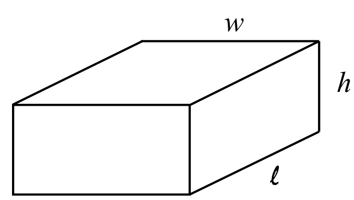
CEMC at Home

Grade 9/10 - Monday, June 1, 2020
Contest Day 5

Today's resource features one question from the recently released 2020 CEMC Mathematics Contests.

2020 Galois Contest, #2

For a rectangular prism with length ℓ, width w, and height h as shown, the surface area is given by the formula $A = 2\ell w + 2\ell h + 2wh$ and the volume is given by the formula $V = \ell wh$.



(a) What is the surface area of a rectangular prism with length 2 cm, width 5 cm, and height 9 cm?

(b) A rectangular prism with height 10 cm has a square base. The volume of the prism is 160 cm^3. What is the side length of the square base?

(c) A rectangular prism has a square base with area 36 cm^2. The surface area of the prism is 240 cm^2. Determine the volume of the prism.

(d) A rectangular prism has length k cm, width $2k$ cm, and height $3k$ cm, where $k > 0$. The volume of the prism is x cm^3. The surface area of the prism is x cm^2. Determine the value of k.

More Info:

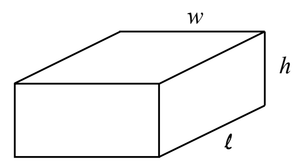
Check out the CEMC at Home webpage on Monday, June 8 for a solution to the Contest Day 5 problem.

CEMC at Home

Grade 9/10 - Monday, June 1, 2020
Contest Day 5 - Solution

A solution to the contest problem is provided below.

2020 Galois Contest, #2



For a rectangular prism with length ℓ, width w, and height h as shown, the surface area is given by the formula $A = 2\ell w + 2\ell h + 2wh$ and the volume is given by the formula $V = \ell wh$.

(a) What is the surface area of a rectangular prism with length 2 cm, width 5 cm, and height 9 cm?

(b) A rectangular prism with height 10 cm has a square base. The volume of the prism is 160 cm^3. What is the side length of the square base?

(c) A rectangular prism has a square base with area 36 cm^2. The surface area of the prism is 240 cm^2. Determine the volume of the prism.

(d) A rectangular prism has length k cm, width $2k$ cm, and height $3k$ cm, where $k > 0$. The volume of the prism is x cm^3. The surface area of the prism is x cm^2. Determine the value of k.

Solution:

(a) The surface area of a rectangular prism is given by the formula $A = 2\ell w + 2\ell h + 2wh$.
Thus, the rectangular prism with length 2 cm, width 5 cm, and height 9 cm has surface area $2(2)(5) + 2(2)(9) + 2(5)(9) = 20 + 36 + 90 = 146$ cm^2.

(b) The volume of a rectangular prism is given by the formula $V = \ell wh$.
If the rectangular prism has a square base, then $\ell = w$ and so $V = \ell^2 h$.
Substituting $V = 160$ cm^3 and $h = 10$ cm, we get $160 = \ell^2(10)$ or $\ell^2 = 16$, and so $\ell = 4$ cm (since $\ell > 0$).
Therefore, the side length of the square base of a rectangular prism with height 10 cm and volume 160 cm^3 is 4 cm.

(c) If a rectangular prism has a square base, then $\ell = w$.
Since the area of the base is 36 cm^2, then $36 = \ell \cdot w = \ell^2$, and so $\ell = w = \sqrt{36} = 6$ cm (since $\ell > 0$).
If the surface area of this prism is 240 cm^2, then substituting the values of ℓ and w, we get $240 = 2(6)(6) + 2(6)h + 2(6)h$ or $240 = 72 + 24h$, and so $h = \dfrac{240 - 72}{24} = 7$ cm.
Thus, the volume of the prism is $\ell wh = (6)(6)(7) = 252$ cm^3.

See the next page for a solution to part (d).

(d) Substituting into the formula for volume, we get $x = k(2k)(3k)$ or $x = 6k^3$.
Substituting into the formula for surface area, we get $x = 2(k)(2k) + 2(k)(3k) + 2(2k)(3k)$ or $x = 4k^2 + 6k^2 + 12k^2 = 22k^2$.
Equating the two expressions that are each equal to x and solving, we get

$$\begin{aligned} 6k^3 &= 22k^2 \\ 6k^3 - 22k^2 &= 0 \\ 2k^2(3k - 11) &= 0 \end{aligned}$$

Since $k > 0$, then $3k - 11 = 0$ and so $k = \dfrac{11}{3}$.

CEMC at Home

Grade 9/10 - Tuesday, June 2, 2020
Famous Mathematicians

Throughout human history, many mathematicians have made significant contributions to the subject. These important historical figures often lead fascinating lives filled with interesting stories. Five of these mathematicians are listed below.

Al-Khwārizmī	He was a 9th century mathematician from Baghdad where he was the head of the library *House of Wisdom*. The title of one of his books gave us the word “algebra”.
Blaise Pascal	He was a 17th French century mathematician whose work laid the foundation for the modern theory of probability. He also contributed greatly to the areas of physics and religion.
William Tutte	He was born in England and was an important codebreaker during World War II. In 1962, he started working at the University of Waterloo where his work greatly shaped the area of graph theory.
Grigori Perelman	A current Russian mathematician who was awarded the Fields Medal in 2006, but declined it. His work in the area of geometry is important for the Poincaré conjecture, a famous result about topology.
Maryam Mirzakhani	She was the first woman to be awarded the prestigious Fields Medal. She was born in Iran and then studied and worked in the United States before breast cancer took her life in 2017.

Choose two of these five mathematicians and for each one you choose:

1. Do some online research to determine an additional interesting fact about the mathematician.
2. Try to find a connection between something you have studied in a recent mathematics class and the mathematical work of this historical figure.
3. If you had the chance to go back in time and meet this mathematician, what question would you ask them?

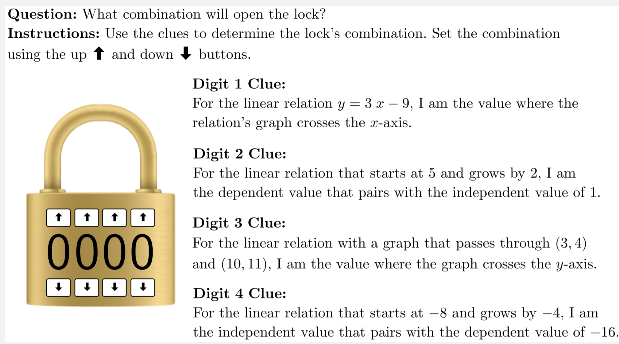
More Info: The CEMC Pascal Math Contest is named in honour of Blaise Pascal.

CEMC at Home

Grade 9/10 - Wednesday, June 3, 2020
Interact with Mathematics

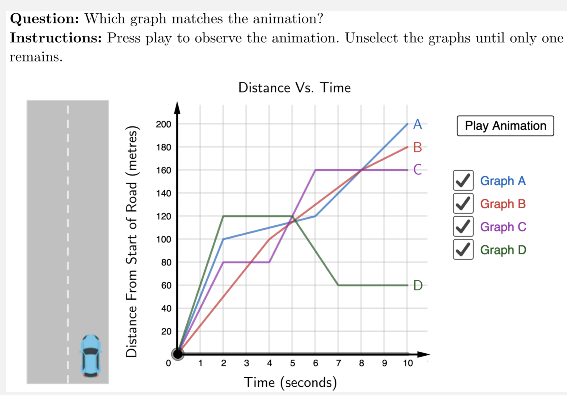
Technology can help us make mathematical discoveries and learn about mathematical objects. Three online examples of this from different areas of mathematics are featured below.

Unknown Linear Values: Use the properties of linear relations to find a lock's combination.



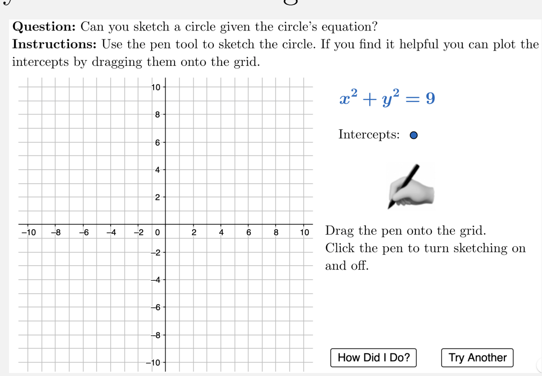
Link to App: https://www.geogebra.org/m/nzvvj3dh

Story Graphs: A car is driving down a road. Identify the graph that describes the car's trip.



Link to App: https://www.geogebra.org/m/n4mtvugh

Sketching a Circle: Try your hand at drawing a freehand circle.



Link to App: https://www.geogebra.org/m/ukedzvnb

More Info: CEMC courseware lessons feature hundreds of interactive mathematics applications. For the Grade 9/10/11 CEMC courseware, an interactive library has been built which allows you to perform a keyword search and/or display only the applications from a given strand, unit or lesson.

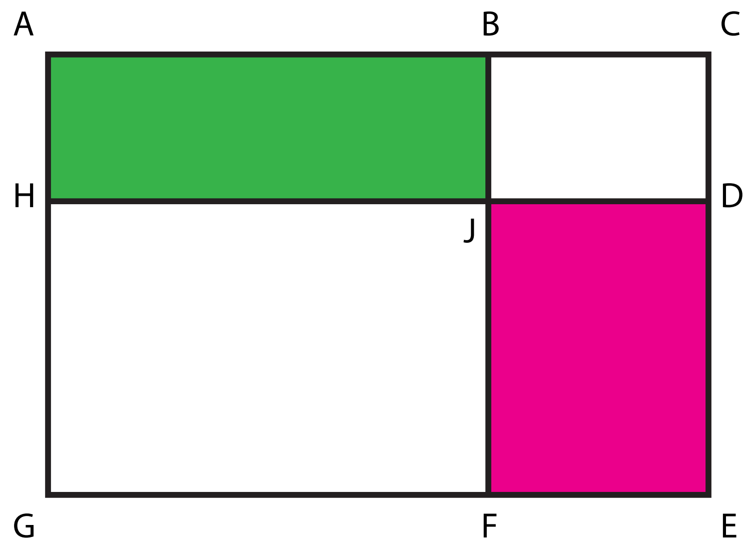
CEMC at Home

Grade 9/10 - Thursday, June 4, 2020
Maximize the Area

Two rectangles, $ABJH$ and $JDEF$, with integer side lengths, share a common corner at J such that HJD and BJF are perpendicular line segments. The two rectangles are enclosed by a larger rectangle $ACEG$, as shown.

The area of rectangle $ABJH$ is 6 cm^2 and the area of rectangle $JDEF$ is 15 cm^2.

Determine the largest possible area of the rectangle $ACEG$. Note that the diagram is not intended to be to scale.



More Info:

Check out the CEMC at Home webpage on Friday, June 5 for a solution to Maximize the Area.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

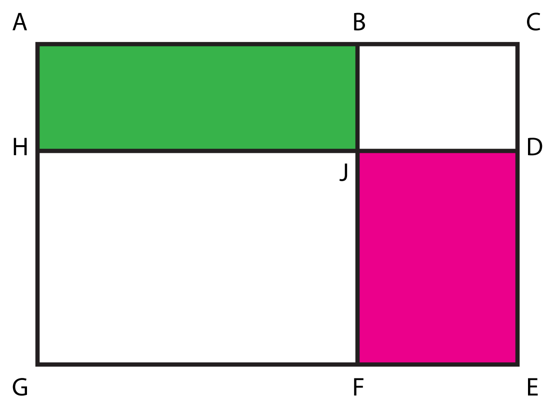
Grade 9/10 - Thursday, June 4, 2020
Maximize the Area - Solution

Problem:

Two rectangles, $ABJH$ and $JDEF$, with integer side lengths, share a common corner at J such that HJD and BJF are perpendicular line segments. The two rectangles are enclosed by a larger rectangle $ACEG$, as shown.

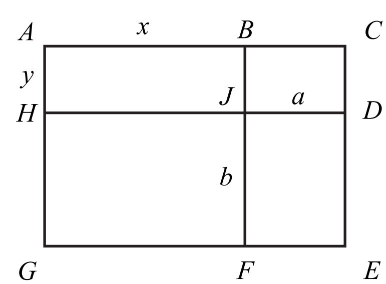
The area of rectangle $ABJH$ is 6 cm^2 and the area of rectangle $JDEF$ is 15 cm^2.

Determine the largest possible area of the rectangle $ACEG$. Note that the diagram is not intended to be to scale.



Solution:

Let $AB = x$, $AH = y$, $JD = a$ and $JF = b$.



Then,

$$\begin{aligned} AB &= HJ = GF = x, \\ AH &= BJ = CD = y, \\ BC &= JD = FE = a, \text{ and} \\ HG &= JF = DE = b. \end{aligned}$$

Also,

$$\begin{aligned} \text{area}(ACEG) &= \text{area}(ABJH) + \text{area}(BCDJ) + \text{area}(JDEF) + \text{area}(HJFG) \\ &= 6 + ya + 15 + xb \\ &= 21 + ya + xb \end{aligned}$$

Since the area of rectangle $ABJH$ is 6 cm^2 and the side lengths of $ABJH$ are integers, then the side lengths must be 1 and 6 or 2 and 3. That is, $x = 1$ cm and $y = 6$ cm, $x = 6$ cm and $y = 1$ cm, $x = 2$ cm and $y = 3$ cm, or $x = 3$ cm and $y = 2$ cm.

Since the area of rectangle $JDEF$ is 15 cm^2 and the side lengths of $JDEF$ are integers, then the side lengths must be 1 and 15 or 3 and 5. That is, $a = 1$ cm and $b = 15$ cm, $a = 15$ cm and $b = 1$ cm, $a = 3$ cm and $b = 5$ cm, or $a = 5$ cm and $b = 3$ cm.

To maximize the area, we need to pick the values of x, y, a, b which make $ya + xb$ as large as possible. We will now break into cases based on the possible side lengths of $ABJH$ and $JDEF$ and calculate the area of $ACEG$ in each case. We do not need to try all 16 possible pairings, because trying $x = 1$ cm and $y = 6$ cm with the four possibilities of a and b will give the same 4 areas, in some order, as trying $x = 6$ cm and $y = 1$ cm with the four possibilities of a and b. Similarly, trying $x = 2$ cm and $y = 3$ cm with the four possibilities of a and b will give the same 4 areas, in some order, as trying $x = 3$ cm and $y = 2$ cm with the four possibilities of a and b. (As an extension, we will leave it to you to think about why this is the case.)

<u>Case 1:</u> $x = 1$ cm, $y = 6$ cm and $a = 1$ cm, $b = 15$ cm

$$\text{area}(ACEG) = 21 + ya + xb = 21 + 6(1) + 1(15) = 42 \text{ cm}^2$$

<u>Case 2:</u> $x = 1$ cm, $y = 6$ cm and $a = 15$ cm, $b = 1$ cm

$$\text{area}(ACEG) = 21 + ya + xb = 21 + 6(15) + 1(1) = 112 \text{ cm}^2$$

<u>Case 3:</u> $x = 1$ cm, $y = 6$ cm and $a = 3$ cm, $b = 5$ cm

$$\text{area}(ACEG) = 21 + ya + xb = 21 + 6(3) + 1(5) = 44 \text{ cm}^2$$

<u>Case 4:</u> $x = 1$ cm, $y = 6$ cm and $a = 5$ cm, $b = 3$ cm

$$\text{area}(ACEG) = 21 + ya + xb = 21 + 6(5) + 1(3) = 54 \text{ cm}^2$$

<u>Case 5:</u> $x = 2$ cm, $y = 3$ cm and $a = 1$, $b = 15$ cm

$$\text{area}(ACEG) = 21 + ya + xb = 21 + 3(1) + 2(15) = 54 \text{ cm}^2$$

<u>Case 6:</u> $x = 2$ cm, $y = 3$ cm and $a = 15$, $b = 1$ cm

$$\text{area}(ACEG) = 21 + ya + xb = 21 + 3(15) + 2(1) = 68 \text{ cm}^2$$

<u>Case 7:</u> $x = 2$ cm, $y = 3$ cm and $a = 3$, $b = 5$ cm

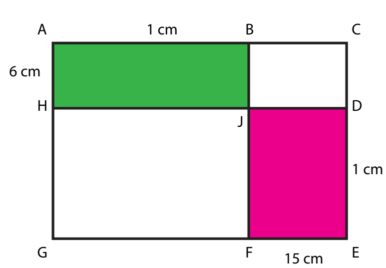
$$\text{area}(ACEG) = 21 + ya + xb = 21 + 3(3) + 2(5) = 40 \text{ cm}^2$$

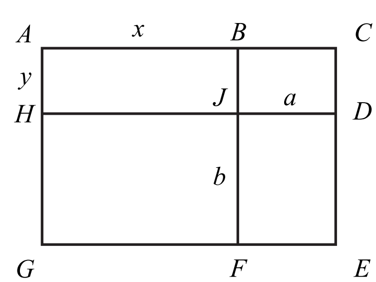
<u>Case 8:</u> $x = 2$ cm, $y = 3$ cm and $a = 5$, $b = 3$ cm

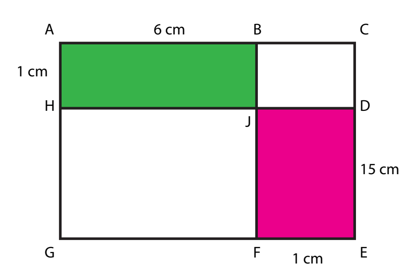
$$\text{area}(ACEG) = 21 + ya + xb = 21 + 3(5) + 2(3) = 42 \text{ cm}^2$$

We see that the maximum area is 112 cm^2, and occurs when $x = 1$ cm, $y = 6$ cm and $a = 15$ cm, $b = 1$ cm. It will also occur when $x = 6$ cm, $y = 1$ cm and $a = 1$ cm, $b = 15$ cm.

The following diagrams show the calculated values placed on the original diagram. The diagram was definitely not drawn to scale! Both solutions produce rectangles with dimensions 7 cm by 16 cm, and area 112 cm^2.







CEMC at Home

Grade 9/10 - Friday, June 5, 2020
Math and CS in the News

Most weeks, our CEMC Homepage provides a link to a story in the media about mathematics and/or computer science. These stories show us how important mathematics and computer science are in today's world. They are a great source for discussions.

Using this article from CBC News, think about the following questions. (URL also provided below.)

1. What do you think someone means when they say that humans are more *intelligent* than other members of the animal kingdom?

2. What is artificial intelligence? Can you name two ways in which you use artificial intelligence yourself?

3. What advantages and disadvantages do you see to artificial intelligence?

4. Predict the future: How will artificial intelligence change in 20 years?

URL of the article:
https://www.cbc.ca/news/technology/artificial-intelligence-human-brain-to-merge-in-2030s-says-futurist-kurzweil-1.3100124

More Info:

A full archive of past posts can be found in our Math and CS in the News Archive. Similar resources for other grades may also be of interest.