CEMC at Home

Grade 7/8 - Monday, March 23, 2020
Multiplication War

You Will Need:

- One full deck of cards
- Two players

Before you Start:

In this game, each card is assigned a value as follows:

- Aces are worth 1, Jacks are worth 11, Queens are worth 12, and Kings are worth 13
- All other cards are assigned the value of the number on the card
- Black cards are all positive numbers
- Red cards are all negative numbers

How to Play:

1. Deal out all cards evenly between two players. Each player puts their cards face down in a pile in front of them.
2. Each player flips over their top two cards at the same time and multiplies them together. *Remember that a negative times a negative equals a positive, and that* -3 *is greater than* -8.
3. The player whose cards have the highest product wins all four cards.
4. If the cards have the same product, then the players each flip over two more cards and calculate their product. This process is repeated until one player has a higher product than the other. This player then wins all face-up cards.
5. The player who ends up with all the cards (or the most cards after a set amount of time) wins.

Variations:

- For a warm-up, try playing without any negative numbers.
- For a challenge, come up with a harder version of this game. You could use fractions, decimals, exponents, etc. We encourage you to share your ideas online using any forum you are comfortable with.

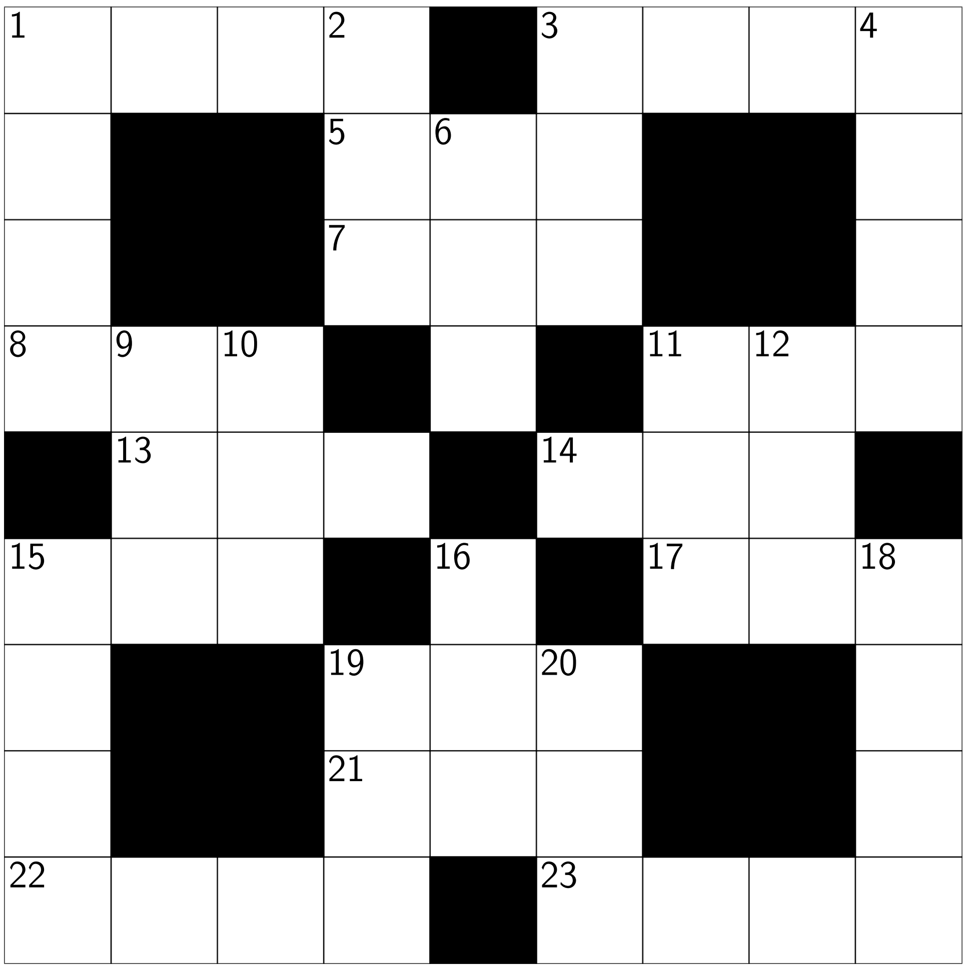
More Info:

Need a reminder on how to multiply positive and negative numbers? Check out this lesson on the CEMC Courseware.

CEMC at Home

Grade 7/8 - Tuesday, March 24, 2020
Crossnumber Puzzle

Use the clues on the next page to complete the crossnumber puzzle below. Each square of the grid will contain exactly one digit. Notice that some answers can be found using only the given clue, and some need the answers from other clues.



More Info:

Check out the CEMC at Home webpage on Wednesday, March 25 for a solution to the Crossnumber Puzzle.

Across

1. The result of $\left(1000 - \boxed{\text{2 DOWN}}\right) \times 103$.
3. A number whose digits are all perfect squares and add to 31.
5. The value of $11 \times 12 + 13$.
7. The number of centimetres in 7.29 metres.
8. A telephone area code in Waterloo, Ontario, Canada.
11. A number whose digits add to a multiple of 9.
13. A rearrangement of the digits in the quotient when $\boxed{\text{10 DOWN}}$ is divided by 3.
14. A prime number less than 200.
15. The sum of $\boxed{\text{9 DOWN}}$ and $\boxed{\text{7 ACROSS}}$.
17. The result when $\boxed{\text{14 ACROSS}}$ is multiplied by 8.
19. The number of days in a leap year.
21. The total value (in cents) of 5 quarters, 3 dimes, and 2 nickels.
22. A number whose digits are all perfect squares and add to 15.
23. The perimeter of a square with side length $\boxed{\text{20 DOWN}}$ units.

Down

1. A multiple of 5 between $\boxed{\text{15 DOWN}}$ and $\boxed{\text{18 DOWN}}$.
2. The result of $\boxed{\text{5 ACROSS}}$ multiplied by 6, then added to 47.
3. A number whose first two digits add to its third digit.
4. The least common multiple of $\boxed{\text{11 ACROSS}}$ and $\boxed{\text{21 ACROSS}}$.
6. A palindrome.
9. The value of $5 \times 5 \times 5$.
10. The largest 3-digit multiple of 24.
11. A number that is $\frac{2}{3}$ of 1212.
12. The number of seconds in 8.5 minutes.
15. $\boxed{\text{14 ACROSS}}$ less than a multiple of 1000.
16. The sum of $\boxed{\text{21 ACROSS}}$ and $\boxed{\text{14 ACROSS}}$.
18. The product of $\boxed{\text{17 ACROSS}}$ and 11.
19. The first three digits in π (pi).
20. A number whose digits multiply to 60.

CEMC at Home

Grade 7/8 - Tuesday, March 24, 2020
Crossnumber Puzzle - Solution

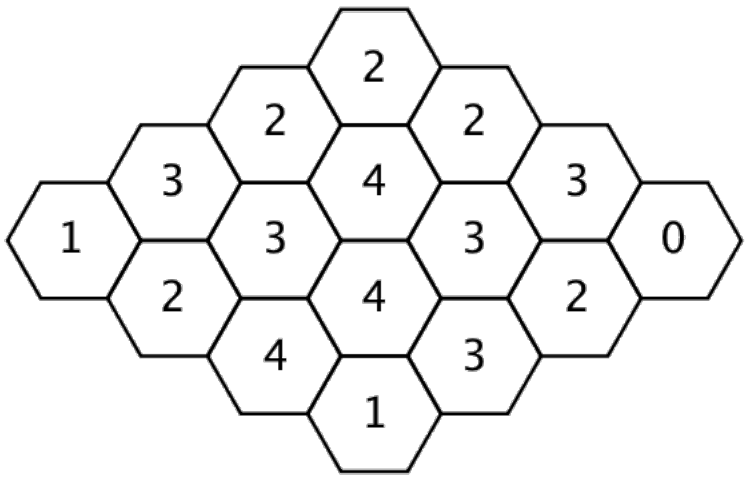
8	5	4	9		4	9	9	9
8			1	4	5			4
9			7	2	9			0
5	1	9		4		8	5	5
	2	8	3		1	0	1	
8	5	4		2		8	0	8
8			3	6	6			8
9			1	6	5			8
9	1	1	4		2	6	0	8

CEMC at Home

Grade 7/8 - Wednesday, March 25, 2020
Beehive

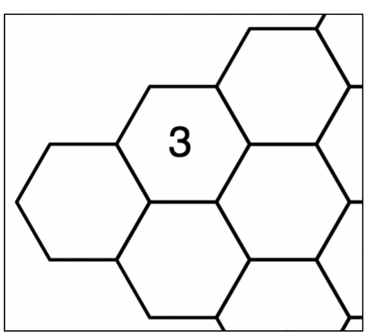
A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many *other* hexagons touching this hexagon contain honey. The results of the bear's study are shown. How many hexagons in the honeycomb contain honey?

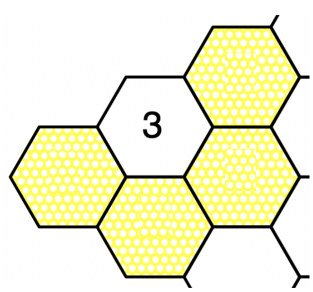
Honeycomb 1



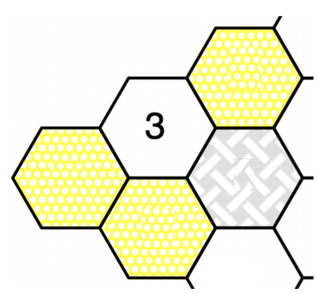
Need Help Getting Started?

Look at the hexagon marked with a "3" near the left corner of the honeycomb above. Notice that there are exactly 4 other hexagons that are touching this hexagon. This number 3 tells us that exactly 3 of those 4 touching hexagons have honey. But can you figure out which 3?





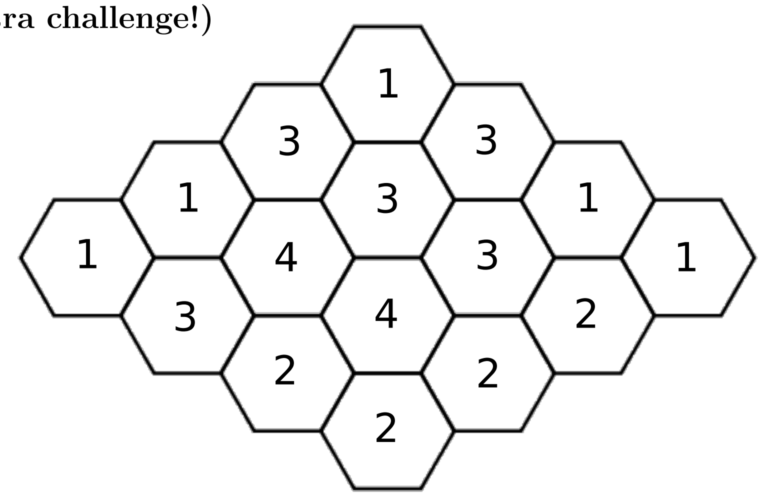
Impossible



Possible

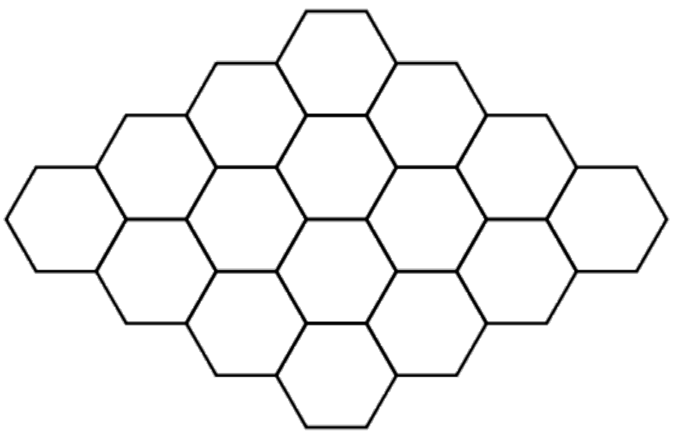
Use the online exploration (https://www.geogebra.org/m/mdbfsjvj) for this question to help you work through the solution. By clicking on a hexagon you can mark whether or not you think it contains honey. You can use this to keep track of what you discover about the hexagons as you go.

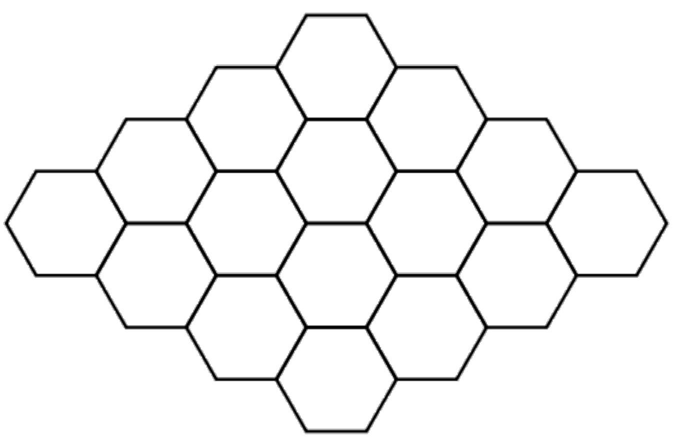
Honeycomb 2 (Extra challenge!)



You might find the second honeycomb harder to figure out. What makes this honeycomb more difficult?

Extension: Use the empty honeycombs below to create your own beehive problems and share them with your friends and family.





More Info:

Check out the CEMC at Home webpage on Thursday, March 26 for a solution to Beehive.

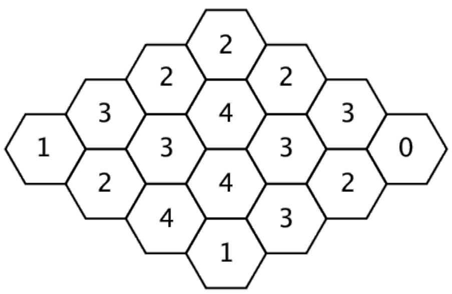
This problem was inspired by a problem on the *Beaver Computing Challenge*. You can find more problems like this on past BCC contests.

CEMC at Home

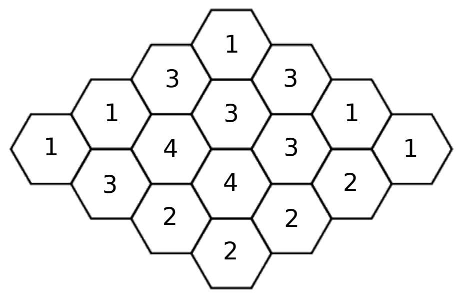
Grade 7/8 - Wednesday, March 25, 2020
Beehive - Solution

Question:
A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many *other* hexagons touching this hexagon contain honey. The results of the bear's study are shown. How many hexagons contain honey?

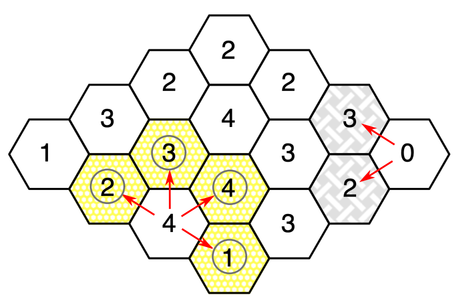
Honeycomb 1



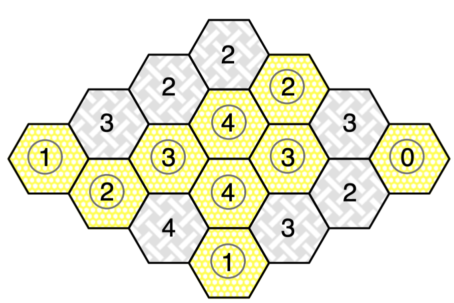
Honeycomb 2



Solution Honeycomb 1:
One way to solve this is to start from a hexagon that contains a zero, because that tells us none of the hexagons touching it contain honey. Another way to solve this is to start from a hexagon that contains a number that is equal to the total number of hexagons touching it, because that tells us all of those touching hexagons contain honey. In this honeycomb we could do either strategy as shown below, where hexagons that contain honey are yellow with a circle around the number, and hexagons that do not contain honey are filled with a grey woven pattern.



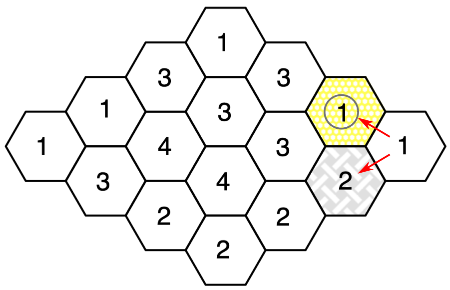
After this first step, we can move through the honeycomb, determining which hexagons contain honey based on the number inside each hexagon and the conclusions we have already made about the hexagons touching it. The completed honeycomb should look like this.



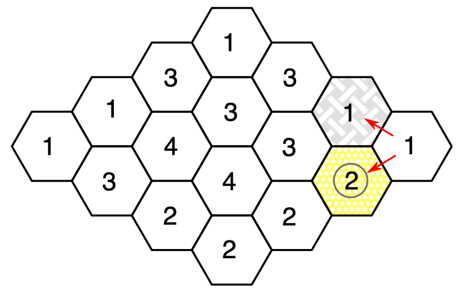
Therefore, 9 hexagons contain honey.

Solution Honeycomb 2:
This honeycomb is more challenging than the first honeycomb because there does not appear to be a good place to start. No hexagon contains a zero, and no hexagon contains a number equal to the total number of hexagons touching it. To solve this honeycomb one strategy is trial and error. Pick a hexagon to start with. Look at all the different options for the hexagons touching it.

For example, suppose we started with the rightmost hexagon in the honeycomb. This might be a good place to start because it is not touching many other hexagons. Since it contains the number one, that tells us that exactly one of the two hexagons touching it contains honey. So we have two options as shown below, where hexagons that contain honey are yellow with a circle around the number, and hexagons that do not contain honey are filled with a grey woven pattern.



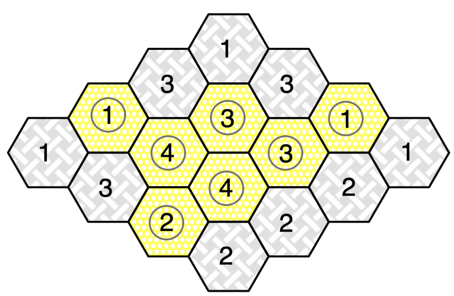
First possibility



Second possibility

We know that one of these pictures must be the correct one, but we cannot be sure which one is correct at this time. So how about we make a guess? For example, we can colour the honeycombs as shown in the second possibility above, and then see if we can move through the honeycomb colouring hexagons from there. If we find that something goes wrong, or we get stuck, then we can always go back to where we made our first choice and make a different choice.

In the end, there is only one way to colour the honeycomb that agrees with all of the numbers in the hexagons. The completed honeycomb should look like this.



Therefore, 7 hexagons contain honey.

Were you able to find this solution on your own? If not, then look at the picture above and check for yourself that this colouring works!

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, March 26, 2020
Ch-ch-changes

Bill made some purchases that totalled \$18.75 and paid for them with a twenty-dollar bill. The cash register has only quarters, dimes and nickels.

In how many different ways can the cashier make change?

"TOONIE" 2 dollar coin	"LOONIE" 1 dollar coin	QUARTER	NICKEL	DIME
200 cents	100 cents	25 cents	5 cents	10 cents

Remember, in this problem, we will only be using quarters, nickels and dimes.

More Info:

Check the CEMC at Home webpage on Thursday, April 2 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 2.

Problem of the Week

Problem C and Solution

Ch-ch-changes

Problem

Bill made some purchases that totalled \$18.75 and paid for them with a twenty-dollar bill. The cash register has only quarters, dimes and nickels. In how many different ways can the cashier make change?

Solution

This is a good problem for applying a systematic approach.

The amount of change required is $\$20 - \$18.75 = \$1.25$ or 125 cents. In order to get to 125 cents, a maximum of 5 quarters are required. Once we determine the amount still required after the value of the quarters has been removed, we can determine the number of different combinations of dimes that can be given. For each of these possibilities, the remainder of the change will be nickels.

If 5 quarters are given as part of the change, the \$1.25 required as change is covered and no other coins are required. There is only 1 possibility for change in which 5 quarters are part of the change.

If 4 quarters are given as part of the change, \$0.25 is still required. There are 3 possibilities for dimes; either 0, 1 or 2 dimes. Therefore, there are 3 different coin combinations possible in which 4 quarters are part of the change.

If 3 quarters are given as part of the change, \$0.50 is still required. There are 6 possibilities for dimes; either 0, 1, 2, 3, 4, or 5 dimes. Therefore, there are 6 different coin combinations possible in which 3 quarters are part of the change.

If 2 quarters are given as part of the change, \$0.75 is still required. There are 8 possibilities for dimes; either 0, 1, 2, 3, 4, 5, 6, or 7 dimes. Therefore, there are 8 different coin combinations possible in which 2 quarters are part of the change.

If 1 quarter is given as part of the change, \$1.00 is still required. There are 11 possibilities for dimes; either 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 dimes. Therefore, there are 11 different coin combinations possible in which 1 quarter is part of the change.

If no quarters are given as part of the change, \$1.25 is still required. There are 13 possibilities for dimes; either 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 dimes. Therefore, there are 13 different coin combinations possible in which no quarters are part of the change.

The cashier can make the required change using $1 + 3 + 6 + 8 + 11 + 13 = 42$ different possible combinations of coins.

The solution is presented in chart form on the following page.

Number of Quarters	Value of Quarters (in cents)	Amount Remaining (in cents)	Number of Dimes	Value of Dimes (in cents)	Amount Remaining (in cents)	Number of Nickels Required
5	125	0	0	0	0	0
4	100	25	2	20	5	1
			1	10	15	3
			0	0	25	5
3	75	50	5	50	0	0
			4	40	10	2
			3	30	20	4
			2	20	30	6
			1	10	40	8
			0	0	50	10
2	50	75	7	70	5	1
			6	60	15	3
			5	50	25	5
			4	40	35	7
			3	30	45	9
			2	20	55	11
			1	10	65	13
			0	0	75	15
1	25	100	10	100	0	0
			9	90	10	2
			8	80	20	4
			7	70	30	6
			6	60	40	8
			5	50	50	10
			4	40	60	12
			3	30	70	14
			2	20	80	16
			1	10	90	18
			0	0	100	20
0	0	125	12	120	5	1
			11	110	15	3
			10	100	25	5
			9	90	35	7
			8	80	45	9
			7	70	55	11
			6	60	65	13
			5	50	75	15
			4	40	85	17
			3	30	95	19
			2	20	105	21
			1	10	115	23
			0	0	125	25

The cashier can make the required change using $1 + 3 + 6 + 8 + 11 + 13 = 42$ different possible combinations of coins.

CEMC at Home

Grade 7/8 - Friday, March 27, 2020
Tessellations

A tessellation (or tiling) is an arrangement of one or more shapes in a repeated pattern without overlaps or gaps. You have probably seen tessellations before without even realizing it! For example, brick walls are tessellations of rectangles, some flooring designs are tessellations of squares, and the honeycombs in a beehive are tessellations of hexagons.

Activity 1: Let's create our own original tessellations!

You Will Need:

- 1 piece of paper (letter size or bigger)
- 1 piece of card stock about the size of a playing card (or regular paper cut to this size)
- A ruler
- Scissors
- Tape
- Coloured markers or pens

Try This:

1. Start with your rectangle cut to the size of a playing card.

2. Make alterations to the top edge of your shape.

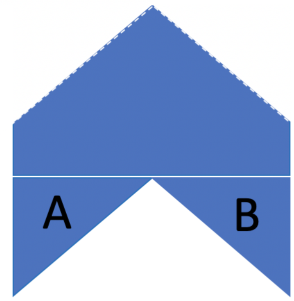
   Interesting alterations make for interesting tessellations! But you might want to make sure that the alteration you choose on your first try is simple enough that you can easily work with your shape.

3. Translate the alterations to the bottom edge of your shape and tape in place.

   Make sure to translate the pieces vertically as shown. Any shifting in the horizontal direction may result in a shape that won't work for you in steps 5 and 6.

4. Trace your new shape on the piece of paper.

5. Repeatedly translate your shape vertically and horizontally on the paper, always fitting it snugly into or against the previously drawn shape, tracing as you go.



6. Colour in your shapes and create a beautiful piece of art!

Activity 2: Let's investigate polygons and tessellations!

We have seen that squares and hexagons both form nice tessellations. These shapes are both examples of what are called *regular polygons*. (A polygon is a closed figure with straight sides. A polygon is regular if all sides are equal in length and all angles are equal in measure.) It is likely that the shapes you formed in Activity 1 were not regular polygons. Can you think of any other regular polygons that can be used to form a tessellation? Are there regular polygons that cannot be used to form a tessellation?

> *Need help getting started with Activity 2?*
>
> Try and cut out a regular polygon and use it as your shape like you did in Activity 1.
>
> If you would like to try something new, then use the following investigation: Explore This!
>
> Here you can use technology to help you explore whether equilateral triangles, squares, pentagons or hexagons can be used to form a tessellation. Can you figure out what features make a polygon good for making a tessellation?

More Info:

If you are interested in learning more about tessellations check out this lesson in the CEMC Courseware. You can find a discussion of Activity 2 there!