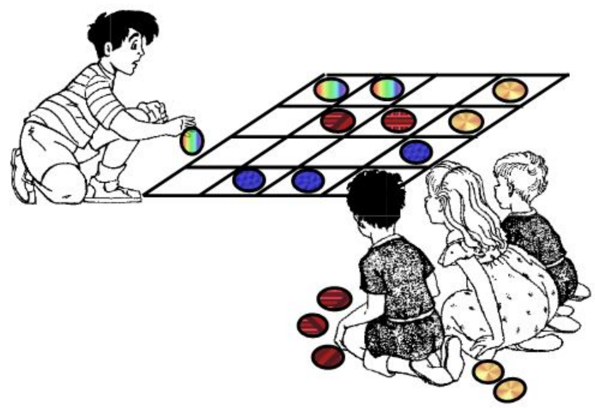
# CEMC at Home

## Grade 4/5/6 - Monday, March 23, 2020
## Sum Bingo

**You Will Need:**



- Two to four players
- The *Number List* (on the next page)
- The *Game Board* (on the next page)
- 10 to 12 "markers" per player

  (These could be small coins, paperclips, or any other small objects, and should be different for each player.)
- A calculator

**How To Play:**

1. Players take turns.
2. On your turn, select two numbers from the Number List that you think will have a sum on the Game Board.
3. Say the two numbers out loud and the number on the Game Board that you think is their sum. No calculators yet!
4. Using a calculator, check their sum.
5. If you are correct and the sum is on the Game Board, place one of your markers on the box containing that sum. If you are not correct, do not place one of your markers.
6. The winner is the first to get four markers in a row in any direction (horizontal, vertical, or diagonal).

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**More Info:**

Check out the CEMC at Home webpage on Monday, March 30 for the solution to Sum Bingo.

If you enjoyed this game and would like to try **Product Bingo**, go to

https://cemc.uwaterloo.ca/resources/invitations-to-math/NumberSense-Grade5.pdf

and scroll down to page 50. Have fun!

*Number List*

23 57 75 43 61 32 95 84 19 115

*Game Board*

| | | | |
|---|---|---|---|
| 80 | 127 | 132 | 94 |
| 159 | 42 | 107 | 93 |
| 98 | 210 | 84 | 156 |
| 199 | 100 | 179 | 62 |

# CEMC at Home

## Grade 4/5/6 - Monday, March 23, 2020
## Sum Bingo - Solution

*Number List*

23 57 75 43 61 32 95 84 19 115

*Game Board Solution*

| | | | |
|---|---|---|---|
| 80<br>$23 + 57 = 80$<br>$19 + 61 = 80$ | 127<br>$32 + 95 = 127$<br>$43 + 84 = 127$ | 132<br>$57 + 75 = 132$ | 94<br>$19 + 75 = 94$ |
| 159<br>$75 + 84 = 159$ | 42<br>$19 + 23 = 42$ | 107<br>$32 + 75 = 107$<br>$23 + 84 = 107$ | 93<br>$32 + 61 = 93$ |
| 98<br>$23 + 75 = 98$ | 210<br>$95 + 115 = 210$ | 84<br>$23 + 61 = 84$ | 156<br>$61 + 95 = 156$ |
| 199<br>$61 + 115 = 199$ | 100<br>$43 + 57 = 100$ | 179<br>$84 + 95 = 179$ | 62<br>$19 + 43 = 62$ |

**Challenge:**

There are 10 numbers in the *Number List* and 16 numbers on the *Game Board*.

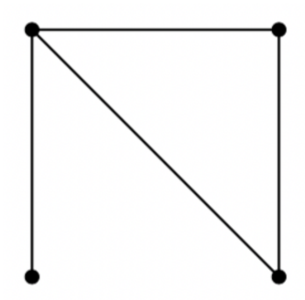
Can you make up an entirely different *Game Board* that uses these same 10 numbers in the *Number List* above?

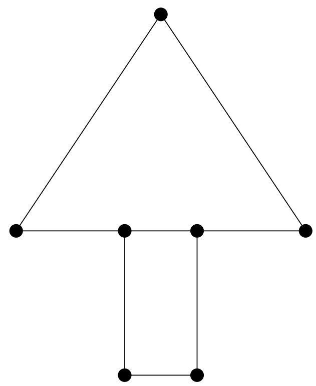
Can you make up an entirely different game (with a different *Number List*)?

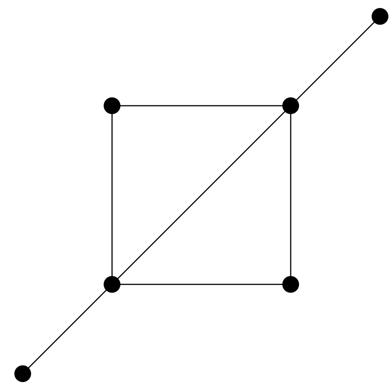
# CEMC at Home

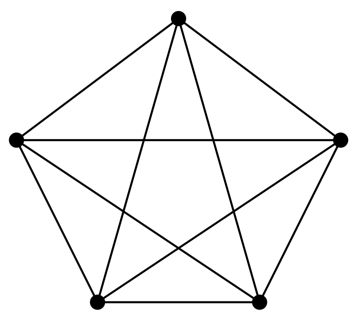
## Grade 4/5/6 - Tuesday, March 24, 2020
## Connect-the-Dots

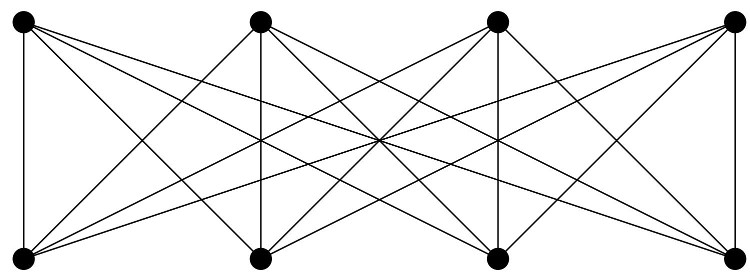
**Question 1:** In each of the following pictures, find a way to trace over every line exactly once, moving from dot to dot without lifting up your pencil.







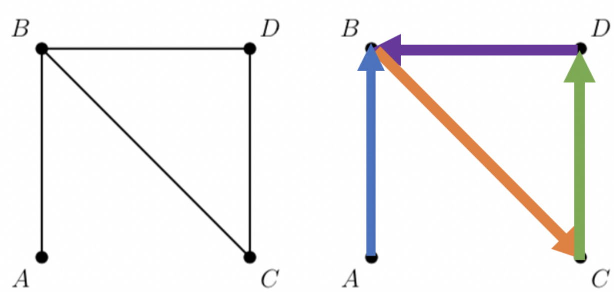




**Notes:** *We are not allowed to trace over the same line twice and we cannot leave any lines out! Take note of all of the dots in the figure. Places where lines meet may or may not correspond to dots. We need to decide at which dot to start our tracing and we do not necessarily need to end our tracing in the same place we started.*

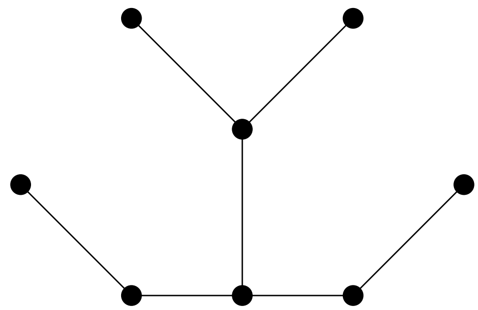
**Here is an explanation of one way to complete the task for the top left figure above:**

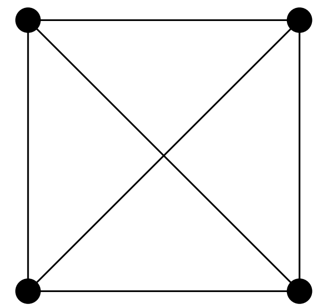
Notice that there are four dots and four lines in the figure. We label each dot with a letter as shown in the figures below. If we start at the letter $A$, then we can trace all of the lines without lifting the pencil.



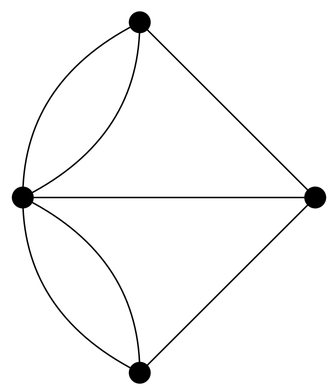

We can trace from $A$ to $B$, then from $B$ to $C$, then from $C$ to $D$, and then from $D$ to $B$ to cover each line exactly once without lifting up our pencil. Can you find different way to trace out the lines?

**Question 2:** For each of the following pictures, explain why it is *impossible* to trace over every line exactly once, moving from dot to dot without lifting up your pencil.





**Question 3:** In the picture below, is it possible to trace over every line exactly once, moving from dot to dot without lifting up your pencil? If yes, show how. If no, explain why not.



**Question 4:** Suppose you are given a picture like the ones in problems 1., 2., 3., with dots connected by a series of lines or curves. Just by looking at the picture, how can you tell whether it is possible to trace over all the lines exactly once without lifting up your pencil?

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**More Info:**

Check out the CEMC at Home webpage on Tuesday, March 31 for the solution to Connect-the-Dots.

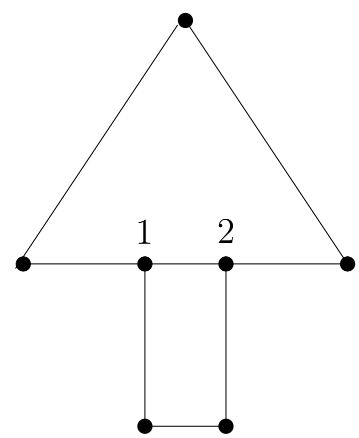
These problems are inspired by the famous *Seven Bridges of Königsberg* problem studied by the mathematician Leonhard Euler. See what you can find out about the Bridges of Königsberg!

# CEMC at Home

## Grade 4/5/6 - Tuesday, March 24, 2020
## Connect-the-Dots - Solution

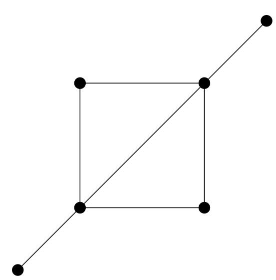
**Question 1:** In each of the following pictures, find a way to trace over every line exactly once, moving from dot to dot without lifting up your pencil.

*Solution:* One way to do this is to start from the dot marked "1" and trace over all the outside lines of the picture in order, until returning to dot 1. Then, finish with the line connecting dot 1 to dot 2. Or, you can start from the dot marked "2" and trace over all the outside lines in the picture, returning to dot 2 and finishing with the line from dot 2 to dot 1.

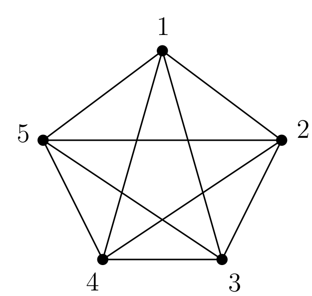



The important thing is that you have to start at either dot 1 or dot 2, since these are the ones with an odd number of lines (or *edges*) coming out of them. Every time you pass through a dot (other than at the beginning or end), you use up an even number of the edges passing through that dot. So, the only way to use up all the edges for a dot (or *vertex*) with an odd number of edges is to make sure that vertex is the first or last vertex!

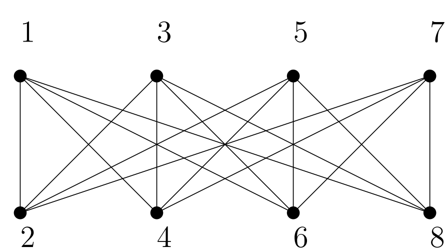
*Solution:* Here, notice there are exactly two dots (vertices) that have just one edge coming out of them. You must start tracing at one of these vertices, and finish at the other – as soon as you get to one of these vertices, you can't get back to the rest without tracing over the same line twice! So, one way to trace this is to start at the bottom-left vertex, move to the bottom-left corner of the square, trace around the perimeter of the square until you return to where you started, and then trace through the remaining diagonal lines to finish at the top-right vertex.



*Solution:* One way to trace out this diagram is to trace the outside (perimeter) of the pentagon, starting and ending at vertex 1, and then to trace over the inside lines in the pentagon in the following order: $1 \to 3 \to 5 \to 2 \to 4 \to 1$.

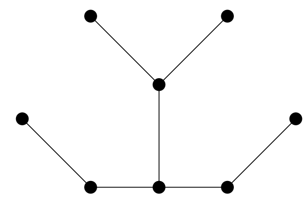



*Solution:* One possible tracing is $1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 5 \to 2 \to 7 \to 4 \to 1 \to 6 \to 3 \to 8 \to 1$.

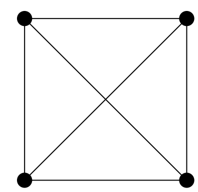


**Question 2:** For each of the following pictures, explain why it is *impossible* to trace over every line exactly once, moving from dot to dot without lifting up your pencil.

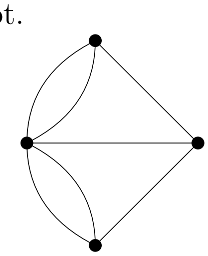
*Solution:* Notice that in this picture, there are four dots (vertices) that have only one edge coming out of them. Unless one of these vertices is your starting point, as soon as you get to that vertex, you can't get back to the rest of the picture without tracing over the same line twice, so you have to finish there. So, after you start tracing, you can only get to one of these dots before you are forced to stop, making it impossible to reach all four of them.



*Solution:* As mentioned earlier, every time you pass through a dot (except at the beginning and end), you use up an even number of its edges. So, if you are going to use up all the edges exactly once, at most two of your vertices (dots) can have an odd number of edges touching it. But in this picture, all four of the dots have an odd number of edges, so there's no way to trace over them all exactly once.



**Question 3:** In the picture below, is it possible to trace over every line exactly once, moving from dot to dot without lifting up your pencil? If yes, show how. If no, explain why not.



*Solution:* No, it is not possible to trace over all the lines in this picture exactly once, since all four of the dots have an odd number of edges coming out of them (so the same argument works as in the last example).

*As the story goes, this particular picture can be used to represent a map of the city of Königsberg back in the 1700s. Each dot represents a landmass, and each edge represents a bridge between the landmasses. The people in this city used to go for walks around town, trying to find a route crossing each bridge exactly once. No one could figure out a way to do this, so they asked the mathematician Leonhard Euler to explain why this was impossible. He came up with the explanation provided here!*

**Question 4:** Suppose you are given a picture like the ones in problems 1., 2., 3., above, with dots connected by a series of lines. Just by looking at the picture, how can you tell whether it is possible to trace over all the lines exactly once without lifting up your pencil?

*Solution:* As mentioned in the previous problem, if there are more than two dots that have an odd number of lines coming out of them, then there is no way to trace over all the lines in the required way. It turns out that there's also no way to draw such a picture where only one dot has an odd number of lines (why?), so this leaves only the case of zero or two dots with an odd number of edges. In both these cases, it turns out that we can always find a tracing of all the lines. The general idea is to break the problem down into smaller problems, essentially finding a tracing for two smaller parts of the picture and then putting those tracings together. If you are interested, you may want to look into *Fleury's algorithm* for more details!

*Mathematicians call pictures like these, with dots and lines (vertices and edges),* graphs. *Aside from modelling walks through a city, or connect-the-dots games, graphs can also be used to represent roads between different intersections in a city, or something as complex as the connections between webpages on the Internet (with each vertex representing a webpage, and each edge representing a link between webpages). Mathematicians use graphs to model and help solve many different types of problems.*

# CEMC at Home

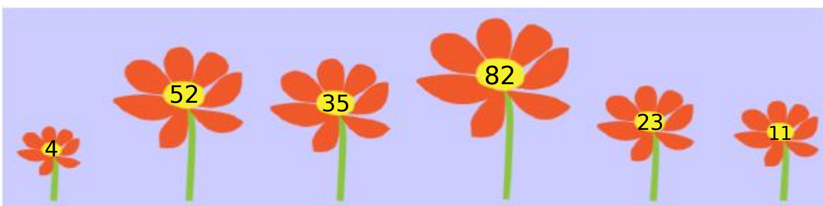
## Grade 4/5/6 - Wednesday, March 25, 2020

## Beever's Choice

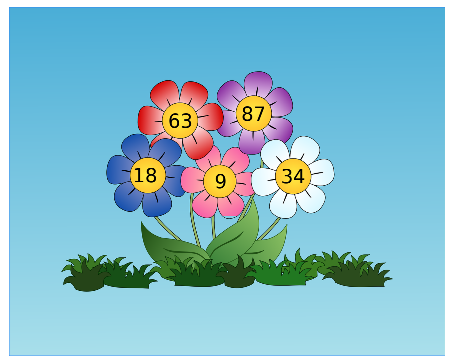
Beever the bee flies to fields of flowers to collect pollen. On each flight, he visits only one flower and can collect up to 10 mg of pollen. He may return to the same flower more than once.

Beever has two favourite flower patches, but visits only one patch daily, making 20 flights per day.

The initial amount of pollen (in mg) in each flower is shown on each flower in the two patches.




**Flower Patch 1**




**Flower Patch 2**

**Question 1:** Suppose that Beever decides to visit Flower Patch 1 today. What is the largest amount of pollen in total that he can collect over 20 flights to this patch?

*If Beever chooses a flower for each trip without thinking carefully about each choice, then he is unlikely to gather the largest amount of pollen overall. For example, if Beever takes his first trip to the flower furthest to the left, then he will only be able to gather 4 mg on his first trip. What would happen if Beever decided to visit the flower furthest to the right for his first two trips?*

**Question 2:** Beever's goal is to collect as much pollen as he can today in 20 flights. Do you think Beever should visit Flower Patch 1 or Flower Patch 2? Why?

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**More info:**

Check out the CEMC at Home webpage on Wednesday, April 1 for the solution to Beever's Choice.

Beever's Choice is related to the computing concept of a *greedy algorithm*, a procedure which tries to optimize each step of a process. (Beever will want to take as much pollen as possible on each flight.)

This problem was inspired by a problem from the *Beaver Computing Challenge*. If you enjoyed this problem, try some similar problems here: 2018 BCC for Grade 5/6.

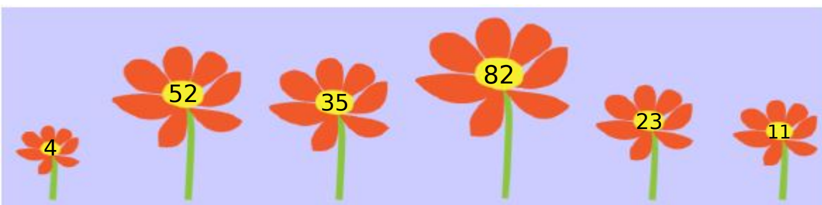
# CEMC at Home

## Grade 4/5/6 - Wednesday, March 25, 2020
## Beever's Choice - Solution

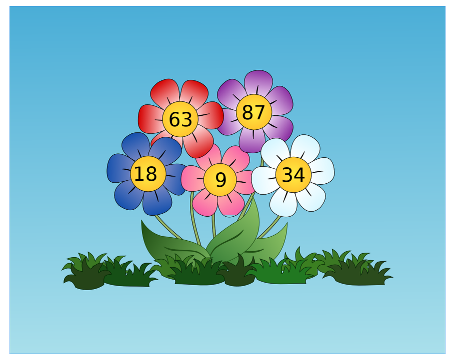
**Problem:**

Beever the bee flies to fields of flowers to collect pollen. On each flight, he visits only one flower and can collect up to 10 mg of pollen. He may return to the same flower more than once. Beever has two favourite flower patches, but visits only one patch daily, making 20 flights per day.

The initial amount of pollen (in mg) in each flower is shown on each flower in the two patches.




**Flower Patch 1**




**Flower Patch 2**

**Question 1:** Suppose that Beever decides to visit Flower Patch 1 today. What is the largest amount of pollen in total that he can collect over 20 flights to this patch?

**Question 2:** Beever's goal is to collect as much pollen as he can today in 20 flights. Do you think Beever should visit Flower Patch 1 or Flower Patch 2? Why?

**Solution 1:** Since Beever can collect up to 10 mg on each flight, he should travel to a flower with at least 10 mg left until it is no longer possible do to so. For example, he can travel five times to the flower with 52 mg, collecting a total of 50 mg, and leaving 2 mg. He can collect the maximum (10 mg) on his first 19 trips totalling

$$50 + 30 + 80 + 20 + 10 = 190 \text{ mg}$$

On his last trip, he will then collect the greatest remaining amount, which is 5 mg from the flower that originally had 35 mg. Thus the largest total amount of pollen he can collect from Flower Patch 1 is $190 + 5 = 195$ mg.

**Solution 2:** Beever should visit Flower Patch 2. We can use similar reasoning to show that the largest amount of pollen that can be collected from Flower Patch 2 is 197 mg which is more than the maximum amount of 195 mg for Flower Patch 1.

# CEMC at Home features Problem of the Week

## Grade 4/5/6 - Thursday, March 26, 2020

## Chip, Chip, Chooray!

At Biscuit Hill Elementary School, Chip and his sister, Charlene, have decided that they want to make cookies for all of the junior students in their school.

The recipe that they found makes enough chocolate chip cookies of 7 cm diameter for 16 people.

**Recipe**

| | |
|---|---|
| 1 cup | butter |
| 1 cup | brown sugar |
| $\frac{1}{2}$ cup | white sugar |
| 2 | eggs |
| 2 tsp | vanilla |
| $2\frac{1}{4}$ cups | flour |
| 1 tsp | baking soda |
| 300 g | chocolate chips |

**Junior Classes**

| | |
|---|---|
| Mrs. Martin | 25 students |
| Mrs. Laing | 26 students |
| Ms. Richmond | 23 students |
| Mrs. Kelter | 24 students |
| Mr. Hallett | 22 students |


a) How many batches should Chip and Charlene make so that they make the exact number of cookies needed for all of the students in the junior classes?

b) They decide to make a whole number of batches so that they have some extra cookies to save for later and one cookie for each teacher. What quantity of each ingredient in the recipe will they need?

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**More Info:**

Check the CEMC at Home webpage on Thursday, April 2 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 2.

This CEMC at Home resource is the current grade 5/6 problem from Problem of the Week (POTW). This problem was developed for students in grades 5 and 6, but is also accessible to students in grade 4. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 3/4 problem and to find many more past problems and their solutions, visit the Problem of the Week webpage.

# Problem of the Week
## Problem B and Solution
## Chip, Chip, Chooray!

### Problem

At Biscuit Hill Elementary School, Chip and his sister, Charlene, have decided that they want to make cookies for all of the junior students in their school.
The recipe that they found makes enough chocolate chip cookies of 7 cm diameter for 16 people.

**Recipe**

| | |
|---|---|
| 1 cup | butter |
| 1 cup | brown sugar |
| $\frac{1}{2}$ cup | white sugar |
| 2 | eggs |
| 2 tsp | vanilla |
| $2\frac{1}{4}$ cups | flour |
| 1 tsp | baking soda |
| 300 g | chocolate chips |

**Junior Classes**

| | |
|---|---|
| Mrs. Martin | 25 students |
| Mrs. Laing | 26 students |
| Ms. Richmond | 23 students |
| Mrs. Kelter | 24 students |
| Mr. Hallett | 22 students |

a) How many batches should Chip and Charlene make so that they make the exact number of cookies needed for all of the students in the junior classes?

b) They decide to make a whole number of batches so that they have some extra cookies to save for later and one cookie for each teacher. What quantity of each ingredient in the recipe will they need?

### Solution

a) There are $25 + 26 + 23 + 24 + 22 = 120$ students in total. Since one recipe makes enough cookies for 16 people, to make exactly enough, Chip and Charlene would need to make $120 \div 16 = 7.5$ batches.

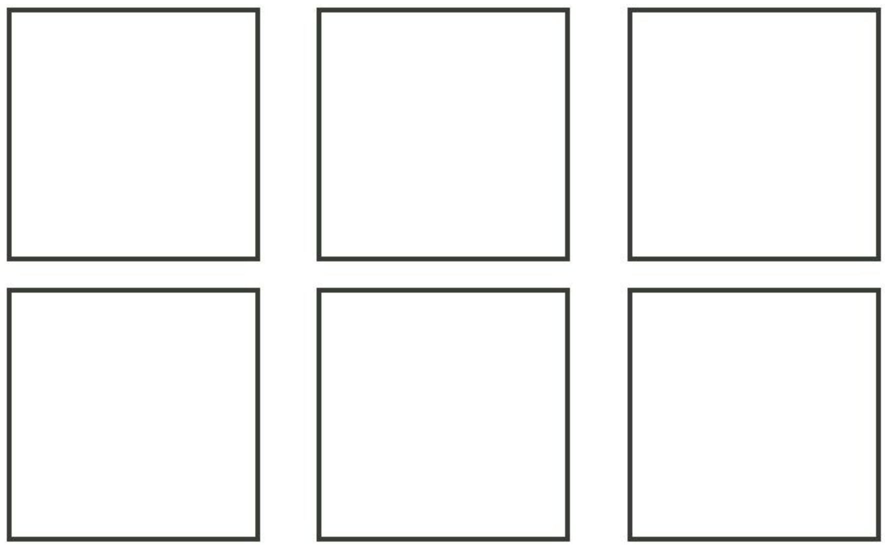
b) Eight batches (128 cookies) will leave 5 for the teachers and 3 to save for later. Thus they will need to multiply all the measurements by eight to get;
$8 \times 1 = 8$ cups butter, $8 \times 1 = 8$ cups brown sugar,
$8 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4$ cups white sugar,
$8 \times 2 = 16$ eggs, $8 \times 2 = 16$ tsp vanilla,
$8 \times (2\frac{1}{4}) = 8 \times 2 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 16 + \frac{8}{4} = 16 + 2 = 18$ cups flour,
$8 \times 1 = 8$ tsp baking soda, and $8 \times 300 = 2400$ g (2.4 kg) of chocolate chips.
(If they want more cookies left over, they will need more batches.)

# CEMC at Home

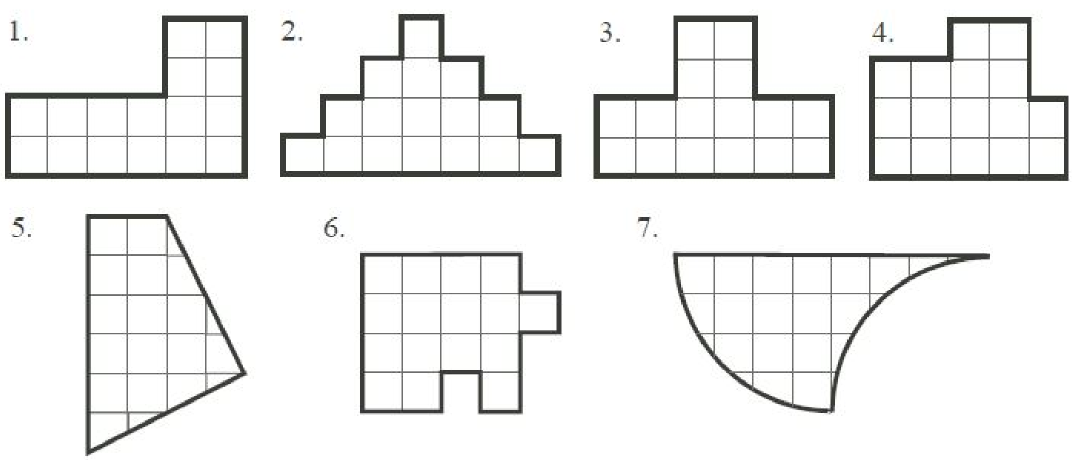
## Grade 4/5/6 - Friday, March 27, 2020
## Quarter Squares

**Question 1:** On the blank squares below, show six different ways to divide a square into four identical pieces, using only straight line segments.

*In math, objects that are identical are called congruent. Congruent shapes have exactly the same shape and size, but may be rotations (turns) and/or reflections (flips) of each other. Can you divide the square into smaller congruent squares or congruent triangles? What other shapes might you use?*



**Question 2:** For each of the templates 1. - 7. given below, discover whether four of these identical shapes could be arranged to create a square. For example, can you arrange four of the shapes from template 1. (of the same size) to form a square? If so, make a sketch on the grid paper on the following page showing how the four identical shapes create a square, and move onto the next template.




Hint: If your visual imagination gets stuck, cut out four copies of the templates to play with. Think about whether you need rotations (turns) and/or reflections (flips) of the pieces.

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**More info:**

Check out the CEMC at Home webpage on Friday, April 3 for the solution to Quarter Squares.

Want another "shape-fitting" activity like this one? Try problem 6 here: 2008 Emmy Noether Circle.

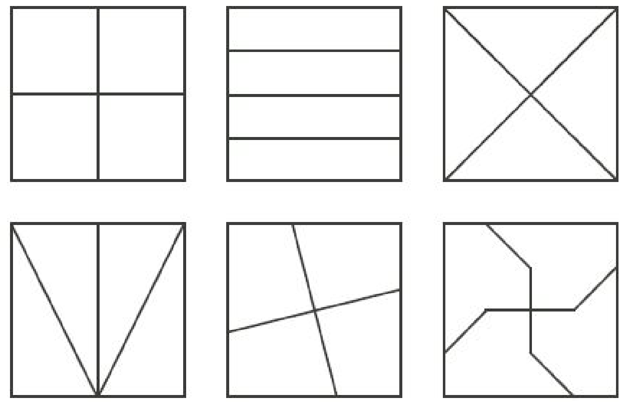
## Grid for Quarter Squares

# CEMC at Home

## Grade 4/5/6 - Friday, March 27, 2020
## Quarter Squares - Solution

**Question 1:** Six ways to divide a square into four identical pieces (or four congruent shapes) are shown to the right.

*Note that there are countless ways to do subdivisions like the last two, using symmetrical divisions of the square with line segments arranged outward from the centre.*



**Question 2:** The diagrams below reveal how templates 1, 2, 3, 4, 5, and 7 can each be used to form squares with a side length of 8 units.