CEMC at Home

Grade 9/10 - Monday, April 27, 2020
Race Track

You Will Need:

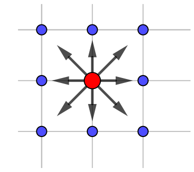
- One to four players
- A Race Track on grid paper
  A Race Track is provided for you on the second to last page. You are also given a blank grid on the last page where you can create your own track!
- A different coloured pen or pencil for each player.
  Since you are likely to play this game multiple times, you may want to place the Race Track inside a sheet protector and then use dry erase markers to play instead. If you do not have a sheet protector, try using clear tape to create an erasable surface for the track.

How to Play:

1. Start with a Race Track.
2. Players take turns. Decide which player will go first, second, and so on.
3. To start the game, each player must place their "car" at a different place on the starting line. Players can do so, one at a time, based on the chosen order of the players.
   Placing your "car" on the starting line actually means drawing a dot on top of one of the grid points lying on the starting line. Each player needs to place their car on a different grid point. You can place your car on the boundary of the track.
4. On each turn, the current player will move their car according to the allowed moves in the game.
   Moving your car means placing a new dot at a new grid point on the track. See below for a description of the rules allowed in the game.
5. The winner is the first player to complete a lap, that is, the first player whose car crosses the finish line.

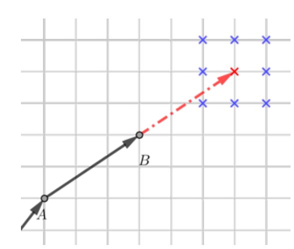
Allowed Moves

All moves must be from one grid point to another grid point. Each grid point has eight neighbouring grid points as shown to the right.



From the starting line, each player's first move must be moving their car to one of the eight neighbours of their starting position.

For all subsequent moves, players must move their car the same distance in the same direction as their previous move, or to one of the eight neighbours of that final position. For example, if arrow AB represents the player's previous move as shown to the right, then on this player's next turn, they can move their car either to the spot marked with a red $\times$, or to any of the eight neighbours of the point with a red $\times$, each marked with a blue $\times$.

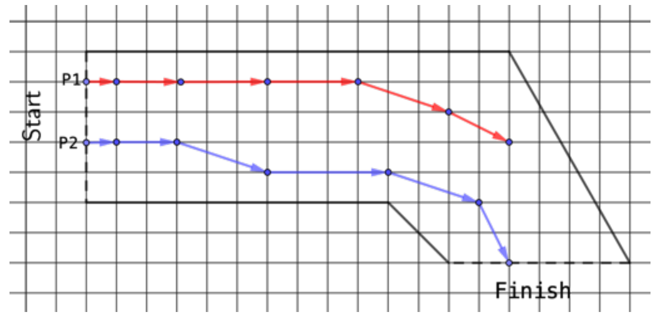


Notice that a move from B to the red $\times$ is represented by an arrow that is the same length and in the same direction as the arrow from A to B.

A car cannot be moved to a grid point where another car is already located.

Here is an example of a two player game on a simple Race Track.

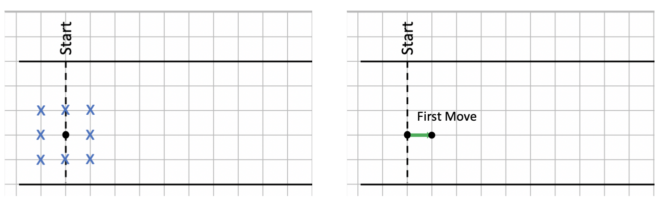
Player 1 (P1) goes first.
Player 2 (P2) wins in 6 moves.



Here is an explanation of Player 2's first four moves in the sample game above.

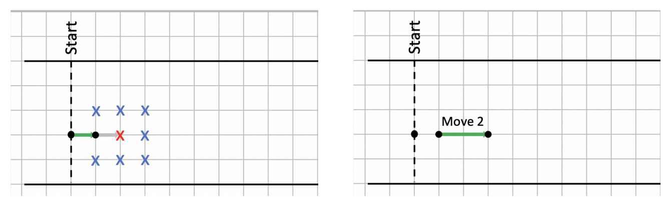
First Move

P2 can move their car to any of the eight locations marked with an ×. P2 chooses to move one grid point to the right. Note that P2 could move backward, but this may not be the best choice if P2 is hoping to complete a lap quickly. (Moving backwards to a finish line does not count!)



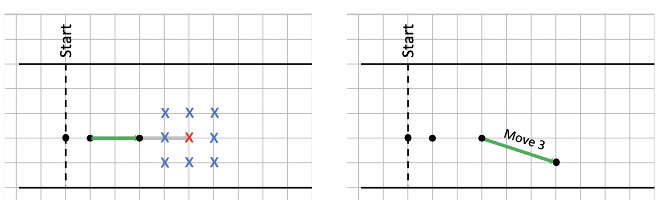
Second Move

Since P2's previous move was one grid point to the right, we place the red × one grid point to the right of P2's current position. P2 can move to this × or any of the eight locations surrounding it (including P2's current position). P2 moves two grid points to the right.



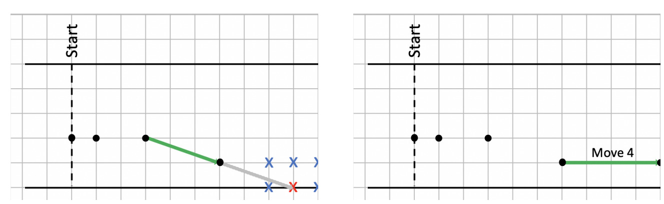
Third Move

Since P2's previous move was two grid points to the right, we place the red × two grid points to the right of P2's current position. P2 can move to this × or any of the eight locations surrounding it. P2 moves three grid points to the right and one grid point down.



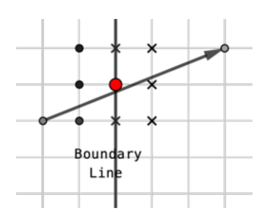
Fourth Move

Since P2's previous move was three to the right and one down, we place the red × three to the right and one down from P2's current position. P2 can move to this × or to *some* of the eight locations surrounding it. P2 moves four grid points to the right.



Dealing with the Boundary

During the game, there may be a time when a player has no choice but to move their car onto or through a boundary line of the Race Track on their turn (as shown to the right). If this happens, then the player places their car at the grid point nearest to where their move touched the boundary (as shown by the red dot). On this player's next turn, they move their car to one of the eight neighbours of their current place (red dot) that *lies inside the track* (shown with black dots).

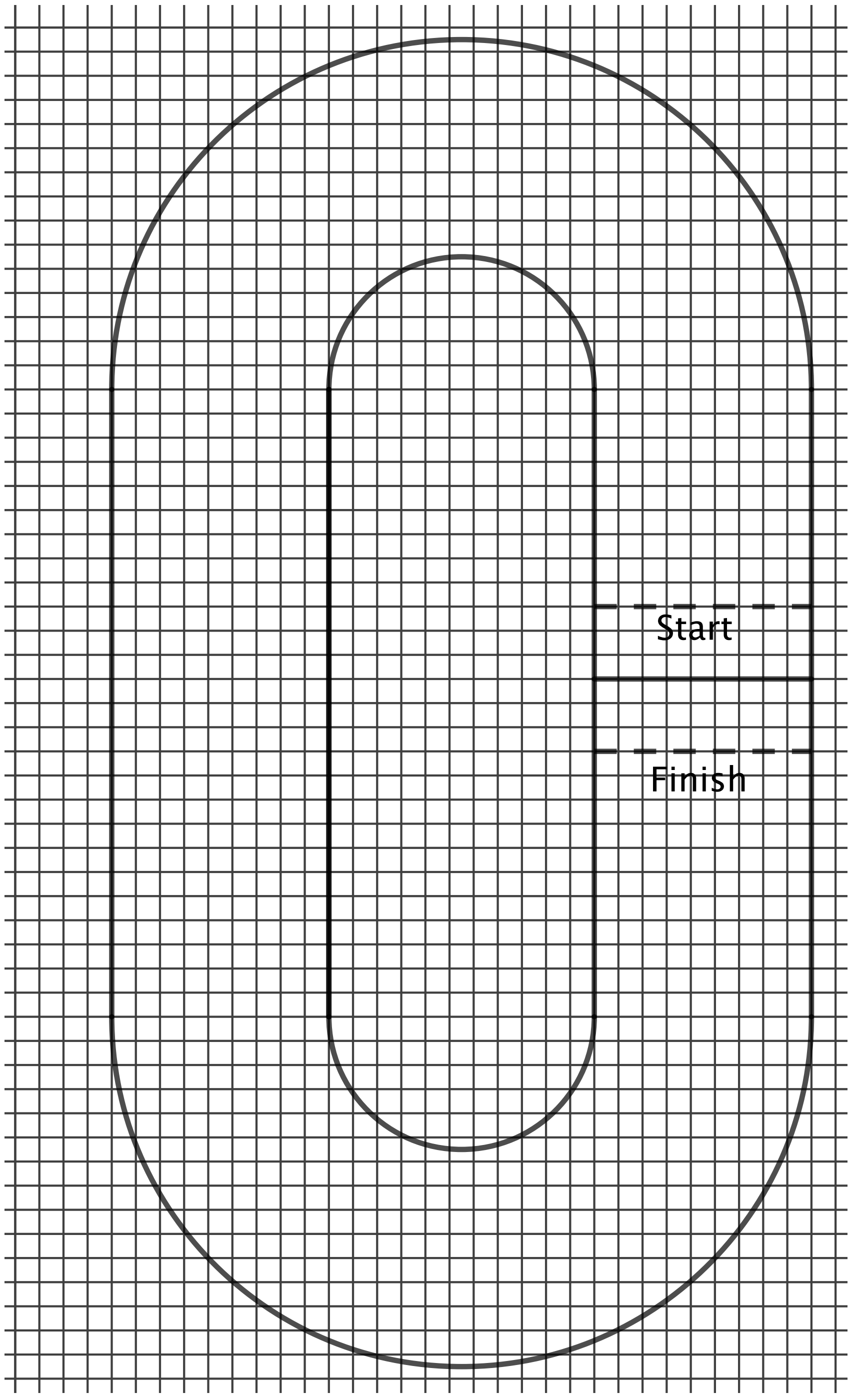


Let's Play!

Play this game a number of times using the track given on the next page. Alternate which player goes first. Were you able to figure out how to avoid hitting a boundary of the Race Track?

More Info: A *vector* is defined as a quantity which has both a magnitude and a direction. In Race Track, each move can be represented by a vector. To learn more about vectors see this Math Circles lesson.

Sample Race Track



Make Your Own Race Track!

You can use your own grid paper or the grid below. Add some sharp corners for an extra challenge!

CEMC at Home

Grade 9/10 - Tuesday, April 28, 2020

More Counting

In some areas of mathematics, we study things called *permutations*. A permutation of a collection of objects is an arrangement of the objects in some order.

For example, consider the integers 1, 2, and 3. There are six different ways to arrange these three objects, in some order, and so there are six permutations of these objects. The permutations are given below:

$$(1,\ 2,\ 3) \qquad (1,\ 3,\ 2) \qquad (3,\ 1,\ 2) \qquad (3,\ 2,\ 1) \qquad (2,\ 1,\ 3) \qquad (2,\ 3,\ 1)$$

In the questions below, we will work with permutations of consecutive integers, and we will think about a particular type of permutation which we will call a VALROBSAR permutation.

> A permutation will be called a **VALROBSAR** permutation if *no* integer in the permutation has two neighbours that both are less than it.
>
> *Two integers in the permutation are neighbours if they appear directly beside each other.*

From our example above, the permutations $(1,\ 3,\ 2)$ and $(2,\ 3,\ 1)$ are *not* VALROBSAR permutations. This is because, in each of these permutations, the integer 3 has a smaller integer immediately to its left and immediately to its right. That is, the integer 3 has two neighbours that are both less than 3.

The other four permutations shown above *are* VALROBSAR permutations. For example, let's look at the permutation $(3,\ 1,\ 2)$. The integer 3 has only one neighbour and so does not have two neighbours less than 3. The integer 2 also has only one neighbour and so does not have two neighbours less than 2. The integer 1 has two neighbours but they are both greater than 1. As a second example, let's look at $(3,\ 2,\ 1)$. The integers 3 and 1 each have only one neighbour and so do not have two neighbours less than themselves, and the integer 2 has two neighbours but only one of them is less than 2. You should work through the remaining two permutations on your own to verify that they are indeed VALROBSAR permutations.

Problems:

1. List all permutations of the integers 1, 2, 3, and 4.
2. How many of the permutations of the integers 1, 2, 3, and 4 are VALROBSAR permutations?
3. How many VALROBSAR permutations are there of the integers 1, 2, 3, 4, and 5?
4. How many VALROBSAR permutations are there of the integers 1, 2, 3, 4, 5, and 6?

Extension: Can you see a pattern forming based on your work in problems 1. to 4.? Suppose that n is a positive integer satisfying $n \geq 2$ and consider the permutations of the integers $1, 2, 3, 4, \ldots, n$. What can you say about the number of VALROBSAR permutations of these integers?

More Info:

Check out the CEMC at Home webpage on Tuesday, May 5 for a solution to More Counting.

CEMC at Home

Grade 9/10 - Tuesday, April 28, 2020
More Counting - Solution

A permutation will be called a **VALROBSAR** permutation if *no* integer in the permutation has two neighbours that both are less than it.

Two integers in the permutation are neighbours if they appear directly beside each other.

Problems:

1. List all permutations of the integers 1, 2, 3, and 4.
2. How many of the permutations of the integers 1, 2, 3, and 4 are VALROBSAR permutations?
3. How many VALROBSAR permutations are there of the integers 1, 2, 3, 4, and 5?
4. How many VALROBSAR permutations are there of the integers 1, 2, 3, 4, 5, and 6?

Solutions:

1. There are 24 permutations of the four integers:

1 2 3 4	1 2 4 3	1 3 2 4	1 3 4 2	1 4 2 3	1 4 3 2
2 1 3 4	2 1 4 3	2 3 1 4	2 3 4 1	2 4 1 3	2 4 3 1
3 1 2 4	3 1 4 2	3 2 1 4	3 2 4 1	3 4 1 2	3 4 2 1
4 1 2 3	4 1 3 2	4 2 1 3	4 2 3 1	4 3 1 2	4 3 2 1

2. There are 8 VALROBSAR permutations. They are the red permutations in the table below:

1 2 3 4	1 2 4 3	1 3 2 4	1 3 4 2	1 4 2 3	1 4 3 2
2 1 3 4	2 1 4 3	2 3 1 4	2 3 4 1	2 4 1 3	2 4 3 1
3 1 2 4	3 1 4 2	3 2 1 4	3 2 4 1	3 4 1 2	3 4 2 1
4 1 2 3	4 1 3 2	4 2 1 3	4 2 3 1	4 3 1 2	4 3 2 1

3. Looking at the solution to 2., you might have noticed that all of the VALROBSAR permutations have the number 4 at one of the two ends. It will be helpful to think about why this must be true. In order to be a VALROBSAR permutation, every number must have at most one neighbour that is smaller than it. Since 4 is the largest among 1, 2, 3, and 4, if a permutation has 4 in one of the two middle positions, then 4 is guaranteed to have two neighbours that are smaller than it. Hence, a permutation cannot be a VALROBSAR permutation unless 4 is at one of the ends. It is worth noting that there are permutations with a 4 at the end that fail to be VALROBSAR permutations, for example, 4132.

   We now focus our attention on the VALROBSAR permutations of 1, 2, 3, 4, and 5. By the same reasoning as in the previous paragraph, a VALROBSAR permutation of these integers must have 5 at one of the ends. The next key observation is that if we "remove" this 5 from the end, what remains will be a VALROBSAR permutation of 1, 2, 3, and 4. This is because removing a number on the end of a permutation does not introduce any new neighbouring pairs, and so cannot cause a failure of the VALROBSAR condition.

   This means all of the VALROBSAR permutations of 1, 2, 3, 4, and 5 take the form $5abcd$ or $abcd5$ where $abcd$ is a VALROBSAR permutation of 1, 2, 3, and 4. On the other hand, if we take a VALROBSAR permutation of 1, 2, 3, and 4 and place a 5 on either end, what results is a VALROBSAR permutation of 1, 2, 3, 4, and 5. To see this, suppose $abcd$ is a VALROBSAR permutation of 1, 2, 3, and 4 and consider the permutation $5abcd$. The neighbours of b, c, and d in $5abcd$ are the same as they are in the permutation $abcd$. Since we are assuming $abcd$ is a VALROBSAR permutation, each of a, b, and c has at most one neighbour smaller than it in $abcd$, and hence, has at most one neighbour smaller than it in $5abcd$. We know that a is equal to one of 1, 2, 3, and 4, so $a < 5$, which means a has at most one neighbour smaller than it in $5abcd$ (namely, b *could* be smaller than a). The number 5 is at the end of the permutation, so it cannot possibly cause a failure of the VALROBSAR condition.

   We are now able to quickly count the number of VALROBSAR permutations of 1, 2, 3, 4, and 5. Using the discussion above, we get all of *these* VALROBSAR permutations by taking a VALROBSAR permutation of 1, 2, 3, and 4 and placing a 5 on one of the two ends. There are 8 VALROBSAR permutations of 1, 2, 3, and 4, so this gives $2 \times 8 = 16$ VALROBSAR permutations of 1, 2, 3, 4, and 5. Furthermore, each of these 16 VALROBSAR permutations must be different. *Can you see why?*

   The VALROBSAR permutations of 1, 2, 3, 4, and 5 are given below:

1 2 3 4 5	2 1 3 4 5	3 1 2 4 5	3 2 1 4 5
4 1 2 3 5	4 2 1 3 5	4 3 1 2 5	4 3 2 1 5
5 1 2 3 4	5 2 1 3 4	5 3 1 2 4	5 3 2 1 4
5 4 1 2 3	5 4 2 1 3	5 4 3 1 2	5 4 3 2 1

   Note: There are more direct ways of counting these permutations without building on the permutations of 1, 2, 3, and 4. (An idea of this form will be discussed in the Extension on the last page.) The method presented above doesn't just give us an easy to count the "next order" of VALROBSAR permutations, but also gives an easy way to list them (based on the "previous list"). When using permutations in mathematics, sometimes we are interested in only the count, and sometimes we are interested in the actual list of permutations. It is often helpful to think about building them in "stages" like we have done here.

4. Similar to the argument in the previous solution, a VALROBSAR permutation of 1, 2, 3, 4, 5, and 6 must have the 6 at one of the ends and what remains after removing the 6 must be a VALROBSAR permutation of 1, 2, 3, 4, and 5. Furthermore, we get a VALROBSAR permutation of 1, 2, 3, 4, 5, and 6 by taking any VALROBSAR permutation of 1, 2, 3, 4, and 5 and adding a 6 to either end of the permutation. For every choice of a VALROBSAR permutation of 1, 2, 3, 4, and 5 and choice of which side to add the 6, we get a VALROBSAR permutation of 1, 2, 3, 4, 5, and 6. There are 16 VALROBSAR permutations of 1, 2, 3, 4, and 5, so this means there are $2 \times 16 = 32$ VALROBSAR permutations of 1, 2, 3, 4, 5, and 6.

Extension: Can you see a pattern forming based on your work in problems 1. to 4.? Suppose that n is a positive integer satisfying $n \geq 2$ and consider the permutations of the integers $1, 2, 3, 4, \ldots, n$. What can you say about the number of VALROBSAR permutations of these integers?

Discussion:

To recap, we found the following counts of the VALROBSAR permutations:

- $n = 4$: $2^3 = 8$ VALROBSAR permutations
- $n = 5$: $2^4 = 16$ VALROBSAR permutations
- $n = 6$: $2^5 = 32$ VALROBSAR permutations

You might guess from this pattern that the number of VALROBSAR permutations of $1, 2, \ldots, n$ is 2^{n-1}, in general. In fact, the arguments in 3. and 4. actually showed why the number of VALROBSAR permutations seemed to double at each stage, and these arguments can be used to justify this formula.

Here is a more direct way to count the number of VALROBSAR permutations of $1, 2, \ldots, n$. Let's build such a permutation by placing each integer in turn and keeping track of how many choices we have at each stage. (This argument could have been used in 3. and 4. as well.)

First, consider n, the largest integer. No matter where you place it in the permutation it will be larger than its neighbours, and so it must have only one neighbour. The integer n must be placed at an end of the permutation, either first or last, which means you have 2 choices.

$$n \,\_\,\_\,\_ \,\ldots\, \_ \qquad\qquad \text{or} \qquad\qquad \_\,\_\,\_ \,\ldots\, \_\, n$$

Once you place n, you have $(n-1)$ places left for the remaining integers $1, 2, \ldots, n-1$. As with n above, you have 2 choices for where to place the integer $n-1$: beside n or at the other end of the permutation.

For example, if we placed n as in the left image shown above, then the two leftmost images below show the choices for where to place $n-1$. *Can you see why $n-1$ must be placed like this in order to form a VALROBSAR permutation?* In the other case above, the choices are shown below on the right.

$$n\ (n-1) \,\_\,\_ \,\ldots\, \_ \quad \text{or} \quad n \,\_\,\_\,\_ \,\ldots\, (n-1) \quad \text{or} \quad (n-1) \,\_\,\_ \,\ldots\, \_\, n \quad \text{or} \quad \_\,\_\,\_ \,\ldots\, (n-1)\ n$$

After each placement of the largest remaining integer, you then have 2 choices for where to place the next largest integer. This continues until you have placed the integer 2, and then the integer 1 must go in the only remaining place.

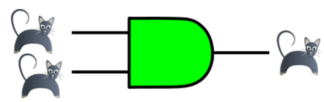
Thus, for each of the first $n-1$ integers, $n, (n-1), (n-2), (n-3), \ldots, 2$, you have 2 choices of where each is placed, and then the 1 goes in the last open place. Thus, the number of VALROBSAR permutations of the integers $1, 2, \ldots, n$ is 2^{n-1}.

CEMC at Home

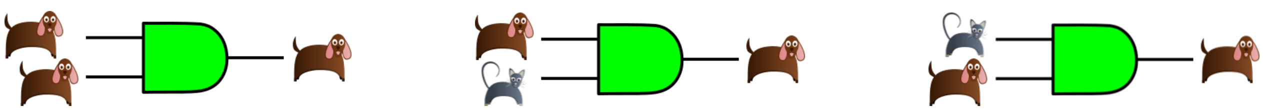
Grade 9/10 - Wednesday, April 29, 2020
Magic Carpets

In a magic castle there are magic carpets and an unlimited supply of toy cats and dogs. The magic carpets behave as follows:

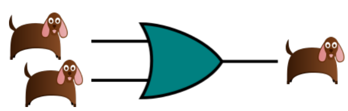
* If two cats walk on a **green** magic carpet, exactly one cat walks off.



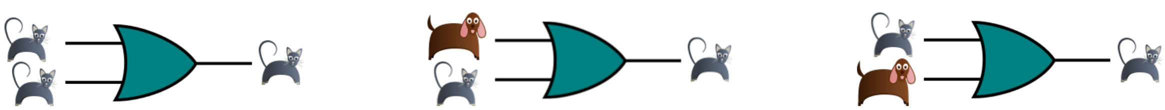
* If any other pair of animals walk on a **green** magic carpet, exactly one dog walks off.



* If two dogs walk on a **teal** magic carpet, exactly one dog walks off.



* If any other pair of animals walk on a **teal** magic carpet, exactly one cat walks off.

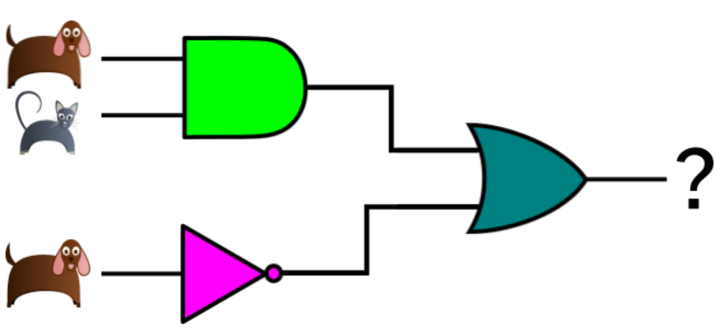


* If a cat walks on a **pink** magic carpet, a dog walks off. If a dog walks on a **pink** magic carpet, a cat walks off.

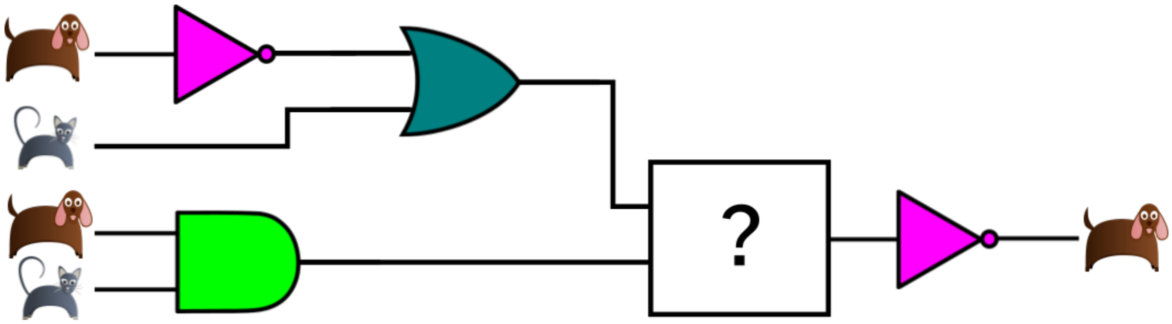
Questions:

Note that in the diagrams in the questions that follow, a line between two carpets means that the animal that walks off the left carpet is the same animal that then walks on the next carpet to the right.

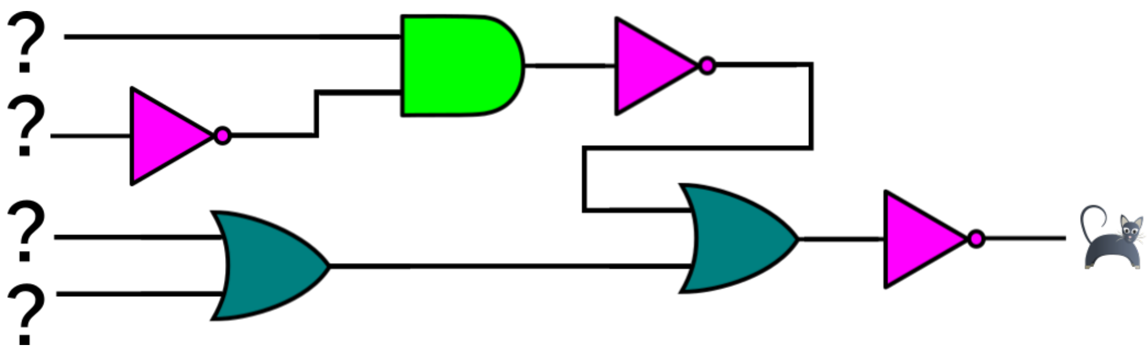
1. On the first floor of the castle, the magic carpets are arranged as shown. If cats and dogs walk on the carpets as indicated, which animal will walk off the rightmost carpet?



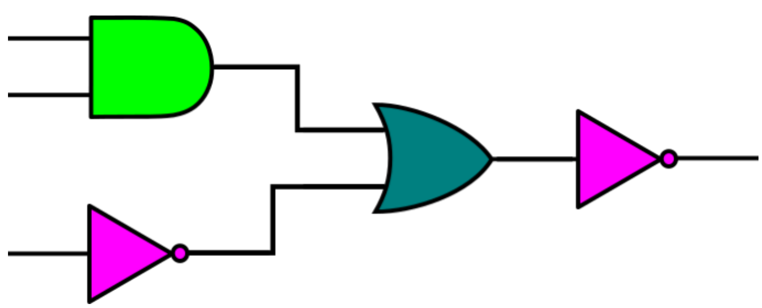
2. On the second floor of the castle, the magic carpets are arranged as shown. If cats and dogs walk on the carpets as indicated, and a dog walks off the rightmost carpet, identify the colour of the missing magic carpet.



3. On the third floor of the castle, the magic carpets are arranged as shown. If a cat walks off the rightmost carpet, which four animals walked onto the carpets?



4. On the fourth floor of the castle, the owner would like to arrange magic carpets as shown below. However, teal magic carpets are incredibly expensive! Suggest a new arrangement, using only green and pink magic carpets, that will have the same behaviour as the owner's desired arrangement. That is, for every possible combination of three animals that could walk across the leftmost carpets, the animal that would walk off the rightmost carpet will be the same in both arrangements.



More Info:

Check out the CEMC at Home webpage on Wednesday, May 6 for a solution to Magic Carpets.

The green, teal, and pink magic carpets *look* and *act* like AND, OR, and NOT gates, respectively. Check out Escape Room on the 2019 Beaver Computing Challenge for a similar problem and more information about gates.

CEMC at Home

Grade 9/10 - Wednesday, April 29, 2020
Magic Carpets - Solution

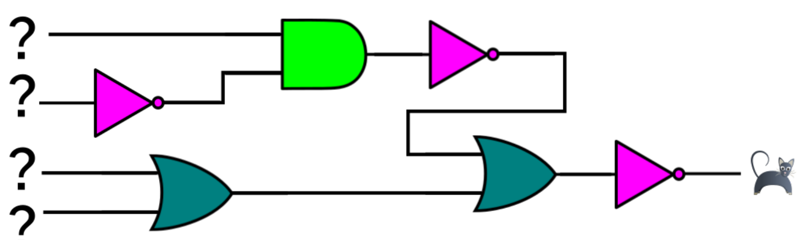
1. A dog will walk off the green carpet and a cat will walk off the pink carpet. Therefore, a dog and a cat will walk on the teal carpet, and so a cat will walk off the teal carpet (rightmost carpet).

2. It must be the case that a cat walks *off* the missing carpet since only a cat walking on the rightmost (pink) carpet will cause a dog to walk off. What animals walk *on* the missing carpet? At the top-left, a cat walks off the pink carpet and so two cats walk on the teal carpet. This means one cat walks off the teal carpet and approaches the missing carpet. At the bottom-left, a dog walks off the green carpet and approaches the missing carpet. So a cat and a dog walk on the missing carpet and a cat walks off. This can only happen if the missing carpet is teal.

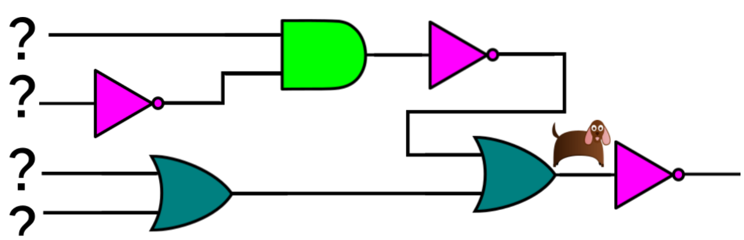
3. From top to bottom the animals are cat, dog, dog, and dog.

   One way to obtain this answer is to try all possible combinations of animals until you find one that results in a cat walking off the rightmost carpet. How many combinations would you have to try? Since there are 4 spots for animals, and each spot could be a cat or a dog, there are $2^4 = 16$ combinations. Testing each combination in turn may not be the best way to proceed.

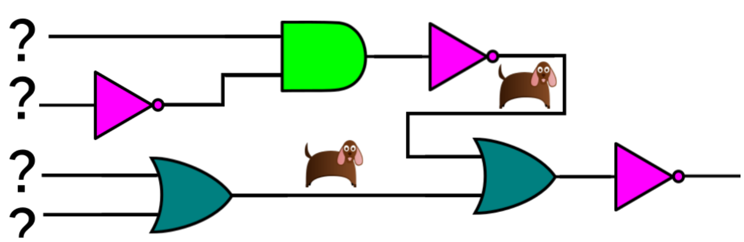
   An alternative way is to work backwards. Imagine that you have a video of the animals walking across the carpets, and you play the video in reverse. Here is a series of images that show the cat on the right moving backwards over the carpets.



The only way a cat can walk off a pink carpet is if a dog walks on.



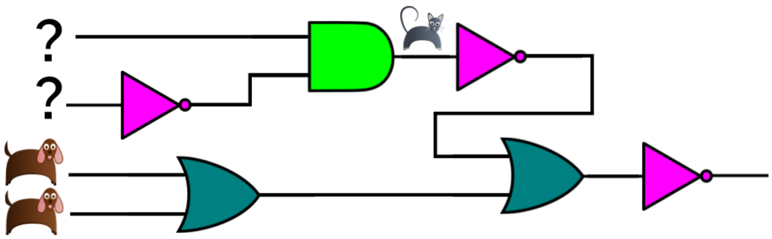
The only way a dog can walk off a teal carpet is if two dogs walk on.



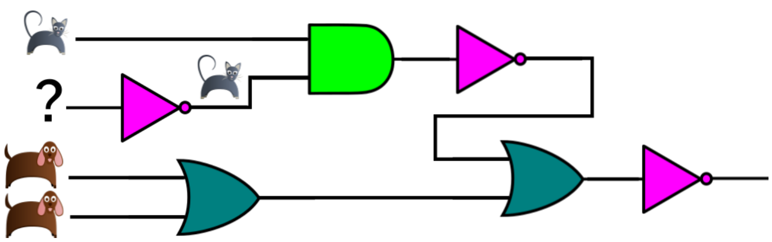
The only way a dog can walk off a pink carpet is if a cat walks on.

AND

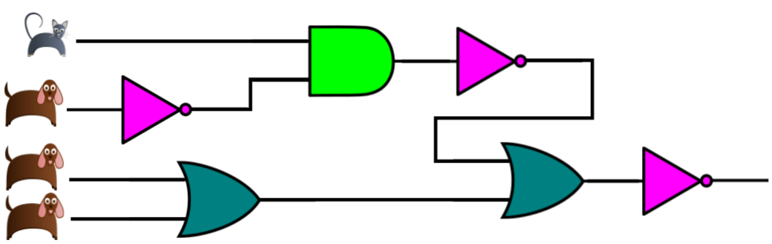
The only way a dog can walk off a teal carpet is if two dogs walk on.



The only way a cat can walk off a green carpet is if two cats walk on.

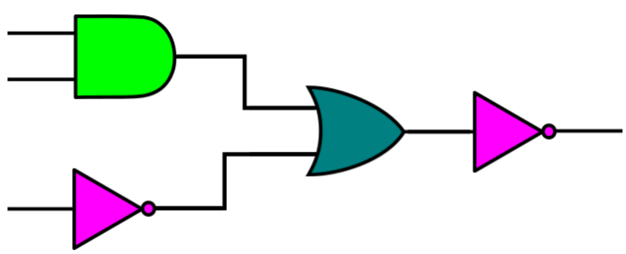


The only way a cat can walk off a pink carpet is if a dog walks on.

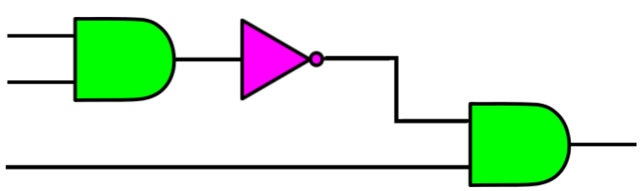


Therefore, this shows the only option for the starting animals.

4. The given arrangement with the teal carpet



is equivalent to the following arrangement that uses no teal carpets:



You may have found a different arrangement than this that also works.

CEMC at Home features Problem of the Week

Grade 9/10 - Thursday, April 30, 2020
A De-Light-Ful Machine

A machine has 2020 lights and 1 button. Each button press changes the state of exactly 3 of the lights. That means if the light is currently on, it turns off, and if the light is currently off, it turns on. Before each button press, the user selects which 3 lights will change their state.

To begin with, all the lights on the machine are off. What is the fewest number of button presses required in order for all the lights to be on?

Hint: Start by thinking about a machine with fewer lights.

More Info:

Check the CEMC at Home webpage on Thursday, May 7 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, May 7.

This CEMC at Home resource is the current grade 9/10 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem D and Solution
A De-Light-Ful Machine

Problem

A machine has 2020 lights and 1 button. Each button press changes the state of exactly 3 of the lights. That means if the light is currently on, it turns off, and if the light is currently off, it turns on. Before each button press, the user selects which 3 lights will change their state. To begin with, all the lights on the machine are off. What is the fewest number of button presses required in order for all the lights to be on?

Hint: Start by thinking about a machine with fewer lights.

Solution

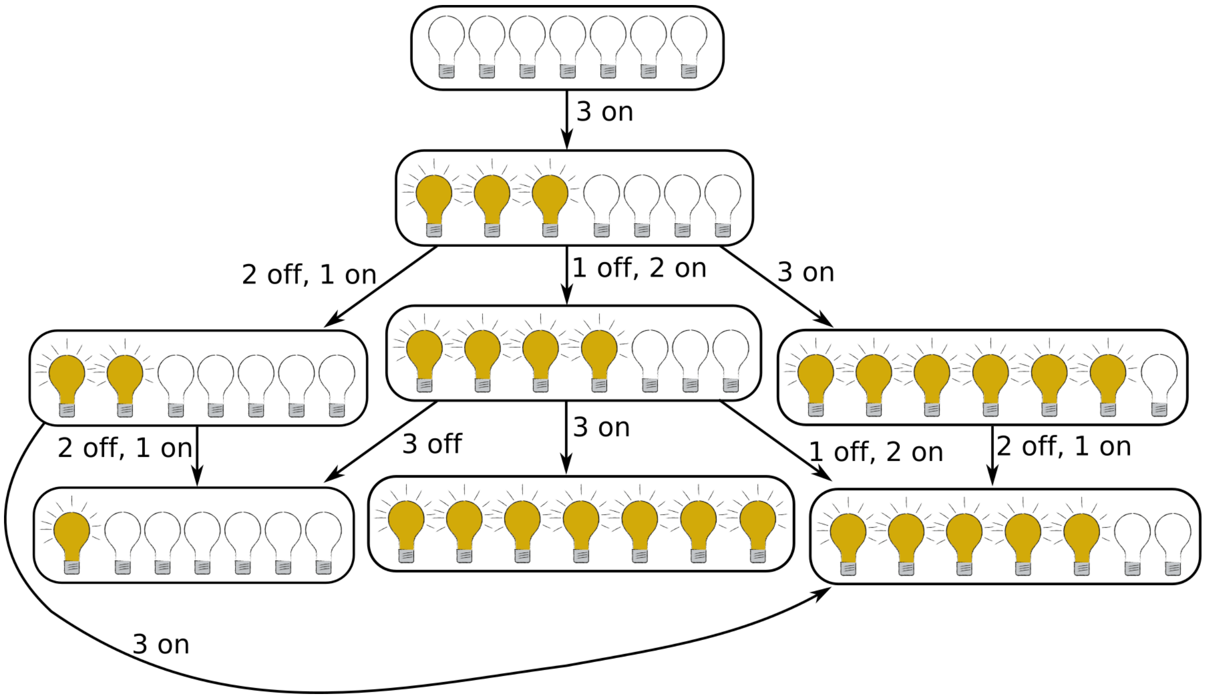
To turn on all the lights with the fewest number of button presses, we should turn on 3 lights with each button press, and not turn any lights off.

- The first button press would turn on 3 lights.
- The second button press would turn on 3 more lights, bringing the total to 6 lights on.
- The third button press would turn on 3 more lights, so now there would be 9 lights on.
- And so on . . .

Continuing in this way we can see that the total number of lights on would always be a multiple of 3. However, since 2020 is not a multiple of 3, this tells us that at least 1 button press must turn some lights off. Since we want to press the button the fewest number of times, that means we want the fewest number of button presses to turn lights off.

Now suppose the button was pressed 671 times, and each time 3 lights turned on. Then there would be $671 \times 3 = 2013$ lights on in total. Let's look at the remaining 7 lights that are still off. We can draw a diagram to show all the possible outcomes for the next button presses until all 7 lights are on.

Note that the order of the lights does not matter. We are interested in how many lights are on, not which particular lights are on. To simplify our diagram, at each step we have moved all of the lights that are on to the left.



Note that if a button press reverses the press that was just made, we did not include this in our diagram, as this will not give us the fewest number of button presses.

We can see in the diagram that the shortest sequence of steps to get all the lights on would be:

1. Turn 3 lights on.
2. Turn 1 light off and 2 lights on.
3. Turn 3 lights on.

This takes 3 steps, which means 3 button presses. If we add this to the 671 button presses to get to this point, that tells us there are $671 + 3 = 674$ button presses in total. We note that only 1 of these 674 button presses turns lights off. Since we know that at least 1 button press must turn some lights off, that tells us we cannot turn all the lights on using fewer button presses.

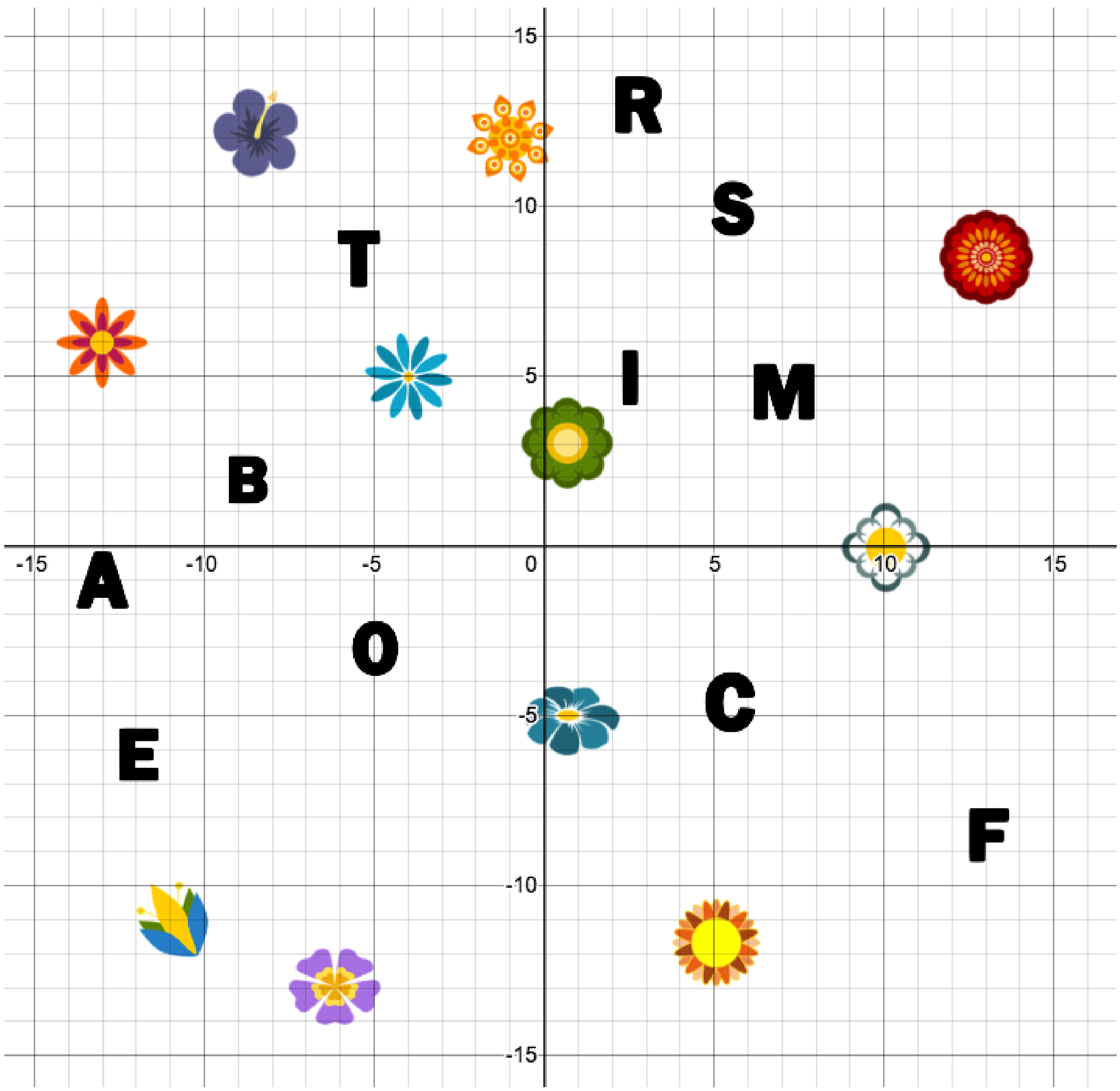
Therefore, 674 button presses are required to turn on all the lights.

CEMC at Home

Grade 9/10 - Friday, May 1, 2020
Flowers, Letters and Lines

Instructions: Eleven lines are described on the next page either by an equation or with other information. Carefully graph these lines on the grid below using a ruler. Each line should pass through *exactly one flower and one letter*. Match each flower to the letter that lies on its line to answer the riddle below. *The table on the next page might be useful to help organize your work.*

Riddle: What does the letter A have in common with a flower?



Answer:

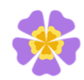
___   ___ ___ ___   ___ ___ ___ ___ ___   ___ ___ ___ ___ ___   ___ ___ !

Lines:

$y = 5x$	$y = \frac{1}{9}x - 10$	$x + 2y - 16 = 0$
$y + 15 = -\frac{1}{3}(x - 15)$	$y = \frac{17}{2}$	$x - 5y - 10 = 0$
A line that has a slope of -2 and a y-intercept of 10	A line that passes through the points $(-15, 40)$ and $(-5, -20)$	A line that has a slope of -1 and passes through the point $(9, -8)$
$y + 4 = 3(x - 1)$	A vertical line that passes through the point $(-13, 0)$	

You may need to re-arrange a given equation or do some additional calculations to make the information about the given line more useful for graphing it.

You may find the table below useful in organizing your work.

Flower											
Letter											

More Info:

Check the CEMC at Home webpage on Friday, May 8 for a solution to Flowers, Letters and Lines.

For more practice with graphing linear equations, check out this lesson in the CEMC Courseware. There are also other lessons you may wish to review in the Linear Relations unit.

CEMC at Home

Grade 9/10 - Friday, May 1, 2020
Flowers, Letters and Lines - Solution

The eleven linear equations in $y = mx + b$ form are:

1. $y = 5x$	2. $y = \frac{1}{9}x - 10$	3. $x + 2y - 16 = 0$ $y = -\frac{1}{2}x + 8$
4. $y + 15 = -\frac{1}{3}(x - 15)$ $y = -\frac{1}{3}x - 10$	5. $y = \frac{17}{2}$	6. $x - 5y - 10 = 0$ $y = \frac{1}{5}x - 2$
7. A line that has a slope of -2 and a y-intercept of 10 $y = -2x + 10$	8. A line that passes through the points $(-15, 40)$ and $(-5, -20)$ $y = -6x - 50$	9. A line that has a slope of -1 and passes through the point $(9, -8)$ $y = -x + 1$
10. $y + 4 = 3(x - 1)$ $y = 3x - 7$	11. A vertical line that passes through the point $(-13, 0)$ $x = -13$	

After graphing each linear equation (see next page for the graph), we see that each line goes through exactly one flower and one letter. Using the graph, we can fill in the table below:

Flower											
Letter	T	O	M	E	S	I	A	C	R	B	F
Equation	5.	6.	3.	4.	10.	7.	11.	9.	1.	8.	2.

Using the information from above, we can answer the riddle *"What does the letter A have in common with a flower?"*

A BEE COMES AFTER IT!

Click on the graph to explore it further!