CEMC at Home

Grade 4/5/6 - Thursday, May 21, 2020
Contest Day 3

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #12

Abdul is 9 years older than Susie, and Binh is 2 years older than Susie. How many years older is Abdul than Binh?

(A) 11 **(B)** 9 **(C)** 14 **(D)** 2 **(E)** 7

2011 Gauss Contest, #10

I bought a new plant for my garden. Anika said it was a red rose, Bill said it was a purple daisy, and Cathy said it was a red dahlia. Each person was correct in stating either the colour or the type of plant. What was the plant that I bought?

(A) purple dahlia **(B)** purple rose **(C)** red dahlia **(D)** yellow rose **(E)** red daisy

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More Info:

Check out the CEMC at Home webpage on Monday, May 25 for solutions to the Contest Day 3 problems.

CEMC at Home

Grade 4/5/6 - Thursday, May 21, 2020
Contest Day 3 - Solution

Solutions to the two contest problems are provided below.

2020 Gauss Contest, #12

Abdul is 9 years older than Susie, and Binh is 2 years older than Susie. How many years older is Abdul than Binh?

(A) 11 **(B)** 9 **(C)** 14 **(D)** 2 **(E)** 7

Solution 1:

Abdul is 9 years older than Susie, and Binh is 2 years older than Susie, and so Abdul is $9 - 2 = 7$ years older than Binh.
For example, if Susie is 10 years old, then Abdul is $9 + 10 = 19$, Binh is $2 + 10 = 12$, and Abdul is $19 - 12 = 7$ years older than Binh.

Solution 2:

We know that Abdul is 9 years older than Susie and Binh is 2 years older than Susie.
Let's say that Susie is 5 years old. That will make Abdul $5 + 9 = 14$ and Binh $5 + 2 = 7$.
This would mean Abdul is $14 - 7 = 7$ years older than Binh.
Let's double-check if this works when Susie is a different age.
This time, let Susie be 8 years old. That will make Abdul $8 + 9 = 17$ and Binh $8 + 2 = 10$.
This would mean Abdul is $17 - 10 = 7$ years older than Binh, which is the same as our previous answer.

ANSWER: (E)

2011 Gauss Contest, #10

I bought a new plant for my garden. Anika said it was a red rose, Bill said it was a purple daisy, and Cathy said it was a red dahlia. Each person was correct in stating either the colour or the type of plant. What was the plant that I bought?

(A) purple dahlia **(B)** purple rose **(C)** red dahlia **(D)** yellow rose **(E)** red daisy

Solution:

Anika said that the plant was a *red* rose.
Cathy said that the plant was a *red* dahlia.
If the plant is not red, then both Anika and Cathy are wrong about the colour, so they would have to be right about the plant type. Unfortunately, they each said a different plant type, so that is not possible. This means the plant must be red.
Bill said that the plant was a purple daisy. Bill is wrong about the colour, so he must be right about the flower type. This means the plant is a red daisy.

ANSWER: (E)

CEMC at Home

Grade 4/5/6 - Friday, May 22, 2020

Toothpick Polyhedrons

You Will Need:

- Toothpicks of the same length (at least 12, but preferably more)
- Some miniature marshmallows, licorice bits, or bits of play dough.
- A flat surface on which to work

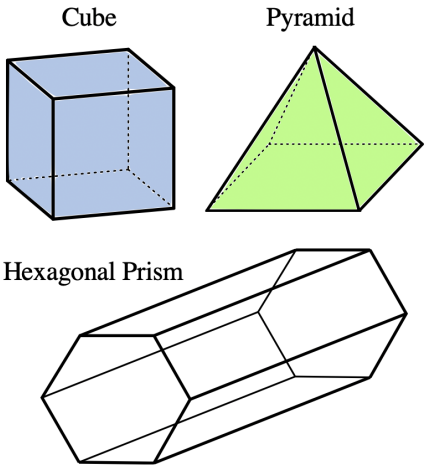
Introduction:

A *polyhedron* is a three-dimensional object with polygons for its faces. Recall that a polygon is a two-dimensional closed shape formed by three or more line segments. Examples of polygons include triangles, quadrilaterals, pentagons, hexagons, and octagons.

A cube is an example of a polyhedron and has six square faces.

A square-based pyramid is another example of a polyhedron, and has one square face and four triangular faces.

We can build models of polyhedrons by building *skeletons* of the objects. A skeleton includes all of the edges of the polyhedron. For example, the skeleton of a hexagonal prism (another type of polyhedron) is shown on the right.



What to Do:

The goal of this activity is to construct models of polyhedrons using toothpicks joined together with miniature marshmallows. Toothpicks cannot be broken, but two or more toothpicks may be joined together to create edges longer than one toothpick ().

Once you have constructed a new polyhedron, sketch the polyhedron carefully and state its name or make up a fun, suitable name.

Activities and Questions to Explore:

1. Construct a model of a square-based pyramid. How many toothpicks did you use? How many marshmallows did you use? Is there more than one way to do this?
2. See how many different polyhedrons you can construct using exactly 12 toothpicks.
   For example, what could you make with a square base? A triangular base? Other bases?
3. Now see how many different polyhedrons you can construct using *fewer* than 12 toothpicks.
4. Can you construct a polyhedron with a hexagonal base using exactly 12 toothpicks? Why, or why not?

More info:

Check out the CEMC at Home webpage on Friday, May 29 for a solution to Toothpick Polyhedrons.

CEMC at Home

Grade 4/5/6 - Friday, May 22, 2020

Toothpick Polyhedrons - Solution

1. Construct a model of a square-based pyramid. How many toothpicks did you use? How many marshmallows did you use? Is there more than one way to do this?

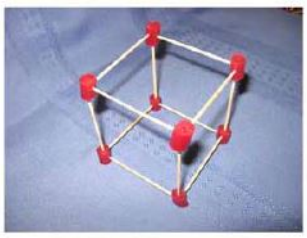
   Solution:

   You can construct a model of a square-based pyramid in more than one way. For example, you can do so using 8 toothpicks and 5 marshmallows, or 12 toothpicks and 9 marshmallows. A diagram of this second way is shown at the end of the solution to 2. below.

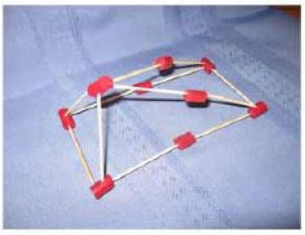
2. See how many different polyhedrons you can construct using exactly 12 toothpicks.

   Solution:

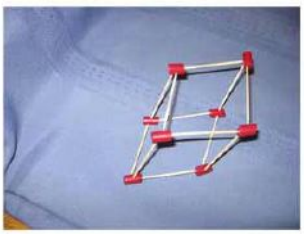
   Here are some photos of models of various polyhedrons which can be built using exactly 12 toothpicks.



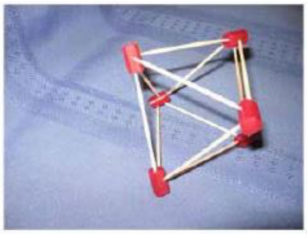
Cube



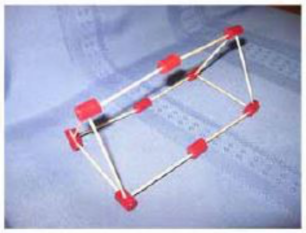
Rectangular-Based Pyramid



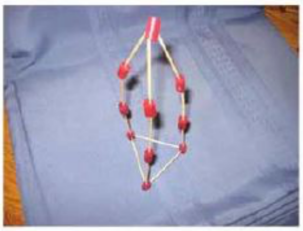
Rhombohedron



Octahedron



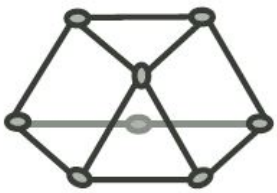
Triangular Prism



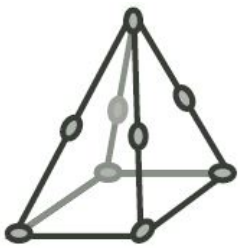
Triangular-Based Pyramid

Notice that the sides of the triangular-based pyramid have sagged a bit under their own weight before the photo was taken. The three toothpicks forming the edges joining at the top vertex should actually be straight.

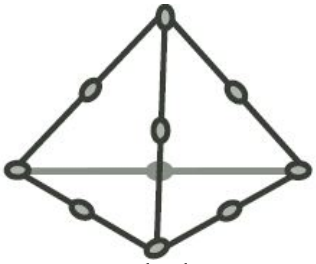
Here are some sketches of a few more polyhedrons you can model with exactly 12 toothpicks.



You name it!



Square-based Pyramid

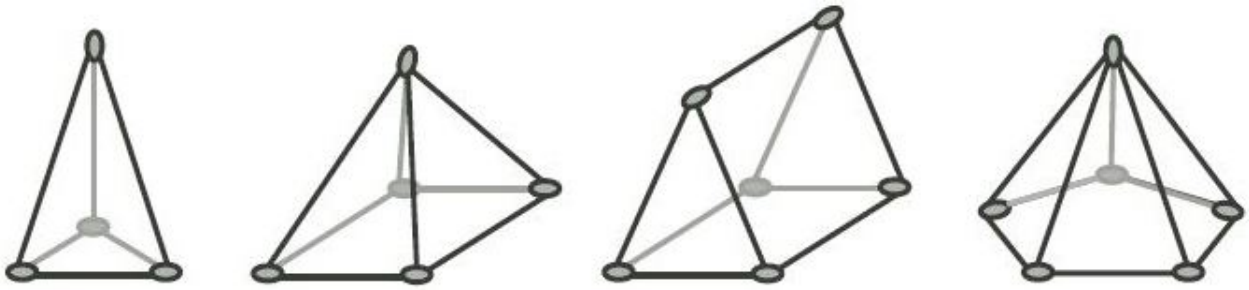


Tetrahedron

3. Now see how many different polyhedrons you can construct using *fewer* than 12 toothpicks.

   Solution:

   Here are some polyhedrons that you can model using *fewer* than 12 toothpicks. Three of them are pyramids and one is a prism. Can you name each of the polyhedrons?



4. Can you construct a polyhedron with a hexagonal base using exactly 12 toothpicks? Why, or why not?

   Solution:

   It is not possible to do this.

   If you try to construct a model of a polyhedron with a hexagonal base, then the base must use at least 6 toothpicks to construct. This would leave you with at most 6 toothpicks to form the other edges. Each of the 6 vertices of the base must have a toothpick coming out, but when you try joining them to form a "peak", you will discover that you actually get a two-dimensional figure!

   This is because a regular hexagon (with equal sides the length of one toothpick) is formed from six equilateral triangles. So your toothpick model would have the shape of the diagram at the right.