CEMC at Home

Grade 9/10 - Thursday, May 21, 2020

Contest Day 3

Today's resource features two questions from the 2020 CEMC Mathematics Contests.

2020 Canadian Team Mathematics Contest, Team Problem #5

What is the smallest eight-digit positive integer that has exactly four digits which are 4?

2020 Canadian Team Mathematics Contest, Team Problem #14

Jeff caught 21 fish, each having a mass of at least 0.2 kg. He noticed that the average mass of the first three fish that he caught was the same as the average mass of all 21 fish. The total mass of the first three fish was 1.5 kg. What is the largest possible mass of any one fish that Jeff could have caught?

More Info:

Check out the CEMC at Home webpage on Monday, May 25 for solutions to the Contest Day 3 problems.

CEMC at Home

Grade 9/10 - Thursday, May 21, 2020
Contest Day 3 - Solution

Solutions to the two contest problems are provided below.

2020 Canadian Team Mathematics Contest, Team Problem #5

What is the smallest eight-digit positive integer that has exactly four digits which are 4?

Solution:

The smallest eight-digit number is 10 000 000.
The smallest 10 000 eight-digit numbers are those of the form $10\,00a\,bcd$ where a, b, c, and d are digits. Among the 10 000 smallest eight-digit numbers, 10 004 444 is the only one that has four digits that are equal to 4.
This means the smallest eight-digit positive integer that has exactly four digits which are 4 is 10 004 444.

2020 Canadian Team Mathematics Contest, Team Problem #14

Jeff caught 21 fish, each having a mass of at least 0.2 kg. He noticed that the average mass of the first three fish that he caught was the same as the average mass of all 21 fish. The total mass of the first three fish was 1.5 kg. What is the largest possible mass of any one fish that Jeff could have caught?

Solution:

Since the total mass of the first three fish is 1.5 kg, the average mass of the first three fish is 0.5 kg.
Let M be the total mass of all of the fish. Since the average mass of the first three fish is the same as the average mass of all of the fish, this means $\frac{M}{21} = 0.5$ kg or $M = 10.5$ kg.
Since the first three fish have a total mass of 1.5 kg, this means the last 18 fish that Jeff caught have a total mass of $10.5 \text{ kg} - 1.5 \text{ kg} = 9 \text{ kg}$.
If 17 of these 18 fish have as small a mass that is as possible, the 18^{th} of these fish will have a mass that is as large as possible.
The smallest possible mass is 0.2 kg, so the total mass of 17 fish, each having as small a mass as possible, is $17 \times 0.2 \text{ kg} = 3.4 \text{ kg}$.
The largest possible mass of any fish that Jeff could have caught is $9 \text{ kg} - 3.4 \text{ kg} = 5.6 \text{ kg}$.

CEMC at Home

Grade 9/10 - Friday, May 22, 2020

Binary Puzzles

A binary puzzle is a type of logic puzzle that is done on an $n \times n$ grid where n is even. Your task is the complete the grid by filling in all empty cells according to the following rules:

1. Each cell in the grid must contain either the digit 0 or the digit 1.
2. The grid cannot have three or more consecutive cells, either horizontally or vertically, that all contain the same digit.
3. Half of the entries in each row of the grid must be 0s and the other half of the entries must be 1s. The same is true of each column.
4. No two rows in the grid can be identical, and no two columns can be identical.

Example: Let's complete the following row in a binary puzzle grid following the first three rules. *To ensure rule 4 is satisfied, we would have to consider all of the rows and columns in the grid at once.*

1	1			0		1	

Notice that the third cell must contain a 0, otherwise the row would have three consecutive cells containing 1s (which would violate the second rule).

1	1	0		0		1	

Now, notice that the fourth cell must contain a 1, otherwise the row would have three consecutive cells containing 0s (which would violate the second rule).

1	1	0	1	0		1	

The row now has four 1s and two 0s. According to the third rule, the remaining two cells must contain 0s.

1	1	0	1	0	0	1	0

Therefore, there is only way to complete this particular row according to the rules.

Note that there are other ways to reason how this row must be completed. For example, we could start by observing that the original partially completed row already has three 1s and so we must use exactly one 1 and three 0s while filling in the row. We could then argue that we have no choice but to place the 1 in the fourth cell. Can you see why? What must happen if we place the 1 in a different cell?

Try the four binary puzzle grids on the next page.

These puzzles increase in difficulty but can all be completed using solid reasoning. Enjoy!

More Info:

Check out the CEMC at Home webpage on Friday, May 29 for solutions to these Binary Puzzles.

The title *Binary Puzzles* is a reference to the binary number system. This system only uses the digits 0 and 1. Certain branches of mathematics as well as many electronics, including computers, make use of the binary number system.

PUZZLE 1

	0			1			
1			0			1	0
1		1					0
		1	1		1		
1					1		
		0		0			1
	1			0			1
0			1				

PUZZLE 2

		1	1			1	
							0
0		0		1		1	1
0					0		
		0				1	
			0				
1				1		1	

PUZZLE 3

			1	1			0
	1						
0		0					
						0	
1		1				1	
	0			1		1	
			1		0		

PUZZLE 4

0			0		1		
		1					1
				0		0	
				0	0		
0				1		1	
	1		1				1

CEMC at Home

Grade 9/10 - Friday, May 22, 2020

Binary Puzzles - Solution

PUZZLE 1

0	0	1	0	1	1	0	1
1	1	0	0	1	0	1	0
1	0	1	1	0	0	1	0
0	0	1	1	0	1	0	1
1	1	0	0	1	1	0	0
1	0	0	1	0	0	1	1
0	1	1	0	0	1	0	1
0	1	0	1	1	0	1	0

PUZZLE 2

0	0	1	1	0	0	1	1
1	1	0	0	1	1	0	0
1	0	1	1	0	1	0	0
0	1	0	0	1	0	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
0	1	1	0	0	1	0	1
1	0	0	1	1	0	1	0

PUZZLE 3

1	0	0	1	1	0	1	0
0	1	1	0	0	1	0	1
0	1	0	1	0	1	1	0
1	0	1	0	1	0	0	1
0	1	0	1	0	1	0	1
1	0	1	0	0	1	1	0
1	0	1	0	1	0	1	0
0	1	0	1	1	0	0	1

PUZZLE 4

0	1	1	0	0	1	1	0
1	0	1	0	1	0	0	1
1	0	0	1	0	1	0	1
0	1	0	1	1	0	1	0
1	0	1	0	1	1	0	0
1	0	0	1	0	0	1	1
0	1	1	0	1	0	1	0
0	1	0	1	0	1	0	1