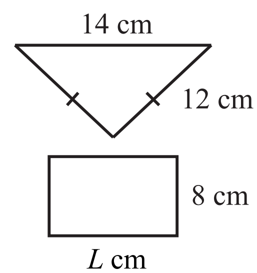
CEMC at Home

Grade 4/5/6 - Monday, May 11, 2020
Contest Day 2

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #9

In the diagram, the perimeter of the triangle is equal to the perimeter of the rectangle. What is the length (L) of the rectangle?



(A) 8 **(B)** 10 **(C)** 11
(D) 14 **(E)** 15

2017 Gauss Contest, #14

When the time in Toronto, ON is 1:00 p.m., the time in Gander, NL is 2:30 p.m. A flight from Toronto to Gander takes 2 hours and 50 minutes. If the flight departs at 3:00 p.m. (Toronto time), what time will the flight land in Gander (Gander time)?

(A) 7:20 p.m. **(B)** 5:00 p.m. **(C)** 6:20 p.m. **(D)** 5:20 p.m. **(E)** 8:50 p.m.

More Info:

Check out the CEMC at Home webpage on Thursday, May 21 for solutions to the Contest Day 2 problems.

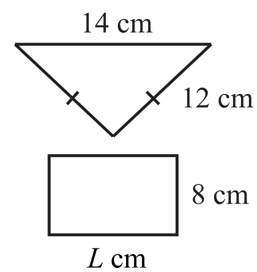
CEMC at Home

Grade 4/5/6 - Monday, May 11, 2020
Contest Day 2 - Solution

Solutions to the two contest problems are provided below, including a video for the first problem.

2020 Gauss Contest, #9

In the diagram, the perimeter of the triangle is equal to the perimeter of the rectangle. What is the length (L) of the rectangle?



(A) 8 **(B)** 10 **(C)** 11
(D) 14 **(E)** 15

Solution:

Before you can solve this problem you have to remember that perimeter means the distance around the outside of a shape, and when a shape has a tick on two sides, it indicates that both sides are the same length. This means the triangle has two sides of length 12 cm and one of length 14 cm. Its perimeter is $12 + 12 + 14 = 38$ cm. The rectangle also has the same perimeter. We know that two of the sides of the rectangle are 8 cm long, so together they add to 16 cm. The remaining two sides combined are therefore $38 - 16 = 22$ cm. Since both of these sides are equal in length, the value of L must be $22 \div 2 = 11$.

Answer: (C)

Video
Visit the following link for a discussion of the solution to the first contest problem:
https://youtu.be/wmLK4eFEcpo

2017 Gauss Contest, #14

When the time in Toronto, ON is 1:00 p.m., the time in Gander, NL is 2:30 p.m. A flight from Toronto to Gander takes 2 hours and 50 minutes. If the flight departs at 3:00 p.m. (Toronto time), what time will the flight land in Gander (Gander time)?

(A) 7:20 p.m. **(B)** 5:00 p.m. **(C)** 6:20 p.m. **(D)** 5:20 p.m. **(E)** 8:50 p.m.

Solution:

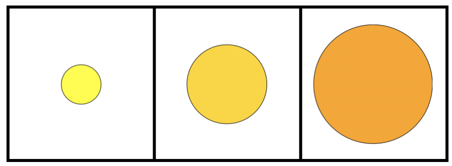
Since the time in Toronto, ON is 1:00 p.m. when the time in Gander, NL is 2:30 p.m., then the time in Gander is 1 hour and 30 minutes ahead of the time in Toronto.
A flight that departs from Toronto at 3:00 p.m. and takes 2 hours and 50 minutes will land in Gander at 5:50 p.m. Toronto time.
When the time in Toronto is 5:50 p.m., the time in Gander is 1 hour and 30 minutes ahead which is 7:20 p.m.

Answer: (A)

CEMC at Home

Grade 4/5/6 - Tuesday, May 12, 2020
Shifting Discs

Three discs, each of a different size, are arranged in a grid as shown below. Each disc starts off in its own square with the discs arranged in increasing order of size, so that the smallest disc is in the leftmost square and the largest disc is in the rightmost square.

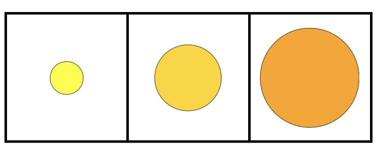


Your goal is to reverse the order of the discs, so that the smallest disc is in the rightmost square and the largest disc is in the leftmost square, however, you must follow certain rules when moving the discs:

- At all times, each square in the grid must contain a single disc, a single stack of discs, or be empty.
- A disc may be moved on top of another disc of larger size, but not of smaller size.
- Any single disc may be moved left or right into any empty square in the grid.
- Discs can only be moved over one square at a time.
  For example, a disc cannot be move directly from the leftmost square to the rightmost square without passing through the middle square.
- Only one disc can be moved at a time. If there is a stack of discs in a square in the grid, then only the top disc in the stack can be moved, not the entire stack at once.

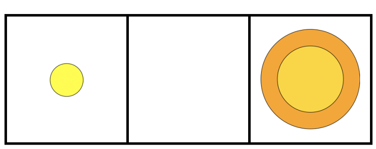
For example, here are three moves performed one after the other that follow the rules:

Start



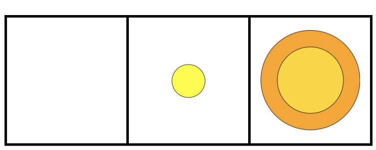
You can either move the small disc on top of the medium disc, or move the medium disc on top of the large disc.

After Move 1



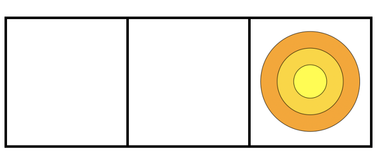
You now cannot move the stack of two discs. You can only move the medium disc back or move the small disc over.

After Move 2



You cannot move the stack of two discs. You cannot move the medium disc as it would have to go on top of the small disc.

After Move 3

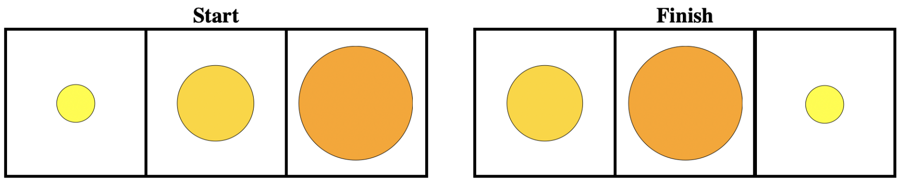


You can only move the small disc from this position.

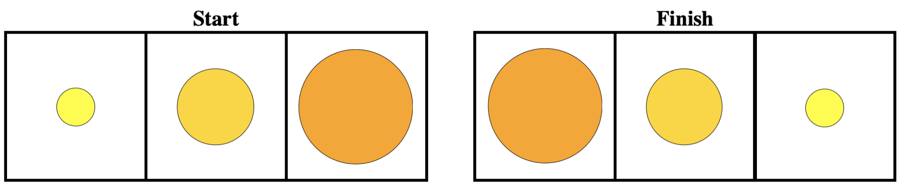
See the next page for some problems to think about while you explore.

Problems:

1. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right.



2. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right, which has the discs in the reverse order.



   Solving this problem is the main goal of the activity!

3. Now suppose you start with four discs instead of three. Just like before, they are all different sizes, and arranged in increasing order of size, with the smallest disc on the left and largest disc on the right. As in 2., you want to reverse the order of the discs, following the same rules. How can you use your solution for moving three discs from 2. to come up with a solution for moving four discs?

4. Building on the previous question, how can you use the solution for four discs to get a solution for the similar puzzle for five discs? In general, if you know how to solve the puzzle for a certain number of discs, how can you use it to solve the similar puzzle with one more disc added?

Extension: Suppose we add one more rule: stacks cannot have more than two discs in them at any time. Do you think there is still a solution to the puzzle with three discs from 2.? Do you think there is still a solution to the similar puzzle with four discs? Explain your answers.

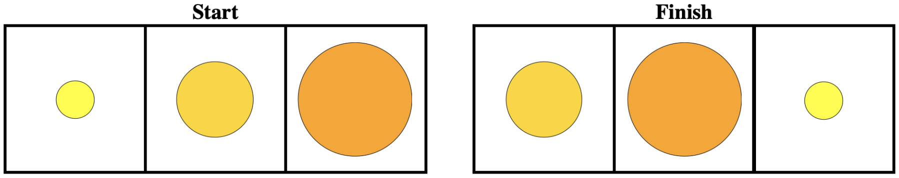
More Info:

Check out the CEMC at Home webpage on Tuesday, May 19 for a solution to Shifting Discs.

CEMC at Home

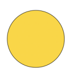
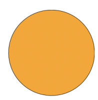
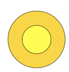
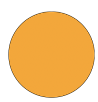
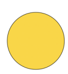
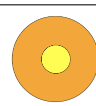
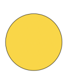
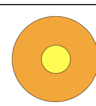
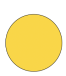
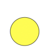
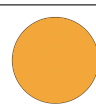
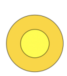
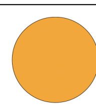
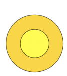
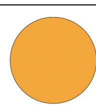
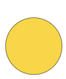
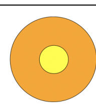
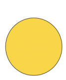
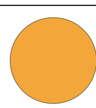
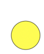
Grade 4/5/6 - Tuesday, May 12, 2020
Shifting Discs - Solution

1. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right.

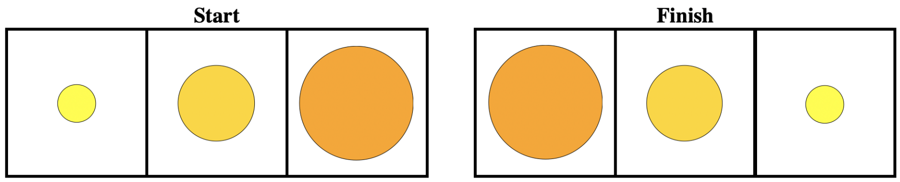


Solution:

The table below shows one possible sequence of moves that produces the correct final arrangement of the discs.

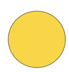
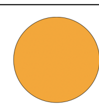
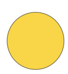
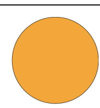
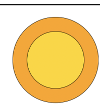
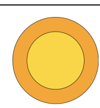
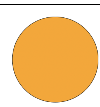
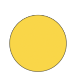
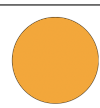
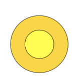
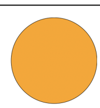
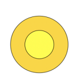
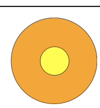
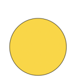
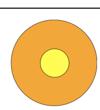
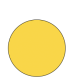
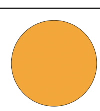
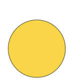
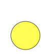
Left Position	Middle Position	Right Position	Explanation
			Starting position
			Move the small disc to the right
			Move the small disc to the right
			Move the medium disc to the left
			Move the small disc to the left
			Move the small disc to the left
			Move the large disc to the left
			Move the small disc to the right
			Move the small disc to the right

2. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right, which has the discs in the reverse order.



Solution:

We start by following the moves from 1. to get the discs into the arrangement medium, large, small. Then we continue as shown the table below to get the final arrangement with the discs reversed. Note that we sometimes do more than one move in each row.

Left Position	Middle Position	Right Position	Explanation
			Starting position
			Final position from 1.
			Move the medium disc to the right
			Move the small disc to the left, **twice**
			Move the medium disc to the right
			Move the small disc to the right, **twice**
			Move the large disc to the left
			Move the small disc to the left, **twice**
			Move the medium disc to the left
			Move the small disc to the right, **twice**

3. Now suppose you start with four discs instead of three. Just like before, they are all different sizes, and arranged in increasing order of size, with the smallest disc on the left and largest disc on the right. As in 2., you want to reverse the order of the discs, following the same rules. How can you use your solution for moving three discs from 2. to come up with a solution for moving four discs?

   Solution:

   Let's label the discs 1, 2, 3, and 4, from smallest to largest. Using this labelling we are tasked with moving the discs from the starting configuration shown below on the left, to the final configuration shown below on the right.

1	2	3	4

4	3	2	1

   We illustrate our solution in the table below. Each row of the table shows where each of the four discs (1, 2, 3, and 4) are located in the grid at each stage. If more than one disc is in the same square in the grid at some time, then these discs will be in a stack.

   We start our solution by ignoring disc 1 and rearranging discs 2, 3 and 4 using the method from the solution to problem 2 to reverse them: | 1 | 2 | 3 | 4 | becomes | 1 | 4 | 3 | 2 |.

Position A	Position B	Position C	Position D	Explanation
1	2	3	4	Starting position
1	4	3	2	Problem 2. with discs 2, 3, and 4
	4	3	1, 2	Move disc 1 all the way to the right
4	3		1, 2	Move disc 4, then disc 3 to the left
4	1, 3		2	Move disc 1 to the left, twice
4	1, 3	2		Move disc 2 to the left
4	3	2	1	Move disc 1 all the way to the right

4. Building on the previous question, how can you use the solution for four discs to get a solution for the similar puzzle for five discs? In general, if you know how to solve the puzzle for a certain number of discs, how can you use it to solve the similar puzzle with one more disc added?

   Solution:

   Once we have a set of moves to solve the four-disc puzzle, such as given above, we can solve the five-disc version of the puzzle using the same type of strategy as in the solution to problem 3. We number the discs 1, 2, 3, 4, and 5, from smallest to largest, and illustrate the solution in the table below.

Pos. A	Pos. B	Pos. C	Pos. D	Pos. E	Explanation
1	2	3	4	5	Starting position
1	5	4	3	2	Problem 3. with discs 2, 3, 4, and 5
	5	4	3	1, 2	Move disc 1 all the way to the right
5	4	3		1, 2	Move disc 5, then disc 4, then disc 3 to the left
5	4	1, 3		2	Move disc 1 to the left, twice
5	4	1, 3	2		Move disc 2 to the left
5	4	3	2	1	Move disc 1 all the way to the right

We can use the same strategy to solve the puzzle for any larger number of discs, too! Can you see how to do this? We can always use our solution to the previous puzzle as the first step in our solution to the next puzzle. How can you use the solution to the five-disc puzzle to get a solution to the six-disc puzzle? How can you get from the six-disc puzzle to the seven-disc puzzle?

Extension: Suppose we add one more rule: stacks cannot have more than two discs in them at any time. Do you think there is still a solution to the puzzle with three discs from 2.? Do you think there is still a solution to the similar puzzle with four discs? Explain your answers.

Solution:

It turns out that it is not possible to solve the three-disc puzzle with this new rule in place. Our solution to the three-disc puzzle (shown earlier) used a stack of three discs, but how can we be sure that there isn't some *other* sequence of moves that avoids any stacks of three?

To see why a stack of three discs is unavoidable, we note that we must start with the discs in the order $1, 2, 3$ and need to move disc 3 all the way to the left.

In order to move disc 3 at all, we must first arrange the discs as follows: | 1, 2 | | 3 |.

Then we can move disc 3 to the left one place: | 1, 2 | 3 | |.

Now, in order to move disc 3 to the left again, we need to somehow move discs 1 and 2 past disc 3 to the right. This cannot be done without forming a stack of three at some point. Can you see why? Here is the idea:

In order to have a chance at moving disc 2, we first need to move disc 1 all the way to the right: | 2 | 3 | 1 |. The problem is that there is now no way to move disc 2 past disc 3 without first moving disc 1 off the rightmost square. The only way to have disc 2 occupy the rightmost square after this point is to have disc 2 and disc 1 somehow "pass" each other, over disc 3. The only way to make this happen is to create a stack of all three discs in the middle (which is not allowed).

On the other hand, it *is* possible to solve the four-disc puzzle with this new rule in place. One way to do it is illustrated in the table below:

Pos. A	**Pos. B**	**Pos. C**	**Pos. D**	**Explanation**
1	2	3	4	Starting position
	2	3	1, 4	Move disc 1 all the way to the right
2	3		1, 4	Move disc 2, then disc 3 to the left
2	1, 3		4	Move disc 1 onto disc 3
2	1, 3	4		Move disc 4 to the left
2	3	4	1	Move disc 1 all the way to the right
	3	2, 4	1	Move disc 2 onto disc 4
3		2, 4	1	Move disc 3 to the left
2, 3		4	1	Move disc 2 all the way to the left
2, 3	4		1	Move disc 4 to the left
3	4	2	1	Move disc 2 two places right
3	1, 4	2		Move disc 1 two places left
3	1, 4		2	Move disc 2 to the right
3	4		1, 2	Move disc 1 all the way to the right
	4	3	1, 2	Move disc 3 two places right
4	3		1, 2	Move disc 4, then disc 3 to the left
4	1, 3		2	Move disc 1 two places left
4	1, 3	2		Move disc 2 to the left
4	3	2	1	Move disc 1 all the way to the right