CEMC at Home

Grade 11/12 - Monday, June 15, 2020
History of Computing

Computers can be found on our desks, in our pockets and even in our refrigerators! This is remarkable because modern computers have been around for less than 100 years. During this time, there has been a constant stream of new discoveries and advances in technology.

Use this online tool to arrange the following list of events in the history of computer science from earliest to most recent.

A. The ASCII is developed to create standard binary codes for 128 different characters.

B. Deep Blue is the first computer program to beat a human world chess champion.

C. Computers are used to determine that a perfect winning strategy does not exist for the game of checkers.

D. A robot named Elektro is built which responds to voice commands.

E. Konrad Zuse designs the Z3 electromechanical computer which is considered the first automatic programmable computer.

F. The Harvard Mark I mechanical computer is built and is used for military purposes during World War II.

G. An algorithm named *Quicksort* is developed to arrange objects in increasing or decreasing order.

H. Keyboard input is introduced as a way of entering data into a computer.

I. The Altair 8800 is the first personal computer to sell in large numbers.

J. Alan Turing uses the *halting problem* to establish a theoretical limit on the power of computers.

K. An international messaging service named Telex allows for the transfer of data and secure communications.

L. NASA and Grumman build the Apollo Guidance Computer, which is used during Apollo space missions.

M. Sun Microsystems develops the Java programming language.

N. Guido van Rossum creates and releases the Python programming language.

O. Animators create Cindy, the first human-like CGI (computer generated imagery) movie character.

More Info:

Our webpage Computer Science and Learning to Program is the best place to find the CEMC's computer science resources.

CEMC at Home

Grade 11/12 - Tuesday, June 16, 2020
Can You Find the Terms?

Can you find all of the given mathematics and computer science terms in the grid? Good Luck!

C	V	O	W	I	Y	J	I	V	L	Q	V	S	M	L	R	B	R	U	O
O	N	S	N	W	R	O	A	Z	S	F	G	M	D	D	G	A	Q	D	U
J	U	Y	S	W	T	Y	G	T	F	E	E	L	H	E	S	R	F	L	H
S	M	X	Y	A	E	V	A	Y	V	M	Q	U	S	R	H	I	N	D	U
U	N	M	C	H	M	Q	N	E	R	Y	Z	O	Z	I	B	L	Q	J	O
H	O	N	N	F	O	P	C	G	X	S	L	H	H	V	E	A	T	R	C
U	I	O	E	U	N	T	L	D	X	O	B	E	Y	A	C	G	O	E	S
K	T	I	I	R	O	O	S	I	G	I	W	X	W	T	N	S	V	T	U
Z	A	S	C	R	G	S	I	A	N	P	M	A	T	I	A	B	Z	E	T
L	R	R	I	M	I	K	R	T	E	G	J	D	J	V	T	H	N	M	J
T	E	U	F	V	R	I	U	R	A	V	L	E	W	E	I	O	A	A	I
G	T	C	F	J	T	W	M	J	J	Z	U	C	A	J	R	M	R	R	T
D	I	E	E	H	G	U	R	K	Y	E	I	I	Q	J	E	M	J	A	L
G	O	R	M	U	T	A	Q	F	U	Q	T	M	O	V	H	R	F	P	S
Y	Z	H	H	A	W	L	C	Q	D	L	Q	A	I	J	N	U	T	K	T
H	L	U	T	R	O	S	K	C	I	U	Q	L	N	T	I	M	W	H	A
M	J	I	P	E	I	R	A	D	I	A	N	R	G	G	P	V	W	I	C
B	O	K	P	U	M	C	B	Y	Z	N	S	C	L	C	E	O	H	J	K
N	N	E	G	T	N	A	N	I	M	I	R	C	S	I	D	N	O	J	E
C	C	U	J	K	C	H	S	K	P	E	J	T	V	P	Y	C	T	I	Y

DISCRIMINANT	OPTIMIZATION	ITERATION	PARAMETER
PERMUTATION	VECTOR	HEXADECIMAL	QUEUE
SAMPLING	LOGARITHM	RECURSION	METHOD
TANGENT	TRIGONOMETRY	STACK	QUICKSORT
DERIVATIVE	RADIAN	EFFICIENCY	INHERITANCE

More Info:

Check the CEMC at Home webpage on Wednesday, June 17 for the solution to Can You Find the Terms?

CEMC at Home

Grade 11/12 - Tuesday, June 16, 2020
Can You Find the Terms? - Solution

C	V	O	W	I	Y	J	I	V	L	Q	V	S	M	L	R	B	R	U	O
O	N	S	N	W	R	O	A	Z	S	F	G	M	D	D	G	A	Q	D	U
J	U	Y	S	W	T	Y	G	T	F	E	E	L	H	E	S	R	F	L	H
S	M	X	Y	A	E	V	A	Y	V	M	Q	U	S	R	H	I	N	D	U
U	N	M	C	H	M	Q	N	E	R	Y	Z	O	Z	I	B	L	Q	J	O
H	O	N	N	F	O	P	C	G	X	S	L	H	H	V	E	A	T	R	C
U	I	O	E	U	N	T	L	D	X	O	B	E	Y	A	C	G	O	E	S
K	T	I	I	R	O	O	S	I	G	I	W	X	W	T	N	S	V	T	U
Z	A	S	C	R	G	S	I	A	N	P	M	A	T	I	A	B	Z	E	T
L	R	R	I	M	I	K	R	T	E	G	J	D	J	V	T	H	N	M	J
T	E	U	F	V	R	I	U	R	A	V	L	E	W	E	I	O	A	A	I
G	T	C	F	J	T	W	M	J	J	Z	U	C	A	J	R	M	R	R	T
D	I	E	E	H	G	U	R	K	Y	E	I	I	Q	J	E	M	J	A	L
G	O	R	M	U	T	A	Q	F	U	Q	T	M	O	V	H	R	F	P	S
Y	Z	H	H	A	W	L	C	Q	D	L	Q	A	I	J	N	U	T	K	T
H	L	U	T	R	O	S	K	C	I	U	Q	L	N	T	I	M	W	H	A
M	J	I	P	E	I	R	A	D	I	A	N	R	G	G	P	V	W	I	C
B	O	K	P	U	M	C	B	Y	Z	N	S	C	L	C	E	O	H	J	K
N	N	E	G	T	N	A	N	I	M	I	R	C	S	I	D	N	O	J	E
C	C	U	J	K	C	H	S	K	P	E	J	T	V	P	Y	C	T	I	Y

DISCRIMINANT	OPTIMIZATION	ITERATION	PARAMETER
PERMUTATION	VECTOR	HEXADECIMAL	QUEUE
SAMPLING	LOGARITHM	RECURSION	METHOD
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CEMC at Home

Grade 11/12 - Wednesday, June 17, 2020
Piecing it Together

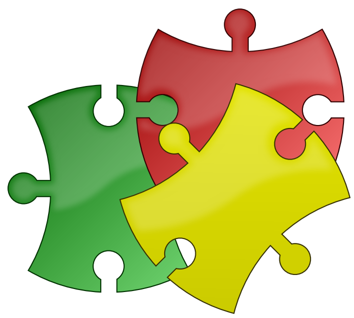
A positive integer is written on the back of each of three puzzle pieces. The numbers are not necessarily different but the sum of the three numbers is 14. Each puzzle piece is placed on a table so that the number cannot be seen. Alpha, Beta and Gamma each select one of the pieces, being careful not to let the other two see the number that is printed on the piece.

Alpha says, after looking at his puzzle piece, "I know that Beta and Gamma have different numbers."

Beta then says, "I already knew that all three numbers were different."

At this point, Gamma confidently exclaims, "I now know what all three of the numbers are!"

What were the numbers and who had which number?



More Info:

Check out the CEMC at Home webpage on Thursday, June 18 for a solution to Piecing it Together.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 11/12 - Wednesday, June 17, 2020
Piecing it Together - Solution

Problem:

A positive integer is written on the back of each of three puzzle pieces. The numbers are not necessarily different but the sum of the three numbers is 14. Each puzzle piece is placed on a table so that the number cannot be seen. Alpha, Beta and Gamma each select one of the pieces, being careful not to let the other two see the number that is printed on the piece.

Alpha says, after looking at his puzzle piece, "I know that Beta and Gamma have different numbers."

Beta then says, "I already knew that all three numbers were different."

At this point, Gamma confidently exclaims, "I now know what all three of the numbers are!"

What were the numbers and who had which number?

Solution:

We want to find the single solution to the problem $x + y + z = 14$ that satisfies the statements offered by Alpha, Beta and Gamma. It turns out that there are 78 different possible sums of three positive integers totalling 14. We could list all of the possible solutions and then proceed through the statements until we determine the required solution. Our approach will be far less exhausting. At the end of the solution, a justification of the existence of 78 possible positive integer solutions to the equation $x + y + z = 14$ will be provided.

The sum of three integers is 14, an even number. To generate an even sum, the three integers must all be even or one of the integers must be even and the other two integers must be odd.

Alpha says, after looking at his puzzle piece, "I know that Beta and Gamma have different numbers." How can Alpha KNOW? If his number is even then Beta and Gamma could both have even numbers or both have odd numbers to generate the sum 14. For example, if Alpha had the number 6, Beta could have 6 and Gamma could have 2 or Beta could have 4 and Gamma could have 4. If his number was even, Alpha would not KNOW that the other two numbers were different. However, if Alpha had an odd number, then one of the others must have an odd number and the other must have an even number. In this case, Alpha would KNOW that Beta and Gamma have different numbers. Therefore, Alpha must have an odd number.

Beta then says, "I already knew that all three numbers were different." Using the same logic as before, since Beta knows Alpha and Gamma have different numbers, Beta must have an odd number (and thus Gamma must have the even number). But how does Beta KNOW that all three numbers are different?

If Beta has a 1, 3 or 5, Alpha could have the same number. Beta would not know that all three numbers are different.

If Beta has a 7, 9, 11 or 13, Alpha could not have the same number in order for the three numbers to sum to 14. Furthermore, if Beta has a 7, Alpha must have a 5 or lower. If Beta has a 9, Alpha must have a 3 or lower. If Beta has an 11, Alpha must have a 1. Beta cannot have a 13 in order for the three numbers to sum to 14.

At this point our list of possible solutions has dropped from 78 to 6.

Beta	Alpha	Gamma
7	1	6
7	3	4
7	5	2
9	1	4
9	3	2
11	1	2

How does Gamma conclude, "I now know what all three of the numbers are!"?

If Gamma has a 4, Beta could have a 7 and Alpha could have a 3 or Beta could have a 9 and Alpha could have a 1. In this case, Gamma would not know what all three numbers are.

If Gamma has a 2, Beta could have a 7 and Alpha could have a 5 or Beta could have a 9 and Alpha could have a 3 or Beta could have an 11 and Alpha could have a 1. In this case, Gamma would not know what all three numbers are.

However, if Gamma has a 6, then Beta must have a 7 and Alpha must have a 1. This is the only possibility in which Gamma's statement is true.

Therefore, Alpha has a 1, Beta has a 7, and Gamma has a 6.

NOTE: It was mentioned at the beginning of the solution that there are 78 solutions to the equation $x+y+z=14$ where $x,\ y,\ z$ are positive integers. We can determine this by systematically counting the solutions:

x	y	z	# of possibilities
1	1	12	
1	2	11	
1	3	10	12
$\vdots$	$\vdots$	$\vdots$	
1	12	1	
2	1	11	
2	2	10	
2	3	9	11
$\vdots$	$\vdots$	$\vdots$	
2	11	1	
3	1	10	
3	2	9	
3	3	8	10
$\vdots$	$\vdots$	$\vdots$	
3	10	1	
$\vdots$	$\vdots$	$\vdots$	
10	1	3	
10	2	2	3
10	3	1	
11	1	2	
11	2	1	2
12	1	1	1

We can see that there are $12+11+10+9+8+7+6+5+4+3+2+1=78$ solutions to the equation $x+y+z=14$ where $x,\ y,\ z$ are positive integers.

CEMC at Home

Grade 11/12 - Thursday, June 18, 2020
Games and Puzzles

The CEMC has created lots of resources that we hope you have found interesting over the last few months. We also know that there are lots of online games and puzzles created by other organizations that make use of mathematics and logic. We've highlighted three examples below that you can explore for more mathematical fun!

Fraction Game from NCTM (https://www.nctm.org)

To make moves in this game, you need to use logic and number sense involving fractions.

Connect Three from NRICH (https://nrich.maths.org)

Use number sense to be the first to place three adjacent counters on the game board.

Kakuro Puzzles by Krazydad (https://krazydad.com)

A Kakuro puzzle is a kind of logic puzzle that is similar to a crossword puzzle. The objective of the puzzle is to insert a digit from 1 to 9 inclusive into each white cell such that the sum of the numbers in each "word" matches the clue associated with it and that no digit is duplicated in any "word".

You can find other interesting mathematics related games and puzzles online. Share your favourites using any forum you are comfortable with.

CEMC at Home

Grade 11/12 - Friday, June 19, 2020
Relay Day - Part 1

As part of the CEMC's Canadian Team Mathematics Contest, students participate in Math Relays. Just like a relay in track, you "pass the baton" from teammate to teammate in order to finish the race, but in the case of a Math Relay, the "baton" you pass is actually a number!

Read the following set of problems carefully.

Problem 1: The numbers $x+5$, 14, x, and 5 have an average of 9. What is the value of x?

Problem 2: *Replace N below with the number you receive.*
Each of the three lines having equations $x+Ny+8=0$, $5x-Ny+4=0$, and $3x-ky+1=0$ passes through the same point. What is the value of k?

Problem 3: *Replace N below with the number you receive.*
Quadrilateral $ABCD$ has vertices $A(0,3)$, $B(0,p)$, $C(N,10)$, and $D(N,0)$, where $p>3$ and $N>0$. The area of quadrilateral $ABCD$ is 50 square units. What is the value of p?

Notice that you can answer Problem 1 without any additional information.

In order to answer Problem 2, you first need to know the mystery value of N. The value of N used in Problem 2 will be the *answer* to Problem 1. (For example, if the answer you got for Problem 1 was 5 then you would start Problem 2 by replacing N with 5 in the problem statement.)

Similarly, you need the answer to Problem 2 to answer Problem 3. The value of N in Problem 3 is the *answer* that you got in Problem 2.

Now try the relay! You can use this tool to check your answers.

Follow-up Activity: Can you come up with your own Math Relay?
What do you have to think about when making up the three problems in the relay?
Can you just find three math problems and put them together to form a relay?

In Part 1 of this resource, you were asked to complete a relay on your own. But, of course, relays are meant to be completed in teams! In a team relay, three different people are in charge of answering the problems. Player 1 answers Problem 1 and passes their answer to Player 2; Player 2 takes Player 1's answer and uses it to answer Problem 2; Player 2 passes their answer to Player 3; and so on.

In Part 2 of this resource, you will find instructions on how to run a relay game for your friends and family. We will provide a relay for you to use, but you can also come up with your own!

CEMC at Home

Grade 4 to 12 - Friday, June 19, 2020
Relay Day - Part 2

Relay for Family and Friends

In Part 1 of this resource, you learned how to do a Math Relay. Now, why not try one out with family and friends!

You can put together a relay team and

- play just for fun, not racing any other team, or
- compete against another team in your household (if you have at least 6 people in total), or
- compete with a team from another family or household by
  - timing your team and comparing times with other teams to declare a winner, or
  - competing live using a video chat.

Here are the instructions for how to play.

Relay Instructions:

1. Decide on a team of three players for the relay. The team will be competing together.
2. Find someone to help administer the relay; let's call them the "referee".
3. Each teammate will be assigned a number: 1, 2, or 3. Player 1 will be assigned Problem 1, Player 2 will be assigned Problem 2, and Player 3 will be assigned Problem 3.
4. The three teammates should not see any of the relay problems in advance and should not talk to each other during the relay.
5. Right before the relay starts, the referee should hand out the correct relay problem to each of the players, with the problem statement face down (not visible).
6. The referee will then start the relay. At this time *all three players* can start working on their problems.

   Think about what Player 2 and Player 3 can do before they receive the value of N (the answer from the previous question passed to them by their teammate).
7. When Player 1 thinks they have the correct answer to Problem 1, they record their answer on the answer sheet and pass the sheet to Player 2. When Player 2 thinks they have the correct answer to Problem 2, they record their answer to the answer sheet and pass the sheet to Player 3. When Player 3 thinks they have the correct answer to Problem 3, they record their answer on the answer sheet and pass the sheet to the referee.

8. If all three answers passed to the referee are correct, then the relay is complete! If at least one answer is incorrect, then the referee passes the sheet back to Player 3.

9. At any time during the relay, the players on the team can pass the answer sheet back and forth between them, as long as they write nothing but their current answers on it and do not discuss anything. (For example, if Player 2 is sure that Player 1's answer must be incorrect, then Player 2 can pass the answer sheet back to Player 1, silently. This is a cue for Player 1 to check their work and try again.)

See the next page for a relay for family and friends! This includes instructions for the referee. You can also come up with your own relays to play. You can find many more relays from past CTMC contests on the CEMC's Past Contests webpage.

Sample answer sheets are provided below for you to use for your relays if you wish.

Answer Sheets:

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Relay For Three

Instructions for the Referee:

1. Multiple questions at different levels of difficulty are given for the different relay positions.
   - Assign one of the first three problems (marked "Problem 1") to Player 1.
   - Assign one of the next three problems (marked "Problem 2") to Player 2.
   - Assign one of the last three problems (marked "Problem 3") to Player 3.

   Choose a problem so that each player is comfortable with the level of their question. The level of difficulty of each question is represented using the following symbols:

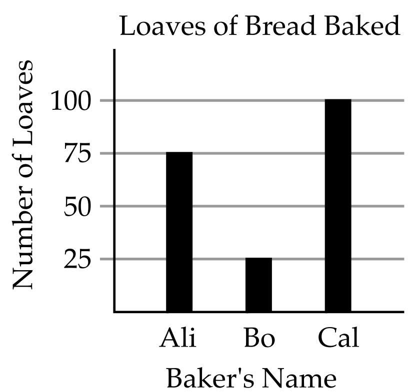
   - ● These questions should be accessible to most students in grade 4 or higher.
   - ■ These questions should be accessible to most students in grade 7 or higher.
   - ◆ These questions should be accessible to most students in grade 9 or higher.

2. Use this tool to find the answers for the relay problems in advance.

Relay Problems (to cut out):

Problem 1 ●

The graph shows the number of loaves of bread that three friends baked. How many loaves did Bo bake?

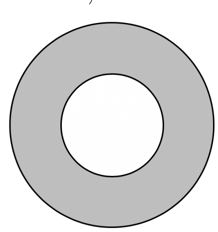


Problem 1 ■

An equilateral triangle has sides of length $x + 4$, $y + 11$, and 20. What is the value of $x + y$?

Problem 1 ◆

In the figure shown, two circles are drawn. If the radius of the larger circle is 10 and the area of the shaded region (in between the two circles) is 75π, then what is the *square* of the radius of the smaller circle?



Problem 2 ●

Replace N below with the number you receive.
Kwame writes the whole numbers in order from 1 to N (including 1 and N). How many times does Kwame write the digit '2'?

Problem 2 ■

Replace N below with the number you receive.
The total mass of three dogs is 43 kilograms. The largest dog has a mass of N kilograms, and the other two dogs have the same mass. What is the mass of each of the smaller dogs?

Problem 2 ◆

Replace N below with the number you receive.
The points $(6, 16)$, $(8, 22)$, and (x, N) lie on a straight line. Find the value of x.

Problem 3 ●

Replace N below with the number you receive.
You have some boxes of the same size and shape. If N oranges can fit in one box, how many oranges can fit in two boxes, in total?

Problem 3 ■

Replace N below with the number you receive.
One morning, a small farm sold 10 baskets of tomatoes, 2 baskets of peppers, and N baskets of zucchini. If the prices are as shown below, how much money, in dollars did the farm earn in total from these sales?

Basket of Tomatoes:	\$0.50
Basket of Peppers:	\$2.00
Basket of Zucchini:	\$1.00

Problem 3 ◆

Replace N with the number you receive.
Elise has N boxes, each containing x apples. She gives 12 apples to her sister. She then gives 20% of her remaining apples to her brother. After this, she has 120 apples left. What is the value of x?