CEMC at Home

Grade 7/8 - Tuesday, April 14, 2020
Balancing the Scale

1. Cindy has two identical small pumpkins, two identical medium pumpkins and two identical large pumpkins.

   - Cindy weighs one small and one medium pumpkin together. Their total mass is 12 kg.
   - Cindy weighs one small and one large pumpkin together. Their total mass is 13 kg.
   - Cindy weighs one medium and one large pumpkin together. Their total mass is 15 kg.

   Can you figure out what the mass of each size of pumpkin is?

   To do this, you could play around with different numbers until you find three that work as the three different masses, or you could gather some objects (toothpicks, nickels, or paperclips) that each represent 1 kg and think about how to distribute these among the pumpkins.

2. Watch a video in the CEMC's Problem Solving and Mathematical Discovery Courseware that walks through a solution to a problem that is very similar to 1.:
   https://courseware.cemc.uwaterloo.ca/40/assignments/1039/1.

   We would encourage you to watch from the beginning to 2:33 and from 4:15 to the end. The section from 2:33 to 4:15 is also worth watching, but may contain more algebra than you have seen before.

3. Maxime has two identical small desks, two identical medium desks and two identical large desks.

   - Maxime weighs one small and one medium desk together. Their total mass is 47 kg.
   - Maxime weighs one small and one large desk together. Their total mass is 80 kg.
   - Maxime weighs one medium and one large desk together. Their total mass is 91 kg.

   Next, Maxime puts all six desks on the scale, two at a time, in the following order: one small and one medium, the second small and one large, and the second medium and the second large.

   (a) What is the total mass of the six desks?

   (b) What is the total mass of one small, one medium and one large desk together?

   (c) How can you use your answer to (b) with the original masses of 47 kg, 80 kg and 91 kg to determine the mass of each size of desk?

   To help solve this problem, you could use three pairs of objects to stand in for these desks as you work. Use a table as the scale and add the objects in the same order as Maxime. Keep track of the total mass on a piece of paper after each new pair of objects is added to the scale.

Extension: Walt has two identical small elephants, two identical medium elephants and two identical large elephants.

- Walt weighs one small and one medium elephant together. Their total mass is 7390 kg.
- Walt weighs one small and one large elephant together. Their total mass is 9039 kg.
- Walt weighs one medium and one large elephant together. Their total mass is 11 051 kg.

Can you determine the mass of each size of elephant?

More Info:

Check out the CEMC at Home webpage on Tuesday, April 21 for a solution to Balancing the Scale.

CEMC at Home

Grade 7/8 - Tuesday, April 14, 2020
Balancing the Scale

1. Suppose that each small pumpkin weighs 5 kg, each medium pumpkin weighs 7 kg, and each large pumpkin weighs 8 kg.
   Then the total mass of one small pumpkin and one medium pumpkin is 5 kg + 7 kg = 12 kg.
   Also, the total mass of one small pumpkin and one large pumpkin is 5 kg + 8 kg = 13 kg.
   Further, the total mass of one medium pumpkin and one large pumpkin is 7 kg + 8 kg = 15 kg.
   These are the correct totals.

2. No solution.

3. (a) Try drawing pictures that show the desks being added as you follow through this solution.
   When the first two desks are put on the scale, the scale reads 47 kg.
   When the next two desks are put on the scale, the scale reads 47 kg + 80 kg.
   When the final two desks are put on the scale, the scale reads 47 kg + 80 kg + 91 kg.
   Therefore, the total mass of the six desks is 47 kg + 80 kg + 91 kg = 218 kg.

   (b) The six desks that have a total mass of 218 kg include two small desks, two medium desks, and two large desks.
   When these desks are divided into equal groups of one small desk, one medium desk, and one large desk each, the mass of each group is identical, and equals (218 kg) ÷ 2 = 109 kg.

   (c) Since the total mass of one small desk, one medium desk, and one large desk is 109 kg, and the total mass of one small desk and one medium desk is 47 kg, then the mass of one large desk is 109 kg − 47 kg = 62 kg. (Imagine having the three desks on the scale and removing the two that you don't want.)
   Since the mass of one large desk is 62 kg and the total mass of one small and one large desk together is 80 kg, then the mass of one small desk is 80 kg − 62 kg = 18 kg.
   Since the mass of one large desk is 62 kg and the total mass of one medium and one large desk together is 91 kg, then the mass of one medium desk is 91 kg − 62 kg = 29 kg.
   Therefore, the mass of each small desk is 18 kg, the mass of each medium desk is 29 kg, and the mass of each large desk is 62 kg.

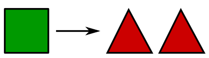
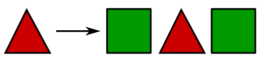
Extension

Try following the steps given in 3. to solve this problem with larger answers.
If you do this, you should find that the total mass of the six elephants is 27 480 kg.
This means that the total mass of one small elephant, one medium elephant, and one large elephant is 13 740 kg.
Can you use this information to determine the mass of each size of elephant? You can check your answers by combining them to see if they match the given information.

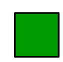
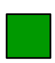
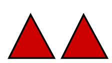
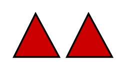
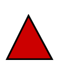
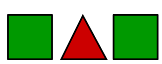
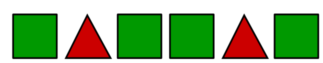
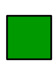
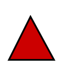
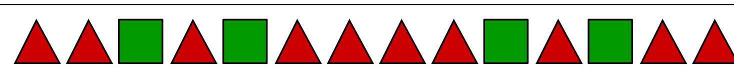
CEMC at Home

Grade 7/8 - Wednesday, April 15, 2020
Replacing Shapes

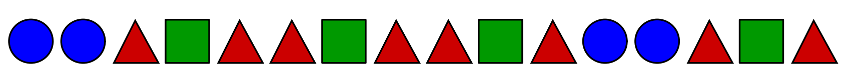
Introduction: Alice plays with cards of different shapes. Alice starts with a single card and builds longer sequences by applying the replacement rules for shapes given below.

Rule	Meaning
	Each square card is replaced by two triangle cards.
	Each triangle card is replaced by one square card, one triangle card, and another square card (in that order).

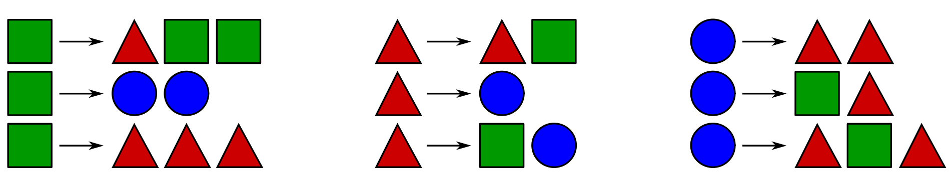
When these replacement rules are applied to a sequence of shapes, each individual shape in the sequence is replaced with a new sequence according to the relevant rule above. If Alice starts with a square card, and applies the replacement rules three times, then Alice builds the following:

Starting sequence *A single square*	
After first application of the rules *Single*  *replaced with* 	
After second application of the rules *Each*  *replaced with* 	
After third application of the rules *Rules for*  *and*  *both applied*	

Problem 1: Boris also plays with cards of different shapes. *Boris starts with one circle*, and plays with a different set of replacement rules. Boris has one replacement rule for each of the following shapes: circle, square, and triangle. The result of applying Boris's rules three times is shown below.



Which three replacement rules from the list below (one from each column) must Boris have used?

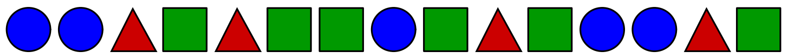


Need help getting started?

Use this online exploration to help you work through the solution. You can select different rules for Boris, and see what the result is after applying the rules once, twice, and three times. See if you can select the three rules that achieve the correct target sequence after three applications.

Problem 2: Cleo also plays with cards of different shapes. Cleo starts with exactly one shape (a square, a triangle, or a circle) and plays with a different set of replacement rules. Two of Cleo's three rules are shown below.

Cleo starts with one shape and applies the replacement rules three times and gets the following result.



Can you figure out Cleo's replacement rule for the square and which shape Cleo started with?

More Info:

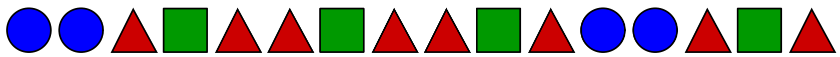
Check out the CEMC at Home webpage on Thursday, April 16 for a solution to Replacing Shapes.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

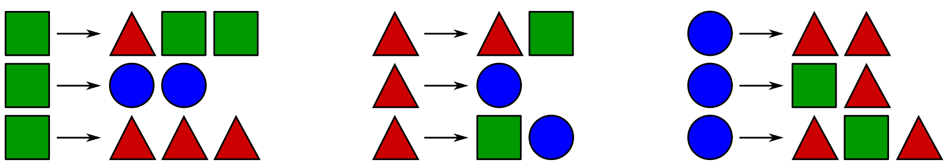
CEMC at Home

Grade 7/8 - Wednesday, April 15, 2020
Replacing Shapes - Solution

Problem 1: Boris starts with one circle, and plays with a set of replacement rules. Boris has one replacement rule for each of the following shapes: circle, square, and triangle. The result of applying Boris's rules three times is shown below.

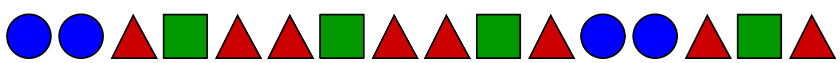


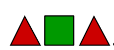
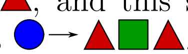
Which three replacement rules from the list below (one from each column) must Boris have used?

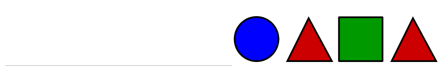


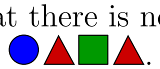
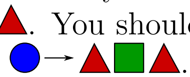
Solution to 1: One way to solve this problem is to try all possible combinations (on paper or using the online exploration) to see which one leads to the correct sequence. There are 27 different combinations and so if you don't stumble upon the right one quickly, this solution could be time consuming! Instead, we will explain how to eliminate some of the choices.

Let's call Boris's final sequence the "target sequence". We are trying to find rules that achieve this target sequence:

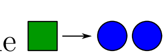
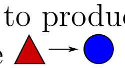


Since the right end of the target sequence is , and this sequence appears to be repeated throughout, it seems likely that the rule for circle is . We can convince ourselves that this *must* be the case by focussing on the right end of the target sequence:



Notice that there is no way to combine the outputs of the *other* rules to produce a sequence ending in exactly . You should spend some time checking this for yourself. This tells us that the circle rule must be .

Now let's focus on the left end of the target sequence:

There are only two ways to produce two circles in a row on the left end: the rule  applied to one square, or the rule  applied to two triangles, side-by-side.

We have narrowed down the choices enough to now do some testing (either on paper or using the online exploration). We proceed on paper.

Suppose that two of the rules are ● → ▲■▲ and ▲ → ●.

Since Boris starts with a circle, the result of applying these replacement rules one time is as follows:

▲■▲

We do not need to know the replacement rule for the square for this step.

Applying these rules a second time will result in a sequence that looks like this:

● __________ ●

The line in the middle of the figure above represents the part of the sequence that is currently unknown to us. We know that the sequence starts and ends in a circle (replacing the two triangles), but without the replacement rule for the square, we cannot be sure of what the middle of the sequence looks like.

Applying these rules a third time will result in a sequence that looks like this:

▲■▲ __________ ▲■▲

This final sequence cannot match the target sequence, regardless of what shapes appear in the middle portion, and so the rule for triangle cannot possibly be ▲ → ●.

Remember that if we do not have the rule ▲ → ● then we *must* have the rule ■ → ●● (since we need a rule that produces the two circles on the left end). This means that two of the three rules must be ● → ▲■▲ and ■ → ●●. All that we have left to do is determine rule for the triangle.

There are now only two options for the full set of three rules:

Option 1: ■ → ●●, ▲ → ■●, ● → ▲■▲

Option 2: ■ → ●●, ▲ → ▲■, ● → ▲■▲

We can check that the rules in *Option 1* produce the correct sequence after three applications, but the rules in *Option 2* do not. After this we can be sure that the three rules Boris used were as follows:

■ → ●●   ▲ → ■●   ● → ▲■▲

Problem 2: Cleo plays with cards of different shapes. Cleo starts with exactly one shape (a square, a triangle, or a circle) and plays with a set of replacement rules. Two of Cleo's three rules are shown below.

▲ → ●●   ● → ■●■

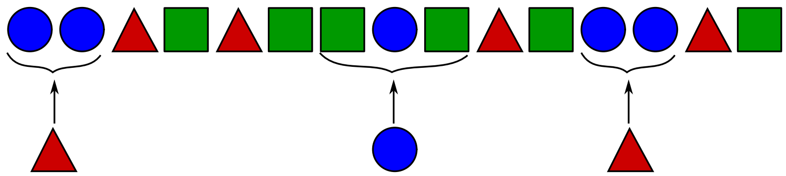
Cleo starts with one shape and applies the replacement rules three times and gets the following result.

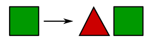
●●▲■▲■■●■▲■●●▲■

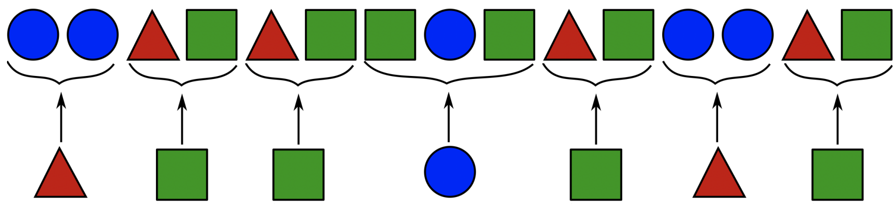
Can you figure out Cleo's replacement rule for the square and which shape Cleo started with?

Solution to 2: Since we do not know what the starting shape is in this problem, let's try and work backwards and see if we can find a solution.

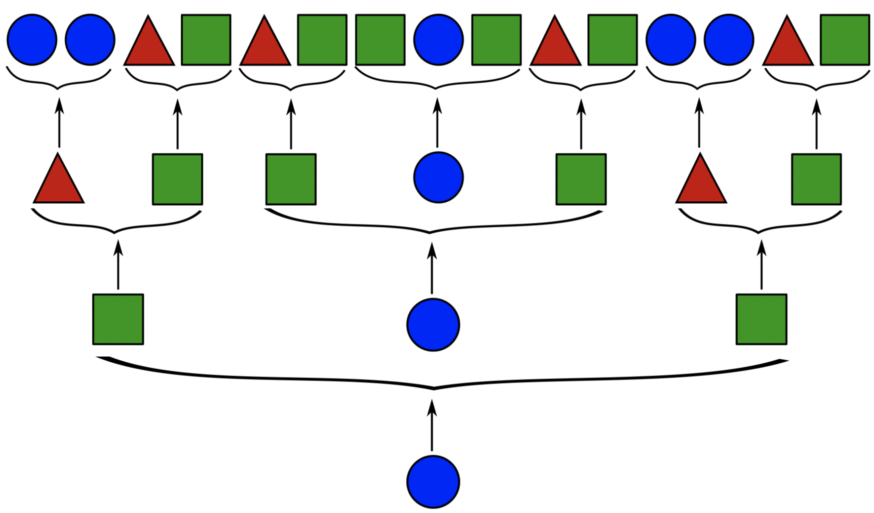
Let's suppose the indicated sections in the final sequence of shapes were obtained by applying the rules for triangle and circle as shown:

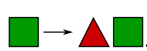


From here we can see that a replacement rule of  could fill in the gaps as shown below.



Working backwards, using the two original rules and the square rule from above, we get the following:



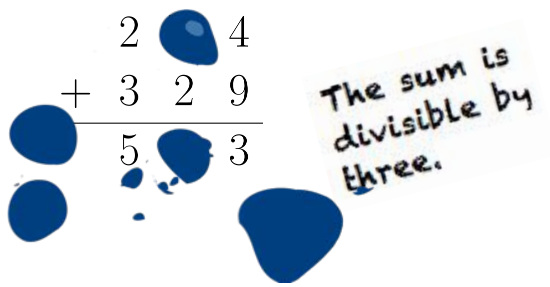
So if Cleo's replacement rule for the square was , and she started with a circle, then Cleo would end up with the sequence given in Problem 2.

It turns out that this is the only possible rule and starting shape that results in the correct final sequence. Can you convince yourself of this? Perhaps you can use reasoning similar to that used in the solution to Problem 1 to determine that there is only one possible answer for Problem 2.

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 16, 2020
What a Mess!

Jing was looking through an old math notebook and found the following mess on one of the pages:



The middle digit of the top and bottom numbers were both smudged and unreadable. Beside the question there was a handwritten note which read: "The sum is divisible by three."

If the missing middle digit in the top number is A and the missing middle digit in the bottom number is B, determine all possible values for A and B.

More Info:

Check the CEMC at Home webpage on Thursday, April 23 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 23.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
What a Mess!

The sum is divisible by three.

Problem

Jing was looking through an old math notebook and found the following question on one of the pages. The middle digit of the top and bottom numbers were both smudged and unreadable. Beside the question there was a handwritten note which read: "The sum is divisible by three." If the missing middle digit in the top number is A and the missing middle digit in the bottom number is B, determine all possible values for A and B.

$$\begin{array}{cccc} & 2 & A & 4 \\ + & 3 & 2 & 9 \\ \hline & 5 & B & 3 \end{array}$$

Solution

Solution 1

First we find the possible values of B. For a number to be divisible by 3, the sum of its digits must be divisible by 3. So $5 + B + 3$ must be divisible by 3. The only possible values for B are thus 1, 4, or 7.

We know that $2A4 + 329 = 5B3$ so $2A4 = 5B3 - 329$.

We can try each of the possible values for B in the equation $2A4 = 5B3 - 329$ to find values of A that make the equation true.

1. If $B = 1$, then $513 - 329 = 184$, which cannot equal $2A4$. So when $B = 1$ there is no A to satisfy the problem.
2. If $B = 4$, then $543 - 329 = 214$, which does equal $2A4$ when $A = 1$. So for $A = 1$ and $B = 4$ there is a valid solution.
3. If $B = 7$, then $573 - 329 = 244$, which does equal $2A4$ when $A = 4$. So for $A = 4$ and $B = 7$ there is a valid solution.

Therefore, when $A = 1$ and $B = 4$ or when $A = 4$ and $B = 7$, the given problem has a valid solution.

Solution 2

$$\begin{array}{cccc} & 2 & A & 4 \\ + & 3 & 2 & 9 \\ \hline & 5 & B & 3 \end{array}$$

When the digits in the unit's column are added together, there is one carried to the ten's column. When the digits in the hundred's column are added together we get $2 + 3 = 5$ so there is no carry from the ten's column. Therefore, when the ten's column is added we get $1 + A + 2 = B$ or $A + 3 = B$.

We can now look at all possible values for A that produce a single digit value for B in the number $5B3$. We can then determine whether or not $5B3$ is divisible by 3.

The following table summarizes the results.

A	$B = A + 3$	$5B3$	Divisible by 3 (*yes*/*no*)?
0	3	533	*no*
1	4	543	*yes*
2	5	553	*no*
3	6	563	*no*
4	7	573	*yes*
5	8	583	*no*
6	9	593	*no*

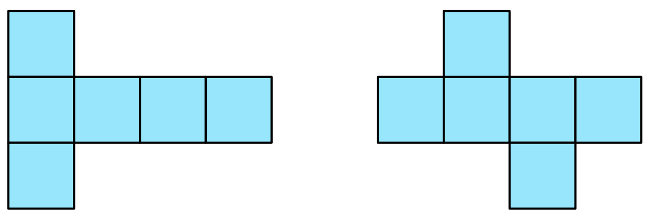
Therefore, when $A = 1$ and $B = 4$ or when $A = 4$ and $B = 7$, the given problem has a valid solution.

CEMC at Home

Grade 7/8 - Friday, April 17, 2020
Cut It Out

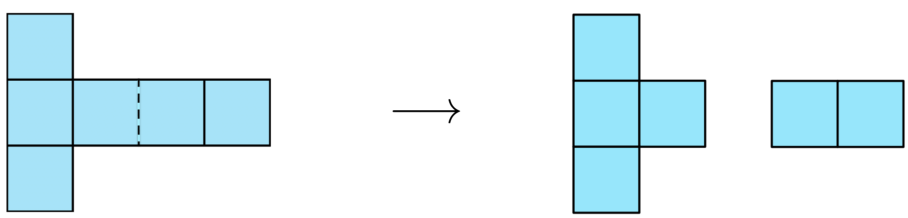
When you look outside your window, many of you will see different buildings or other three-dimensional structures. If you look closely, you can start to break these buildings down into familiar shapes. Understanding how to compose and decompose two-dimensional and three-dimensional shapes is an important skill in mathematics.

In this activity, we are going to explore this idea by looking at nets of three-dimensional objects. A net is a two-dimensional representation of a three-dimensional object, and is what the object would look like if its surface was opened up and laid flat. Look at the two nets below.



Even though these two nets are different, they can both be folded along the lines shown and used to form the surface of a cube. These are two examples of the many different nets of a cube.

Activity 1: Imagine that the first net of a cube given above has been cut in two, as shown below. In how many different ways can you attach these two pieces together in order to form a net of a cube?



To get started, try cutting out the two net pieces and putting them together in different ways.

Follow-up to Activity 1:

- Can you draw a few different nets of a cube that *cannot* be formed by attaching the pieces from Activity 1? (Note that one such net can be found above Activity 1.)
- Can you attach the two pieces from Activity 1 together (matching up edges) in a way that creates a net for an object that is *not* a cube?

Activity 2: Nets of seven different three-dimensional objects have been cut in two and scattered. Find the correct pairs and match them with the appropriate three-dimensional object.

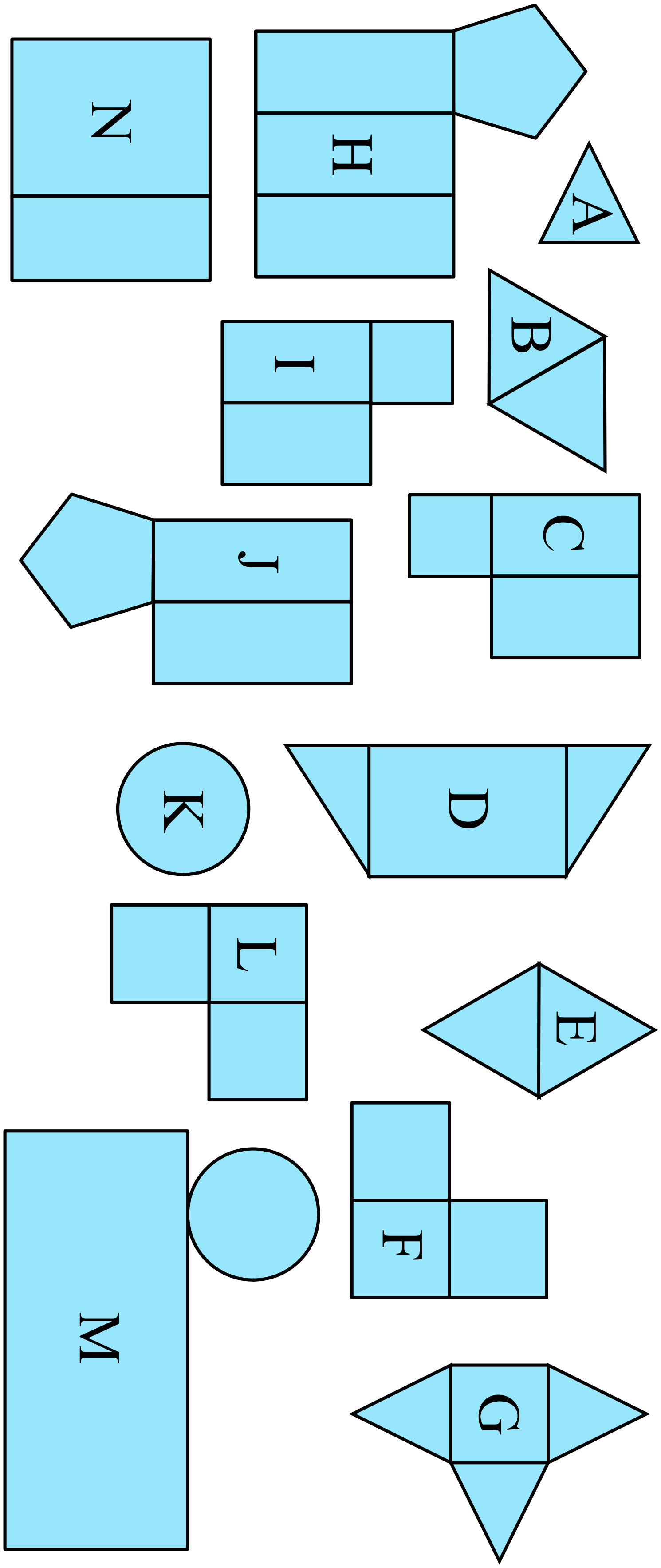
See the next page for the shapes for Activity 2 and a table to record your findings.
To get started, try cutting out the net pieces and putting them together in different ways.

More Info:

Check out the CEMC at Home webpage on Monday, April 20 for a solution to Cut It Out.

This problem was inspired by an activity from NRICH Maths.

For more practice using nets, check out this lesson in the CEMC Courseware.



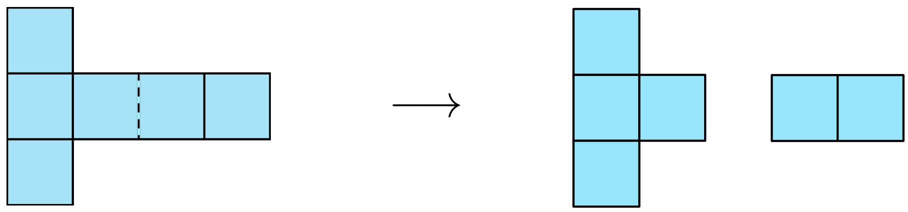
Record your answers to Activity 2 in the table below.

Net Pieces	3D Shape
	cube
	rectangular prism
	triangular prism
	pentagonal prism
	cylinder
	square pyramid
	triangular pyramid

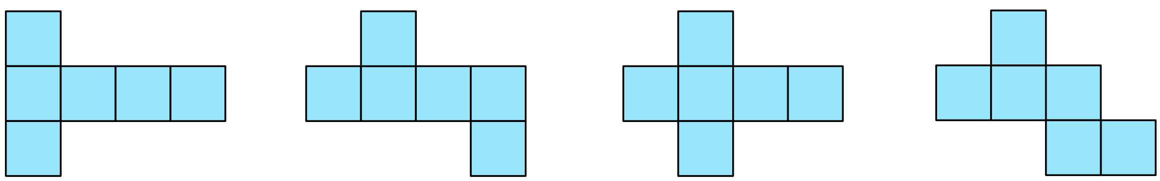
CEMC at Home

Grade 7/8 - Friday, April 17, 2020
Cut It Out - Solutions

Activity 1: Imagine that the following net of a cube given has been cut in two, as shown below. In how many different ways can you attach these two pieces together in order to form a net of a cube?

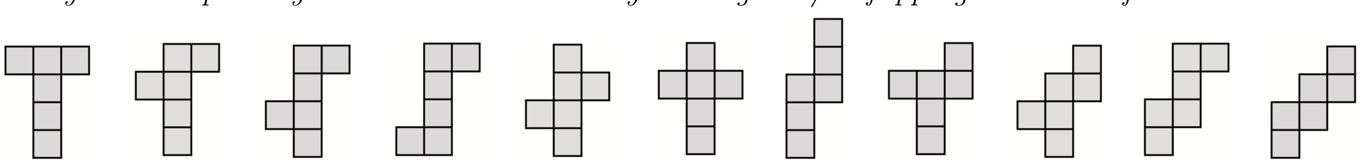


Answer: You can put the pieces together in the following ways to make four different nets of a cube.



The net shown above furthest to the left is the net that we started with (before the cut in Activity 1), and the other three nets are new.

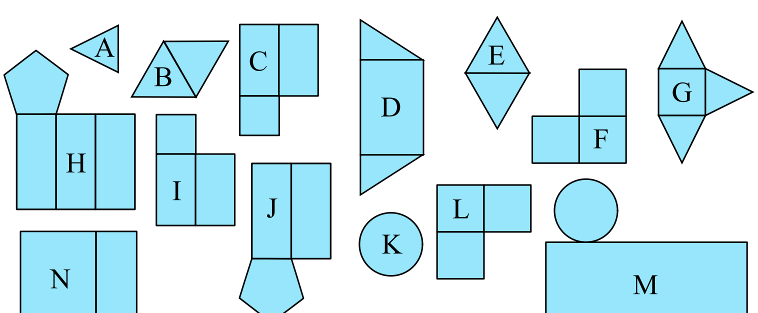
Note that there are eleven different nets of a cube and these are shown below. Every net of a cube that you could possibly draw can be obtained by turning and/or flipping over one of the nets below.



Four of these eleven nets can be made by attaching the pieces from Activity 1 in some way, and the remaining seven cannot. Can you explain why?

Activity 2: The nets of seven different three-dimensional objects have been cut in two and scattered. Find the correct pairs and match them with the appropriate three-dimensional object.

Net pieces:



Answers:

Net Pieces	3D Shape
F and L	cube
C and I	rectangular prism
D and N	triangular prism
H and J	pentagonal prism
K and M	cylinder
A and G	square pyramid
B and E	triangular pyramid