CEMC at Home

Grade 4/5/6 - Monday, June 8, 2020

Contest Day 6

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2013 Gauss Contest, #13

Jack, Kelly, Lan, Mihai, and Nate are sitting in the 5 chairs around a circular table. Lan and Mihai are sitting beside each other. Jack and Kelly are not sitting beside each other. The 2 people who are seated on either side of Nate are

(A) Jack and Lan **(B)** Jack and Kelly **(C)** Kelly and Mihai

(D) Lan and Mihai **(E)** Mihai and Jack

2020 Gauss Contest, #15

Emil and Olivia ran a race. Their race times totalled 1 hour 52 minutes. If Emil's time was 4 minutes less than Olivia's time, how many minutes did it take Olivia to run the race?

(A) 78 **(B)** 56 **(C)** 58 **(D)** 74 **(E)** 55

More Info:

Check out the CEMC at Home webpage on Monday, June 15 for solutions to the Contest Day 6 problems.

CEMC at Home

Grade 4/5/6 - Monday, June 8, 2020

Contest Day 6 - Solution

Solutions to the two contest problems are provided below.

2013 Gauss Contest, #13

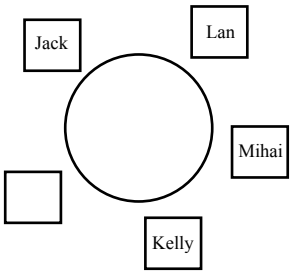
Jack, Kelly, Lan, Mihai, and Nate are sitting in the 5 chairs around a circular table. Lan and Mihai are sitting beside each other. Jack and Kelly are not sitting beside each other. The 2 people who are seated on either side of Nate are

(A) Jack and Lan **(B)** Jack and Kelly **(C)** Kelly and Mihai

(D) Lan and Mihai **(E)** Mihai and Jack

Solution:

Since Lan and Mihai are seated beside each other, while Jack and Kelly are not, the only possible location for the remaining chair (Nate's chair) is between Jack and Kelly. Therefore, the 2 people who are seated on either side of Nate are Jack and Kelly.



ANSWER: (B)

2020 Gauss Contest, #15

Emil and Olivia ran a race. Their race times totalled 1 hour 52 minutes. If Emil's time was 4 minutes less than Olivia's time, how many minutes did it take Olivia to run the race?

(A) 78 **(B)** 56 **(C)** 58 **(D)** 74 **(E)** 55

Solution:

There are 60 minutes in 1 hour, and so there are $60 + 52 = 112$ minutes in 1 hour 52 minutes.
If Emil's race time was 54 minutes, then Olivia's race time was 4 minutes more, or 58 minutes.
In this case, their race times total $54 + 58 = 112$ minutes, as required.
Therefore, it took Olivia 58 minutes to run the race.

ANSWER: (C)

CEMC at Home

Grade 4/5/6 - Tuesday, June 9, 2020

Doggy Description

An unknown dog wanders the neighbourhood every night, scavenging in people's garbage pails and making a mess. Four of the kids in the neighbourhood (Mathias, Li Jing, Rajiv, and Olivia) think they have each seen the guilty dog, but the culprit is hard to see clearly in the dark.

Below are their descriptions, each giving exactly four details about the dog: the dog's colour, hair type, collar colour, and tail length.

Each witness has exactly one of the four details correct in their description. Each detail is described correctly by exactly one of the four witnesses.

1. Mathias says the dog is white, fluffy, wears a red collar, and has a long tail.
2. Li Jing says the dog is black, has short hair, wears a red collar, and has a long tail.
3. Rajiv says the dog is brown, has long hair, wears a blue collar, and has a long tail.
4. Olivia says the dog is spotted, fluffy, wears a red collar, and has a short tail.

Determine the correct description of the guilty dog.

Completing the table below may help to sort out which details are correct in the descriptions.

Witness	Colour	Hair Type	Collar Colour	Tail Length
Mathias				
Li Jing				
Rajiv				
Olivia				

HINTS:

1. Start by thinking about the length of the dog's tail.
2. If more than one description says the dog is fluffy, could that be correct?

Challenge:

Work with a friend or family member to make up a different set of four witness descriptions that could also be used to determine the correct description of the dog. (Remember that each witness should have exactly one detail correct and each detail should be described correctly by exactly one witness.)

More info:

Check out the CEMC at Home webpage on Tuesday, June 16 for a solution to Doggy Description.

CEMC at Home

Grade 4/5/6 - Tuesday, June 9, 2020

Doggy Description - Solution

Problem: An unknown dog wanders the neighbourhood every night, scavenging in people's garbage pails and making a mess. Four of the kids in the neighbourhood (Mathias, Li Jing, Rajiv, and Olivia) think they have each seen the guilty dog, but the culprit is hard to see clearly in the dark.

Below are their descriptions, each giving exactly four details about the dog: the dog's colour, hair type, collar colour, and tail length.

Each witness has exactly one of the four details correct in their description. Each detail is described correctly by exactly one of the four witnesses.

1. Mathias says the dog is white, fluffy, wears a red collar, and has a long tail.
2. Li Jing says the dog is black, has short hair, wears a red collar, and has a long tail.
3. Rajiv says the dog is brown, has long hair, wears a blue collar, and has a long tail.
4. Olivia says the dog is spotted, fluffy, wears a red collar, and has a short tail.

Determine the correct description of the guilty dog.

Solution:

Here is the completed table of details.

Child	Colour	Hair Type	Collar Colour	Tail Length
Mathias	white [4]	fluffy	red	long
Li Jing	black	short [3]	red	long
Rajiv	brown	long	blue [2]	long
Olivia	spotted	fluffy	red	short [1]

Noting that each witness can only be right about one detail, and each detail is described correctly by only one witness, we reason as follows (in the order indicated by the numbered items in the table).

1. Since exactly one of the witnesses has the tail length correct, it must be Olivia's description of the tail length that is correct. This means the dog must have a short tail.
2. Since exactly one of the witnesses has the collar colour correct, it must be Rajiv's description of the collar colour that is correct. This means the dog must have a blue collar.
3. Since Rajiv and Olivia can only be correct about one detail each, based on the work above we know they most both be wrong about the hair type of the dog. This means that the dog's hair cannot be long and cannot be fluffy. Since one of the witnesses has the hair type correct, it must be Li Jiang who has it correct, and so the dog's hair must be short.
4. Based on the work above, we know Matias is wrong about tail length, collar colour, and hair type. Since Matias must have one detail correct, it must be the colour. This means the dog must be white.

Thus we conclude that the dog is a white, short-haired dog with a short tail, wearing a blue collar.

CEMC at Home

Grade 4/5/6 - Wednesday, June 10, 2020

Treasure Hunt

You found a treasure map that contains information to find hidden treasures. The map is a grid with rows labelled with letters and columns labelled with numbers. Each square of the grid is identified by a unique name such as **B3** or **D5** and contains either a number or a ◇.

Under some of the ◇ spots on the grid, there is hidden treasure. To find the treasure, you must know the rules of the map and be given a correct starting position.

Here are the rules:

- You will be given a starting position including: a starting square that contains a number and a starting direction which indicates how you start moving in the map (e.g., **A3** ↓).
- From the starting position, move through the grid in the given direction, accumulating the sum of the numbers in the squares that you pass through (including the one on the starting square). Let's call this accumulated sum **S**.

Partial Map:

	1	2	3	4	5	6
A	◇	15	13	3	6	◇
B	2	5	15	16	10	7
C	7	9	◇	1	4	◇
D	◇	11	5	3	17	4

  - When you reach a square containing a ◇, there are four possibilities:
    - If **S** is even and its leftmost digit is even (e.g., **S** = 24), then you make a quarter turn (90°) counterclockwise and continue accumulating sums along this new path.
    - If **S** is even and its leftmost digit is odd (e.g., **S** = 34), then you make a quarter turn clockwise and continue accumulating sums along this new path.
    - If **S** is odd and its leftmost digit is even (e.g., **S** = 45), then you will keep moving in the same direction and continue accumulating sums along the same path.
    - If **S** is odd and its leftmost digit is odd (e.g., **S** = 125), then you have found a treasure!
  - While you continue searching for treasure, the accumulated sum **S** continues to grow. Keep moving through the map, changing directions at the squares containing a ◇ symbol when necessary, until you find a treasure.

Example: Suppose you have the *Partial Map* above and you are given the starting position **A3** ↓. This is the path that you would take through the map:

Starting position

	1	2	3	4	5	6
A	◇	15	13	3	6	◇
B	2	5	15	16	10	7
C	7	9	◇	1	4	◇
D	◇	11	5	3	17	4

	1	2	3	4	5	6
A	◇	15	13	3	6	◇
B	2	5	15	16	10	7
C	7	9	◇	1	4	◇
D	◇	11	5	3	17	4

You start at square **A3** and move downwards in the map.
You move from **A3** to **B3** to **C3** and accumulate a sum of **S** = 13 + 15 = 28 when you reach your first ◇ spot.
Since 28 is even and its leftmost digit is even, you make a quarter turn counterclockwise, making your new direction →.

From square **C3** you move to the right in the map.
You move from **C3** to **C4** to **C5** to **C6** and accumulate a sum of **S** = 28 + 1 + 4 = 33 when you reach your second ◇ spot.
Since 33 is odd and its leftmost digit is odd, you have found a treasure at square **C6**!

Here is the complete treasure map.

	1	2	3	4	5	6	7	8	9	10
A	◇	15	13	3	6	◇	12	4	◇	9
B	2	5	15	16	10	7	6	14	3	8
C	7	9	◇	1	4	◇	7	2	◇	18
D	◇	11	5	3	17	4	6	4	9	19
E	2	7	15	13	◇	8	7	◇	10	◇
F	13	14	10	17	1	24	6	12	3	12
G	◇	5	◇	21	1	◇	11	◇	12	◇
H	7	3	10	4	◇	3	11	12	1	5
I	17	6	◇	5	9	5	◇	14	◇	8
J	1	7	15	2	16	6	3	9	10	11

Problem 1

The starting position **J6** ↑ will lead you to a second treasure. Find where this treasure is located.

Problem 2

The starting position **C1** → will lead you to a third treasure. Find where this treasure is located.

Let the treasure hunt begin!

Feel free to explore other starting positions on the grid. If from some starting position you happen to reach the end of the grid without an instruction to turn (and without finding a treasure), then stop and start again with another starting position.

More Info:

Check out the CEMC at Home webpage on Wednesday, June 17 for a solution to Treasure Hunt.

CEMC at Home

Grade 4/5/6 - Wednesday, June 10, 2020

Treasure Hunt - Solution

The starting position **J6** ↑ will lead you to a treasure at **G10**.

	1	2	3	4	5	6	7	8	9	10
A	◇	15	13	3	6	◇	12	4	◇	9
B	2	5	15	16	10	7	6	14	3	8
C	7	9	◇	1	4	◇	7	2	◇	18
D	◇	11	5	3	17	4	6	4	9	19
E	2	7	15	13	◇	8	7	◇	10	◇
F	13	14	10	17	1	24	6	12	3	12
G	◇	5	◇	21	1	◇	11	◇	12	◇
H	7	3	10	4	◇	3	11	12	1	5
I	17	6	◇	5	9	5	◇	14	◇	8
J	1	7	15	2	16	6	3	9	10	11

- **J6** to **I6** to **H6** to **G6**:
  S is $6 + 5 + 3 = 14$ which is even with leftmost digit odd (turn clockwise).
- **G6** to **G7** to **G8**:
  S is $14 + 11 = 25$ which is odd with leftmost digit even (continue straight).
- **G8** to **G9** to **G10**:
  S is $25 + 12 = 37$ which is odd with leftmost digit odd. There is treasure at **G10**!

The starting position **C1** → will lead you to a treasure at **C9**.

	1	2	3	4	5	6	7	8	9	10
A	◇	15	13	3	6	◇	12	4	◇	9
B	2	5	15	16	10	7	6	14	3	8
C	7	9	◇	1	4	◇	7	2	◇	18
D	◇	11	5	3	17	4	6	4	9	19
E	2	7	15	13	◇	8	7	◇	10	◇
F	13	14	10	17	1	24	6	12	3	12
G	◇	5	◇	21	1	◇	11	◇	12	◇
H	7	3	10	4	◇	3	11	12	1	5
I	17	6	◇	5	9	5	◇	14	◇	8
J	1	7	15	2	16	6	3	9	10	11

- **C1** to **C2** to **C3**:
  S is $7 + 9 = 16$ which is even with leftmost digit odd (turn clockwise).
- **C3** to **D3** to **E3** to **F3** to **G3**:
  S is $16+5+15+10 = 46$ which is even with leftmost digit even (turn counterclockwise).
- **G3** to **G4** to **G5** to **G6**:
  S is $46 + 21 + 1 = 68$ which is even with leftmost digit even (turn counterclockwise).
- **G6** to **F6** to **E6** to **D6** to **C6**:
  S is $68+24+8+4 = 104$ which is even with leftmost digit odd (turn clockwise).
- **C6** to **C7** to **C8** to **C9**:
  S is $104+7+2 = 113$ which is odd with leftmost digit odd. There is treasure at **C9**!

CEMC at Home

Grade 4/5/6 - Thursday, June 11, 2020

Amazing Grids

If possible, for each maze find your way from the top left square to the bottom right square, moving only horizontally or vertically to adjacent squares that satisfy the property given in the title.

Multiples of 4, in increasing order

4	10	14	18	22	26	30	34	38	42
8	14	24	28	32	36	40	44	46	50
12	16	20	28	30	38	42	46	50	54
16	18	22	26	34	42	70	66	62	58
20	24	26	28	38	46	50	62	66	62
22	28	30	32	42	50	54	58	70	66
36	32	34	36	46	54	58	62	74	70
40	34	36	38	50	54	58	66	70	74
44	48	52	56	60	62	66	70	74	78
46	50	54	56	64	68	72	76	80	84

Multiples of 7, in increasing order

7	14	21	27	34	41	48	55	62	69
14	20	28	33	40	47	54	61	70	76
21	27	35	42	49	54	61	68	77	83
28	35	40	47	56	61	63	75	84	90
34	41	47	50	63	68	75	82	90	97
41	48	54	58	70	77	84	89	97	104
49	55	61	65	72	79	91	96	104	111
56	62	68	72	79	86	98	103	111	118
63	69	75	82	89	96	105	112	119	125
70	76	83	89	96	103	110	117	126	133

Factors of 48, in any order

48	2	4	12	24	48	3	8	6	10
1	9	13	15	20	40	13	22	14	12
24	12	6	4	1	2	16	18	22	24
5	8	10	14	18	22	26	20	28	16
6	3	14	7	1	4	3	15	20	10
10	16	22	5	20	2	28	18	16	14
8	12	26	9	13	6	3	8	22	24
14	2	4	13	20	1	17	24	3	5
16	7	6	1	4	12	14	26	22	1
8	4	20	5	18	24	2	6	8	3

Composite numbers, in any order

45	1	41	16	15	27	33	18	9	11
22	15	7	11	25	33	37	20	23	32
3	18	13	41	13	40	7	17	8	12
17	9	7	43	2	12	18	21	28	35
23	12	33	27	7	35	11	23	42	40
32	2	40	48	13	19	21	13	16	25
37	16	19	11	12	43	23	24	32	40
9	25	18	14	8	25	41	11	46	50
16	31	4	1	2	30	51	28	24	17
25	43	6	22	27	29	38	19	63	42

Factors

The whole number *factors* of any whole number N are the whole numbers which divide evenly into N. For example, 12 has six whole number factors. They are 1, 2, 3, 4, 6, and 12.

Composite numbers

A *composite number* is a whole number that has whole number factors in addition to 1 and itself. For example, 6 is a composite number because it has whole number factors 1, 2, 3, and 6. The factors 2 and 3 are in addition to 1 and 6 (the number itself). However, the number 1 is not composite because its only whole number factor is 1, and the number 7 is not composite because its only whole number factors are 1 and 7 (the number itself).

More Info:

Check out the CEMC at Home webpage on Friday, June 12 for a solution to Amazing Grids.

CEMC at Home

Grade 4/5/6 - Thursday, June 11, 2020

Amazing Grids - Solution

Problem:

If possible, for each maze find your way from the top left square to the bottom right square, moving only horizontally or vertically to adjacent squares that satisfy the property given in the title.

Multiples of 4, in increasing order

4	10	14	18	22	26	30	34	38	42
8	14	24	28	32	36	40	44	46	50
12	16	20	28	30	38	42	46	50	54
16	18	22	26	34	42	70	66	62	58
20	24	26	28	38	46	50	62	66	62
22	28	30	32	42	50	54	58	70	66
36	32	34	36	46	54	58	62	74	70
40	34	36	38	50	54	58	66	70	74
44	48	52	56	60	62	66	70	74	78
46	50	54	56	64	68	72	76	80	84

Multiples of 7, in increasing order

7	14	21	27	34	41	48	55	62	69
14	20	28	33	40	47	54	61	70	76
21	27	35	42	49	54	61	68	77	83
28	35	40	47	56	61	63	75	84	90
34	41	47	50	63	68	75	82	90	97
41	48	54	58	70	77	84	89	97	104
49	55	61	65	72	79	91	96	104	111
56	62	68	72	79	86	98	103	111	118
63	69	75	82	89	96	105	112	119	125
70	76	83	89	96	103	110	117	126	133

Factors of 48, in any order

48	2	4	12	24	48	3	8	6	10
1	9	13	15	20	40	13	22	14	12
24	12	6	4	1	2	16	18	22	24
5	8	10	14	18	22	26	20	28	16
6	3	14	7	1	4	3	15	20	10
10	16	22	5	20	2	28	18	16	14
8	12	26	9	13	6	3	8	22	24
14	2	4	13	20	1	17	24	3	5
16	7	6	1	4	12	14	26	22	1
8	4	20	5	18	24	2	6	8	3

Composite numbers, in any order

45	1	41	16	15	27	33	18	9	11
22	15	7	11	25	33	37	20	23	32
3	18	13	41	13	40	7	17	8	12
17	9	7	43	2	12	18	21	28	35
23	12	33	27	7	35	11	23	42	40
32	2	40	48	13	19	21	13	16	25
37	16	19	11	12	43	23	24	32	40
9	25	18	14	8	25	41	11	46	50
16	31	4	1	2	30	51	28	24	17
25	43	6	22	27	29	38	19	63	42

Solution:

Solutions are given on the first three mazes. The final maze has no solution; two (of many possible) attempts are shown with dotted lines, both working forward and backward, but both dead-end.

CEMC at Home

Grade 4/5/6 - Friday, June 12, 2020

It's in the Cards

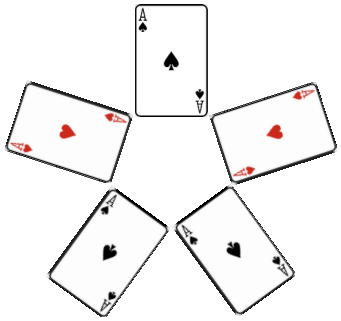
Good friends Bahaa and Helena are each working on some top-secret government projects. They would like to discuss the projects they both work on, but due to secrecy rules, must not reveal that they are working on a particular project unless their friend is also working on that same project.

Together, the two friends have devised a cooperative game using playing cards to tell them what they need to know. Here is how the game works:

- Bahaa and Helena identify a particular project; let's call it Project X.
- Each of Bahaa and Helena takes one Ace of Hearts and one Ace of Spades. An additional Ace of Spades is placed face down in front of them. (All cards of the same type should be identical.)
- Bahaa goes first. Bahaa will place his two cards face down on top of the card already on the table. If Bahaa is working on the project, he first puts down the Ace of Spades and then the Ace of Hearts. If Bahaa is not working on the project, then he puts down the Ace of Hearts, then the Ace of Spades.
- Next, it's Helena's turn. Helena will place her two cards face down on top of the three cards already on the table. If Helena is working on the project, she first puts down the Ace of Hearts and then the Ace of Spaces. If Helena is not working on the project, she first puts down the Ace of Spades and then the Ace of Hearts.

  Notice that she does the opposite of what Bahaa did.
- Now Bahaa and Helena ask a mutual friend to pick up the deck. While Bahaa and Helena close their eyes, the friend places the cards from the deck face up, in order, so that they form a circle.

  The friend is not aware of the rules of the game and must place the cards in order, but so that Bahaa and Helena cannot tell which card was on the top of the pile by looking at the circle of cards.



- Looking at the circle of cards, Bahaa and Helena now know what information about the project it is safe to share with one another.

Activity: Let's explore how Bahaa's and Helena's game works!

Decide which person will play the role of Bahaa and which player will play the role of Helena. You can enlist a friend to secretly place the cards in a circle for you, but you do not need to. You can also use homemade cards if you do not have three identical decks of playing cards.

Play four different rounds of this game, and complete the table below:

Round	1	2	3	4
Bahaa on Project X?	Yes	No	No	Yes
Helena on Project X?	No	Yes	No	Yes
Resulting card circle				

To learn how Bahaa and Helena can use this game to find out which projects they both work on, think about the questions:

1. The four rounds from the activity show what the result of the game would be in all four possible cases for Project X. Three of the four cases result in card circles that can be rotated so that they all look the same. The remaining case results in a card circle that looks different from the other three, no matter how the circle is rotated. Which three cases result in card circles that look the same (when rotated)?

2. If Bahaa is working on Project X, then he can use the card circle to determine whether or not Helena is also working on Project X. Can you explain why?

3. If Bahaa is *not* working on Project X, then he *cannot* use the card circle to determine whether or not Helena is working on Project X. Can you explain why?

Try this!

Make up your own table for four different projects.

Player's Name	Project 1	Project 2	Project 3	Project 4

Player's Name	Project 1	Project 2	Project 3	Project 4

Each player should fill in their name, and place a "Yes" or "No" for each project, but keep their answers hidden from the other player. Play the game again and see if you can use what you have learned about the game to determine which projects you are both working on.

Note that the role of the mutual friend can be replaced by the players taking turns "cutting the cards" a few times after they have all been placed in the pile. To cut a pile of cards, you split the pile in two, lifting the upper part of the pile from the top and placing the lower part of the pile on top of it.

Remember the goal of the game is to achieve the following:

- *If you are working on a project, then you can use the card circle to determine whether or not the other person is also working on the project, and*
- *if you are <u>not</u> working on a project, then you <u>cannot</u> use the card circle to determine whether or not the other person is working on the project.*

More Info:

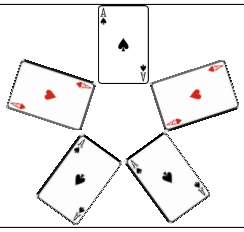
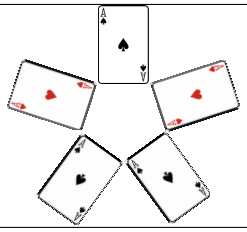
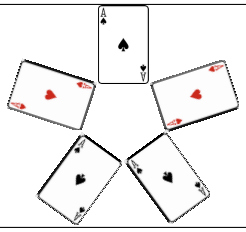
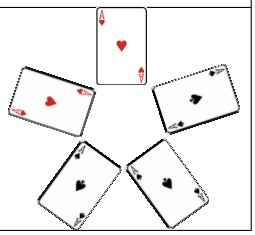
Check out the CEMC at Home webpage on Friday, June 19 for a solution to It's in the Cards.

CEMC at Home

Grade 4/5/6 - Friday, June 12, 2020

It's in the Cards - Solution

Activity Solution:

Round	1	2	3	4
Bahaa on Project X?	Yes	No	No	Yes
Helena on Project X?	No	Yes	No	Yes
Resulting card circle				

Your answers should be the card circles shown above or some rotation of these card circles.

Questions

1. The four rounds from the activity show what the result of the game would be in all four possible cases for Project X. Three of the four cases result in card circles that can be rotated so that they all look the same. The remaining case results in a card circle that looks different from the other three, no matter how the circle is rotated. Which three cases result in card circles that look the same (when rotated)?

   Solution: The first three rounds all have card circles that look the same (when rotated). These all have the Aces of Hearts separated by an Ace of Spades. The final round has a card circle that is different. This one has the Aces of Hearts beside each other in the circle.

   Can you explain why the cards end up like this? Think about the rules for how Bahaa and Helena place their cards in the pile.

2. If Bahaa is working on Project X, then he can use the card circle to determine whether or not Helena is also working on Project X. Can you explain why?

   Solution: If Bahaa is working on Project X, then we know we are in the situation of either "Round 1" or "Round 4" of the Activity. If Bahaa observes a final card circle that has the two Aces of Hearts separated, then he can be sure that Helena is *not* working on Project X. If Bahaa observes a final card circle that has the two Aces of Hearts beside each other in the circle, then he can be sure that Helena *is* working on Project X (and so knows it is safe to reveal that he is too).

3. If Bahaa is *not* working on Project X, then he *cannot* use the card circle to determine whether or not Helena is working on Project X. Can you explain why?

   Solution: If Bahaa is *not* working on Project X, then we know we are in the situation of either "Round 2" or "Round 3" of the Activity. Bahaa would observe a card circle with the Aces of Hearts separated, regardless of whether Helena is working on Project X or not. This means he cannot tell which is the case. (If Helena is indeed working on Project X, then the game will not give this away.)