CEMC at Home

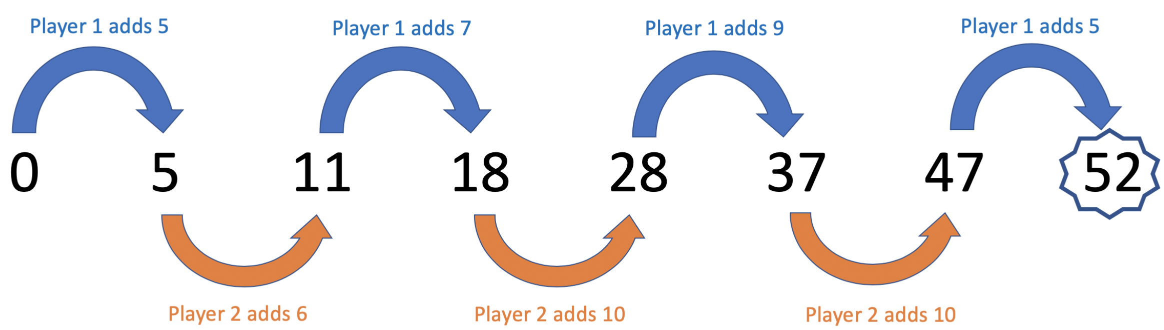
Grade 9/10 - Monday, March 23, 2020
Addition Magician

You Will Need:

- Two players
- A piece of paper and a pencil

How to Play:

1. Start with a total of 0 (on the paper).
2. The two players will alternate turns changing the total. Decide which player will go first.
3. On your turn, you can add 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 to the total.
   Numbers may be used more than once throughout the game.
4. The player who brings the total to 52 wins the game!



Play this game a number of times. Can you come up with a strategy that will allow you to win most of the time? What about every time? Is it better to go first or second or does this not matter?

Variations:

- How would your strategy change if the game was played to 55 instead of 52?
- What would your strategy be if the players are allowed to use the numbers from 1 to 15 but must play to 300 (instead of to 52)?
- What would your strategy be if the players are allowed to use the numbers from 1 to n, with $n > 1$, but must play to a total of T where T is some positive integer larger than $3n$?

More Info:

Check out the CEMC at Home webpage on Monday, March 30 for a discussion of a strategy for this game. We encourage you to discuss your ideas online using any forum you are comfortable with.

We sometimes put games on our math contests! Check out Question 2 on the 2003 Hypatia Contest for another game where we are looking for a strategy.

CEMC at Home
Grade 9/10 - Monday, March 30, 2020
Addition Magician - Solution

The Strategy

You likely noticed that the player that brings the total to 42, 43, 44, 45, 46, 47, 48, 49, 50, or 51 generally loses the game on the next turn. The next player can reach 52 by adding 10, 9, 8, 7, 6, 5, 4, 3, 2, or 1, respectively, and so will win the game as long as they choose the correct number. Therefore, the player that brings the total to 41 is guaranteed to be able to bring the total to 52 on their next turn.

Using similar reasoning, the player that brings the total to 30 is guaranteed to be able to bring the total to 41 on their next turn. Also the player that brings the total to 19 is guaranteed to be able to bring the total to 30 on their next turn, and the player that brings the total to 8 is guaranteed to be able to bring the total to 19 on their next turn.

Putting all of this together, we see that there is a strategy that guarantees a win for the first player, regardless of what the second player does. (This is what is called a *winning strategy* for the game.) The first player starts by adding 8 to the total of 0. In the turns that follow, the first player will add whatever is needed to bring the totals to 19, 30, 41, and then 52. Our analysis above explains why this is always possible within the rules of the game.

Notice that the target numbers 8, 19, 30, 41, and 52 all differ by 11. We can describe the strategy more concisely as follows: Go first and start by adding 8. For all turns that follow, if the other player adds n, then you add $11 - n$.

The Variations

- In the first variation, the winning total is 55 which is a multiple of 11. A winning strategy in this variation is to go second and, on each turn, if the other player adds n, then add $11-n$, so that the total changes by 11 in total over the two turns. For example, if they add 4 then you add 7. This way the second player will bring the total to 11, 22, 33, 44, and then 55 to win.

- In the second variation, the player that brings the total to a number between 285 and 299 inclusive will generally lose the game since the next player can reach 300. Since the allowable numbers in this variation are 1 to 15, we focus on multiples of 16. Since 300 is 12 more than a multiple of 16, the winning strategy is to go first and start with 12. Then, if the other player adds n, you add $16 - n$, so that the total changes by 16 over the two turns. For example, if they add 7, you add 9. The first player can always bring the total to the next number that is 12 more than a multiple of 16, eventually reaching 300 to win.

- In the third variation, we need to consider different cases for T:

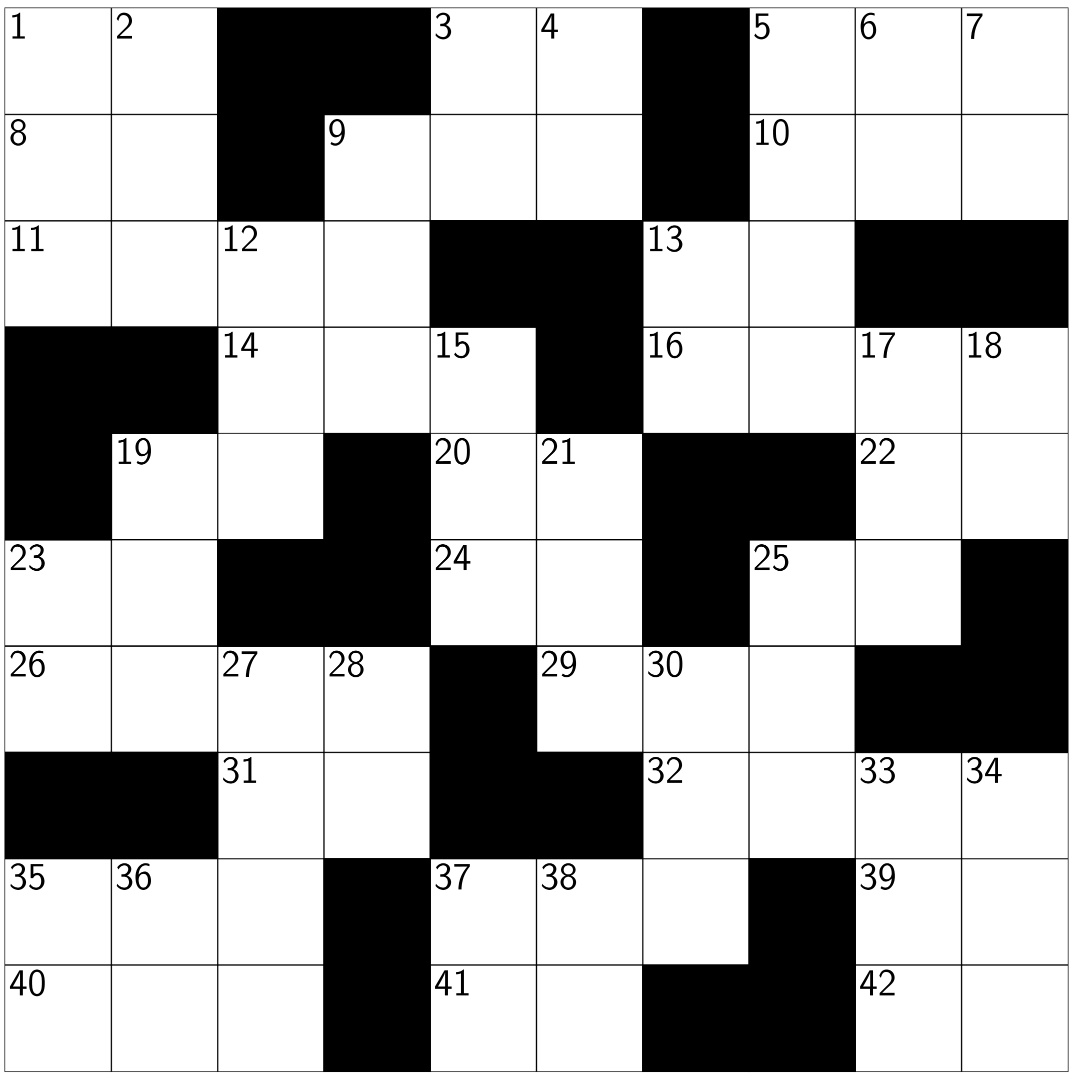
  If T is a multiple of $n + 1$, then go second. Whatever number the other player chooses, you choose the number that totals $n + 1$ when summed with their chosen number. This means you will bring the total to each multiple of $n+1$, in turn, eventually reaching T. (The first variation above is an instance of this case.)

  If T is not a multiple of $n + 1$, then go first. Find the remainder when T is divided by $n + 1$ and start with this number. Whatever number the other player chooses, choose the number that totals $n + 1$ when summed with their chosen number. Eventually you will bring the total to T. (The second variation above is an instance of this case.)

CEMC at Home

Grade 9/10 - Tuesday, March 24, 2020
Crossnumber Puzzle

Use the clues on the next page to complete the crossnumber puzzle below. Each square of the grid will contain exactly one digit. Notice that some answers can be found using only the given clue, and some need the answers from other clues.
You may need to do a bit of research before you can figure out some of the clues!



More Info:

Check out the CEMC at Home webpage on Tuesday, March 31 for a solution to the Crossnumber Puzzle. We encourage you to spend some time discussing and investigating the references in this puzzle that are new to you.

Across

1. The sum of the squares of the first three primes.
3. The number of years the Grinch put up with the Whos' Christmas cheer.
5. A perfect cube.
8. With 31 ACROSS , a factor pair of 832.
9. 6!
10. A Mersenne prime.
11. The Hardy-Ramanujan number.
13. 25% of 300.
14. How much you spent if you received $4.17 in change from $10.
16. The prime factorization of 140.
19. A Fibonacci number.
20. A multiple of 11.
22. The smallest number in this grid.
23. The third side of a right triangle with hypotenuse 18 DOWN and other side 39 ACROSS .
24. A triangular number.
25. The number of bits in 5 bytes.
26. The number 9 in binary.
29. A palindrome.
31. The freezing point of water in degrees Fahrenheit.
32. MMDXIII.
35. 32 ACROSS - 16 ACROSS .
37. The balance after investing $100 at 3% simple interest for 8 years.
39. The same digit repeated.
40. The 11th, 12th, and 13th digits of pi.
41. ASCII value of lowercase b.
42. Atomic number of silver.

Down

1. Consecutive digits in decreasing order.
2. 1000 less than the year of Canada's Confederation.
3. The number of clues in this puzzle.
4. The number of edges in an icosahedron.
5. The digits of 32 ACROSS in reverse order.
6. The least common multiple of 6 and 7.
7. The number 55 in hexadecimal.
9. The last digit is the average of the first two digits.
12. The middle digit is the sum of the other two digits.
13. The smallest Achilles number.
15. The number of legs on a farm that has 24 chickens, 18 pigs, and 33 spiders.
17. The number of years in 5 centuries.
18. Sheldon Cooper's favourite number.
19. The sum of the interior angles of a triangle.
21. 9 ACROSS + 29 ACROSS .
23. The number of Mozart's last symphony.
25. The total value (in cents) of 9 quarters, 12 dimes, and 14 nickels.
27. Consecutive multiples of 3.
28. The number of sides in a dodecagon.
30. A perfect square.
33. Blaise Pascal's year of birth subtracted from Carl Gauss' year of birth.
34. Consecutive odd numbers.
35. The greatest common divisor of 13 ACROSS and 17 DOWN .
36. The number of minutes in 3480 seconds.
37. The sum of the digits of 11 ACROSS .
38. The number of days in 2 fortnights.

CEMC at Home

Grade 9/10 - Tuesday, March 24, 2020
Crossnumber Puzzle - Solution

3	8			5	3		3	4	3
2	6		7	2	0		1	2	7
1	7	2	9			7	5		
		5	8	3		2	2	5	7
	1	3		8	8			0	3
4	8			4	5		4	0	
1	0	0	1		1	3	1		
		3	2			2	5	1	3
2	5	6		1	2	4		5	5
5	8	9		9	8			4	7

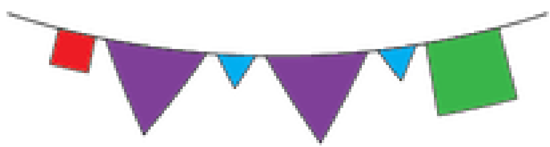
CEMC at Home

Grade 9/10 - Wednesday, March 25, 2020
Build A Banner

A computer program can be used to draw banners consisting of squares and triangles. The program makes use of the following five instructions:

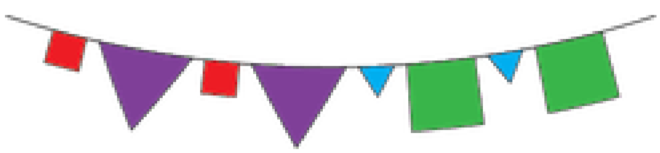
Instruction	Meaning
`S`	Draw a large square
`s`	Draw a small square
`T`	Draw a large triangle
`t`	Draw a small triangle
`N[I]`	Repeat the instructions, `I`, exactly `N` times

For example, the program `s 2[T t] S` draws the following banner:

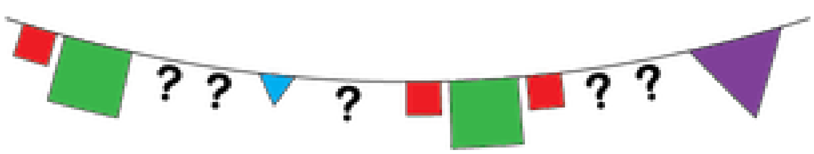


Questions:

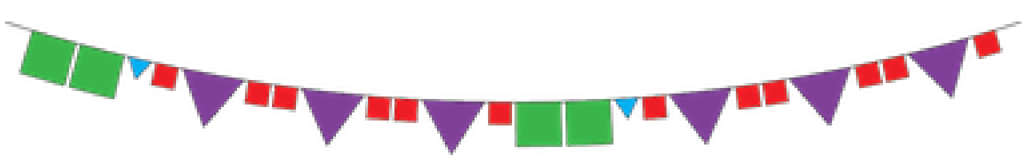
1. Given the program `t 4[s] T 3[t S]`, draw the corresponding banner.
2. Create two different programs that will draw the following banner:



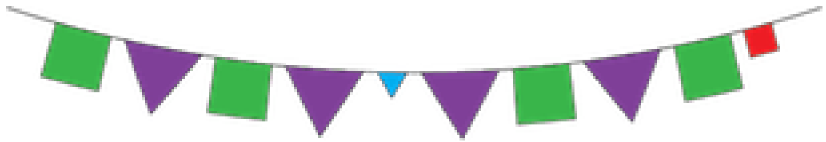
3. Given the program `2[2[s S] t T]`, draw the missing shapes in the following banner:



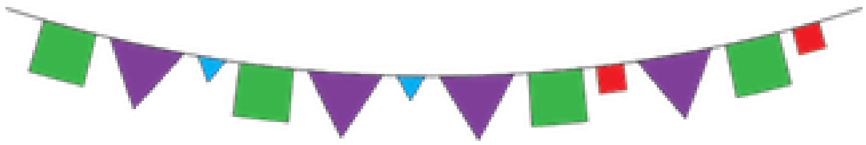
4. Given the incomplete program `?[2[?] t ?[s T ?]]`, complete the missing instructions in order to draw the following banner:



5. Suppose you want to draw the following banner:



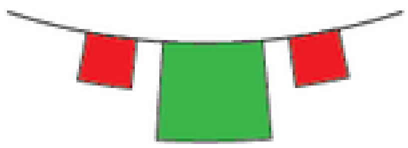
You create the program `2[S T t] 2[T S s]` which incorrectly draws this banner:



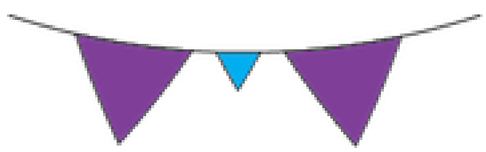
What are the mistakes in your program?

6. A new instruction named **if** is now available to you. The instruction `(a:b/c)` means that *if* the previous shape drawn was `a`, then the next shape drawn is `b`. *If* the previous shape drawn was **not** `a`, then the next shape drawn is `c`.

For example, the program `s (s:S/t) (t:T/s)` draws the following banner:



For each program in parts (a) to (f), decide whether or not it will draw the following banner:



(a) `2[T (t:T/t)]`

(b) `T (T:t/s) (t:T/S)`

(c) `T 2[(t:T/t)]`

(d) `t (t:T/s) (s:S/t)`

(e) `T (T:t/S) (S:s/T)`

(f) `3[(T:t/T)]`

7. Try creating your own new instructions. Perhaps add new shapes, or new capabilities such as chaining shapes vertically. Swap programs with a friend or family member and try to draw each other's banners.

More Info:

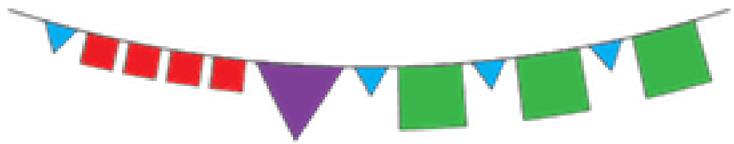
Check out the CEMC at Home webpage on Wednesday, April 1 for the solutions to these questions.

This task exercises your computational thinking muscles! For more information on how this task relates to computer science, check out Chain on the 2016 Beaver Computing Challenge.

CEMC at Home

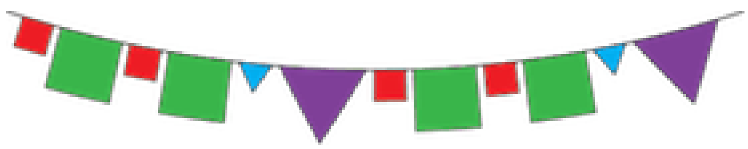
Grade 9/10 - Wednesday, March 25, 2020
Build a Banner - Solution

1. `t 4[s] T 3[t S]`



2. One possible program is `s T s T t S t S`.
   Another possible program is `2[s T] 2[t S]`.

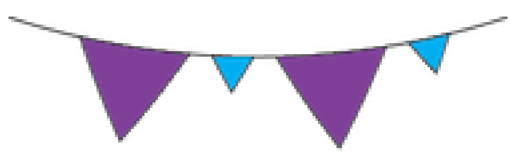
3. `2[2[s S] t T]`



4. `2[2[S] t 3[s T s]]`

5. The small triangle instruction, `t`, and the small square instruction, `s`, should be moved outside of their repeating blocks. The correct program is `2[S T] t 2[T S] s`.

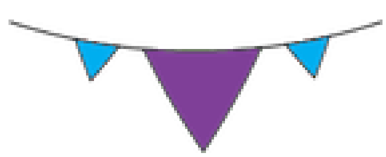
6. (a) No. The program draws this banner:



   (b) Yes

   (c) Yes

   (d) No. The program draws this banner:



   (e) Yes

   (f) No. The program is invalid and will not draw any banner. The first instruction is the new **if** instruction, but there is no previously drawn shape that it can use in order to make a decision about what to draw next.

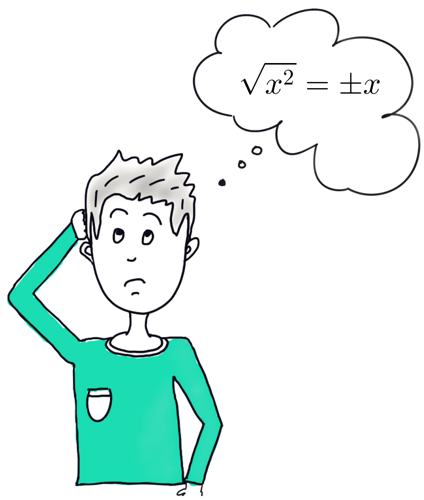
CEMC at Home features Problem of the Week

Grade 9/10 - Thursday, March 26, 2020
What's that Total?

We know the following about the numbers a, b and c:

$$(a+b)^2 = 9,\ (b+c)^2 = 25, \text{ and } (a+c)^2 = 81.$$

If $a+b+c \geq 1$, determine the **number** of possible values for $a+b+c$.

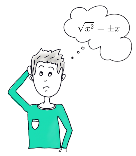


More Info:

Check the CEMC at Home webpage on Thursday, April 2 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 2.

This CEMC at Home resource is the current grade 9/10 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php



Problem of the Week
Problem D and Solution
What's that Total?

Problem

We know the following about the numbers a, b and c:

$$(a+b)^2 = 9,\ (b+c)^2 = 25,\text{ and } (a+c)^2 = 81.$$

If $a+b+c \geq 1$, determine the **number** of possible values for $a+b+c$.

Solution

Since $(a+b)^2 = 9$, $a+b = \pm 3$. Since $(b+c)^2 = 25$, $b+c = \pm 5$. And since $(a+c)^2 = 81$, $a+c = \pm 9$.

Now $(a+b)+(b+c)+(a+c) = 2a+2b+2c = 2(a+b+c)$. This quantity is two times the value of the quantity we are looking for.

The following chart summarizes all possible combinations of values for $a+b$, $b+c$, and $a+c$ and the resulting values of $2a+2b+2c$ and $a+b+c$. The final column of the chart states a yes or no answer to whether the value of $a+b+c$ is ≥ 1.

$a+b$	$b+c$	$a+c$	$2a+2b+2c$	$a+b+c$	$a+b+c \geq 1$? (yes / no)
3	5	9	17	8.5	yes
3	5	−9	−1	−0.5	no
3	−5	9	7	3.5	yes
3	−5	−9	−11	−5.5	no
−3	5	9	11	5.5	yes
−3	5	−9	−7	−3.5	no
−3	−5	9	1	0.5	no
−3	−5	−9	−17	−8.5	no

Therefore, there are three possible values of $a+b+c$ such that $a+b+c \geq 1$.

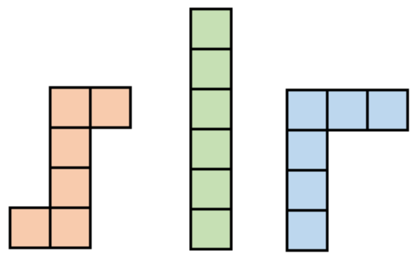
It should be noted that for each of the three possibilities, values for a, b, and c which produce each value can be determined but that was not the question asked.

CEMC at Home

Grade 9/10 - Friday, March 27, 2020

Hexominoes

A **hexomino** is a geometric shape composed of six equal-sized squares which are connected at one or more edges. Below are a few examples of hexominoes. Try drawing a few others yourself.

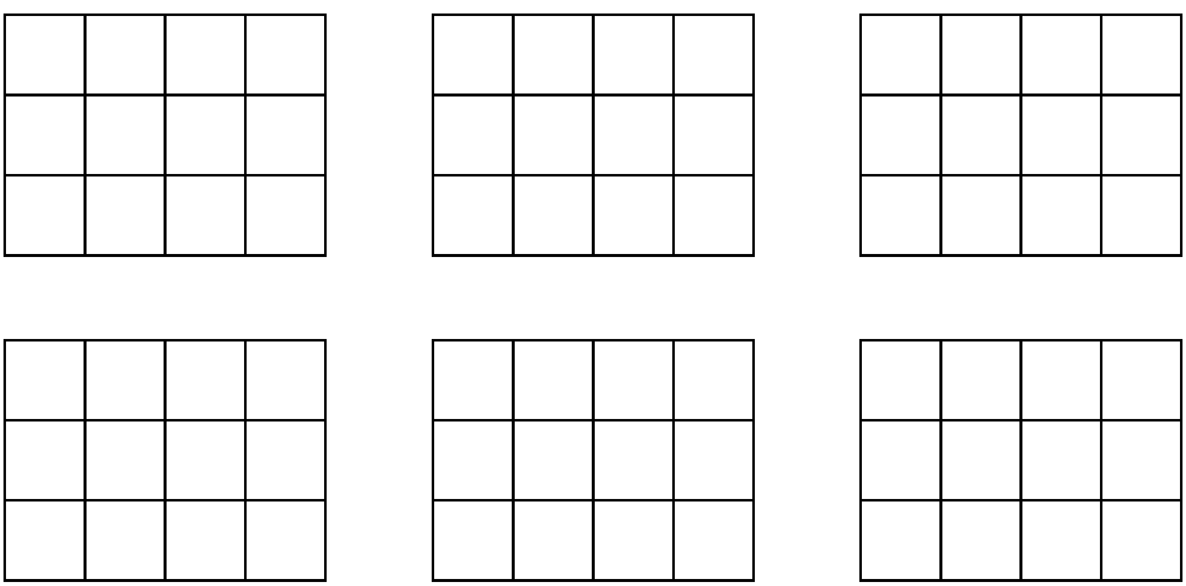


On the last page you will find 35 hexominoes drawn and numbered. Every other hexomino can be obtained by translating, rotating, or reflecting one of these 35 hexominoes, possibly using a combination of these transformations. In the following activities, you will be free to translate, rotate, and reflect the 35 shapes as needed to complete the tasks. The collection of shapes that we will be working with are sometimes called the 35 *free* hexominoes.

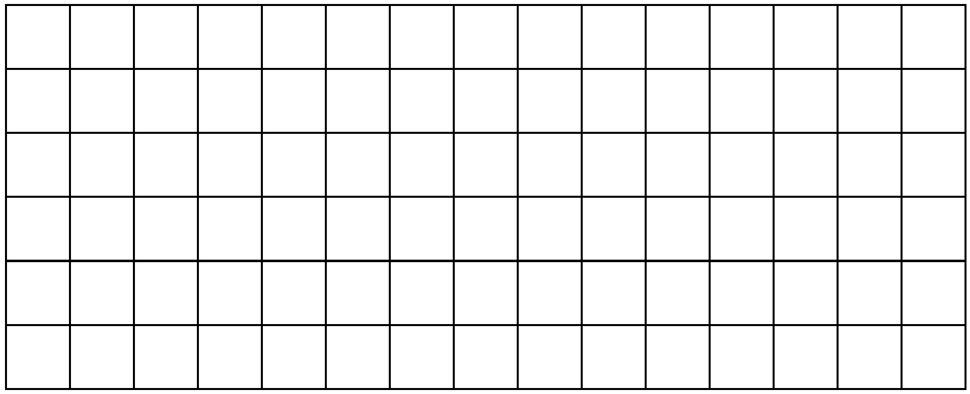
Questions:

1. Which of the 35 hexominoes represent the net of a cube? In other words, which hexominoes can be folded up into a cube? To help visualize this, you can print the hexominoes onto paper, cut them out, and fold them. Magnetic tiles would also work really well.

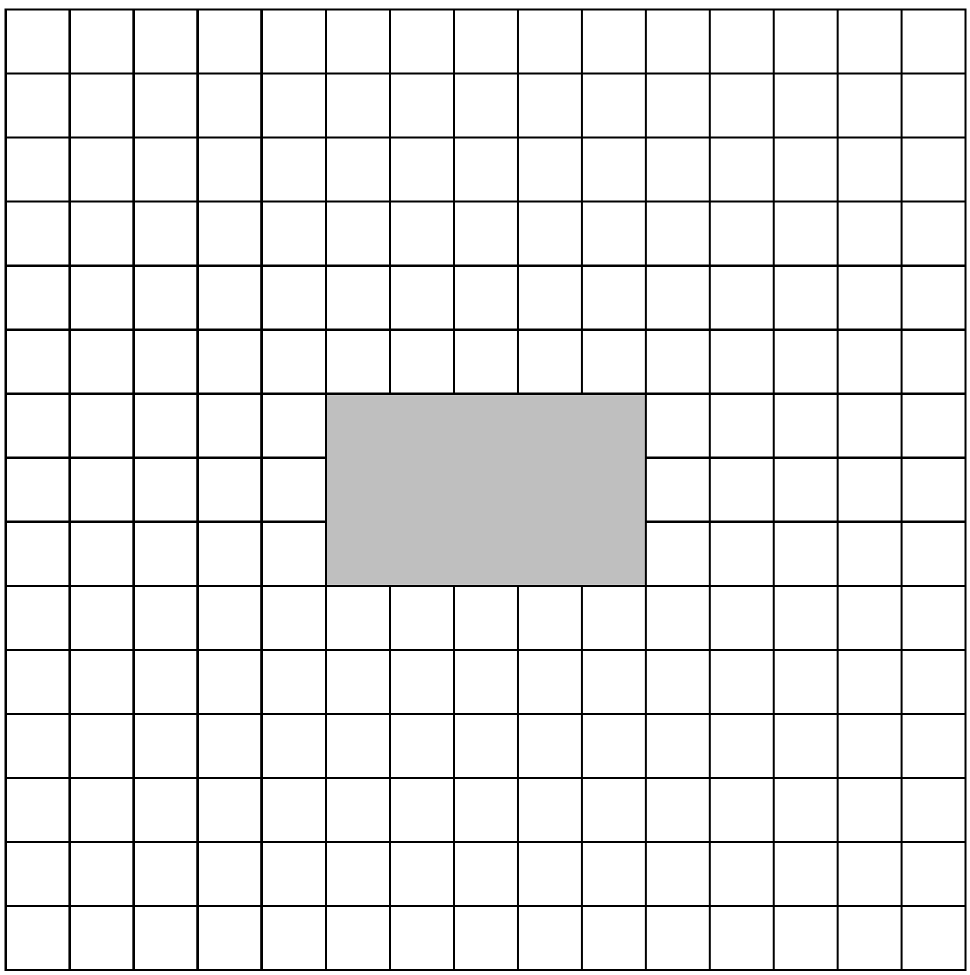
2. Cover a 3×4 rectangle using two copies of any single hexomino. How many different solutions can you come up with? (Remember you are free to translate, rotate, and/or reflect the shape.)

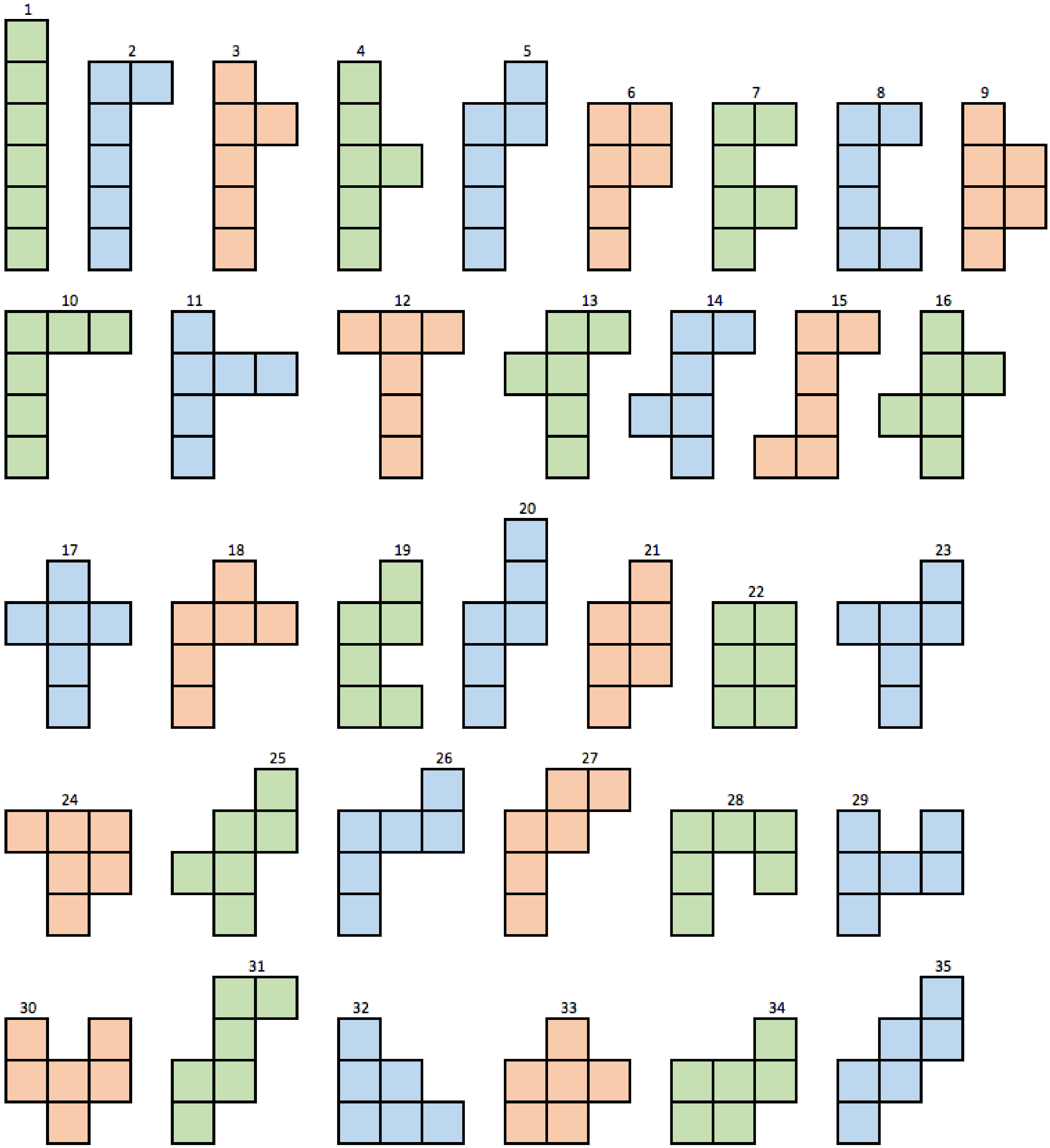


3. Cover the 6×15 rectangle below using any combination of hexominoes. Can you do so using each hexomino at most once?



4. Take a 15×15 square and cut out a 3×5 rectangle from the middle. Cover the remaining white squares using each of the 35 hexominoes exactly once.





More Info:

Check out the CEMC at Home webpage on Friday, April 3 for the solution to Hexominoes.

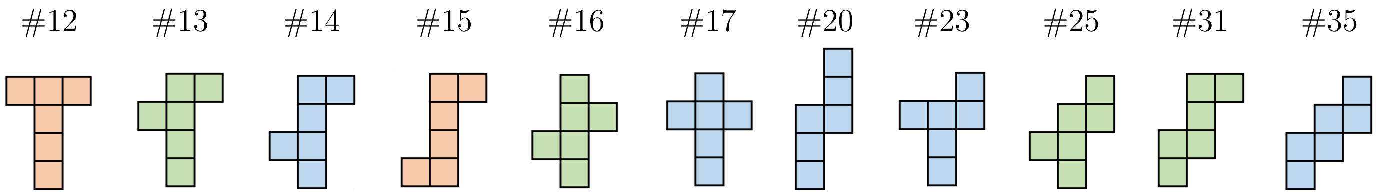
When four equal-sized squares are used instead of six, the geometric shapes are called **tetrominoes**. Tetrominoes are the building blocks of the original *Tetris* game.

CEMC at Home

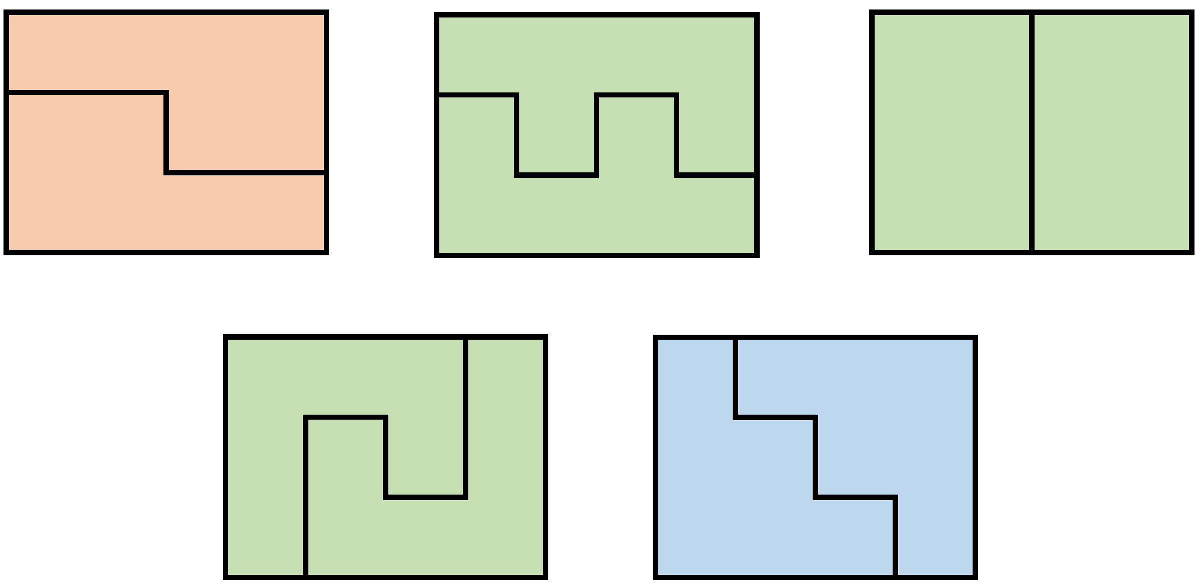
Grade 9/10 - Friday, March 27, 2020
Hexominoes - Solution

Answers:

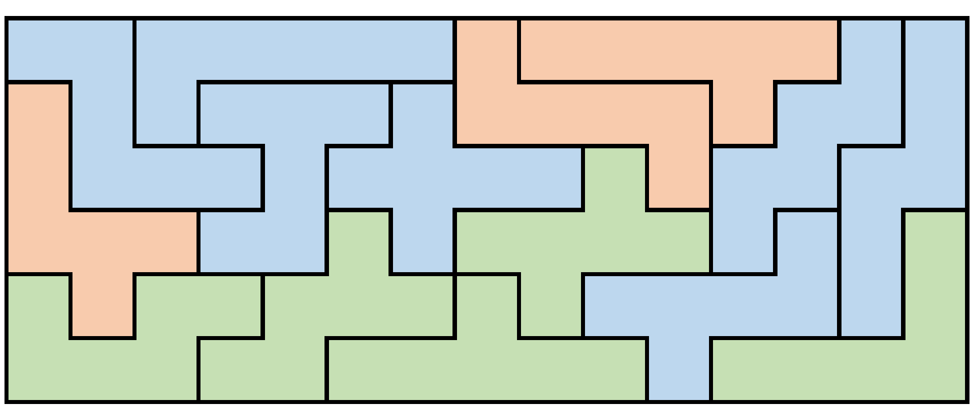
1. The following 11 hexominoes represent the net of a cube.



2. There are five different solutions using hexominoes #6, #7, #22, #28, and #32.



3. Here is one possible solution.



4. Here is one possible solution.