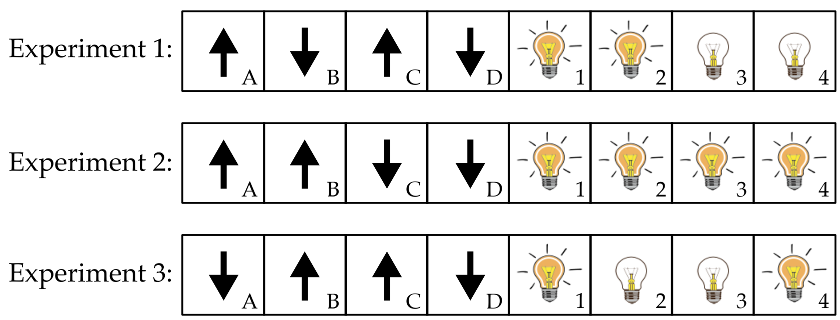
A combination of reasoning and some trial and error is a good approach for this problem!

CEMC at Home

Grade 7/8 - Wednesday, May 27, 2020
Sneaky Switches

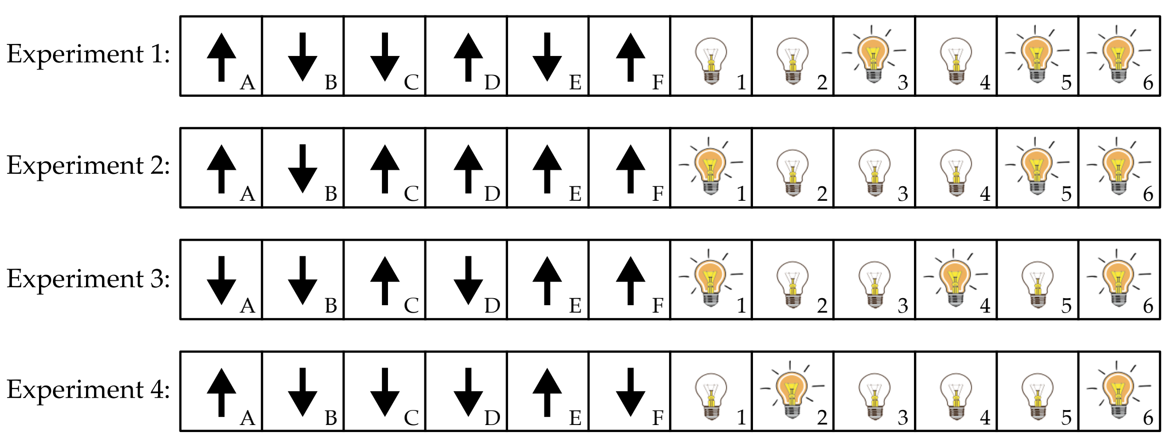
As a practical joke, Rachel connected light bulbs to switches so that each switch operates exactly one light bulb but nobody knows which one. Each switch can be either up or down, but we do not know which position corresponds to the connected bulb being on and which position corresponds to the connected bulb being off. To make matters worse, this could be different for different switches.

Problem 1: Rachel connected four switches (marked A, B, C, and D) to four light bulbs (numbered 1, 2, 3, and 4). Three experiments were conducted to determine which switch is connected to which light bulb. The position of each switch and the on/off status of each light bulb in each of the experiments is shown below. Which switch is connected to which light bulb?



To start, can you determine which switch must be connected to bulb 1?

Problem 2: Rachel connected six switches (marked A, B, C, D, E, and F) to six light bulbs (numbered 1, 2, 3, 4, 5, and 6). Four experiments were conducted to determine which switch is connected to which light bulb. The position of each switch and the on/off status of each light bulb in each of the experiments is shown below. Which switch is connected to which light bulb?



Extension: For each problem above, replace the last experiment with a different experiment that could have been used instead of the one given, and would have still allowed you to solve the problem.

More Info:

Check out the CEMC at Home webpage on Thursday, May 28 for a solution to Sneaky Switches.

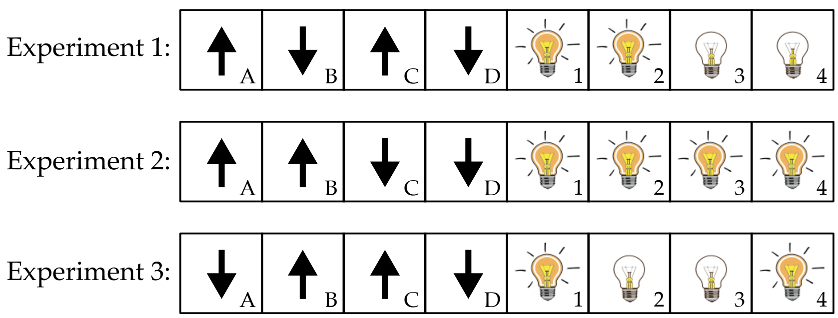
A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

CEMC at Home

Grade 7/8 - Wednesday, May 27, 2020
Sneaky Switches - Solution

Set Up: Rachel connected light bulbs to switches so that each switch operates exactly one light bulb but nobody knows which one. Each switch can be either up or down, but we do not know which position corresponds to the connected bulb being on and which position corresponds to the connected bulb being off. To make matters worse, this could be different for different switches.

Problem 1 Summary: Rachel connected four switches (marked A, B, C, and D) to four light bulbs (numbered 1, 2, 3, and 4). Three experiments were conducted and the results are shown below. Which switch is connected to which light bulb?



Solution:

The only switch that is in the same position in all three experiments is switch D. The only light bulb that has the same on/off status in all three experiments is light bulb 1. This means switch D must be connected to light bulb 1.

Between the first and second experiments, the only switches that change positions are B and C and the only lights that change on/off status are 3 and 4. This means that switches B and C must be connected to light bulbs 3 and 4, in some order.

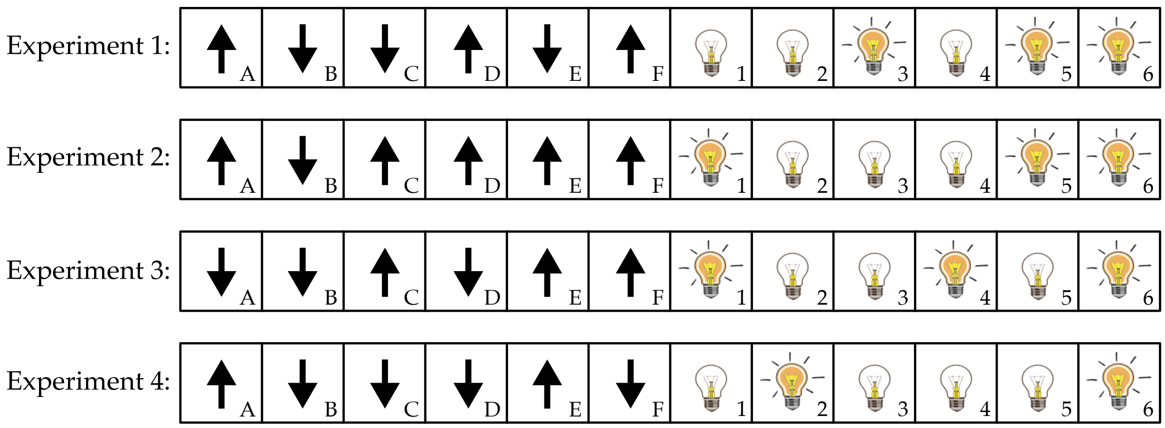
Between the second and third experiments, the only switches that change positions are A and C and the only lights that change on/off status are 2 and 3. This means that switches A and C must be connected to light bulbs 2 and 3, in some order.

Using the two observations above, we can conclude that switch C must be connected to light bulb 3. It follows that switch B is connected to light bulb 4 and switch A is connected to light bulb 2.

In summary, the switches and light bulbs are connected as follows.

Switch	A	B	C	D
Light Bulb	2	4	3	1

Problem 2 Summary: Rachel connected six switches (marked A, B, C, D, E, and F) to six light bulbs (numbered 1, 2, 3, 4, 5, and 6). Four experiments were conducted and the results are shown below. Which switch is connected to which light bulb?



Solution:

Between the first and second experiments, the only switches that change positions are C and E, and the only lights that change on/off status are 1 and 3. This means that switches C and E must be connected to light bulbs 1 and 3, in some order. This is our first observation.

Between the second and third experiments, the only switches that change positions are A and D, and the only lights that change on/off status are 4 and 5. This means that switches A and D must be connected to light bulbs 4 and 5, in some order. This is our second observation.

Between the third and fourth experiments, the only switches that change positions are A, C, and F, and the only lights that change on/off status are 1, 2, and 4. This means that switches A, C, and F must be connected to light bulbs 1, 2, and 4, in some order. This is our third observation.

We know from our first observation that switch C is connected to either light bulb 1 or light bulb 3. Using this along with our third observation, we can conclude that switch C must be connected to light bulb 1. It follows that switch E is connected to light bulb 3. Similarly, using our second observation with our third observation, we can conclude that switch A must be connected to light bulb 4. It follows that switch D is connected to light bulb 5 and switch F is connected to light bulb 2.

We are now left with switch B and light bulb 6, which must be connected. We can see that switch B is in the same position during all four experiments and that light bulb 6 is "on" during all four experiments, and so this is indeed a correct match. (Note that we could have started our solution by observing that switch B must be connected to light bulb 6.)

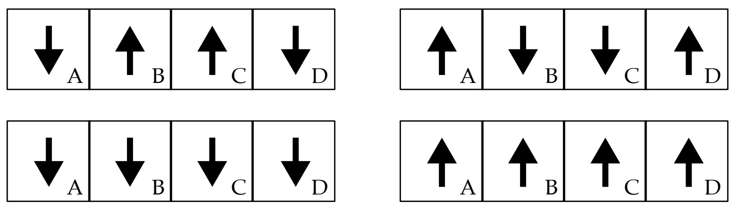
In summary, the switches and light bulbs are connected as follows.

Switch	A	B	C	D	E	F
Light Bulb	4	6	1	5	3	2

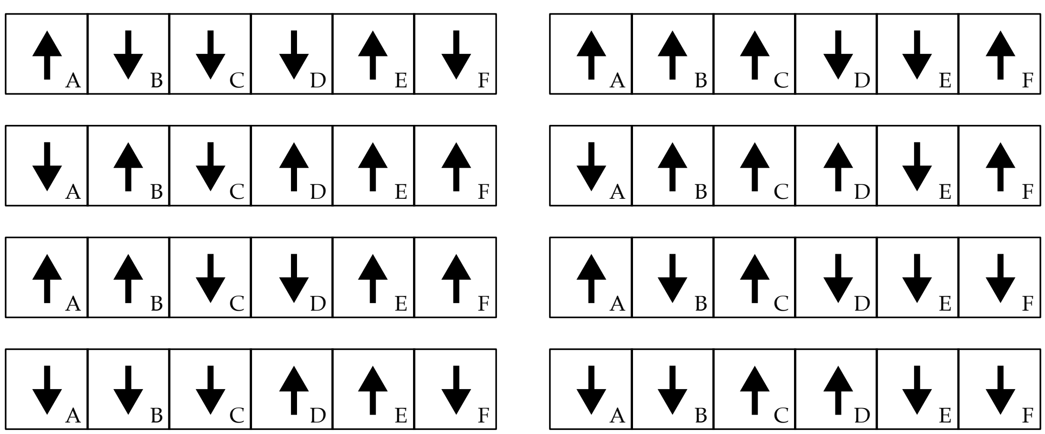
Extension: For each problem above, replace the last experiment with a different experiment that could have been used instead of the one given, and would have still allowed you to solve the problem.

Solution:

In Problem 1, without the final experiment you can determine that switches B and C are connected to light bulbs 3 and 4 (in some order), and thus, switches A and D must be connected to light bulbs 1 and 2 (in some order). So a final experiment needs to distinguish between switches A and D, and switches B and C. This can be achieved by changing two switches from the second experiment: switch A or D and switch B or C. There are four ways to do this, and they are shown below. Notice that the top left experiment shown is actually "Experiment 3" from the original problem. This experiment can be replaced with any of the other three experiments given below.



In Problem 2, without the final experiment you can determine that switches C and E are connected to light bulbs 1 and 3 (in some order), switches A and D are connected to light bulbs 4 and 5 (in some order), and switches B and F are connected to light bulbs 2 and 6 (in some order). So the final experiment needs to distinguish between switches C and E, switches A and D, and switches B and F. This can be achieved by changing three switches from the third experiment: switch C or E, switch A or D, and switch B or F. There are eight ways to do this, and they are shown below. Notice that the top left experiment shown is actually "Experiment 4" from the original problem. This experiment can be replaced with any of the other seven experiments given below.



CEMC at Home

Grade 7/8 - Thursday, May 28, 2020
Just Your Average Sequence

In a sequence of six numbers, every number after the first two is the average of the previous two numbers.

The 4^{th} number in the sequence is 22 and the 6^{th} number in the sequence is 45.

Determine all six numbers in the sequence.

?	?	?	22	?	45

More Info:

Check out the CEMC at Home webpage on Friday, May 29 for two different solutions to Just Your Average Sequence.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 7/8 - Thursday, May 28, 2020
Just Your Average Sequence - Solution

Problem:

In a sequence of six numbers, every number after the first two is the average of the previous two numbers.

The 4^{th} number in the sequence is 22 and the 6^{th} number in the sequence is 45.

Determine all six numbers in the sequence.

?	?	?	22	?	45

Solution:

If x is the average of two numbers y and z, then $\dfrac{y+z}{2} = x$.

It follows that $y + z = 2 \times x$. This idea is used in both Solution 1 and Solution 2 below.

Solution 1

In this solution, we solve the problem by working backwards.

Since the 6^{th} number in the sequence is equal to the average of the two previous numbers, the 6^{th} number must be the average of the 4^{th} and 5^{th} numbers.
So the sum of the 4^{th} and 5^{th} numbers must be 2 times the 6^{th} number, or $2 \times 45 = 90$. Therefore, the 5^{th} number is $90 - 22 = 68$.

We now determine the 3^{rd} number. The 5^{th} number in the sequence is the average of the 3^{rd} and 4^{th} numbers. So the sum of the 3^{rd} and 4^{th} numbers is 2 times the 5^{th} number, or $2 \times 68 = 136$. Therefore, the 3^{rd} number is $136 - 22 = 114$.

We now determine the 2^{nd} number. The 4^{th} number in the sequence is the average of the 2^{nd} and 3^{rd} numbers. So the sum of the 2^{nd} and 3^{rd} numbers is 2 times the 4^{th} number, or $2 \times 22 = 44$. Therefore, the 2^{nd} number is $44 - 114 = -70$.

We now determine the 1^{st} number. The 3^{rd} number in the sequence is the average of the 1^{st} and 2^{nd} numbers. So the sum of the 1^{st} and 2^{nd} numbers is 2 times the 3rd number, or $2 \times 114 = 228$. Therefore, the 1^{st} number is $228 - (-70) = 228 + 70 = 298$.

The sequence of six numbers is $298, -70,\ 114,\ 22,\ 68,\ 45$.

We can indeed check that in this sequence each number after the first two is equal to the average of the previous two numbers.

Solution 2

We will now present a similar, but more algebraic solution.

Let the sequence be

a	b	c	22	d	45

where a represents the 1st number, b represents the 2nd number, c represents the 3rd number and d represents the 5th number in the sequence.

We again solve this problem by working backwards.

Since the 6th number in the sequence is equal to the average of the 4th and 5th numbers, we have $45 = \dfrac{22+d}{2}$.
Multiplying both sides by 2, we obtain $22 + d = 45 \times 2 = 90$. Rearranging, $d = 90 - 22 = 68$. Therefore, the 5th number in the sequence is 68.

We now determine the 3rd number. Since the 5th number in the sequence is equal to the average of the 3rd and 4th numbers, we have $68 = \dfrac{c+22}{2}$.
Multiplying both sides by 2, we obtain $c + 22 = 68 \times 2 = 136$. Rearranging, $c = 136 - 22 = 114$. Therefore, the 3rd number in the sequence is 114.

We now determine the 2nd number. Since the 4th number in the sequence is equal to the average of the 2nd and 3rd numbers, we have $22 = \dfrac{b+114}{2}$.
Multiplying both sides by 2, we obtain $b + 114 = 22 \times 2 = 44$. Rearranging, $b = 44 - 114 = -70$. Therefore, the 2nd number in the sequence is -70.

We now determine the 1st number. Since the 3rd number in the sequence is equal to the average of the 1st and 2nd numbers, we have $114 = \dfrac{a+(-70)}{2}$.
Multiplying both sides by 2, we obtain $a+(-70) = 114\times 2 = 228$. Rearranging, $a = 228+70 = 298$. Therefore, the 1st number in the sequence is 298.

Therefore, the sequence of six numbers is $298, -70,\ 114,\ 22,\ 68,\ 45$.

We can indeed check that in this sequence each number after the first two is equal to the average of the previous two numbers.

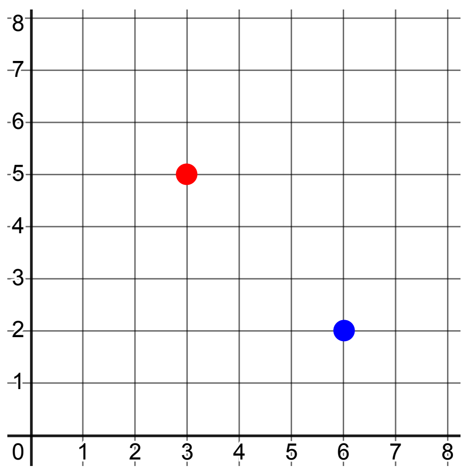
298	-70	114	22	68	45

CEMC at Home

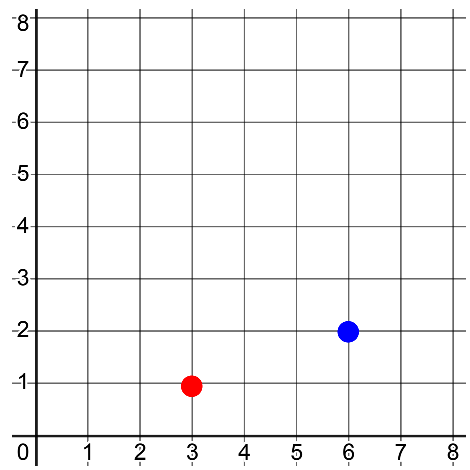
Grade 7/8 - Friday, May 29, 2020
Gridiron Expert

In the activities given below, you will be making different types of shapes by plotting points on a grid.

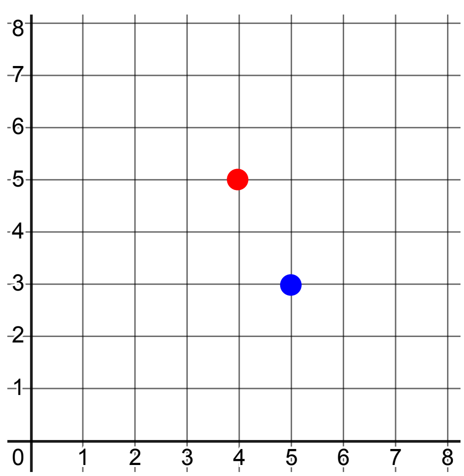
Activity 1: The points $(3, 5)$ and $(6, 2)$ are plotted on the grid below. Plot a third point on the grid so that the three points are the vertices of a right-angled triangle.



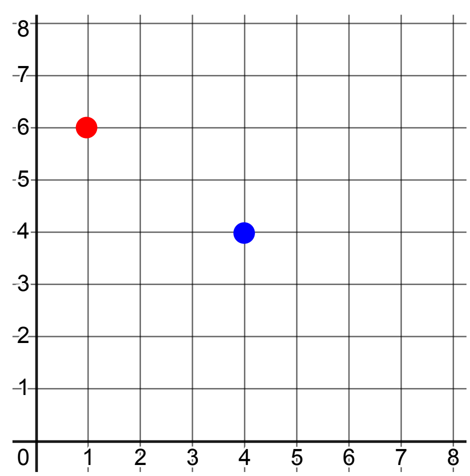
Activity 2: The points $(3, 1)$ and $(6, 2)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a rectangle that is not a square.



Activity 3: The points $(4, 5)$ and $(5, 3)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a quadrilateral with all four sides equal in length.



Activity 4: The points $(1, 6)$ and $(4, 4)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a trapezoid that is not a parallelogram.



There is more than one way to construct the shape described in each of the activities above. Spend some time thinking about how many different ways you could plot the points in each activity. Can you explain why you have the right type of shape each time?

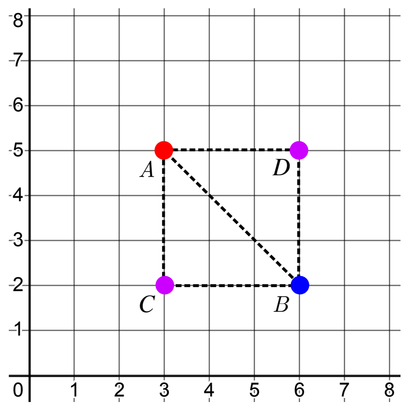
More Info: Check the CEMC at Home webpage on Monday, June 1 for a solution to Gridiron Expert.

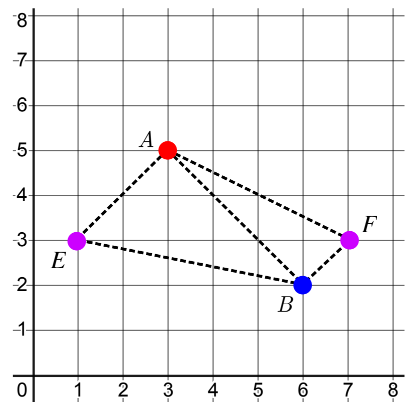
CEMC at Home

Grade 7/8 - Friday, May 29, 2020
Gridiron Expert - Solution

Activity 1: The points $(3, 5)$ and $(6, 2)$ are plotted on the grid. Plot a third point on the grid so that the three points are the vertices of a right-angled triangle.

Solution: We label the original points as A and B as shown. You can plot the third point in several different ways. Four different ways are shown below.





If you plot the third point $C(3, 2)$, then $\triangle ACB$ is a right-angled triangle. Since line segment AC is vertical and line segment CB is horizontal, these line segments are perpendicular and so there is a right angle at C.

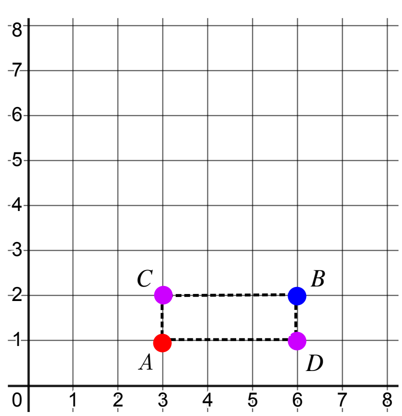
If you instead plot the third point $D(6, 5)$, then you get $\triangle ADB$ with a right angle at D.

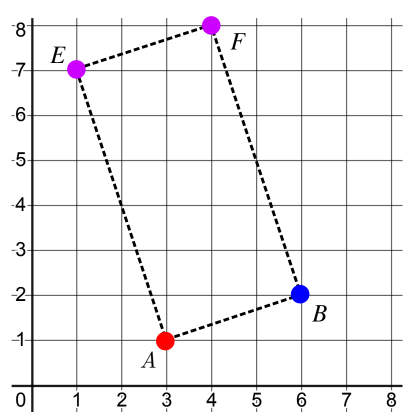
If you plot the third point $E(1, 3)$, then $\triangle EAB$ has right angle $\angle EAB$. If you use a protractor to measure this angle, then you will see that the measure of the angle is around 90°. Do you know how to justify that the line segments EA and AB are indeed perpendicular by just looking at the grid? (Can you explain why $\angle EAB$ is made up of two 45° angles? Think about the diagonals of a square.)

If you plot the third point $F(7, 3)$, then $\triangle ABF$ has right angle $\angle ABF$. Can you explain why?

Activity 2: The points $(3, 1)$ and $(6, 2)$ are plotted on the grid. Plot two more points on the grid so that the four points are the vertices of a rectangle that is not a square.

Solution: We label the original points as A and B as shown. You can plot the two additional points in several different ways. Two different ways are shown below.



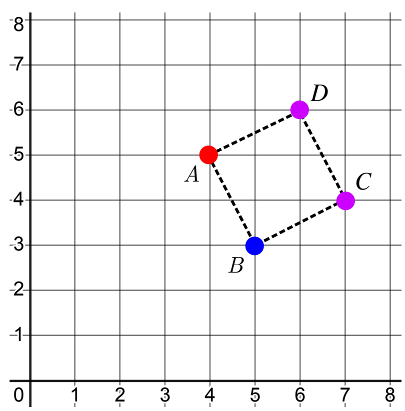


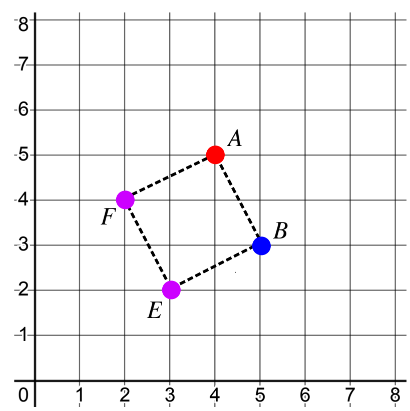
If you plot the two additional points $C(3, 2)$ and $D(6, 1)$, then quadrilateral $ACBD$ is a rectangle, but not a square. Since opposite sides AC and BD are vertical, and opposite sides CB and AD are horizontal, we know that the shape has four right angles. We can also see that the vertical sides have length 1 unit, and the horizontal sides have length 3 units, and so opposite sides are equal in length, but not all sides are equal in length.

If you instead plot the two additional points $E(1, 7)$ and $F(4, 8)$, then quadrilateral $ABFE$ is also a rectangle, but not a square. Since opposite sides AB and EF are diagonals of identical 1×3 rectangles, they must be equal in length. Since opposite sides AE and BF are diagonals of identical 2×6 rectangles, they must be equal in length as well. But, we can see that not all four sides are equal in length. (In fact, side AE is twice the length of side AB. Can you see why?) If you use a protractor to measure the four angles, then you will see that the measure of each angle is around 90°. Do you know how to justify, for example, that the line segments EA and AB are indeed perpendicular by just looking at the grid?

Activity 3: The points $(4, 5)$ and $(5, 3)$ are plotted on the grid. Plot two more points on the grid so that the four points are the vertices of a quadrilateral with all four sides equal in length.

Solution: We label the original points as A and B as shown. You can plot the two additional points in more than one way. Two different ways are shown below.





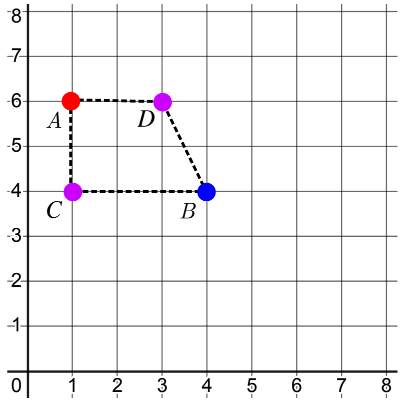
If you plot the two additional points $C(7, 4)$ and $D(6, 6)$, then quadrilateral $ABCD$ has all four sides equal in length. We know that sides AB, BC, CD, and DA are all equal in length because they are all diagonals of identical rectangles. (Each rectangle is 1×2, but they are not all oriented in the same way.)

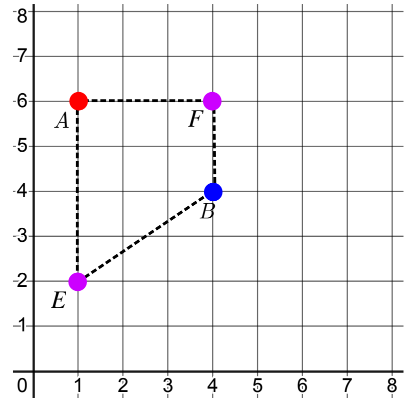
You get a similar situation if you instead plot the two additional points $E(3, 2)$ and $F(2, 4)$ to form quadrilateral $ABEF$.

A quadrilateral with all four sides equal in length is called a rhombus. In addition, quadrilaterals $ABCD$ and $ABEF$ each have four right angles and so they are actually squares. Can you see why?

Activity 4: The points $(1, 6)$ and $(4, 4)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a trapezoid that is not a parallelogram.

Solution: We label the original points as A and B as shown. You can plot the two additional points in several different ways. Two different ways are shown below.





If you plot the two additional points $C(1, 4)$ and $D(3, 6)$, then quadrilateral $ACBD$ is a trapezoid that is not a parallelogram. Since sides AD and CB are horizontal, these opposite sides are parallel, which means $ACBD$ is a trapezoid. Since AC is vertical but DB is not, these opposite sides are not parallel, which means $ACBD$ is not a parallelogram.

You get a similar situation if you plot the two additional points $E(1, 2)$ and $F(4, 6)$ to form quadrilateral $ABEF$.

Can you find some other ways to plot the two additional points in this activity? In particular, try to plot some trapezoids that have line segment AB as a side.