CEMC at Home

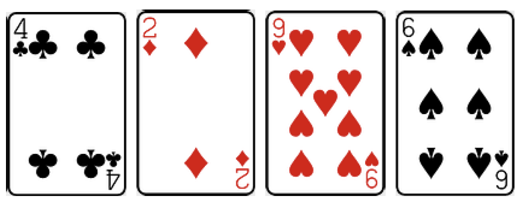
Grade 7/8 - Monday, April 27, 2020
The Game of 24

You Will Need:

- Two or more players
- A standard deck of playing cards, with all Jacks, Queens, and Kings removed (leaving 40 cards in total)
- Some paper and a pencil for score-keeping

How to Play:

1. First, shuffle the deck of 40 cards and put it face down in front of the players.
2. The game has 10 rounds. On the paper, start a column for keeping a running tally of each player's score.
3. Flip the top four cards of the deck face up on the playing surface.
4. Using the numbers on the four cards exactly once each, along with any of the operations of addition, subtraction, multiplication and division, each player privately tries to make a number that is as close to 24 as possible. Aces have a value of 1.
   For example, what numbers can you make with following four cards?

   You can add up the four numbers to get $4 + 2 + 9 + 6 = 21$ *which is pretty close to* 24*, but you can do better! It might be tempting to make exactly* 24 *by calculating* $4 \times 6 = 24$ *but this move is not allowed because this calculation does not use each of the four numbers exactly once. However, we can make exactly* 24 *by introducing parentheses and calculating*

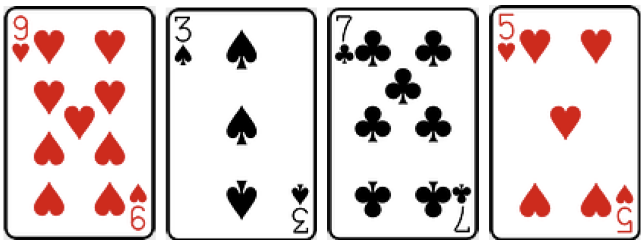
   $$(4 \times 9) - (6 \times 2) = 36 - 12 = 24$$

   This means we first calculate $4 \times 9 = 36$ *and* $6 \times 2 = 12$ *and then subtract these numbers to get* $36 - 12 = 24$.
5. When everyone is finished, all players reveal their calculations and the round is scored as follows: Each player's score is the difference between their calculated number and the number 24. Record each player's score for the round and add this to their running tally for the game.
   For example, if a player finds a way to get 24 *exactly, then their score for the round is* 0*. If a player does not find a way to get* 24*, but manages to make* 23 *or* 25*, then their score is* 1*. If a player makes* 22 *or* 26 *then their score is* 2*, and the scoring continues like this.*
6. Discard the current four cards and then flip the next four cards of the deck face up to play the next round with the same rules.
7. Keep playing until all cards have been used up (a total of 10 rounds). At the end of the game, the winner is the player with the *smallest* tally!

If you would like to practice different ways to combine four whole numbers using operations and parentheses, then think about the examples shown below before you play the game. The calculations follow the order of operations given here:

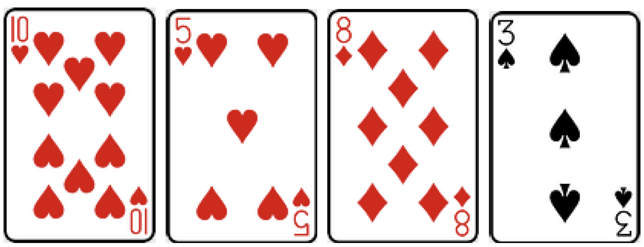
- Multiplication and division are performed before addition and subtraction.
- Any calculations between parentheses $(\cdots)$ are done first, before the calculations outside the parentheses.

Examples:



In this case, we can get 24 exactly by adding:

$$9 + 3 + 7 + 5 = 24$$

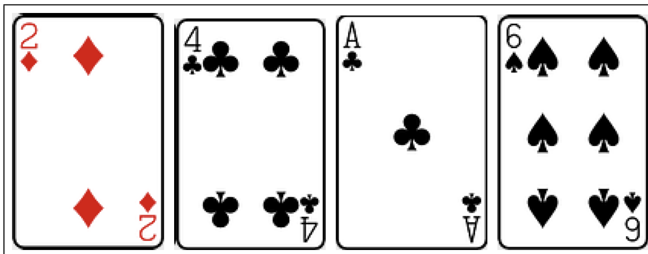


In this case, we can get 26 in at least two different ways:

$$8 + 3 + 10 + 5 = 26$$

$$8 \times 3 + 10 \div 5 = 24 + 2 = 26$$

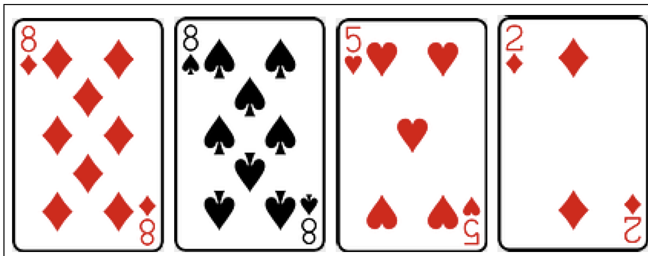
With the help of parentheses, we can get 25 by calculating $(10 - 5) \times (8 - 3) = 5 \times 5 = 25$.



In this case, we can get 24 exactly with the help of parentheses:

$$6 \times 4 \times (2 - 1) = 6 \times 4 \times 1 = 24$$

(Remember that Aces count as 1 in this game.)



In this case, we can get very close to 24 by adding:

$$8 + 8 + 5 + 2 = 23$$

With the help of parentheses, we can get 24 exactly:

$$(8 \times 5) - (8 \times 2) = 40 - 16 = 24$$

Variation:

Change the rules for how the game is scored to the following:

On each round, the first player to find a way to make 24 with the four cards earns 1 point for the round. No other players can score a point on the round.

If no player can find a way to make 24 with the current set of four cards, then shuffle the four cards back into the deck, and place four new cards face up.

In this variation, the winner is the player who has the most points when the deck has been used up!

CEMC at Home

Grade 7/8 - Tuesday, April 28, 2020
Missing Digits

Each problem below contains a positive integer that has at least one digit missing. A square is used in place of each missing digit.

Problem 1: The two-digit number $5\square$ is divisible by 2. How many possibilities are there for this two-digit number?

Problem 2: The four-digit number $6\square 53$ is divisible by 3. How many possibilities are there for this four-digit number?

Since there are only 10 possibilities for the missing digit, you could solve this problem by substituting each digit, in turn, and checking if the resulting number is divisible by 3. Can you use the following fact to answer this question without actually dividing all 10 resulting numbers by 3?

> **Did You Know?**
>
> A number is divisible by 3 exactly when the sum of its digits is divisible by 3.
> For example, 27 is divisible by 3 because $2+7=9$ and 9 is divisible by 3, and 38 is *not* divisible by 3 because $3+8=11$ and 11 is *not* divisible by 3.

Problem 3: The four-digit number $4\square 3\square$ is divisible by 6. How many possibilities are there for this four-digit number?

Note that the two missing digits don't need to be the same.

> **Did You Know?**
>
> A number is divisible by 6 exactly when it is divisible by both 2 and 3.
> For example, 48 is divisible by 6 because of the following:
>
> - The last digit is 8, which is an even number. This means 48 is divisible by 2.
> - The sum of the digits is $4+8=12$ and 12 is divisible by 3. This means 48 is divisible by 3.

Challenge Problem: The five-digit number $\square 5\square\square 2$ is less than 30 000 and is divisible by 12. How many possibilities are there for this five-digit number?

> **Did You Know?**
>
> A number is divisible by 12 exactly when it is divisible by both 3 and 4.
> Also, a number is divisible by 4 exactly when its last two digits (tens and units, in order) form a two-digit number that is divisible by 4.
> For example, 236 is divisible by 4 because the two-digit number 36 is divisible by 4, but 534 is not divisible by 4 because the two-digit number 34 is not divisible by 4.

More Info:

Check out the CEMC at Home webpage on Wednesday, April 29 for a solution to Missing Digits.

Try this lesson in the CEMC Courseware for more practice with multiples.

CEMC at Home

Grade 7/8 - Tuesday, April 28, 2020
Missing Digits - Solution

Each problem below contains a positive integer that has at least one digit missing. A square is used in place of each missing digit.

To help us solve these problems, we will use various divisibility tests as outlined in the problem statement:

- A whole number is divisible by 3 exactly when the sum of its digits is divisible by 3.
- A whole number is divisible by 4 exactly when its last two digits (tens and units, in order) form a two-digit number that is divisible by 4.
- A whole number is divisible by 6 exactly when it is divisible by both 2 and 3.
- A whole number is divisible by 12 exactly when it is divisible by both 3 and 4.

Problem 1: The two-digit number 5☐ is divisible by 2. How many possibilities are there for this two-digit number?

Solution: Since the number 5☐ is divisible by 2, its units (ones) digit must be 0, 2, 4, 6, or 8. So there are five possibilities for the given two-digit number. They are 50, 52, 54, 56, and 58.

Problem 2: The four-digit number 6☐53 is divisible by 3. How many possibilities are there for this four-digit number?

Solution: There are 10 possibilities for the digit in the box, and these result in the following four-digit numbers:

$$6053, 6153, 6253, 6353, 6453, 6553, 6653, 6753, 6853, 6953$$

One quick way to check which of these numbers is divisible by 3 is to calculate the sum of their digits and check whether or not this sum is divisible by 3.

Since $6 + 0 + 5 + 3 = 14$ and 14 is *not* divisible by 3, the number 6053 is *not* divisible by 3.

On the other hand, $6 + 1 + 5 + 3 = 15$ and 15 *is* divisible by 3, and so the number 6153 *is* divisible by 3.

Checking the sum of the digits of the other eight numbers, we find that only the numbers 6153, 6453, and 6753 have the sum of their digits divisible by 3.

This means there are three possibilities for the given number. They are 6153, 6453, and 6753.

Problem 3: The four-digit number 4☐3☐ is divisible by 6. How many possibilities are there for this four-digit number? *Note that the two missing digits don't need to be the same.*

Solution: Since the number 4☐3☐ is divisible by 6, it must be divisible by both 2 and 3. The fact that it is divisible by 2 tells us that its units digit must be 0, 2, 4, 6, or 8. Let's look at each of these cases. *See the next page for the case work.*

- Case 1: The units digit is 0.

  In this case the number looks like 4□30. Since the number is also divisible by 3, we know that the sum of its digits must be divisible by 3.

  - If the missing digit is 2, then the sum of the digits is 9, which is divisible by 3.
  - If the missing digit is 5, then the sum of the digits is 12, which is divisible by 3.
  - If the missing digit is 8, then the sum of the digits is 15, which is divisible by 3.

  None of the other possible digits result a sum that is divisible by 3.

- Case 2: The units digit is 2.

  In this case the number looks like 4□32. Since the number is also divisible by 3, we know that the sum of its digits must be divisible by 3.

  - If the missing digit is 0, then the sum of the digits is 9, which is divisible by 3.
  - If the missing digit is 3, then the sum of the digits is 12, which is divisible by 3.
  - If the missing digit is 6, then the sum of the digits is 15, which is divisible by 3.
  - If the missing digit is 9, then the sum of the digits is 18, which is divisible by 3.

  None of the other possible digits result a sum that is divisible by 3.

- Case 3: The units digit is 4.

  In this case the number looks like 4□34. Since the number is also divisible by 3, we know that the sum of its digits must be divisible by 3.

  - If the missing digit is 1, then the sum of the digits is 12, which is divisible by 3.
  - If the missing digit is 4, then the sum of the digits is 15, which is divisible by 3.
  - If the missing digit is 7, then the sum of the digits is 18, which is divisible by 3.

  None of the other possible digits result a sum that is divisible by 3.

- Case 4: The units digit is 6.

  In this case the number looks like 4□36.

  Doing similar work as the other cases, we find that only the digits 2, 5, and 8 produce digit sums that are divisible by 3.

- Case 5: The units digit is 8.

  In this case the number looks like 4□38.

  Doing similar work as the other cases, we find that only the digits 0, 3, 6, 9 produce digit sums that are divisible by 3.

We count that there are $3 + 4 + 3 + 3 + 4 = 17$ possibilities for the four-digit number 4□3□.

If we would like, we can also list them:

$$4230, 4530, 4830, 4032, 4332, 4632, 4932, 4134, 4434, 4734, 4236, 4536, 4836, 4038, 4338, 4638, 4938$$

Challenge Problem: The five-digit number □5□□2 is less than 30 000 and is divisible by 12. How many possibilities are there for this five-digit number?

Solution: Since the number □5□□2 is less than 30 000, the first digit must be either 1 or 2.

Since the number □5□□2 is divisible by 12, it must be divisible by both 3 and 4.

Since it is divisible by 4, the last two digits must form a two-digit number that is divisible by 4. The two-digit numbers of the form □2 that are divisible by 4 are 12, 32, 52, 72, and 92.

So we know that the number must "start" in one of two ways: with the digits 15 or the digits 25. We also know that the number must "end" in one of five ways: with the digits 12, 32, 52, 72, or 92.

For example, one possibility is that the number is of the following form:

15□12

There are many different numbers that satisfy these properties! Our goal is to find the ones that are *also* divisible by 3.

Notice that all of these integers must have one of the following 10 forms shown in the table below. (Can you see why?) Similar to the previous problem, we determine which values when substituted for the missing digit result in the sum of the five digits in the number being divisible by 3.

Form of the number	**Digits resulting in a number divisible by 3**	**Count**
15□12	0, 3, 6, 9	4
15□32	1, 4, 7	3
15□52	2, 5, 8	3
15□72	0, 3 ,6 ,9	4
15□92	1, 4, 7	3
25□12	2, 5, 8	3
25□32	0, 3, 6, 9	4
25□52	1, 4, 7	3
25□72	2, 5, 8	3
25□92	0, 3, 6, 9	4

Adding up how many numbers we get in each of the 10 cases above, we see that there are

$$4 \times 4 + 6 \times 3 = 34$$

possibilities for the given integer.

This is probably more numbers than you would want to write out!

Note: We solved this challenge problem which involves checking whether five-digit numbers are divisible by 12 without actually attempting to divide a single five-digit number by 12. This shows the power of the divisibility tests! If you work through all of the details it takes to complete the table given above, then you will see that these tests allow us to replace questions like

"Is 25 992 divisible by 3?"

with questions like

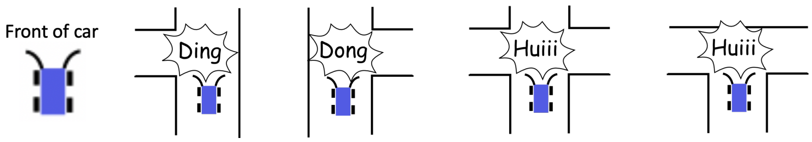
"Is the sum $2 + 5 + 9 + 9 + 2 = 27$ divisible by 3?"

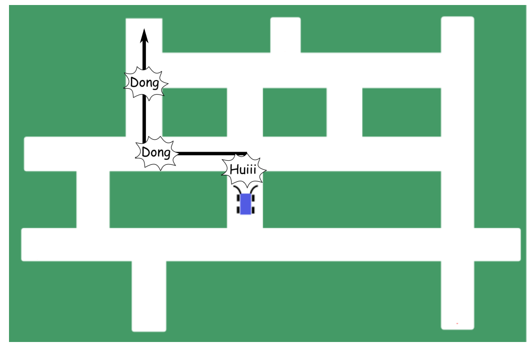
CEMC at Home

Grade 7/8 - Wednesday, April 29, 2020

No Road Blocks

A robotic car has sensors that detect intersections. It produces a different sound depending on whether it is possible to turn left only, turn right only, or turn in both directions at an intersection. The sounds that the car makes at various types of intersections is shown below.



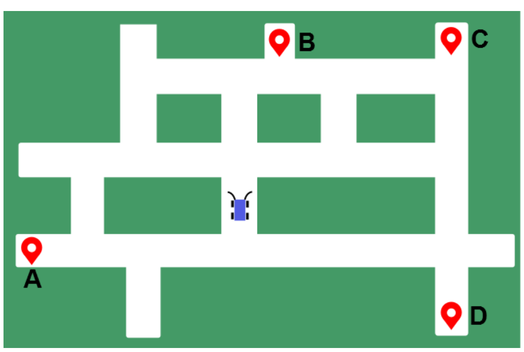


For example, if the car follows the route shown on the map to the right, it would make the sounds Huiii Dong Dong*, in that order.*

Note that the sound the car makes has nothing to do with the direction the car actually turns at the intersection. The sound just indicates the type of intersection the car has encountered.

The robotic car can go straight through an intersection (when possible), turn right (when possible), or turn left (when possible). The robotic car cannot make U-turns and cannot reverse. It automatically stops when it senses an obstacle in front of it (like a dead end).

Problem: The robotic car takes three different trips. On each trip, the car starts off in the position shown in the figure below, and ends up at one of the destinations marked A, B, C, or D. The sequence of sounds produced by the car on each of these trips is shown in the table below. What is the final destination of the robotic car on each trip?



Trip	Sounds Produced	Destination
1	Huiii Ding Huiii Ding	
2	Huiii Dong Dong Dong Ding	
3	Huiii Dong Ding Huiii	

To get started, try drawing a few paths from the starting position to one of the final destinations. Looking at the intersections encountered along one of these paths, determine the list of sounds the car would have made during this trip. Does this match one of the sequences in the table?

Extension: Is it possible for the robotic car to produce exactly the same sequence of sounds on two trips from its starting position (from the Problem above) that have different final destinations (from the list A, B, C, D)? If so, write down this sequence of sounds and the two final destinations. If not, explain why this is not possible.

More Info:

Check out the CEMC at Home webpage on Thursday, April 30 for a solution to No Road Blocks.

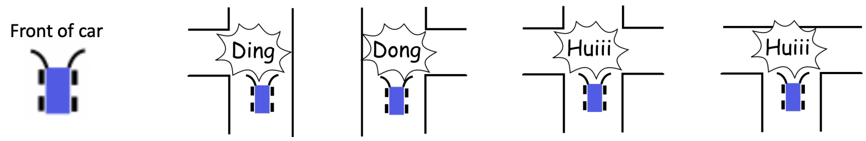
A variation of this problem appeared on a past Beaver Computing Challenge (BCC).

CEMC at Home

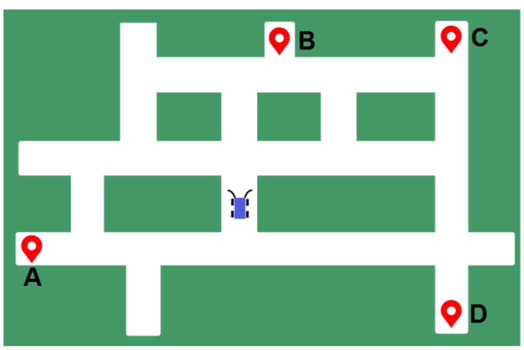
Grade 7/8 - Wednesday, April 29, 2020
No Road Blocks - Solution

Problem Summary and Answers

Here are the sounds made by the car at each type of intersection that it encounters. Remember that the car cannot make U-turns and cannot reverse.



The robotic car takes three different trips. On each trip, the car starts off in the position shown in the figure below and ends up at one of the destinations marked A, B, C, or D. The sequence of sounds produced by the car on each of these trips is shown in the table below. What is the final destination of the robotic car on each trip?



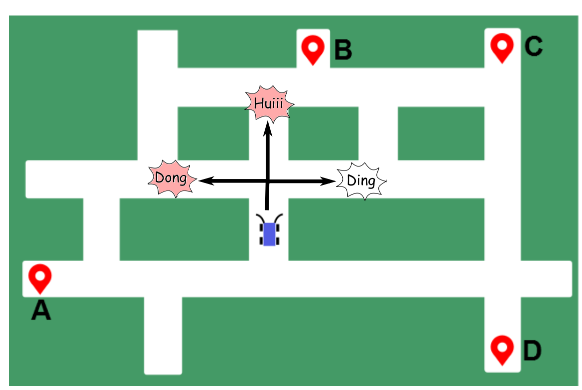
Trip	Sounds Produced	Destination
1	Huiii Ding Huiii Ding	**Answer: C**
2	Huiii Dong Dong Dong Ding	**Answer: B**
3	Huiii Dong Ding Huiii	**Answer: A**

Solution: The final destination of the car on each of the three trips is shown in the table above. Let's explain why these are the only possible final destinations based on the sounds made by the car.

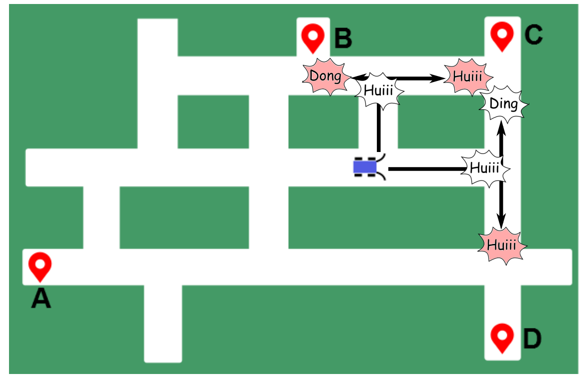
Based on the starting position of the car, just after it starts moving forward it will encounter an intersection where it can turn both left and right and so it will make a Huiii sound first. This will happen no matter what route the car takes on the trip.

During Trip 1, the car made the sounds Huiii Ding Huiii Ding. We can determine which way the car moved at each intersection based on the other three sounds it made before reaching its destination: Ding Huiii Ding.

Once the car reached the first intersection it had the choice of turning left, turning right, or going straight as shown in the map on the right. Notice that the only way the car could have made a Ding sound second is if it turned right at this first intersection. If the car turned left it would have next encountered an intersection with no left turn (Dong) and if it went straight then it would have next encountered an intersection with both a right and a left turn (Huiii). So we know that the car must have made a right turn at this first intersection.



After the car made this first right turn and moved to its second intersection, it had the choice of turning left or going straight as shown in the map on the right. Notice that no matter what choice the car made at this intersection, it would have made the correct third sound (Huiii) at the next intersection. This means we cannot be sure yet which choice it made.

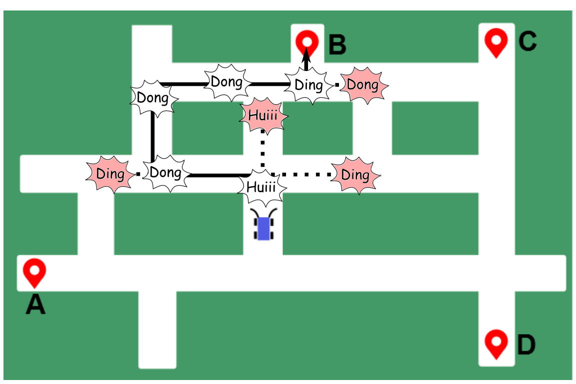


Suppose that the car turned left (travelling upward on the map). Notice that in this case, the only two options for the fourth sound the car could make are Dong (if it turned left next) and Huiii (if it turned right next). Neither of these is the correct fourth sound.

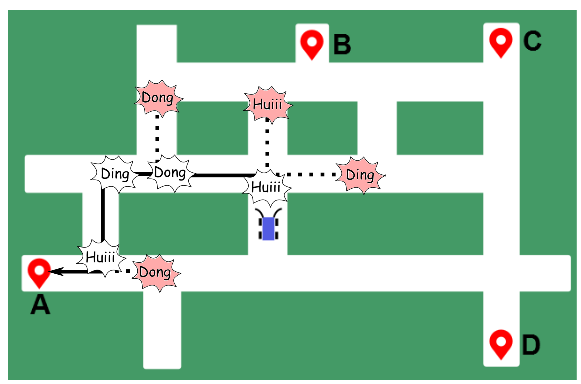
This means the car must have gone straight at this second intersection (travelling instead to the right on the map). Following similar reasoning as earlier, we can see that the car must then have turned left when it reached the next intersection in order to produce a Ding as the fourth sound (rather than a Huiii). Since the car did not make any more sounds during this trip, we know it must not have encountered another intersection, and so it must have travelled straight to destination C.

We can use similar reasoning to find the final destination on the remaining two trips. The two trips are shown in the maps below. Again, we indicate the various choices the car could have made at each intersection and eliminate the possibilities (using the sounds) until we find the correct route.

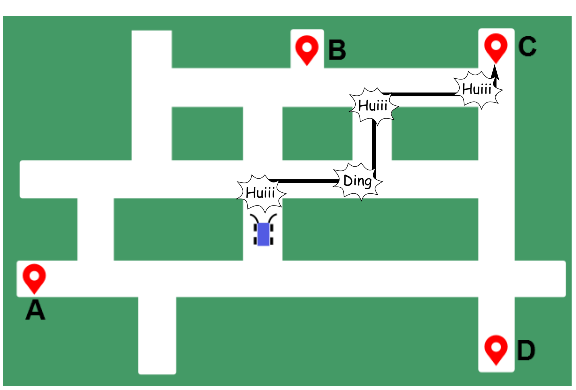
Trip 2: Huiii Dong Dong Dong Ding

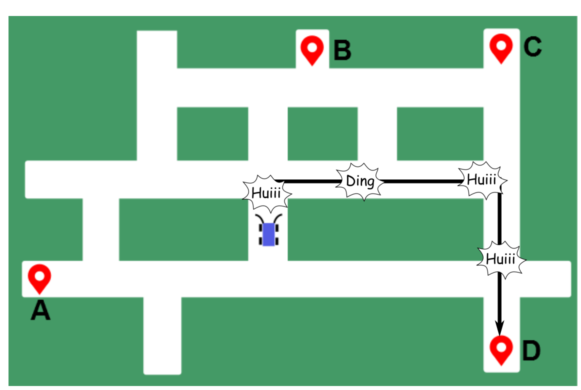


Trip 3: Huiii Dong Ding Huiii



Extension: It *is* possible for the car to produce exactly the same sequence of sounds but arrive at two different destinations on this map. As shown in the maps below, the car would produce the sounds Huiii Ding Huiii Huiii during each of the trips indicated while ending up at C on one trip and D on the other.





CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 30, 2020
Digit Swapping

Ali programs three buttons in a machine to swap some digits in a 4-digit number.

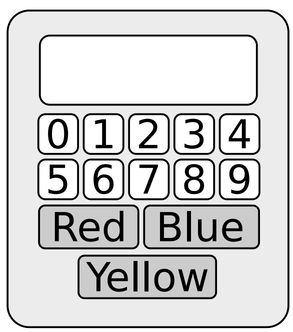
Red button:	swaps the thousands and tens digits
Blue button:	swaps the thousands and hundreds digits
Yellow button:	swaps the hundreds and units (ones) digits

Ali types a 4-digit number into the machine. She then presses the following sequence of buttons to produce 6943 as the output.

[Red] [Yellow] [Blue] [Red] [Yellow]

What would the output have been if Ali had instead pressed the following sequence of buttons after typing in her original number?

[Blue] [Red] [Yellow] [Blue]

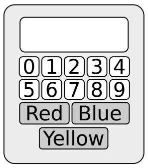


More Info:

Check the CEMC at Home webpage on Thursday, May 7 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, May 7.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php



Problem of the Week

Problem C and Solution

Digit Swapping

Problem

Ali programs three buttons in a machine to swap some digits in a 4-digit number.

Red button:	swaps the thousands and tens digits
Blue button:	swaps the thousands and hundreds digits
Yellow button:	swaps the hundreds and units (ones) digits

Ali types a 4-digit number into the machine. She then presses the following sequence of buttons to produce 6943 as the output.

$$\boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}}$$

What would the output have been if Ali had instead pressed the following sequence of buttons after typing in her original number?

$$\boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}}$$

Solution

First we need to know what number Ali initially typed into the machine in order to produce 6943 as the output.

$$???? \rightarrow \boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}} \rightarrow 6943$$

We can determine this by working backwards. This means we will start with 6943 and go through the sequence of buttons in the opposite order.

$$6943 \rightarrow \boxed{\text{Yellow}} \rightarrow 6349$$
$$6349 \rightarrow \boxed{\text{Red}} \rightarrow 4369$$
$$4369 \rightarrow \boxed{\text{Blue}} \rightarrow 3469$$
$$3469 \rightarrow \boxed{\text{Yellow}} \rightarrow 3964$$
$$3964 \rightarrow \boxed{\text{Red}} \rightarrow 6934$$

So Ali typed 6934 into the machine. Now we want to know the output after pressing the second sequence of buttons.

$$6934 \rightarrow \boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}} \rightarrow ????$$

We can go through the second button sequence to determine the new output.

$$6934 \rightarrow \boxed{\text{Blue}} \rightarrow 9634 \rightarrow \boxed{\text{Red}} \rightarrow 3694 \rightarrow \boxed{\text{Yellow}} \rightarrow 3496 \rightarrow \boxed{\text{Blue}} \rightarrow 4396$$

Therefore, the output would have been 4396.

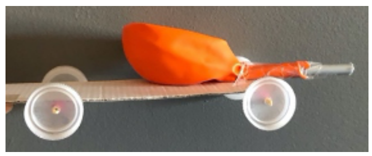
CEMC at Home

Grade 7/8 - Friday, May 1, 2020
RACECAR Backwards is RACECAR

In today's activity we will make a car powered by a balloon, and solve problems involving speeds.

Activity

Make a balloon car using household items! This car needs wheels on axles, a body, and an attached balloon for power. We can get the car to run by filling the balloon with air and letting it go.



See the next page for possible instructions for how to make a balloon car for the activity.

See how far you can make your balloon car go and time how long your car is in motion before it comes to a stop. Once your car comes to a stop, measure the distance travelled. Use the distance and time to calculate the *average speed* of your car on this trip. What might you do to increase the speed of your car? Try some adjustments and keep track of what happens to the speed of the car.

The average speed of a car on a trip is the total distance travelled by the car during the trip divided by the total time it takes the car to complete the trip.

> Remember that speed is a unit rate, written as meters per second (m/s), kilometres per hour (km/hr), etc. If your car travelled 45 centimetres in 5 seconds, then to calculate the average speed as a unit rate, you need to figure out how far your car travelled in 1 second, on average. This means you need to figure out what value should go in the box below.
>
> $$\frac{45\text{ cm}}{5\text{ sec}} = \frac{\square\text{ cm}}{1\text{ sec}}$$

Try these problems involving speeds!

1. Tiegan is travelling at a constant speed of 85 km/h. If Tiegan is halfway through a 510 km trip, how much longer will the trip take?

2. A bicycle travels at a constant speed of 15 km/h. A bus starts 195 km behind the bicycle and catches up to the bicycle in 3 hours. What is the average speed of the bus in km/h?

   To get started, determine how far the bicycle has travelled.

3. Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many metres was Greg behind?

   You will likely find this problem more challenging than the first two. Can you use ratios to help you solve this problem?

How to Make a Balloon Car

You Will Need:

- Four plastic bottle caps

  These should have roughly the same diameter.
- One wooden skewer
- One straw
- One pen
- One balloon
- Cardboard
- Tape

Instructions:

1. Cut the wooden skewer in half.
2. Cut the straw into two pieces, each slightly shorter than the wooden skewer pieces.

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3. Cut a piece of cardboard to make the body of the car. The width should be about the same as the length of each straw piece.
4. Poke a hole in the centre of each bottle cap so that the skewer can fit through.

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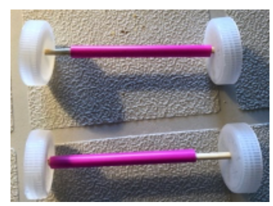
5. Press the bottle cap onto one end of the skewer. Slide the straw around the skewer and then press another bottle cap onto the other end of the skewer so that the straw can rotate. Repeat this process with the other skewer, straw, and bottle caps to form your wheels.
6. Attach the cardboard to the straws using tape.



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7. Put your car down on a flat surface and give it a push. Make sure the car rolls easily and coasts for a bit before stopping. Make any necessary adjustments.
8. Take the pen apart so you are left with just the empty plastic case. Tape the neck of the balloon around one end of the pen case. Wrap the tape very tightly so that no air can escape.

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9. Tape the pen case onto the cardboard. Make sure that when the balloon is inflated, it does not touch either wheel.
10. Inflate the balloon and then quickly press your thumb on the end of the pen case. Place the car gently on the ground and then remove your thumb to watch your balloon car go.

More Info:

Check the CEMC at Home webpage on Monday, May 4 for a solution to these problems.

For more practice with speeds and other rates, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Friday, May 1, 2020
RACECAR Backwards is RACECAR - Solution

1. Tiegan is travelling at a constant speed of 85 km/h. If Tiegan is halfway through a 510 km trip, how much longer will the trip take?

   Solution 1: First we will calculate how long it would take Tiegan to complete the entire trip. Since Tiegan is travelling at a constant speed, we know that she will travel 85 km every 1 hr. How long will it take Tiegan to travel 510 km?

   ×6

   $$\frac{85\text{ km}}{1\text{ h}} = \frac{510\text{ km}}{\square\text{ h}}$$

   ×6

   Since 510 km $= 6 \times 85$ km, the entire trip will take 6×1 h $= 6$ h. Since Tiegan is halfway through the trip, the remainder of the trip will take 6 h $\div 2 = 3$ h.

   Solution 2: Since Tiegan is halfway through a 510 km trip, that means she has $510 \div 2 = 255$ km left to travel. Since Tiegan is travelling at a constant speed, we know that she will travel 85 km every 1 hr. How long will it take her to travel 255 km?

   ×3

   $$\frac{85\text{ km}}{1\text{ h}} = \frac{255\text{ km}}{\square\text{ h}}$$

   ×3

   Since 255 km $= 3 \times 85$ km, it will take her 3×1 h $= 3$ h to travel the remaining 255 km.

2. A bicycle travels at a constant speed of 15 km/h. A bus starts 195 km behind the bicycle and catches up to the bicycle in 3 hours. What is the average speed of the bus in km/h?

   Solution: Since the bicycle travels at a constant speed, we know it travels 15 km every 1 h. How far will it travel in 3 h?

   ×3

   $$\frac{15\text{ km}}{1\text{ h}} = \frac{\square\text{ km}}{3\text{ h}}$$

   ×3

   Since 3×15 km $= 45$ km, the bicycle will travel a distance of 45 km in 3 h. At the start, the bicycle was 195 km ahead of the bus. Therefore, in order to catch up to the bicycle, the bus must travel the 195 km plus the additional 45 km that the bicycle travels, or $195 + 45 = 240$ km. So the bus travels 240 km in 3 hours. What is the average speed of the bus as a unit rate?

   ÷3

   $$\frac{240\text{ km}}{3\text{ h}} = \frac{\square\text{ km}}{1\text{ h}}$$

   ÷3

   Since $240 \div 3 = 80$, the bus must have travelled at an average speed of 80 km/h.

3. Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many metres was Greg behind?

   Solution 1: In the first race, when Azarah crossed the finish line, Charlize was 20 m behind or Charlize had run 80 m.
   Since Azarah and Charlize each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or $100 : 80$.
   Similarly in the second race, when Charlize crossed the finish line, Greg was 10 m behind or Greg had run 90 m.
   Since Charlize and Greg each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or $100 : 90$.
   Let A, C and G represent Azarah's, Charlize's and Greg's speeds, respectively.
   Then, $A : C = 100 : 80 = 25 : 20$ and $C : G = 100 : 90 = 20 : 18$.
   Therefore, $A : C : G = 25 : 20 : 18$ and $A : G = 25 : 18 = 100 : 72$.
   Over equal times, the ratio of their speeds is equal to the ratio of their distances travelled.
   Therefore, when Azarah travels 100 m, Greg travels 72 m.
   When Azarah crossed the finish line, Greg was $100 - 72 = 28$ m behind.

   Solution 2: In the first race, when Azarah crossed the finish line, Charlize was 20 m behind or Charlize had run 80 m.
   Since Azarah and Charlize each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or $100 : 80$.
   That is, Charlize's speed is 80% of Azarah's speed.
   Similarly, Greg's speed is 90% of Charlize's speed.
   Therefore, Greg's speed is 90% of Charlize's speed which is 80% of Azarah's speed, or Greg's speed is 90% of 80% of Azarah's speed.
   Since 90% of 80% is equivalent to $0.90 \times 0.80 = 0.72$ or 72%, then Greg's speed is 72% of Azarah's speed.
   When Azarah ran 100 m (crossed the finish line), Greg ran 72% of 100 m or 72 m in the same amount of time.
   When Azarah crossed the finish line, Greg was $100 - 72 = 28$ m behind.