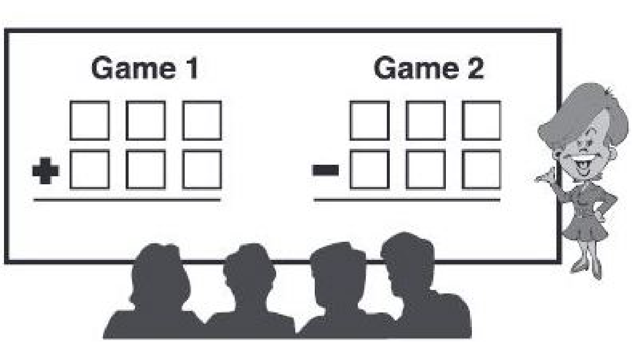
CEMC at Home

Grade 4/5/6 - Monday, March 30, 2020
This Game is Really Sum-thing!

For this activity, you will need to think about *optimizing* sums and differences. This means you will need to think about how to find maximum (greatest) values and minimum (least) values.

Try this example: Let's use the four digits 2, 3, 4, and 5, each exactly once, to form two numbers and then add them. There are many different *sums* you can get by doing this. For example, we could form the numbers 25 and 34 and then add them to get the sum $25 + 34 = 59$. What is the largest possible result that you can get? Think this over and then try out the following games.

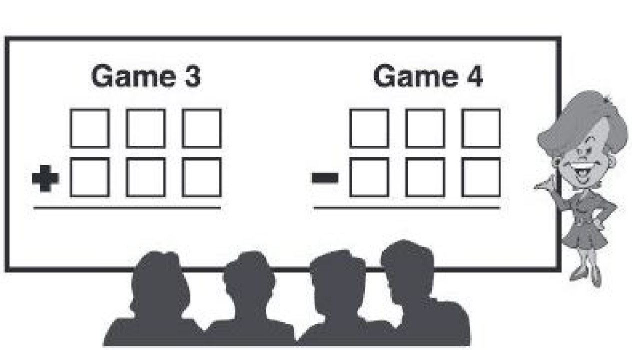
Games 1 and 2: In each game below, you need to place each of the six digits 4, 5, 6, 7, 8, 9, one in each box. In Game 1, the goal is to place the digits in such a way that you obtain the *greatest possible sum*. In Game 2, the goal is to place the digits in such a way that you obtain the *greatest possible difference*.



Digits		
4	5	6
7	8	9

Once you finish Game 1 and Game 2, take a moment to think about the strategies you used to obtain the *greatest* possible sum and difference. (These are strategies for *maximization*!) How will these strategies change if you wish to obtain the *least* possible sum or difference? (These would be strategies for *minimization*!)

Games 3 and 4: In each game below, you need to place each of the six digits 4, 5, 6, 7, 8, 9, one in each box. In Game 3, the goal is to place the digits in such a way that you obtain the *least possible sum*. In Game 4, the goal is to place the digits in such a way that the top number is larger than the bottom number and you obtain the *least possible difference*.



Digits		
4	5	6
7	8	9

More info:

Check out the CEMC at Home webpage on Monday, April 6 for the solutions to these games.

If you'd like more fun with digits and sums, try Problem 1 here: 2007/2008 Emmy Noether Circle 2

CEMC at Home

Grade 4/5/6 - Monday, March 30, 2020
This Game is Really Sum-thing! - Solution

Problem:

In Games 1 and 2, the goal is to maximize the sum and the difference of two three-digit numbers formed from the digits 4, 5, 6, 7, 8, and 9.

In Games 3 and 4, the goal is to minimize the sum and the difference, using the same six digits.

Solution:

GAME 1: The key strategy to obtain the greatest possible sum is to use the greatest digits (8 and 9) in the hundreds column, the next greatest digits (6 and 7) in the tens column, and the other two digits (4 and 5) in the ones column. It does not matter which of the two digits you place in the top or bottom box. Can you see why? The greatest possible sum is 1839, and here are a few ways to place the digits to get this sum.

$$\begin{array}{cccc} & \boxed{9} & \boxed{7} & \boxed{5} \\ + & \boxed{8} & \boxed{6} & \boxed{4} \\ \hline 1 & 8 & 3 & 9 \end{array} \qquad \begin{array}{cccc} & \boxed{9} & \boxed{6} & \boxed{4} \\ + & \boxed{8} & \boxed{7} & \boxed{5} \\ \hline 1 & 8 & 3 & 9 \end{array} \qquad \begin{array}{cccc} & \boxed{8} & \boxed{7} & \boxed{4} \\ + & \boxed{9} & \boxed{6} & \boxed{5} \\ \hline 1 & 8 & 3 & 9 \end{array} \qquad \begin{array}{cccc} & \boxed{8} & \boxed{6} & \boxed{5} \\ + & \boxed{9} & \boxed{7} & \boxed{4} \\ \hline 1 & 8 & 3 & 9 \end{array}$$

GAME 2: To obtain a large difference, we want to subtract a small number from a large number. The largest three-digit number we can form is 987 and the smallest three-digit number we can form (with these digits) is 456. To get the largest possible difference we subtract 456 from 987 to get 531.

$$\begin{array}{cccc} & \boxed{9} & \boxed{8} & \boxed{7} \\ - & \boxed{4} & \boxed{5} & \boxed{6} \\ \hline & 5 & 3 & 1 \end{array}$$

GAME 3: To obtain the least possible sum, we use the least two digits in the hundreds column, the next least digits in the tens column, and the remaining two digits in the ones column. The least possible sum is 1047, and here are a few ways to place the digits to get this sum.

$$\begin{array}{cccc} & \boxed{4} & \boxed{6} & \boxed{8} \\ + & \boxed{5} & \boxed{7} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array} \qquad \begin{array}{cccc} & \boxed{4} & \boxed{7} & \boxed{8} \\ + & \boxed{5} & \boxed{6} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array} \qquad \begin{array}{cccc} & \boxed{5} & \boxed{7} & \boxed{8} \\ + & \boxed{4} & \boxed{6} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array} \qquad \begin{array}{cccc} & \boxed{5} & \boxed{6} & \boxed{8} \\ + & \boxed{4} & \boxed{7} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array}$$

GAME 4: To obtain the least possible difference we want to make two numbers that are as close as possible in value. To start, we want to make sure the hundreds digits differ by one. This gives us a few possibilities, remembering that we want the top number to be larger than the bottom number:

$$\begin{array}{cccc} & \boxed{5} & \boxed{} & \boxed{} \\ - & \boxed{4} & \boxed{} & \boxed{} \\ \hline \end{array} \quad \begin{array}{cccc} & \boxed{6} & \boxed{} & \boxed{} \\ - & \boxed{5} & \boxed{} & \boxed{} \\ \hline \end{array} \quad \begin{array}{cccc} & \boxed{7} & \boxed{} & \boxed{} \\ - & \boxed{6} & \boxed{} & \boxed{} \\ \hline \end{array} \quad \begin{array}{cccc} & \boxed{8} & \boxed{} & \boxed{} \\ - & \boxed{7} & \boxed{} & \boxed{} \\ \hline \end{array} \quad \begin{array}{cccc} & \boxed{9} & \boxed{} & \boxed{} \\ - & \boxed{8} & \boxed{} & \boxed{} \\ \hline \end{array}$$

Some more work, possibly involving some trial and error, will hopefully lead you to find that the least possible difference is 47, which is found by placing the digits as shown below:

$$\begin{array}{cccc} & \boxed{7} & \boxed{4} & \boxed{5} \\ - & \boxed{6} & \boxed{9} & \boxed{8} \\ \hline & & 4 & 7 \end{array}$$

CEMC at Home

Grade 4/5/6 - Tuesday, March 31, 2020
Happy Campers

The 48 members of the Junior Division are camped in six tents along Golden Pond. Each tent is a different colour, to help them find their own "home away from home".

Your goal is to discover how many campers reside in each tent. Here are some clues.

1. The tent with the smallest group has 6 campers.
2. The orange tent has the largest group, with 10 campers.
3. The yellow and green tents are the only two tents with the same number of campers.
4. There are a total of 13 campers in the red and blue tents, one of which has the least number of campers.
5. The purple tent has 2 more campers than the blue tent.

Tent	**Number of Campers**
Orange	10
Red	
Blue	
Yellow	
Green	
Purple	

Hint 1: What do clues 1. and 4. tell you if you consider them together?

Hint 2: Don't forget that there are 48 campers in total!

Extension: If you'd like a bit more of this problem, try and figure out whether 11 campers could reside in the orange tent if clues 1., 3., 4, and 5. were still true.

More info:

Check out the CEMC at Home webpage on Tuesday, April 7 for the solution to Happy Campers.

CEMC at Home

Grade 4/5/6 - Tuesday, March 31, 2020
Happy Campers - Solution

Problem:

The 48 members of the Junior Division are camped in six tents along Golden Pond. Each tent is a different colour, to help them find their own "home away from home".

Your goal is to discover how many campers reside in each tent. Here are some clues.

1. The tent with the smallest group has 6 campers.
2. The orange tent has the largest group, with 10 campers.
3. The yellow and green tents are the only two tents with the same number of campers.
4. There are a total of 13 campers in the red and blue tents, one of which has the least number of campers.
5. The purple tent has 2 more campers than the blue tent.

Solution:

Clues 1 and 4 tell us that there are 6 campers in one of the red or blue tents, and 7 in the other. Since there must be at least 6 campers in each tent (clue 1), and 13 in these two tents in total (clue 4), these are the only possibilities. Clues 2 and 4 tell us that there are $10+13 = 23$ campers in total in the orange, red, and blue tents. Since there are 48 campers over all, we see there must be $48 - 23 = 25$ campers in total in the yellow, green, and purple tents.

Since there are either 6 or 7 campers in the blue tent, clue 5 tells us that there are either $6 + 2 = 8$ or $7 + 2 = 9$ campers in the purple tent. This means that there must be either $25 - 8 = 17$ or $25 - 9 = 16$ campers in total in the yellow and green tents.

Tent	Number of Campers
Orange	10
Red	6
Blue	7
Yellow	8
Green	8
Purple	9

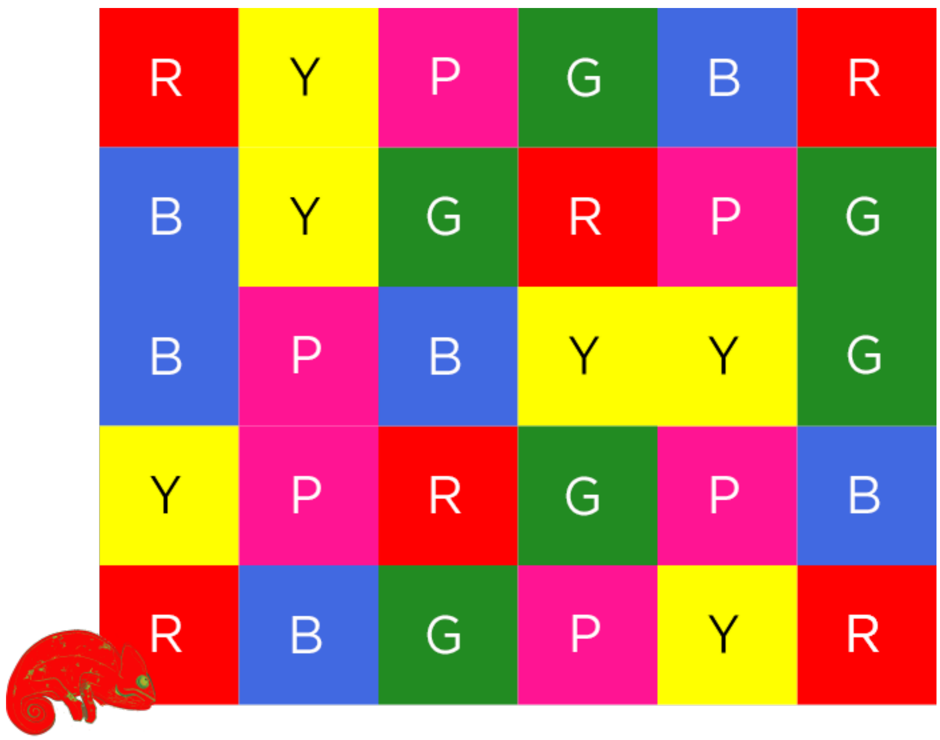
Clue 3 tells us that there are the same number of campers in each of the yellow and green tents which means the total number of campers in these tents must be an even number. This tells us that there must be 16 campers in total in these tents, rather than 17.

We can now be sure that there are 8 campers in each of the yellow and green tents, 9 campers in the purple tent, 7 campers in the blue tent, 6 campers in the red tent, and 10 campers in the orange tent.

CEMC at Home

Grade 4/5/6 - Wednesday, April 1, 2020
Chameleon Gridwalk

Chester the chameleon travels on the grid below, moving from cell to cell. As Chester passes through each cell, his colour changes to match the colour of the cell that he is currently on.



Question 1: Chester travels from the lower left corner of the grid to the upper right corner of the grid, moving between adjacent cells either horizontally or vertically. There are many possible paths he could take, and these will result in him taking on different sequences of colours while he travels. What is the minimum number of different colours that Chester could take on during his trip?

Question 2: Chester travels from the lower left corner of the grid to the upper right corner of the grid, now moving between adjacent cells horizontally, vertically, *or diagonally*. What is the minimum number of different colours that Chester could take on during his trip?

More info:

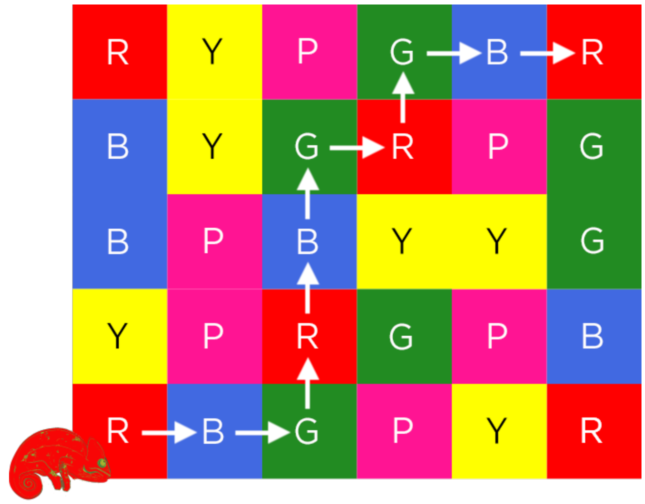
Check the CEMC at Home webpage on Wednesday, April 8 for the solution to Chameleon Gridwalk.

CEMC at Home

Grade 4/5/6 - Wednesday, April 1, 2020
Chameleon Gridwalk - Solution

Solution to Question 1

The minimum number of different colours that Chester could take on while travelling from the lower left corner to the upper right corner, using only horizontal or vertical moves, is 3. The image displayed on the right shows one route that has Chester taking on exactly 3 different colours: red, blue, and green.

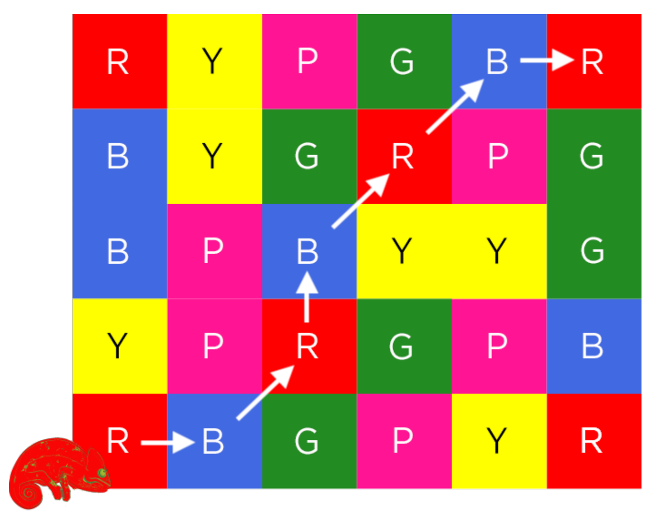


How can we be sure that 3 is the minimum number of different colours that Chester could possibly take on? To show that 3 is the minimum, we have to explain why it is not possible for Chester to walk a path that will have him take on fewer than 3 colours (which means either 1 or 2 colours).

Since Chester's colour is red to begin with, and the next cell he moves to must either be yellow or blue, there is no way that Chester can make it to the right corner without taking on at least 2 colours. If Chester decides to move to the yellow cell above, then we see that there is no path that he can follow from there that only uses red and yellow cells. (The only way to stay on red and yellow cells on his next move would be to move back down to where he started.) So any path will have Chester taking on at least one more colour, making the total number at least 3. If Chester instead decides to move to the blue cell to the right, then we have a similar situation. Again, there is no way to continue from there without picking up a third colour along the way. No matter which path Chester takes from the bottom left corner, he will have to take on at least 3 colours along the the way to the top red corner.

Solution to Question 2

The minimum number of different colours that Chester could take on while travelling from the lower left corner to the upper right corner, using horizontal, vertical, *or diagonal* moves, is 2. The image displayed on the right shows one route that has Chester taking on exactly 2 different colours: red and blue.



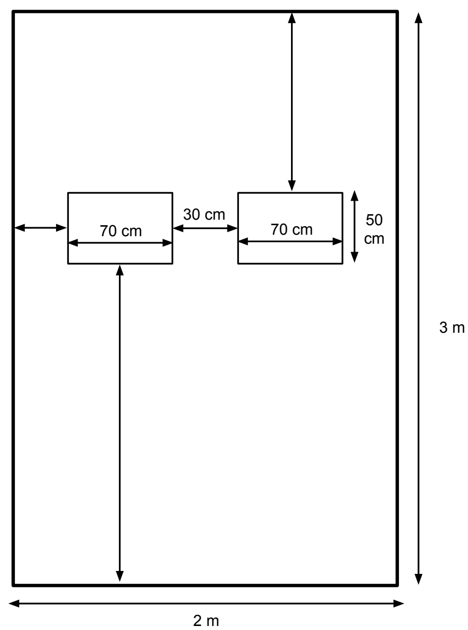
It is not possible for Chester to make the trip taking on fewer than 2 colours (which means only 1 colour). Chester's colour is red to begin with, and he will have to take on a second colour (yellow, pink, or blue) as soon as he makes his first move toward the top right corner.

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, April 2, 2020
Picture Perfect

Two pictures, each 70 cm wide and 50 cm tall, are hung on a rectangular wall so that the top of each picture is 2 m above the floor. The wall is 3 m tall and 2 m wide. The pictures are to be hung so that the horizontal distance from the outside edge of each picture to the nearest wall is the same. Two ways of arranging the pictures are being considered.

a) If the pictures are 30 cm apart on the wall, complete the labels on the following diagram showing where the pictures will be placed on the wall. Note that the diagram is not drawn to scale.



b) If the horizontal distance from the outside edge of each picture to the nearest wall is to be equal to the distance between the two pictures, how far apart should the pictures be?

More Info:

Check the CEMC at Home webpage on Thursday, April 9 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 9.

This CEMC at Home resource is the current grade 3/4 problem from Problem of the Week (POTW). This problem was developed for students in grades 3 and 4, but is also appropriate for students in grades 5 and 6. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 5/6 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem A and Solution
Picture Perfect

Problem

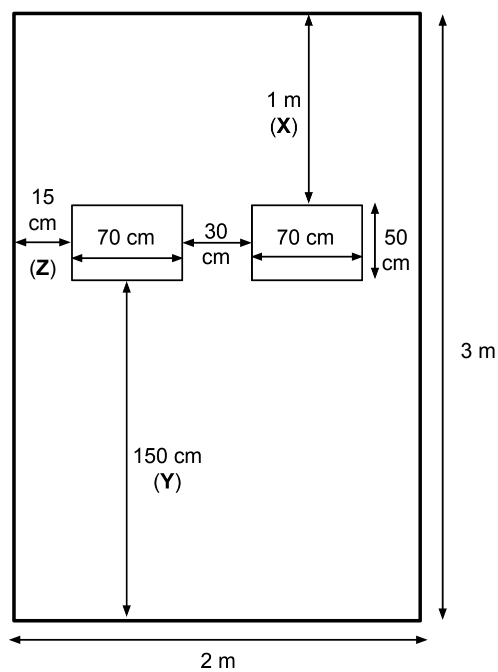
Two pictures, each 70 cm wide and 50 cm tall, are hung on a rectangular wall so that the top of each picture is 2 m above the floor. The wall is 3 m tall and 2 m wide. The pictures are to be hung so that the horizontal distance from the outside edge of each picture to the nearest wall is the same. Two ways of arranging the pictures are being considered.

a) If the pictures are 30 cm apart on the wall, complete the labels on the following diagram showing where the pictures will be placed on the wall. Note that the diagram is not drawn to scale.

b) If the horizontal distance from the outside edge of each picture to the nearest wall is to be equal to the distance between the two pictures, how far apart should the pictures be?

Solution

A) Looking at the original diagram, there are three values missing. Here is the completed diagram:



Here is how we can calculate the missing values:

- (Label **X**) From the problem description we know that the top of each picture is 2 m above the floor. Since the wall is 3 m, then the distance from the ceiling to the top of the picture is $3 - 2 = 1$ m.
- (Label **Y** in the solution) If the distance from the top of the pictures to the floor is 2 m, this is equal to 200 cm. Since the height of each picture is 50 cm, then the distance from the floor to the bottom of the picture is $200 - 50 = 150$ cm.
- (Label **Z**) If the pictures are centred on the wall, then the space on the left side must match the space on the right side. Horizontally, the width of the pictures and the space between them is $70 + 30 + 70 = 170$ cm. Since the wall is 2 m wide and this is equal to 200 cm, the leftover space is $200 - 170 = 30$ cm. If we want the space to be equal on either side, then there must be $30 \div 2 = 15$ cm from the left edge of the picture on the the left to the edge of the wall.

B) The widths of the pictures take up $70 + 70 = 140$ cm of space on the wall. This leaves $200 - 140 = 60$ cm of horizontal space. If we want to equally distribute that space to the left, centre, and right of the pictures we need $60 \div 3 = 20$ cm of space between the two pictures.

Teacher's Notes

Creating and/or reading a diagram properly is a fundamental skill in mathematics. Although a diagram drawn to scale is helpful (or possibly necessary) in some cases, most of the time it is more important that the diagram includes clearly labelled, critical information, and it is unnecessary to have precise measurements. Identifying what is important information is also a useful skill. However, it is a good idea to start a diagram with **any** known information. It is possible you include values that end up being superfluous to the problem, but it is better to have access to extra information than be missing important details.

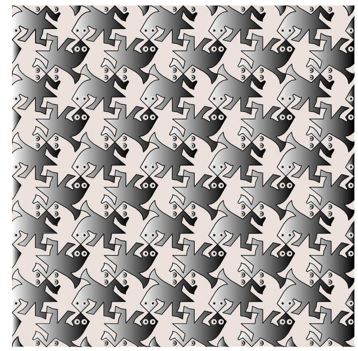
Once you have the initial information labelled, you can infer other values using deductive logic. Logical thinking and *formal logic* are important in the study of mathematics and computer science. In these contexts, we look for precise ways to state our argument that will justify conclusions. A good diagram can be very helpful in this pursuit.

CEMC at Home

Grade 4/5/6 - Friday, April 3, 2020
Dotty Tessellations

A tessellation (or tiling) is an arrangement of one or more shapes in a repeated pattern without overlaps or gaps. Tessellations occur in nature, as illustrated by the honeycomb of bees (a tessellation of hexagons), in masonry (in the walls and floors of buildings), and in artwork.





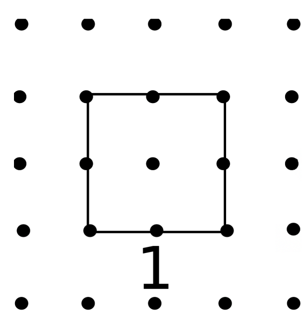
In this activity, we will explore how to make our own tessellations using dot paper.

You Will Need:

- Dot paper (which can be found on the last page)
- A pencil
- An eraser
- A ruler
- Something to colour with (coloured pencils, paints, etc.)

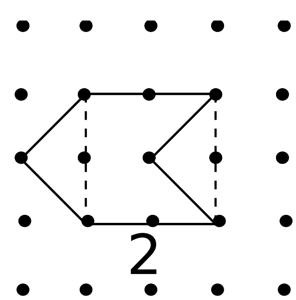
Step 1: Start by drawing a single square, rectangle, or parallelogram on the dot paper using a pencil.

Make sure that it is at least two units long and two units wide, but not so large that it covers too much of the dot paper. An example is shown in image 1.



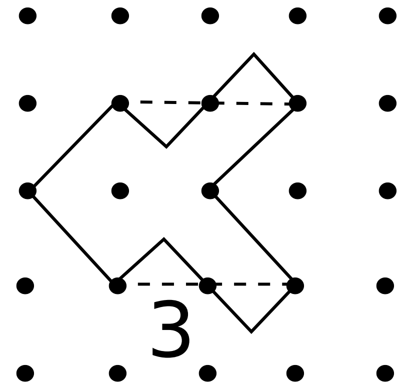
Step 2: "Add" a simple shape to one of the sides of your shape from from Step 1, and then "remove" an identical shape from the opposite side. Erase any lines that are no longer used to form the perimeter of the shape.

In image 2, a triangle is added to the left side of the square by drawing two diagonal lines. An identical triangle is removed from the right side of the square, also by drawing two diagonal lines. The dashed lines in the image show the two vertical edges of the original shape that can now be erased as they are no longer on the perimeter of the shape.



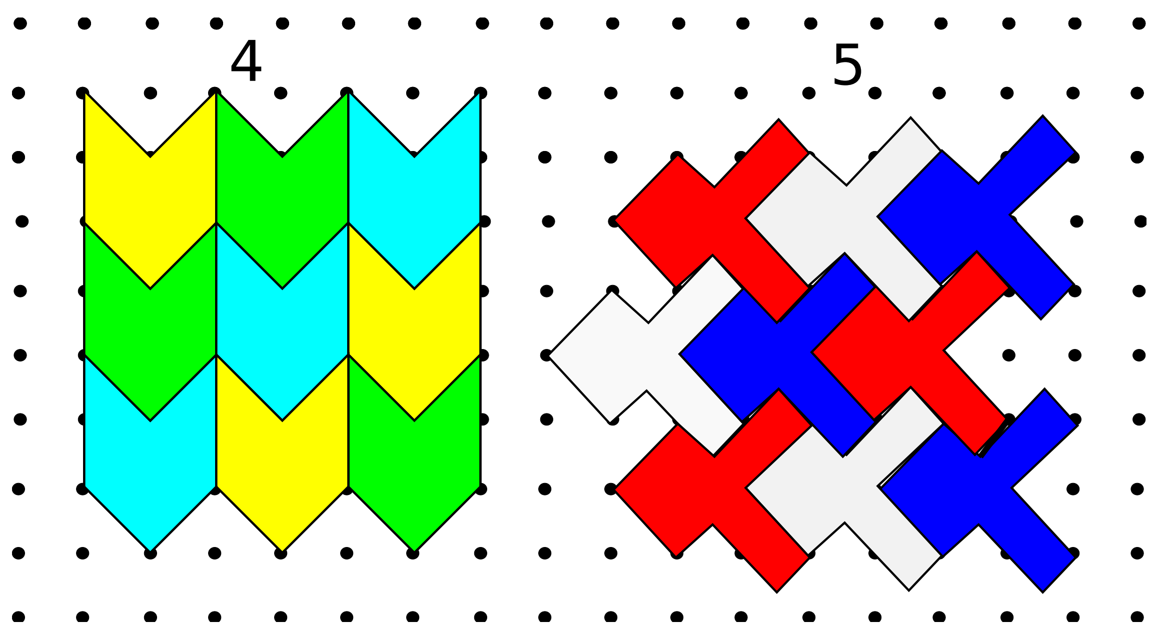
Step 3 (Optional): Go further, if you wish, by now making alterations to the other pair of opposite sides. You can try and use a similar idea as Step 2 (by adding and removing an identical shape) or explore a more complicated alteration.

In image 3, we have altered the shape using four identical triangles, with two triangles being "added" and two triangles being "removed". It turns out that this final shape can be used in a tessellation but it might not be immediately clear how to do it! If you choose to do Step 3, and use a more imaginative alteration, then you may want to think about exactly what changes you can and can't make if your goal is to produce a shape that can be used in a tessellation.



Step 4: Draw identical copies of the final shape created in either Step 2 or Step 3 throughout the dot paper. Make sure that your shapes do not overlap and there are no gaps between your shapes. Once you have your tessellation, colour it as imaginatively as you can!

Image 4 shows the final shape from Step 2 tessellated, and image 5 shows the final shape from Step 3 tessellated.



On the next page, you will find some dot paper to work on. You will also find some shapes that have been created for you and can be used in a tessellation. It is easiest if you make your first few shapes by only making alterations that involve straight lines between two dots, but once you get the hang of it, try and make some more complex shapes. Can you make shapes with a mix of straight edges and curved edges that can be used in a tessellation?

More info:

Many works of art are inspired by mathematics! To see some exceptional tessellations, check out the work of the world famous Dutch graphic artist M. C. Escher.

If you would like to explore tessellations further, check out this Math Circles lesson.

Dot Paper for Tessellations