CEMC at Home

Grade 7/8 - Monday, March 23, 2020
Multiplication War

You Will Need:

- One full deck of cards
- Two players

Before you Start:

In this game, each card is assigned a value as follows:

- Aces are worth 1, Jacks are worth 11, Queens are worth 12, and Kings are worth 13
- All other cards are assigned the value of the number on the card
- Black cards are all positive numbers
- Red cards are all negative numbers

How to Play:

1. Deal out all cards evenly between two players. Each player puts their cards face down in a pile in front of them.
2. Each player flips over their top two cards at the same time and multiplies them together. *Remember that a negative times a negative equals a positive, and that* -3 *is greater than* -8*.*
3. The player whose cards have the highest product wins all four cards.
4. If the cards have the same product, then the players each flip over two more cards and calculate their product. This process is repeated until one player has a higher product than the other. This player then wins all face-up cards.
5. The player who ends up with all the cards (or the most cards after a set amount of time) wins.

Variations:

- For a warm-up, try playing without any negative numbers.
- For a challenge, come up with a harder version of this game. You could use fractions, decimals, exponents, etc. We encourage you to share your ideas online using any forum you are comfortable with.

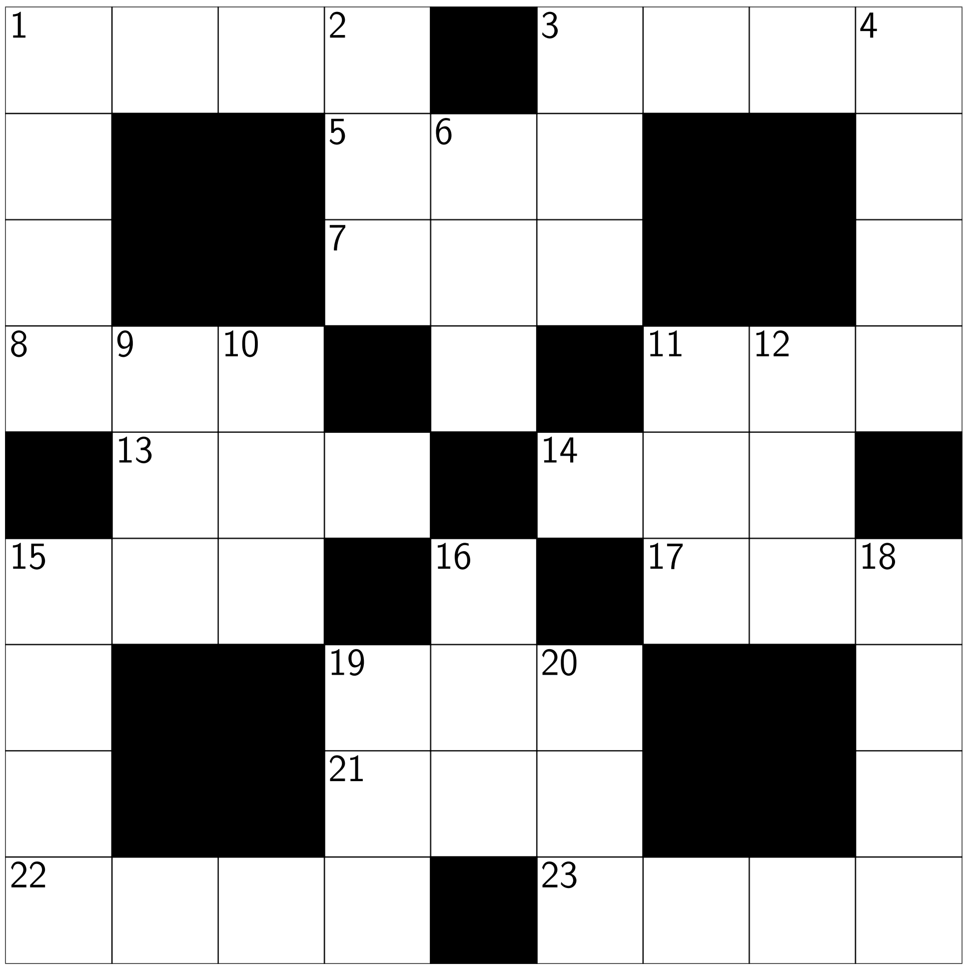
More Info:

Need a reminder on how to multiply positive and negative numbers? Check out this lesson on the CEMC Courseware.

CEMC at Home

Grade 7/8 - Tuesday, March 24, 2020
Crossnumber Puzzle

Use the clues on the next page to complete the crossnumber puzzle below. Each square of the grid will contain exactly one digit. Notice that some answers can be found using only the given clue, and some need the answers from other clues.



More Info:

Check out the CEMC at Home webpage on Wednesday, March 25 for a solution to the Crossnumber Puzzle.

Across

1. The result of $\left(1000 - \boxed{2 \text{ DOWN}}\right) \times 103$.
3. A number whose digits are all perfect squares and add to 31.
5. The value of $11 \times 12 + 13$.
7. The number of centimetres in 7.29 metres.
8. A telephone area code in Waterloo, Ontario, Canada.
11. A number whose digits add to a multiple of 9.
13. A rearrangement of the digits in the quotient when $\boxed{10 \text{ DOWN}}$ is divided by 3.
14. A prime number less than 200.
15. The sum of $\boxed{9 \text{ DOWN}}$ and $\boxed{7 \text{ ACROSS}}$.
17. The result when $\boxed{14 \text{ ACROSS}}$ is multiplied by 8.
19. The number of days in a leap year.
21. The total value (in cents) of 5 quarters, 3 dimes, and 2 nickels.
22. A number whose digits are all perfect squares and add to 15.
23. The perimeter of a square with side length $\boxed{20 \text{ DOWN}}$ units.

Down

1. A multiple of 5 between $\boxed{15 \text{ DOWN}}$ and $\boxed{18 \text{ DOWN}}$.
2. The result of $\boxed{5 \text{ ACROSS}}$ multiplied by 6, then added to 47.
3. A number whose first two digits add to its third digit.
4. The least common multiple of $\boxed{11 \text{ ACROSS}}$ and $\boxed{21 \text{ ACROSS}}$.
6. A palindrome.
9. The value of $5 \times 5 \times 5$.
10. The largest 3-digit multiple of 24.
11. A number that is $\frac{2}{3}$ of 1212.
12. The number of seconds in 8.5 minutes.
15. $\boxed{14 \text{ ACROSS}}$ less than a multiple of 1000.
16. The sum of $\boxed{21 \text{ ACROSS}}$ and $\boxed{14 \text{ ACROSS}}$.
18. The product of $\boxed{17 \text{ ACROSS}}$ and 11.
19. The first three digits in π (pi).
20. A number whose digits multiply to 60.

CEMC at Home

Grade 7/8 - Tuesday, March 24, 2020
Crossnumber Puzzle - Solution

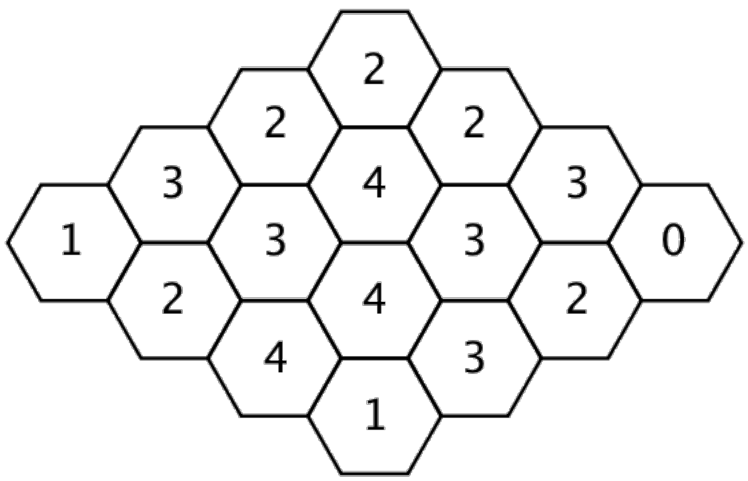
8	5	4	9		4	9	9	9
8			1	4	5			4
9			7	2	9			0
5	1	9		4		8	5	5
	2	8	3		1	0	1	
8	5	4		2		8	0	8
8			3	6	6			8
9			1	6	5			8
9	1	1	4		2	6	0	8

CEMC at Home

Grade 7/8 - Wednesday, March 25, 2020
Beehive

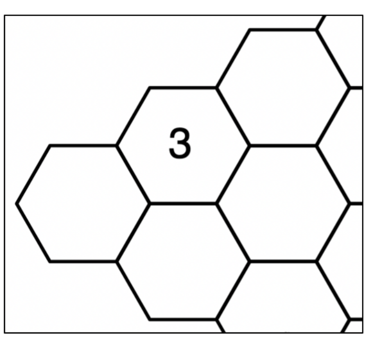
A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many *other* hexagons touching this hexagon contain honey. The results of the bear's study are shown. How many hexagons in the honeycomb contain honey?

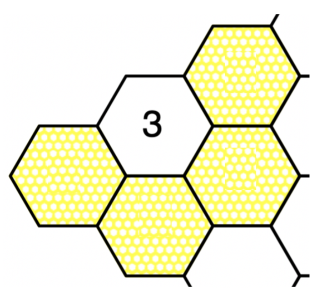
Honeycomb 1



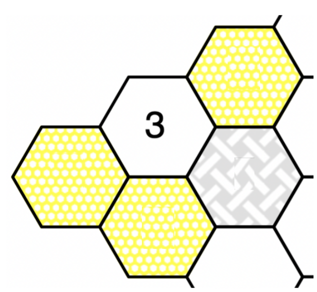
Need Help Getting Started?

Look at the hexagon marked with a "3" near the left corner of the honeycomb above. Notice that there are exactly 4 other hexagons that are touching this hexagon. This number 3 tells us that exactly 3 of those 4 touching hexagons have honey. But can you figure out which 3?





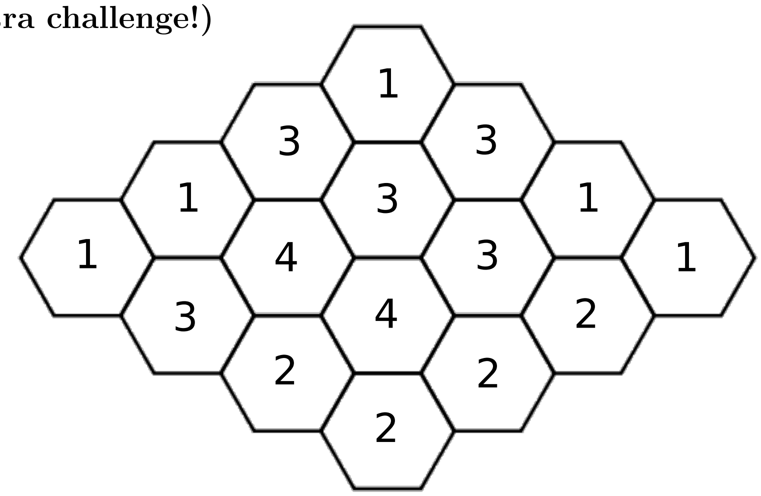
Impossible



Possible

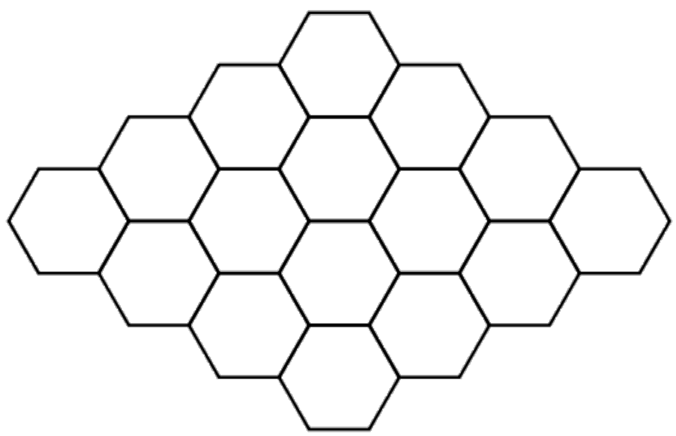
Use the online exploration (https://www.geogebra.org/m/mdbfsjvj) for this question to help you work through the solution. By clicking on a hexagon you can mark whether or not you think it contains honey. You can use this to keep track of what you discover about the hexagons as you go.

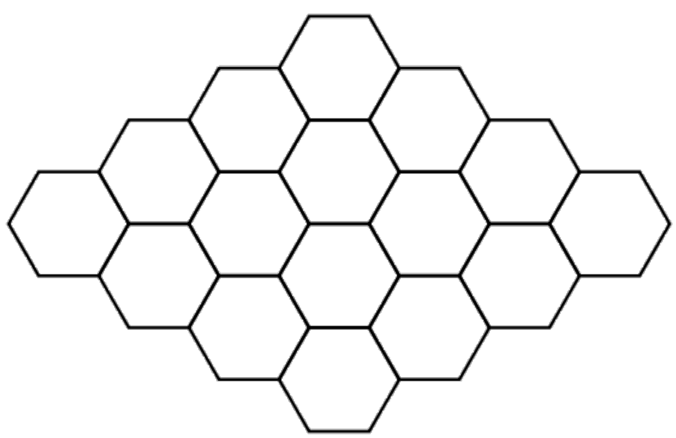
Honeycomb 2 (Extra challenge!)



You might find the second honeycomb harder to figure out. What makes this honeycomb more difficult?

Extension: Use the empty honeycombs below to create your own beehive problems and share them with your friends and family.





More Info:

Check out the CEMC at Home webpage on Thursday, March 26 for a solution to Beehive.

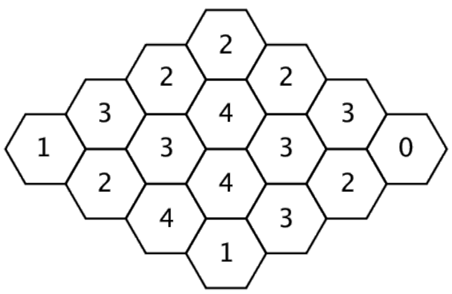
This problem was inspired by a problem on the *Beaver Computing Challenge*. You can find more problems like this on past BCC contests.

CEMC at Home

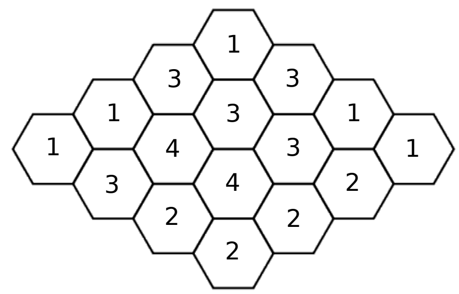
Grade 7/8 - Wednesday, March 25, 2020
Beehive - Solution

Question:
A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many *other* hexagons touching this hexagon contain honey. The results of the bear's study are shown. How many hexagons contain honey?

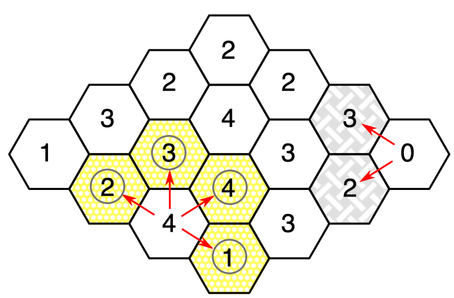
Honeycomb 1



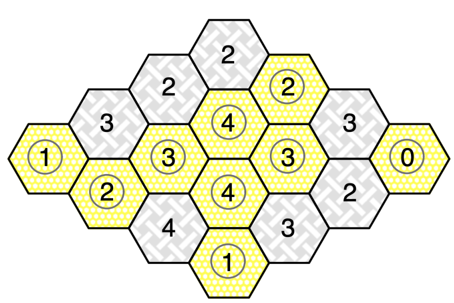
Honeycomb 2



Solution Honeycomb 1:
One way to solve this is to start from a hexagon that contains a zero, because that tells us none of the hexagons touching it contain honey. Another way to solve this is to start from a hexagon that contains a number that is equal to the total number of hexagons touching it, because that tells us all of those touching hexagons contain honey. In this honeycomb we could do either strategy as shown below, where hexagons that contain honey are yellow with a circle around the number, and hexagons that do not contain honey are filled with a grey woven pattern.



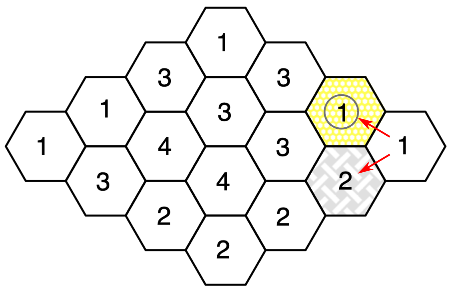
After this first step, we can move through the honeycomb, determining which hexagons contain honey based on the number inside each hexagon and the conclusions we have already made about the hexagons touching it. The completed honeycomb should look like this.



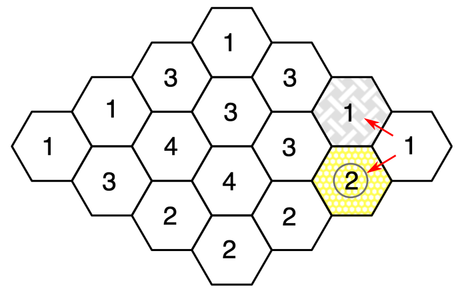
Therefore, 9 hexagons contain honey.

Solution Honeycomb 2:
This honeycomb is more challenging than the first honeycomb because there does not appear to be a good place to start. No hexagon contains a zero, and no hexagon contains a number equal to the total number of hexagons touching it. To solve this honeycomb one strategy is trial and error. Pick a hexagon to start with. Look at all the different options for the hexagons touching it.

For example, suppose we started with the rightmost hexagon in the honeycomb. This might be a good place to start because it is not touching many other hexagons. Since it contains the number one, that tells us that exactly one of the two hexagons touching it contains honey. So we have two options as shown below, where hexagons that contain honey are yellow with a circle around the number, and hexagons that do not contain honey are filled with a grey woven pattern.



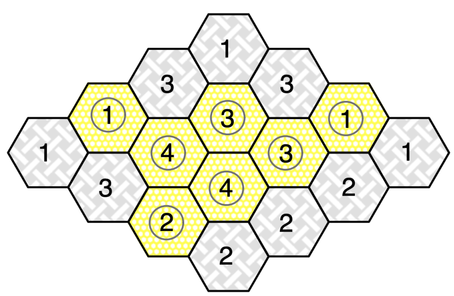
First possibility



Second possibility

We know that one of these pictures must be the correct one, but we cannot be sure which one is correct at this time. So how about we make a guess? For example, we can colour the honeycombs as shown in the second possibility above, and then see if we can move through the honeycomb colouring hexagons from there. If we find that something goes wrong, or we get stuck, then we can always go back to where we made our first choice and make a different choice.

In the end, there is only one way to colour the honeycomb that agrees with all of the numbers in the hexagons. The completed honeycomb should look like this.



Therefore, 7 hexagons contain honey.

Were you able to find this solution on your own? If not, then look at the picture above and check for yourself that this colouring works!

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, March 26, 2020

Ch-ch-changes

Bill made some purchases that totalled \$18.75 and paid for them with a twenty-dollar bill. The cash register has only quarters, dimes and nickels.

In how many different ways can the cashier make change?

"TOONIE" 2 dollar coin	"LOONIE" 1 dollar coin	QUARTER	NICKEL	DIME
200 cents	100 cents	25 cents	5 cents	10 cents

Remember, in this problem, we will only be using quarters, nickels and dimes.

More Info:

Check the CEMC at Home webpage on Thursday, April 2 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 2.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
Ch-ch-changes

Problem

Bill made some purchases that totalled \$18.75 and paid for them with a twenty-dollar bill. The cash register has only quarters, dimes and nickels. In how many different ways can the cashier make change?

Solution

This is a good problem for applying a systematic approach.

The amount of change required is $\$20 - \$18.75 = \$1.25$ or 125 cents. In order to get to 125 cents, a maximum of 5 quarters are required. Once we determine the amount still required after the value of the quarters has been removed, we can determine the number of different combinations of dimes that can be given. For each of these possibilities, the remainder of the change will be nickels.

If 5 quarters are given as part of the change, the \$1.25 required as change is covered and no other coins are required. There is only 1 possibility for change in which 5 quarters are part of the change.

If 4 quarters are given as part of the change, \$0.25 is still required. There are 3 possibilities for dimes; either 0, 1 or 2 dimes. Therefore, there are 3 different coin combinations possible in which 4 quarters are part of the change.

If 3 quarters are given as part of the change, \$0.50 is still required. There are 6 possibilities for dimes; either 0, 1, 2, 3, 4, or 5 dimes. Therefore, there are 6 different coin combinations possible in which 3 quarters are part of the change.

If 2 quarters are given as part of the change, \$0.75 is still required. There are 8 possibilities for dimes; either 0, 1, 2, 3, 4, 5, 6, or 7 dimes. Therefore, there are 8 different coin combinations possible in which 2 quarters are part of the change.

If 1 quarter is given as part of the change, \$1.00 is still required. There are 11 possibilities for dimes; either 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 dimes. Therefore, there are 11 different coin combinations possible in which 1 quarter is part of the change.

If no quarters are given as part of the change, \$1.25 is still required. There are 13 possibilities for dimes; either 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 dimes. Therefore, there are 13 different coin combinations possible in which no quarters are part of the change.

The cashier can make the required change using $1 + 3 + 6 + 8 + 11 + 13 = 42$ different possible combinations of coins.

The solution is presented in chart form on the following page.

Number of Quarters	Value of Quarters (in cents)	Amount Remaining (in cents)	Number of Dimes	Value of Dimes (in cents)	Amount Remaining (in cents)	Number of Nickels Required
5	125	0	0	0	0	0
4	100	25	2	20	5	1
			1	10	15	3
			0	0	25	5
3	75	50	5	50	0	0
			4	40	10	2
			3	30	20	4
			2	20	30	6
			1	10	40	8
			0	0	50	10
2	50	75	7	70	5	1
			6	60	15	3
			5	50	25	5
			4	40	35	7
			3	30	45	9
			2	20	55	11
			1	10	65	13
			0	0	75	15
1	25	100	10	100	0	0
			9	90	10	2
			8	80	20	4
			7	70	30	6
			6	60	40	8
			5	50	50	10
			4	40	60	12
			3	30	70	14
			2	20	80	16
			1	10	90	18
			0	0	100	20
0	0	125	12	120	5	1
			11	110	15	3
			10	100	25	5
			9	90	35	7
			8	80	45	9
			7	70	55	11
			6	60	65	13
			5	50	75	15
			4	40	85	17
			3	30	95	19
			2	20	105	21
			1	10	115	23
			0	0	125	25

The cashier can make the required change using $1+3+6+8+11+13=42$ different possible combinations of coins.

CEMC at Home

Grade 7/8 - Friday, March 27, 2020
Tessellations

A tessellation (or tiling) is an arrangement of one or more shapes in a repeated pattern without overlaps or gaps. You have probably seen tessellations before without even realizing it! For example, brick walls are tessellations of rectangles, some flooring designs are tessellations of squares, and the honeycombs in a beehive are tessellations of hexagons.

Activity 1: Let's create our own original tessellations!

You Will Need:

- 1 piece of paper (letter size or bigger)
- 1 piece of card stock about the size of a playing card (or regular paper cut to this size)
- A ruler
- Scissors
- Tape
- Coloured markers or pens

Try This:

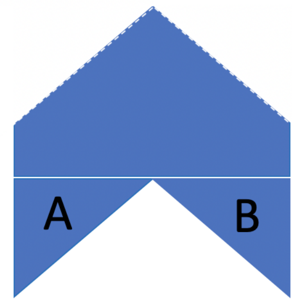
1. Start with your rectangle cut to the size of a playing card.

2. Make alterations to the top edge of your shape.

   Interesting alterations make for interesting tessellations! But you might want to make sure that the alteration you choose on your first try is simple enough that you can easily work with your shape.

3. Translate the alterations to the bottom edge of your shape and tape in place.

   Make sure to translate the pieces vertically as shown. Any shifting in the horizontal direction may result in a shape that won't work for you in steps 5 and 6.



4. Trace your new shape on the piece of paper.

5. Repeatedly translate your shape vertically and horizontally on the paper, always fitting it snugly into or against the previously drawn shape, tracing as you go.

6. Colour in your shapes and create a beautiful piece of art!

Activity 2: Let's investigate polygons and tessellations!

We have seen that squares and hexagons both form nice tessellations. These shapes are both examples of what are called *regular polygons*. (A polygon is a closed figure with straight sides. A polygon is regular if all sides are equal in length and all angles are equal in measure.) It is likely that the shapes you formed in Activity 1 were not regular polygons. Can you think of any other regular polygons that can be used to form a tessellation? Are there regular polygons that cannot be used to form a tessellation?

> *Need help getting started with Activity 2?*
>
> Try and cut out a regular polygon and use it as your shape like you did in Activity 1.
>
> If you would like to try something new, then use the following investigation: Explore This!
>
> Here you can use technology to help you explore whether equilateral triangles, squares, pentagons or hexagons can be used to form a tessellation. Can you figure out what features make a polygon good for making a tessellation?

More Info:

If you are interested in learning more about tessellations check out this lesson in the CEMC Courseware. You can find a discussion of Activity 2 there!

CEMC at Home

Grade 7/8 - Monday, March 30, 2020
Sum the Dice

You Will Need:

- At least two players
- Two dice
- Paper and a pencil for keeping score

How to Play:

1. Players alternate turns rolling the two dice at once.
2. On your turn, you start by rolling the dice once. Here is how the score for this turn is decided:
   - If you rolled a 1 on either die, then your turn is over and you score 0 on this turn.
   - If you did not roll a 1 on either die, then you add up the two numbers rolled and record this number.
   - You now decide if you would like to roll again, or take the this recorded number as your final score for this turn.
   - If you decide to roll again, then you keep adding the sum of the two dice on each roll to your running total, until you either decide to stop or you roll a 1 on either die.
   - If you decide to stop on your own, then you take the total you have accumulated as your score for this turn, but if you roll a 1 at any time, then your turn is over and you score a 0 on this turn.
3. The first player whose total score (on all of their turns) reaches 100 wins the game!

Play the game a number of times. Can you come up with a good strategy for this game?

Follow-up Questions (*You can use the aids on the next page to help you answer these questions.*)

1. Let's say you decide in advance that you will stop after one roll no matter what you see on the dice. In this case, there are 10 different possible scores that you could achieve on this turn. We will call these scores the *possible outcomes*. Fill in the boxes below with the possible outcomes.

2. Explain why the 10 possible outcomes in 1. are not all *equally likely* to occur.
3. Write the outcomes in 1. in order from *most likely* to *least likely* to occur. Explain your thinking.

most likely ⟶ **least likely**

4. Does this exploration lead you to believe that you should change your strategy for this game?

More Info:

Check out the CEMC at Home webpage on Tuesday, March 31 for a solution to Sum the Dice.

Aids for the follow-up questions:

Each cell in the table below represents a possible result when two dice are rolled. Notice that the row and column with a 1 have already been filled in. This is because if you roll a 1 on either die, then your score for the turn is zero.

Fill in the rest of the table with the score for each possible roll and then use it to answer the follow-up questions.

		Die #1 Roll					
		1	2	3	4	5	6
Die #2 Roll	1	0	0	0	0	0	0
	2	0					
	3	0					
	4	0					
	5	0					
	6	0					

An *outcome* is the result of a probability experiment, such as rolling a die.
Outcomes are considered *equally likely* if they have the same chance of happening.

If you want more practice exploring outcomes in a game or an experiment, then check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Monday, March 30, 2020
Sum the Dice - Solution

		Die #1					
		1	2	3	4	5	6
Die #2	1	0	0	0	0	0	0
	2	0	4	5	6	7	8
	3	0	5	6	7	8	9
	4	0	6	7	8	9	10
	5	0	7	8	9	10	11
	6	0	8	9	10	11	12

Follow-up Questions:

1. Let's say you decide in advance that you will stop after one roll no matter what you see on the dice. In this case, there are 10 different possible scores that you could achieve on this turn. We will call these scores the *possible outcomes*. Fill in the boxes below with the possible outcomes.

0	4	5	6	7	8	9	10	11	12

2. Explain why the 10 possible outcomes in 1. are not all *equally likely* to occur.

   This is because there are more ways to get some outcomes than others. For example, there are two ways to get a score of 5 but only one way to get a score of 4.

3. Write the outcomes in 1. in order from *most likely* to *least likely* to occur. Explain your thinking.

0	8	7 & 9	6 &10	5 & 11	4 & 12

most likely ⟶ **least likely**

Using the table, we can count the number of ways we can roll each score. The greater the number of ways we can roll a particular score, the more likely that score is. We noticed that some scores are equally likely (for example 7 and 9), so we wrote them in the same box.

There are 11 ways to roll a score of 0, so it is the most likely. There are 5 ways to roll a score of 8. There are 4 ways to roll a score of 7 or a score of 9. There are 3 ways to roll a score of 6 or a score of 10. There are two ways to roll a score of 5 or a score of 11. There is only 1 way to roll a score of 4 or a score of 12, so those are the least likely to be rolled.

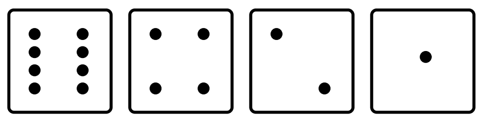
CEMC at Home

Grade 7/8 - Tuesday, March 31, 2020
Building Numbers

In this activity we will explore a way to build the counting numbers. Let's think about the following sequence of numbers:

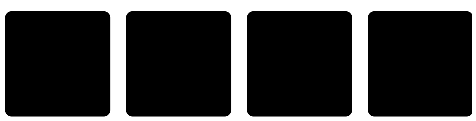
$$1, 2, 4, 8$$

Notice that each number is twice as big as the number to its left. (You may recognize these numbers as powers of 2.) We will represent each of the numbers in the list using a card with the correct number of dots on its face. Notice that we have placed the cards in order from most to fewest dots.

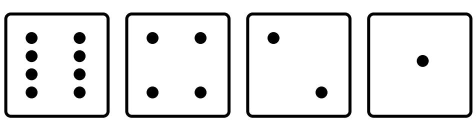


We will build different numbers by choosing which of these cards to put face up and which of these cards to put face down. The number represented by the cards will be the total number of dots that are showing.

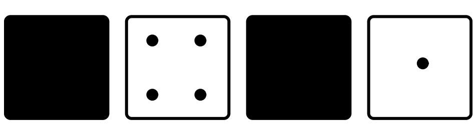
Let's look at some examples.



If we put all of the cards face down, this would represent the number 0 because there are no dots showing.



If we put all of the cards face up, this would represent the number 15 because there are $8+4+2+1 = 15$ dots showing in total.



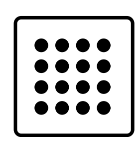
This would represent the number 5 because there are $4 + 1 = 5$ dots showing in total.

Question 1: How many different numbers can you represent using these four cards? List all of the numbers that can be represented and show how to represent each one using the cards.

See the next page for tools to help organize your solution.

Question 2: Now add a fifth card that has 16 dots on its face. Can you determine which numbers can be represented using the five cards (following the same rules)?

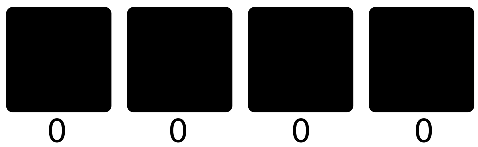
How might you use your work in Question 1 to help answer Question 2?



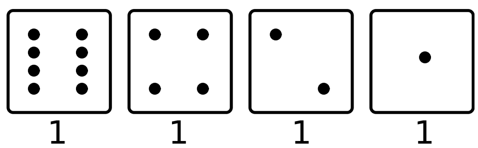
Variation: Make four cards with 1, 3, 9, and 27 dots on them. What numbers can you make with these cards? What do you notice about building numbers with these cards that is different from building numbers with the original cards 1, 2, 4, 8?

On the last page, you will find cards you can cut out and use while you explore these questions.

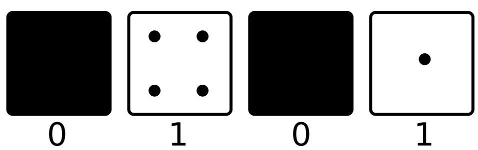
Drawing the cards for each number will take a lot of time, so let's use a simple "code" to keep track of our work. We will indicate that a card is face up by a 1, and indicate that a card is face down with a 0. This means we can communicate what our cards look like using just 0s and 1s.



The code 0000 represents all cards being face down (the number 0).



The code 1111 represents all cards being face up (the number 15).



The code 0101 represents this configuration of cards (the number 5).

Record all of the numbers you can represent here.

Code	Number
0000	0

Code	Number

Code	Number

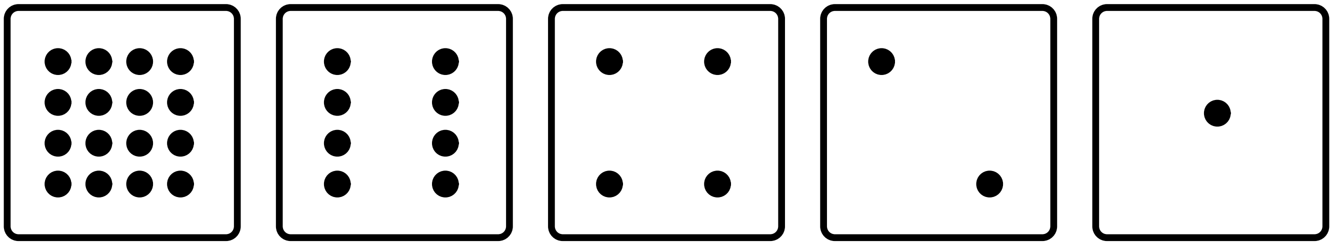
Code	Number
1111	15

More Info:

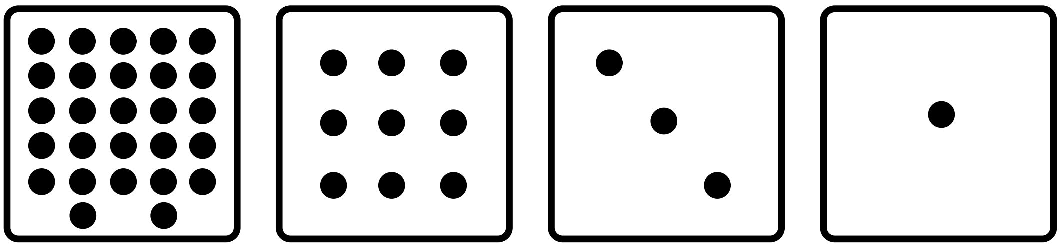
Check out the CEMC at Home webpage on Wednesday, April 1 for the solution to Building Numbers.

We will explore Building Numbers further on Wednesday, April 1 with Secret Messages.

Cards for the main activity



Cards for the variation



CEMC at Home

Grade 7/8 - Tuesday, March 31, 2020
Building Numbers - Solution

Solution to Question 1: We can represent each of the integers from 0 to 15 using the four cards. Here are the codes for these 16 integers.

Code	Number
0000	0
0001	1
0010	2
0011	3

Code	Number
0100	4
0101	5
0110	6
0111	7

Code	Number
1000	8
1001	9
1010	10
1011	11

Code	Number
1100	12
1101	13
1110	14
1111	15

Solution to Question 2: We can represent each of the integers from 0 to 31 using the five cards. We can represent each of the numbers from Question 1 by using the same configuration for the cards 1, 2, 4, and 8, as given above and then placing the new card (with 16 dots) face down. We can make the numbers 16 through 31 in a similar way but by placing the new card face up. For example, since 1001 is the code for 9 in Question 1, 01001 will be the code for 9 in Question 2, and 11001 will be the code for $9+16 = 25$ in Question 2. Here are the codes for all 32 integers that can be represented.

Code	Number
00000	0
00001	1
00010	2
00011	3
00100	4
00101	5
00110	6
00111	7

Code	Number
01000	8
01001	9
01010	10
01011	11
01100	12
01101	13
01110	14
01111	15

Code	Number
10000	16
10001	17
10010	18
10011	19
10100	20
10101	21
10110	22
10111	23

Code	Number
11000	24
11001	25
11010	26
11011	27
11100	28
11101	29
11110	30
11111	31

Variation: Using the cards with 1, 3, 9, and 27 dots, you can make the following numbers:

$$0, 1, 3, 4, 9, 10, 12, 13, 27, 28, 30, 31, 36, 37, 39, 40$$

Notice that we can represent exactly 16 numbers just like we could with the cards with 1, 2, 4, and 8 dots. The main difference here is that there are gaps in our list. For example, we cannot represent any number from 14 to 26. Is it surprising that there are gaps in this list, while the lists in Questions 1 and 2 had no gaps?

CEMC at Home

Grade 7/8 - Wednesday, April 1, 2020
Secret Messages

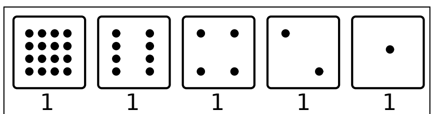
In yesterday's activity Building Numbers, we used cards and codes to represent counting numbers using only the digits 0 and 1. (If you didn't do this activity, please try it now. You may want the cards for today's activity too.) Today we will use these same ideas to write secret messages!

Our secret messages will have two "levels of secrecy" which are explained below.

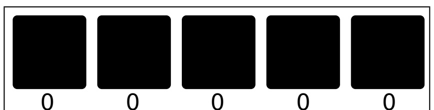
First, we pair each letter of the alphabet with an integer from 1 to 26 as shown in the table.

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
1	2	3	4	5	6	7	8	9	10	11	12	13
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
14	15	16	17	18	19	20	21	22	23	24	25	26

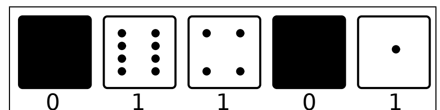
Second, we pair each number between 1 and 26 with a sequence of five digits, all either 0 or 1. We can set up the five cards as in Building Numbers to help us with this part of the code. Remember that to determine the number represented by the cards, we count the total number of dots showing.



11111 codes the number 31



00000 codes the number 0



01101 codes the number 13

Activity 1: Decode the following secret messages.

1. $\boxed{01001}$ $\boxed{00001}\boxed{01101}$ $\boxed{10111}\boxed{10010}\boxed{01001}\boxed{10100}\boxed{01001}\boxed{01110}\boxed{00111}$
   $\boxed{01001}\boxed{01110}$ $\boxed{00011}\boxed{01111}\boxed{00100}\boxed{00101}$.

2. $\boxed{00011}\boxed{00001}\boxed{01110}$ $\boxed{11001}\boxed{01111}\boxed{10101}$
   $\boxed{10010}\boxed{00101}\boxed{00001}\boxed{00100}$ $\boxed{10100}\boxed{01000}\boxed{01001}\boxed{10011}$?

Activity 2: Write your own secret messages for your friends and family to decode. Can they read your messages without knowing your coding plan? Explain to them how to decode your messages.

Extension: Can you make a similar coding scheme instead using the cards with 1, 3, and 9 dots from Building Numbers? You may notice that you cannot represent all of the integers from 1 to 26 by placing these cards face up or face down as usual. Can you fix this problem by using two copies of each card?

More Info:

Check out the CEMC at Home webpage on Thursday, April 2 to see a solution to Secret Messages.

Did you know that the codes we have been using (the sequences of 0s and 1s) are called *binary numbers*? Every counting number can be represented using a binary number and binary numbers are used by computers to store and share information.

Cryptography is the study of reading and writing secret messages. To learn more about cryptography, check out this Math Circles lesson.

CEMC at Home

Grade 7/8 - Wednesday, April 1, 2020
Secret Messages - Solution

Answers for Activity 1: Here are the secret messages decoded.

1. 01001 00001 01101 10111 10010 01001 10100 01001 01110 00111 01001 01110 00011 01111 00100 00101.

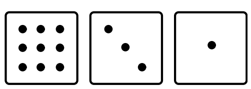
   I AM WRITING IN CODE.

2. 00011 00001 01110 11001 01111 10101 10010 00101 00001 00100 10100 01000 01001 10011?

   CAN YOU READ THIS?

Discussion of the Extension:

You can only represent eight numbers using the following three cards.

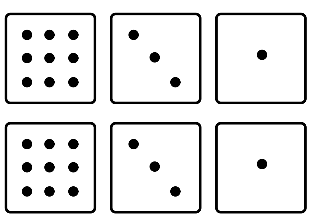


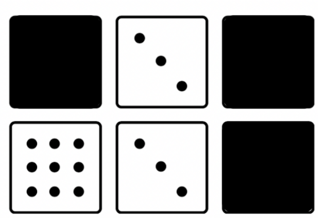
Remember that the digit 1 indicates that the card is face up and 0 indicates that the card is face down.

Code	Number
000	0
001	1
010	3
011	4

Code	Number
100	9
101	10
110	12
111	13

This is not enough to assign a different code to each letter in the alphabet, but we can fix this issue if we instead use two of each card to make our codes.





1 2 0

Again, let's use sequences of three digits to represent numbers, but this time we will use the digits 0, 1, and 2 according to the following rules:

- If both cards (of the same type) are face down, then the digit is 0.
- If exactly one card (of the two cards of the same type) is face up, then the digit is 1.
- If both cards (of the same type) are face up then the digit is 2.

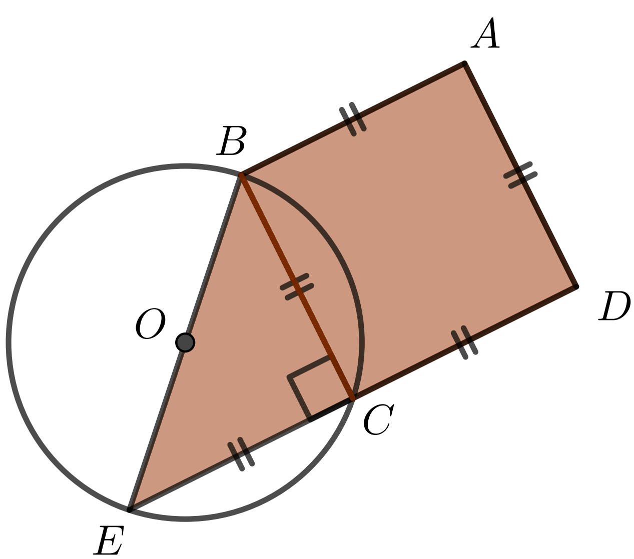
In the example above, we see that the code 120 represents the number $1 \times 9 + 2 \times 3 + 0 \times 1 = 15$. Can you represent all of the integers from 1 to 26 using this new coding strategy?

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 2, 2020
A Circle and Other Shapes

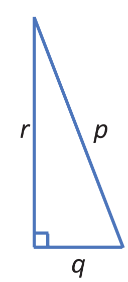
Quadrilateral $ABED$ is made up of square $ABCD$ and right isosceles $\triangle BCE$. BE is a diameter of the circle with centre O. Point C is also on the circle.

If the area of $ABED$ is 24 cm^2, what is the length of BE?



The *Pythagorean Theorem* states, "In a right triangle, the square of the length of hypotenuse (the side opposite the right angle) equals the sum of the squares of the lengths of the other two sides".

In the right triangle to the right, $p^2 = r^2 + q^2$.

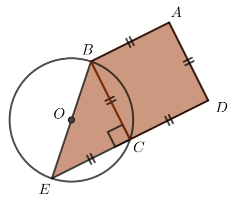


More Info:

Check the CEMC at Home webpage on Thursday, April 9 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 9.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

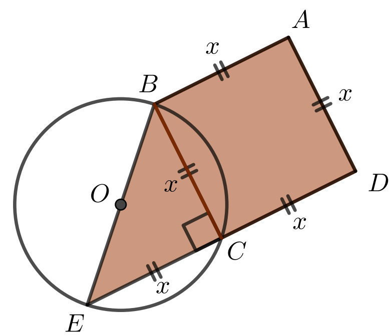


Problem of the Week
Problem C and Solution
A Circle and Other Shapes

Problem

Quadrilateral $ABED$ is made up of square $ABCD$ and right isosceles $\triangle BCE$. BE is a diameter of the circle with centre O. Point C is also on the circle. If the area of $ABED$ is 24 cm^2, what is the length of BE?

Solution



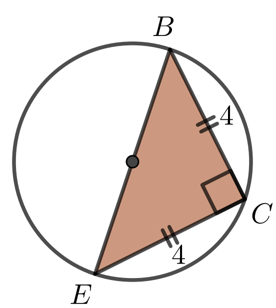
Let $AB = AD = DC = CB = CE = x$.
Therefore, the area of square $ABCD$ is x^2 and the area of $\triangle BCE$ is $\frac{1}{2}(x)(x) = 0.5x^2$.

Therefore,

$$\begin{aligned} \text{total area of quadrilateral } ABED &= \text{area of square } ABCD + \text{ area of } \triangle BCE \\ &= x^2 + 0.5x^2 \\ &= 1.5x^2 \end{aligned}$$

Now we also know that the area of $ABED$ is 24 cm^2. Therefore,

$$\begin{aligned} 1.5x^2 &= 24 \\ \frac{1.5x^2}{1.5} &= \frac{24}{1.5} \\ x^2 &= 16 \\ x &= 4, \text{ since } x > 0 \end{aligned}$$



Now let's look at $\triangle BCE$.
We know $BC = CE = 4$.
Using the Pythagorean Theorem,

$$\begin{aligned} BE^2 &= 4^2 + 4^2 \\ &= 16 + 16 \\ &= 32 \\ BE &= \sqrt{32}, \text{ since } BE > 0 \end{aligned}$$

Therefore, $BE = \sqrt{32}$ cm, or approximately 5.7 cm.

CEMC at Home

Grade 7/8 - Friday, April 3, 2020
Toothpick Patterns

A sequence is a list of numbers or other objects. In math, we often study sequences that follow a pattern rule. A pattern rule is used to determine how to form or continue a sequence. This activity will explore the importance of having a clear pattern rule.

Before you start:

Look at the first three terms of a sequence with six terms in total: $2, 3, 5$.
Are you confident that you know which three numbers come next in this sequence? Why or why not?

Without the context (or a pattern rule), we cannot be sure exactly how a sequence was meant to continue. Here are a few continuations of this sequence based on some possible pattern rules. See if the sequence you first imagined is among these:

- $2, 3, 5, \mathbf{8}, \mathbf{13}, \mathbf{21}$

  Rule: Add the two previous terms to get the next term.

- $2, 3, 5, \mathbf{9}, \mathbf{17}, \mathbf{33}$

  Rule: Double the previous term and subtract 1 to get the next term.

- $2, 3, 5, \mathbf{8}, \mathbf{12}, \mathbf{17}$

  Rule: Add 1 to the first term to get the second term, add 2 to the second term to get the third term, and continue in this way, adding n to the n^{th} term to get the term after that.

These are just three of the many different pattern rules that could describe the sequence. Can you come up with some more? We will explore the idea of pattern rules further in the following activities.

You Will Need: As many toothpicks as you can find!

Any small straight objects, similar in length, can be used in place of toothpicks if needed.

Question 1: Use toothpicks to build the following two figures, side by side as shown.

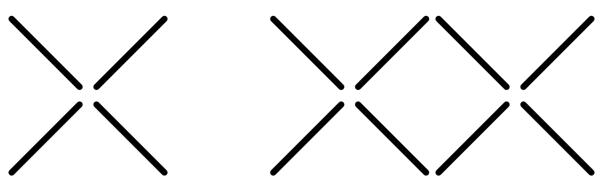


Figure 1 Figure 2

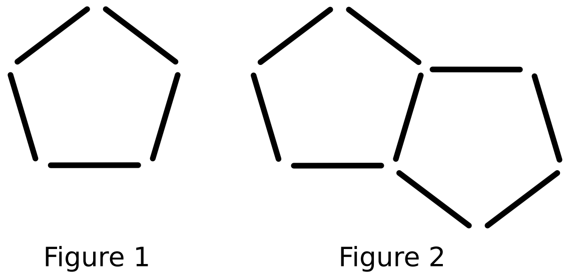
(a) Describe a possible pattern rule for a sequence of figures starting with Figure 1 and Figure 2. *We are looking for a pattern rule that describes how to create the next shapes in the sequence.*

(b) Use your pattern rule to build Figure 3 and Figure 4 in the sequence.

(c) Without sharing your pattern rule, have a friend or family member build what they think Figure 3 and Figure 4 in the sequence should be. Are they the same as your figures?

(d) You may have found that everyone you asked in (c) built exactly the figures you expected. Does this mean that the pattern rule is clear? Can you come up with a few different possible pattern rules that also make sense with the first two figures? Try and be as creative as possible!

Question 2: Use toothpicks to build the following three figures, side by side as shown.



(a) Determine a possible pattern rule for a sequence of figures starting with Figures 1, 2, and 3.

(b) Use your pattern rule to build Figures 4, 5, and 6 in the sequence.

(c) Without sharing your pattern rule, have a friend or family member build what they think Figures 4, 5, and 6 in the sequence should be. Are they the same as your figures?

(d) Using your pattern rule, can you determine

- how many toothpicks are needed to build Figure 10 in the sequence?
- how many toothpicks are needed to build Figure 25 in the sequence?

Extension: Create a toothpick pattern activity yourself!

Come up with a new idea for a pattern that can be formed using toothpicks. For an extra twist, why not throw in two or three different types of objects to be used in the pattern you build? Make the first few figures in the sequence with your pattern rule, and see if your friends and family can figure our your intended pattern. If they get it wrong on their first attempt then ask them to try again, or help them out by adding one extra figure and have them try again!

CEMC at Home

Grade 7/8 - Monday, April 6, 2020

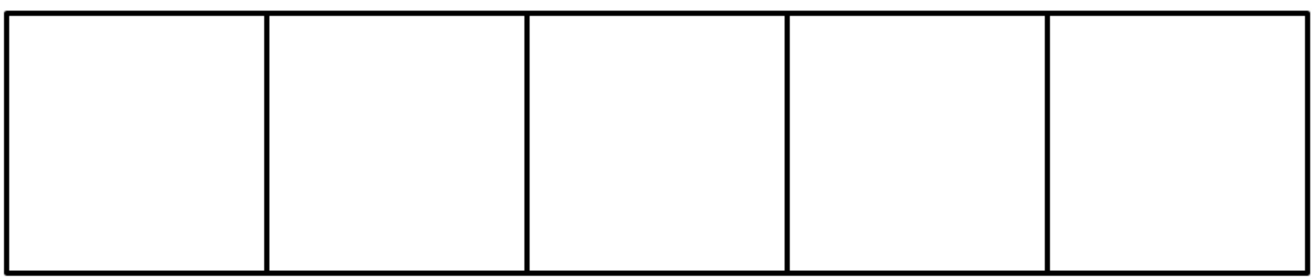
Five Square

You Will Need:

- Two players
- The grid below
- Four matching game pieces for each player (coins, paper clips, etc.)

How to Play:

1. Players alternate turns. Decide which player will go first.
2. On each turn, a player can either
    - put one of their game pieces on an empty square, or
    - put two of their game pieces on two empty squares, as long as the two empty squares are right next to each other.
3. The player to put a game piece on the last empty square wins the game!



Play this game a number of times. Can you come up with a strategy that will allow you to win most of the time? What about every time? Is it better to go first or second or does this not matter?

Variation: Add another square so that the game grid now has six squares. Keep the rules of the game the same. How does this change your strategy for the game?

More Info:

Check the CEMC at Home webpage on Tuesday, April 7 for a discussion of a strategy for Five Square.

CEMC at Home

Grade 7/8 - Monday, April 6, 2020
Five Square - Solution

Hopefully you thought about a strategy for this game while you played the game a number of times. Maybe you found a strategy that helped you win the game more often than you lost. Maybe you found a strategy that helped you win every time. It turns out that this game has what is called a *winning strategy*. This is a strategy that allows you to always win, regardless of what the other player does! Let's talk about a strategy.

Winning Strategy

We will play first and start by placing one game piece on the middle square (square 3).

1	2	3 X	4	5

Then our opponent has two different options for the type of move they play on their turn.

Option 1: Place two game pieces on two empty squares that are right beside each other.

Option 2: Place one game piece on an empty square.

We need to explain how we will win the game no matter what they choose to do on their next turn.

If they choose Option 1, then they will have to place game pieces either on squares 1 and 2 or on squares 3 and 4. If they place game pieces on squares 1 and 2, then on our next move, we will place game pieces on squares 3 and 4 to win the game. If they place game pieces on squares 3 and 4, then on our next move, we will place game pieces on squares 1 and 2 to win the game.

So to have a chance to win, it seems our opponent must go with Option 2 instead of Option 1.

If they choose Option 2, then they will have to place one game piece on one of squares 1, 2, 4, or 5. In this case, on our next move, we will place one game piece on the "other side" of the grid: If they place a game piece on square 1 or 2, then we place one on square 4 or 5, and if they place a game piece on square 4 or 5, then we place one on square 1 or 2. Either way, after these turns, the grid will have exactly two empty squares left, but they will not be right beside each other. Since it is our opponent's turn, they will have to place one game piece only, leaving us the opportunity to fill he grid to win the game!

From this explanation, we see that no matter what our opponent does, we can follow this plan and guarantee a win each game.

Exploring Other Strategies

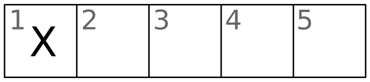
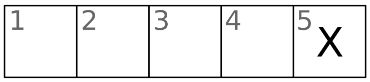
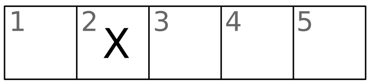
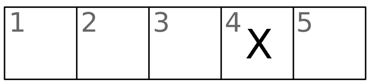
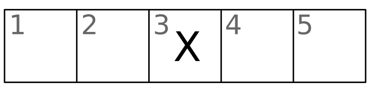
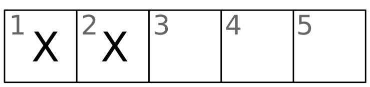
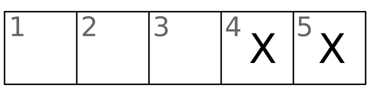
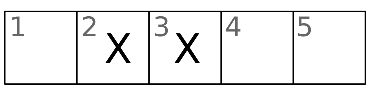
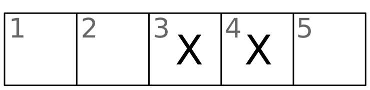
The success of the strategy described above depends on the fact that we were playing as the "first player". There are a few questions that we might ask ourselves:

(1) Is there a different winning strategy for the first player that starts with a different first move?

(2) Is it possible that the second player has a winning strategy as well?

We will explore these questions on the next page.

(1) Let's consider all possible first moves that the first player could make and explore what may happen next in the game.

First Move	Explanation
 or 	If we start by placing one game piece on square 1, then our opponent can place two game pieces on squares 3 and 4. We are then forced to choose between square 2 and square 5, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing one game piece on square 5, we can get a similar result.
 or 	If we start by placing one game piece on square 2, then our opponent can place two game pieces on squares 4 and 5. We are then forced to choose between square 1 and square 3, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing one game piece on square 4, we can get a similar result.
	If we start by placing one game piece on square 3, then we already know we are guaranteed a win in this game. *This was the first move in the winning strategy discussed on the previous page.*
 or 	If we start by placing two game pieces on squares 1 and 2, then our opponent can place one game piece on square 4. We are then forced to choose between square 3 and square 5, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing two game pieces on squares 4 and 5, we can get a similar result.
 or 	If we start by placing two game pieces on squares 2 and 3, then our opponent can place one game piece on square 4. We are then forced to choose between square 1 and square 5, and then our opponent will take the other square and win the game. By symmetry, we can see that if we start by placing two game pieces on squares 3 and 4, we can get a similar result.

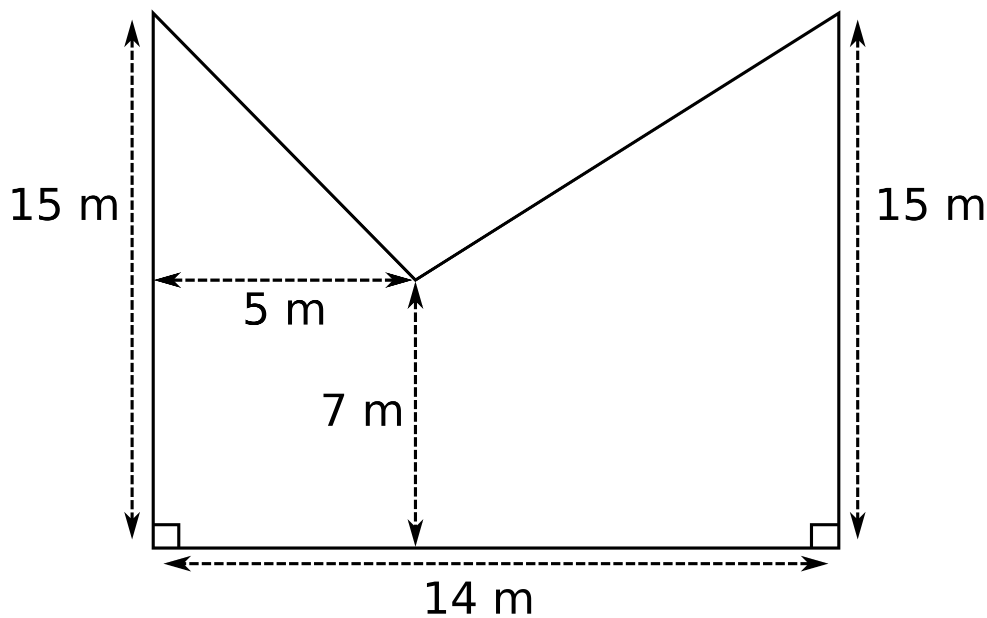
We can argue that every winning strategy for the first player must start with placing one piece on square 3. If the first player starts with any other move, then the second player can win the game as explained in the work above.

(2) The second player cannot also have a winning strategy for this game. It is definitely possible for the second player to win the game (and you may have won games as the second player), but the second player cannot guarantee a win. They have to hope that the first player either does not know the winning strategy described earlier or makes a mistake along the way.

CEMC at Home

Grade 7/8 - Tuesday, April 7, 2020
Floor Plan Areas

Shay asked his two sons, Dexter and Bennett, to find the area of their basement floor. Dexter and Bennett measured their basement and noted the dimensions shown below. What is the area of their basement?



Follow-up Questions:

1. There is more than one way to decompose (break up) the shape above into familiar shapes. Can you find a few different ways to do this?
2. Let's pretend that Dexter and Bennett forgot to measure and label the 5 m dimension in their diagram. Can they still determine the area of their basement without this measurement?
3. Make a floor plan for a room, an apartment, or one floor of a house. Measure any dimensions that you want to use to calculate the area of the floor. Did you actually need all of the dimensions that you measured, or could you have calculated the area with fewer dimensions known?

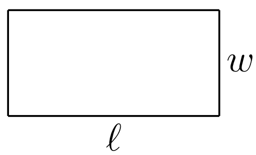
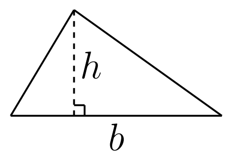
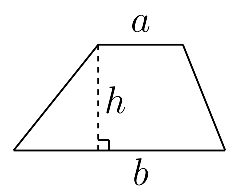
See the next page for a reminder of the area formulas for some familiar shapes, and some extra problems for practice.

More Info:

Check out the CEMC at Home webpage on Wednesday, April 8 for a solution to Floor Plan Areas.

For more practice calculating the area of composite shapes (like the shape above), check out this lesson in the CEMC Courseware.

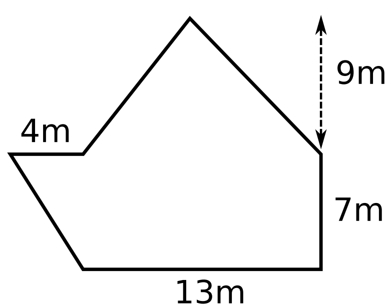
Area Formulas:

Rectangle	Triangle	Trapezoid
		
$A = \ell \times w$	$A = \dfrac{1}{2}(b \times h)$	$A = \dfrac{1}{2}(a + b) \times h$

More Practice:

1. Brie-Ann wants to place patio stones on a rectangular pathway towards her house. The pathway has dimensions of 14 m by 4 m. If each patio stone measures 0.5 m by 0.5 m, how many patio stones are needed to cover the pathway?

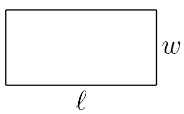
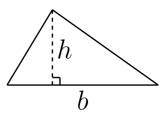
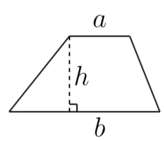
2. Can you calculate the area of the following shape given only the information below? Is there any extra information that you need?



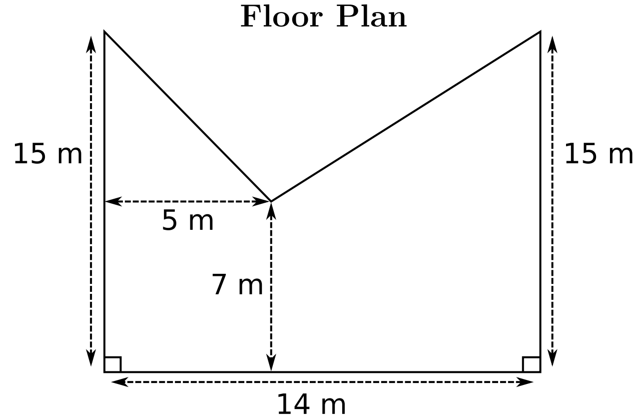
CEMC at Home

Grade 7/8 - Tuesday, April 7, 2020
Floor Plan Areas - Solution

Here is a reminder of the floor plan and the area formulas.

Rectangle	Triangle	Trapezoid
		
$A = \ell \times w$	$A = \frac{1}{2}(b \times h)$	$A = \frac{1}{2}(a+b) \times h$

Floor Plan



Answer: The total area of the floor is 154 m^2.

We will show different ways to calculate this area in the discussion that follows.

Follow-up Question 1: There are many different ways to decompose the shape above into familiar shapes like triangles, rectangles and trapezoids. Two different ways to do this are given here. Did you find these? Did you find others?

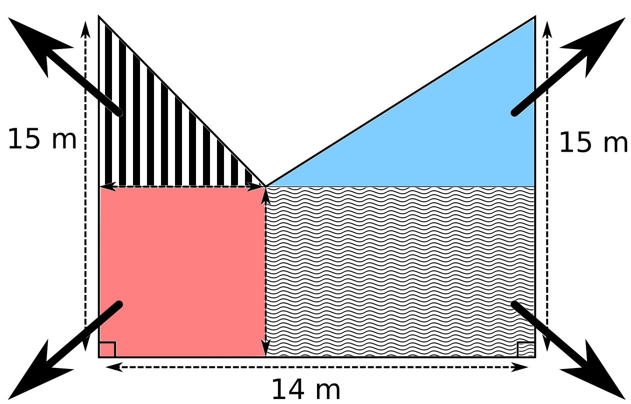
Method 1: Break the shape into two triangles and two rectangles.

Triangle 1

The base is 5 m.
The height is $15 - 7 = 8$ m.

Triangle 2

The base is $14 - 5 = 9$ m.
The height is $15 - 7 = 8$ m.



Rectangle 1

The length is 5 m and the width is 7 m.

Rectangle 2

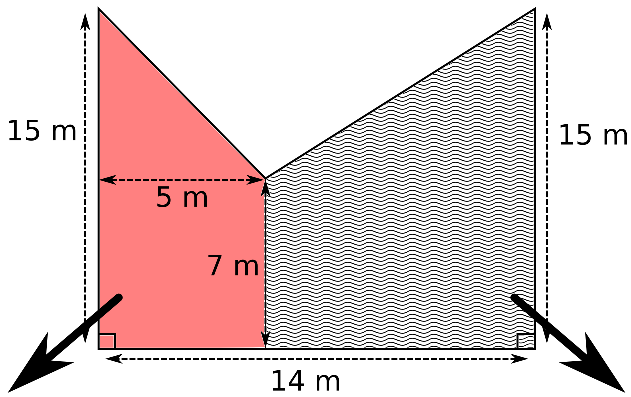
The width is 7 m.
The length is $14 - 5 = 9$ m.

The area of the composite shape is the sum of the areas of the two triangles and the two rectangles.

Shape	Area
Triangle 1	$\frac{1}{2}(b \times h) = \frac{1}{2}(5 \times 8) = 20$
Triangle 2	$\frac{1}{2}(b \times h) = \frac{1}{2}(9 \times 8) = 36$
Rectangle 1	$\ell \times w = 5 \times 7 = 35$
Rectangle 2	$\ell \times w = 9 \times 7 = 63$

Since $20 + 36 + 35 + 63 = 154$, the total area of the floor is 154 m^2.

Method 2: Break the shape into two trapezoids.



Trapezoid 1

The parallel sides are 15 m and 7 m long.
The height is 5 m.

Trapezoid 2

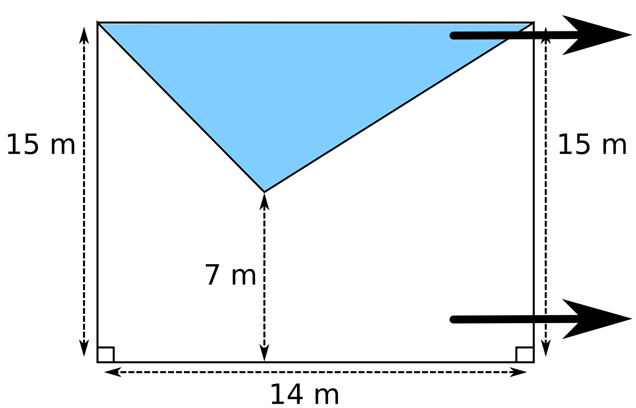
The parallel sides are 15 m and 7 m long.
The height is $14 - 5 = 9$ m.

The area of the composite shape is the sum of the areas of the two trapezoids.

Shape	Area
Trapezoid 1	$\frac{1}{2}(a + b) \times h = \frac{1}{2}(15 + 7) \times 5 = 55$
Trapezoid 2	$\frac{1}{2}(a + b) \times h = \frac{1}{2}(15 + 7) \times 9 = 99$

Since $55+99 = 154$, the total area of the floor is 154 m^2.

Follow-up Question 2: It is still possible to determine the area of the basement without knowing the measurement of 5 m. To do this, we think of the shape as a big rectangle with a triangle cut out.



Triangle

The base is 14 m.
The height is $15 - 7 = 8$ m.

Rectangle

The length is 14 m.
The width is 15 m.

The area of the composite shape is the area of the rectangle minus the area of the triangle.

Shape	Area
Triangle	$\frac{1}{2}(b \times h) = \frac{1}{2}(14 \times 8) = 56$
Rectangle	$\ell \times w = 14 \times 15 = 210$

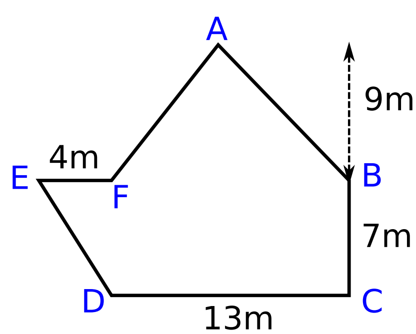
Since $210 - 56 = 154$, the total area of the floor is 154 m^2.

More Practice:

1. Brie-Ann wants to place patio stones on a rectangular pathway towards her house. The pathway has dimensions of 14 m by 4 m. If each patio stone measures 0.5 m by 0.5 m, how many patio stones are needed to cover the pathway?

Answer: Brie-Ann needs 224 patio stones.

2. Can you calculate the area of the following shape given only the information below? Is there any extra information that you need?



Answer: Notice that we added labels to the corners of the shape. We could calculate its area if we knew the following were all true.

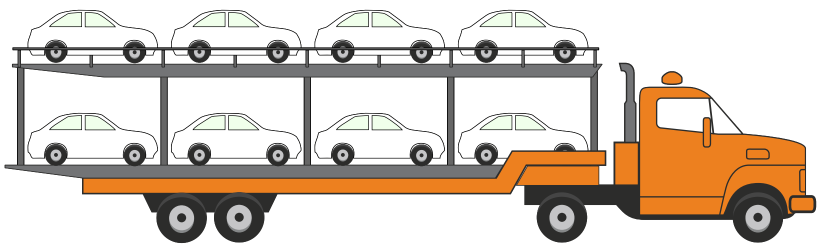
- $\angle BCD$ is a right angle.
- The line connecting E and B passes through F and is parallel to line DC.
- $\angle EFD$ is a right angle.

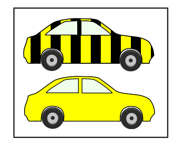
CEMC at Home

Grade 7/8 - Wednesday, April 8, 2020
Car Transportation

Question 1: A car manufacturer produces cars in solid colours as well as patterns. A new striped car comes from a manufacturing line every 7 minutes. A new solid yellow car comes from another manufacturing line every 4 minutes. Both manufacturing lines start working at the same time. A driver parks the cars on the back of a large transport truck in the order the cars leave their respective manufacturing lines. The top floor of the transport truck is loaded first.

Which of the cars in the loaded transport truck below are striped and which are yellow?

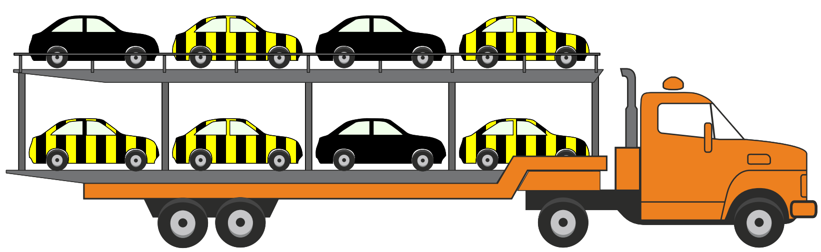




Need help getting started?

Try using objects like coins, paper clips, or pieces of paper to represent the cars leaving their respective manufacturing lines. This is called a simulation. Create a model or a table to summarize what the loading truck looks like at various time intervals.

Question 2: A new solid black car comes from a third manufacturing line every n minutes, where n is an integer between 5 and 10, inclusive. A second transport truck is loaded with only black cars and striped cars using the same method as in Question 1. After the truck is fully loaded it looks like the truck below. What is the value of n?



More Info:

Check out the CEMC at Home webpage on Thursday, April 9 for a solution to Car Transportation.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

Many industries, including car manufacturing, are highly *automated*, and this automation relies on computers to control and coordinate production. As such, the production needs to be carefully planned and synchronized so that various demands (availability of transport trucks, the need for particularly designed items to go on a particular truck, etc.) can be managed. The need to understand, create, manage, and improve automated systems is a real-world example of computer science being applied.

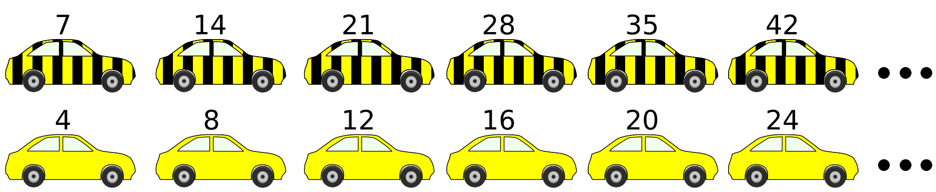
CEMC at Home

Grade 7/8 - Wednesday, April 8, 2020
Car Transportation - Solution

Solution to Question 1

We will start by recording the times at which the first few cars leave their respective manufacturing lines. We will record the elapsed time, starting our clock at the time both lines start working.

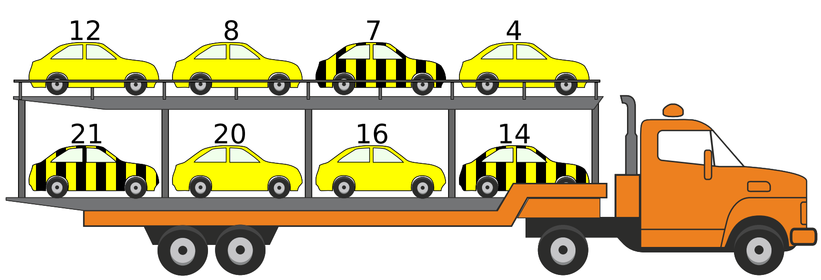
The striped cars come from the manufacturing line every 7 minutes. This means there will be a striped car at 7 minutes, another striped car at 14 minutes, another one at 21 minutes, and so on. The yellow cars come from the manufacturing line every 4 minutes. This means there will be a yellow car at 4 minutes, another yellow car at 8 minutes, another one at 12 minutes, and so on.



The cars are loaded onto the transport truck, filling the top level first and then the bottom level. We can determine the order in which these cars go into the truck by looking at the number (representing the time, in minutes) appearing above each car. From our work above, we can be sure that the first eight times at which a car is released are as follows:

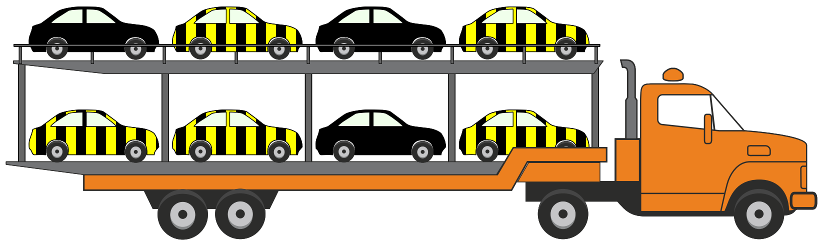
Time	4	7	8	12	14	16	20	21
Colour	Yellow	Striped	Yellow	Yellow	Striped	Yellow	Yellow	Striped

Keeping track of the colour of the car attached to each of the times above, we see that the transport truck will end up looking like this:



Solution to Question 2

Remember that the striped cars come from the manufacturing line every 7 minutes and that the black cars come from the manufacturing line every n minutes. Notice that the first car that was loaded into the truck in Question 2 is a striped car.

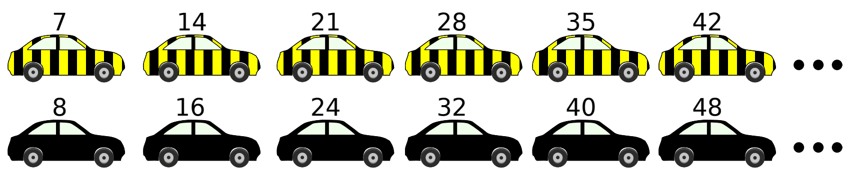


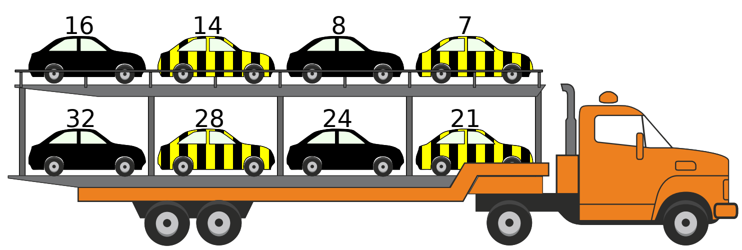
This means that n cannot be less than 7, as otherwise the first car would have to be black. Since n is an integer from the list $5, 6, 7, 8, 9, 10$, we know that n must be equal to 7, 8, 9, or 10.

The case where $n = 7$ is special, and we will discuss this at the end.

Now that we have narrowed our answer down to only a few possibilities, we may choose to proceed by checking what would happen in each of the different cases and see which values of n lead to the correct picture. We do similar work as in the solution to Question 1 for each possible value of n.

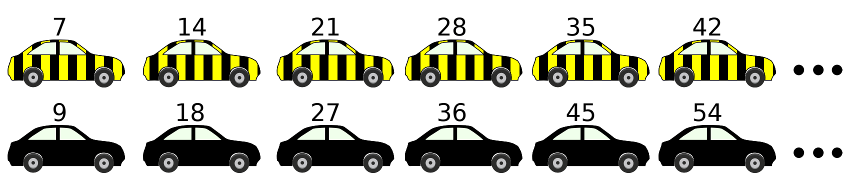
Option 1: $n = 8$

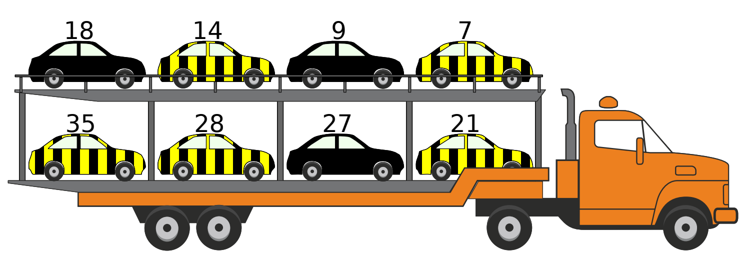




This is not the picture we are looking for, so n cannot be equal to 8.

Option 2: $n = 9$

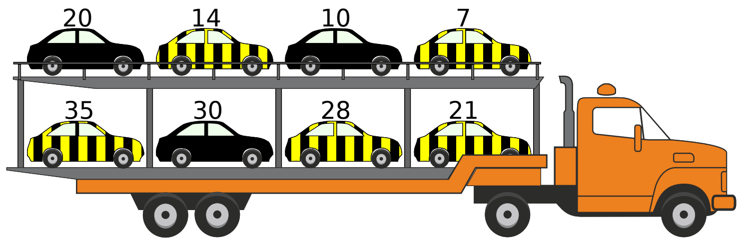




This shows us that $n = 9$ does produce the correct picture.

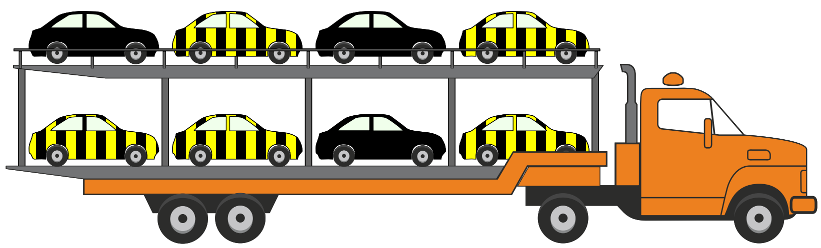
Option 3: $n = 10$

We can verify that $n = 10$ also results in the wrong final picture, so n cannot be equal to 10.



Option 4: $n = 7$

Notice that, in this case, a car comes from both manufacturing lines every 7 minutes. We are not told which line will get priority when two cars arrive at the same time, but no matter what rule you make, you cannot end up with the correct final picture (which is shown again below).



If $n = 7$, then the first eight cars must be split evenly with four striped and four black, which is not the case in the picture for Question 2 above. Therefore, n cannot be equal to 7.

From our work, we conclude that the value of n must be 9, and so the black cars come from the third manufacturing line every 9 minutes.

Follow-up Question: Can you explain why $n = 9$ is the only positive integer n that will result in the given transport truck picture? We have explained why 9 is the only number from the list $5, 6, 7, 8, 9, 10$ that gives the correct picture, but can you explain why no integer outside of this list can work either?

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 9, 2020

What's Not to Love?

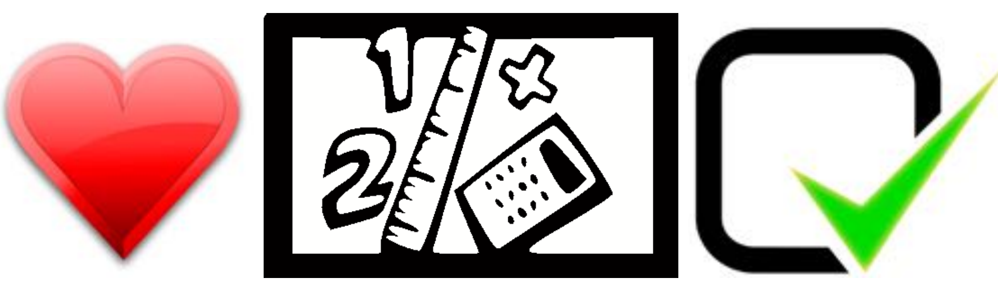
At the start of the school year, students in Mr. Pi's class were asked the following question: "Do you love Math?" They were only allowed to answer "yes" or "no", and everyone had to answer. Of the 30 students in the class, 21 answered "yes" and 9 answered "no".

That day, with every student present, the probability of randomly selecting a student who answered the question "yes" was $\frac{21}{30} = \frac{7}{10}$ and the probability of randomly selecting a student who answered the question "no" was $\frac{9}{30} = \frac{3}{10}$.

However, on one particular morning later in the year, the following information was known about the class:

- at least one of the students who had answered "yes" was absent and at least one of the students who had answered "no" was absent;
- more than half of the class was present; and
- the probability of randomly selecting a student who had answered the question "yes" was $\frac{3}{4}$.

Is there enough information to determine how many students were absent that particular morning? If yes, how many students were absent? If no, explain why not.



More Info:

Check the CEMC at Home webpage on Thursday, April 16 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 16.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
What's Not to Love?

Problem

At the start of the school year, students in Mr. Pi's class were asked the following question: "Do you love Math?" They were only allowed to answer "yes" or "no", and everyone had to answer. Of the 30 students in the class, 21 answered "yes" and 9 answered "no". So, with every student present, the probability of randomly selecting a student who answered the question "yes" was $\frac{21}{30} = \frac{7}{10}$ and the probability of randomly selecting a student who answered the question "no" was $\frac{9}{30} = \frac{3}{10}$.

However, on one particular morning later in the year, the following information was known about the class: at least one of the students who had answered "yes" was absent and at least one of the students who had answered "no" was absent; more than half of the class was present; and the probability of randomly selecting a student who had answered the question "yes" was $\frac{3}{4}$.

Is there enough information to determine how many students were absent that particular morning? If yes, how many students were absent? If no, explain why not.

Solution

Since at least one student from each of the two groups was absent, there were at least 2 students absent and at most 28 students present. Also, the maximum number of students who said "yes" would be $21 - 1 = 20$ and the maximum number of students who said "no" would be $9 - 1 = 8$. More than half the class was present so at least 16 students were present.

Since the probability of randomly selecting a student who answered "yes" was $\frac{3}{4}$, then the probability of randomly selecting a student who answered "no" was $\frac{1}{4}$.

We are looking for any number from 16 to 28 which is divisible by 4, so that when we find $\frac{3}{4}$ and $\frac{1}{4}$ of this number our result is a whole number. The numbers from 16 to 28 that are divisible by 4 are 16, 20, 24 and 28.

The following chart shows the results which are possible using the given information. There are 3 valid solutions that satisfy the given information. Therefore, there is not enough given information to determine the number of students who were absent that particular morning. The last solution in the chart in not valid. If the number present was 28, then 21 of those present would have answered the question "yes". But at least one student who answered "yes" was absent so the maximum number of students who answered "yes" would have been 20.

Number Present	Number Absent	Number who said "yes"	Number who said "no"	Valid / Not Valid
16	$30 - 16 = 14$	$\frac{3}{4} \times 16 = 12$	$\frac{1}{4} \times 16 = 4$	Valid
20	$30 - 20 = 10$	$\frac{3}{4} \times 20 = 15$	$\frac{1}{4} \times 20 = 5$	Valid
24	$30 - 24 = 6$	$\frac{3}{4} \times 24 = 18$	$\frac{1}{4} \times 24 = 6$	Valid
28	$30 - 28 = 2$	$\frac{3}{4} \times 28 = 21$	$\frac{1}{4} \times 28 = 7$	Not Valid

To Think About: Is there another piece of information that Mr. Pi could have provided so that two of the three valid answers could be eliminated leaving only one valid answer?

CEMC at Home

Grade 7/8 - Tuesday, April 14, 2020
Balancing the Scale

1. Cindy has two identical small pumpkins, two identical medium pumpkins and two identical large pumpkins.

- Cindy weighs one small and one medium pumpkin together. Their total mass is 12 kg.
- Cindy weighs one small and one large pumpkin together. Their total mass is 13 kg.
- Cindy weighs one medium and one large pumpkin together. Their total mass is 15 kg.

Can you figure out what the mass of each size of pumpkin is?

To do this, you could play around with different numbers until you find three that work as the three different masses, or you could gather some objects (toothpicks, nickels, or paperclips) that each represent 1 kg and think about how to distribute these among the pumpkins.

2. Watch a video in the CEMC's Problem Solving and Mathematical Discovery Courseware that walks through a solution to a problem that is very similar to 1.:
`https://courseware.cemc.uwaterloo.ca/40/assignments/1039/1`.

We would encourage you to watch from the beginning to 2:33 and from 4:15 to the end. The section from 2:33 to 4:15 is also worth watching, but may contain more algebra than you have seen before.

3. Maxime has two identical small desks, two identical medium desks and two identical large desks.

- Maxime weighs one small and one medium desk together. Their total mass is 47 kg.
- Maxime weighs one small and one large desk together. Their total mass is 80 kg.
- Maxime weighs one medium and one large desk together. Their total mass is 91 kg.

Next, Maxime puts all six desks on the scale, two at a time, in the following order: one small and one medium, the second small and one large, and the second medium and the second large.

(a) What is the total mass of the six desks?

(b) What is the total mass of one small, one medium and one large desk together?

(c) How can you use your answer to (b) with the original masses of 47 kg, 80 kg and 91 kg to determine the mass of each size of desk?

To help solve this problem, you could use three pairs of objects to stand in for these desks as you work. Use a table as the scale and add the objects in the same order as Maxime. Keep track of the total mass on a piece of paper after each new pair of objects is added to the scale.

Extension: Walt has two identical small elephants, two identical medium elephants and two identical large elephants.

- Walt weighs one small and one medium elephant together. Their total mass is 7390 kg.
- Walt weighs one small and one large elephant together. Their total mass is 9039 kg.
- Walt weighs one medium and one large elephant together. Their total mass is 11 051 kg.

Can you determine the mass of each size of elephant?

More Info:

Check out the CEMC at Home webpage on Tuesday, April 21 for a solution to Balancing the Scale.

CEMC at Home

Grade 7/8 - Tuesday, April 14, 2020
Balancing the Scale

1. Suppose that each small pumpkin weighs 5 kg, each medium pumpkin weighs 7 kg, and each large pumpkin weighs 8 kg.
   Then the total mass of one small pumpkin and one medium pumpkin is 5 kg + 7 kg = 12 kg.
   Also, the total mass of one small pumpkin and one large pumpkin is 5 kg + 8 kg = 13 kg.
   Further, the total mass of one medium pumpkin and one large pumpkin is 7 kg + 8 kg = 15 kg.
   These are the correct totals.

2. No solution.

3. (a) Try drawing pictures that show the desks being added as you follow through this solution.
   When the first two desks are put on the scale, the scale reads 47 kg.
   When the next two desks are put on the scale, the scale reads 47 kg + 80 kg.
   When the final two desks are put on the scale, the scale reads 47 kg + 80 kg + 91 kg.
   Therefore, the total mass of the six desks is 47 kg + 80 kg + 91 kg = 218 kg.

   (b) The six desks that have a total mass of 218 kg include two small desks, two medium desks, and two large desks.
   When these desks are divided into equal groups of one small desk, one medium desk, and one large desk each, the mass of each group is identical, and equals (218 kg) $\div$ 2 = 109 kg.

   (c) Since the total mass of one small desk, one medium desk, and one large desk is 109 kg, and the total mass of one small desk and one medium desk is 47 kg, then the mass of one large desk is 109 kg − 47 kg = 62 kg. (Imagine having the three desks on the scale and removing the two that you don't want.)
   Since the mass of one large desk is 62 kg and the total mass of one small and one large desk together is 80 kg, then the mass of one small desk is 80 kg − 62 kg = 18 kg.
   Since the mass of one large desk is 62 kg and the total mass of one medium and one large desk together is 91 kg, then the mass of one medium desk is 91 kg − 62 kg = 29 kg.
   Therefore, the mass of each small desk is 18 kg, the mass of each medium desk is 29 kg, and the mass of each large desk is 62 kg.

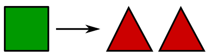
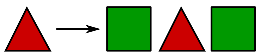
Extension

Try following the steps given in 3. to solve this problem with larger answers.
If you do this, you should find that the total mass of the six elephants is 27 480 kg.
This means that the total mass of one small elephant, one medium elephant, and one large elephant is 13 740 kg.
Can you use this information to determine the mass of each size of elephant? You can check your answers by combining them to see if they match the given information.

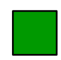
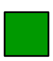
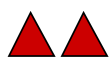
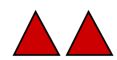
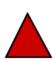
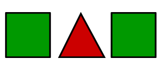
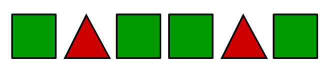
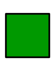
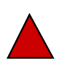
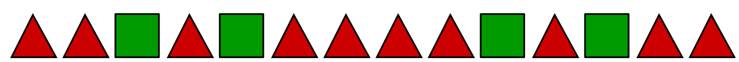
CEMC at Home

Grade 7/8 - Wednesday, April 15, 2020
Replacing Shapes

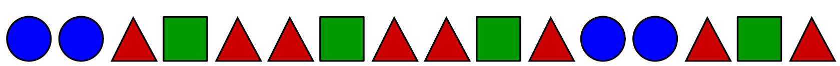
Introduction: Alice plays with cards of different shapes. Alice starts with a single card and builds longer sequences by applying the replacement rules for shapes given below.

Rule	Meaning
	Each square card is replaced by two triangle cards.
	Each triangle card is replaced by one square card, one triangle card, and another square card (in that order).

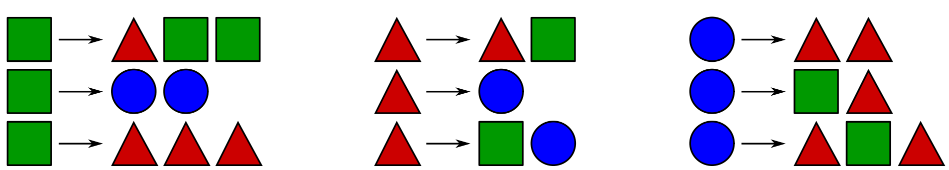
When these replacement rules are applied to a sequence of shapes, each individual shape in the sequence is replaced with a new sequence according to the relevant rule above. If Alice starts with a square card, and applies the replacement rules three times, then Alice builds the following:

Starting sequence *A single square*	
After first application of the rules *Single*  *replaced with* 	
After second application of the rules *Each*  *replaced with* 	
After third application of the rules *Rules for*  *and*  *both applied*	

Problem 1: Boris also plays with cards of different shapes. *Boris starts with one circle*, and plays with a different set of replacement rules. Boris has one replacement rule for each of the following shapes: circle, square, and triangle. The result of applying Boris's rules three times is shown below.



Which three replacement rules from the list below (one from each column) must Boris have used?

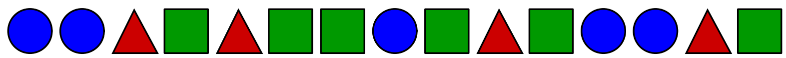


Need help getting started?

Use this online exploration to help you work through the solution. You can select different rules for Boris, and see what the result is after applying the rules once, twice, and three times. See if you can select the three rules that achieve the correct target sequence after three applications.

Problem 2: Cleo also plays with cards of different shapes. Cleo starts with exactly one shape (a square, a triangle, or a circle) and plays with a different set of replacement rules. Two of Cleo's three rules are shown below.

Cleo starts with one shape and applies the replacement rules three times and gets the following result.



Can you figure out Cleo's replacement rule for the square and which shape Cleo started with?

More Info:

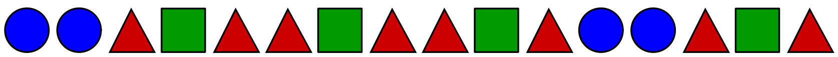
Check out the CEMC at Home webpage on Thursday, April 16 for a solution to Replacing Shapes.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

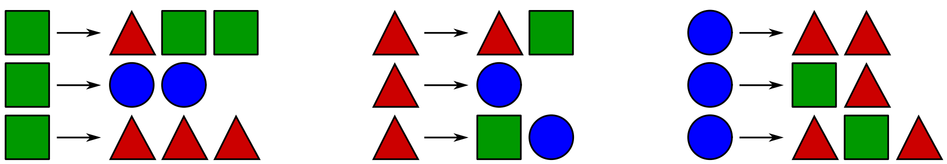
CEMC at Home

Grade 7/8 - Wednesday, April 15, 2020
Replacing Shapes - Solution

Problem 1: Boris starts with one circle, and plays with a set of replacement rules. Boris has one replacement rule for each of the following shapes: circle, square, and triangle. The result of applying Boris's rules three times is shown below.

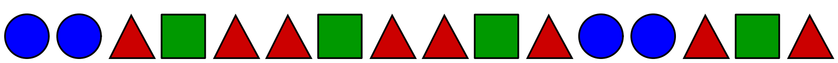


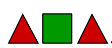
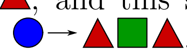
Which three replacement rules from the list below (one from each column) must Boris have used?

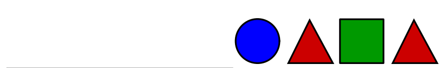


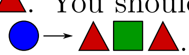
Solution to 1: One way to solve this problem is to try all possible combinations (on paper or using the online exploration) to see which one leads to the correct sequence. There are 27 different combinations and so if you don't stumble upon the right one quickly, this solution could be time consuming! Instead, we will explain how to eliminate some of the choices.

Let's call Boris's final sequence the "target sequence". We are trying to find rules that achieve this target sequence:

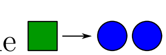
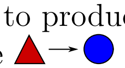


Since the right end of the target sequence is , and this sequence appears to be repeated throughout, it seems likely that the rule for circle is . We can convince ourselves that this *must* be the case by focussing on the right end of the target sequence:



Notice that there is no way to combine the outputs of the *other* rules to produce a sequence ending in exactly . You should spend some time checking this for yourself. This tells us that the circle rule must be .

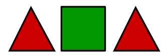
Now let's focus on the left end of the target sequence:

There are only two ways to produce two circles in a row on the left end: the rule  applied to one square, or the rule  applied to two triangles, side-by-side.

We have narrowed down the choices enough to now do some testing (either on paper or using the online exploration). We proceed on paper.

Suppose that two of the rules are ● → ▲■▲ and ▲ → ●.

Since Boris starts with a circle, the result of applying these replacement rules one time is as follows:



We do not need to know the replacement rule for the square for this step.

Applying these rules a second time will result in a sequence that looks like this:

The line in the middle of the figure above represents the part of the sequence that is currently unknown to us. We know that the sequence starts and ends in a circle (replacing the two triangles), but without the replacement rule for the square, we cannot be sure of what the middle of the sequence looks like.

Applying these rules a third time will result in a sequence that looks like this:

This final sequence cannot match the target sequence, regardless of what shapes appear in the middle portion, and so the rule for triangle cannot possibly be ▲ → ●.

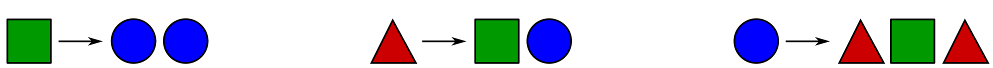
Remember that if we do not have the rule ▲ → ● then we *must* have the rule ■ → ●● (since we need a rule that produces the two circles on the left end). This means that two of the three rules must be ● → ▲■▲ and ■ → ●●. All that we have left to do is determine rule for the triangle.

There are now only two options for the full set of three rules:

Option 1: ■ → ●●, ▲ → ■●, ● → ▲■▲

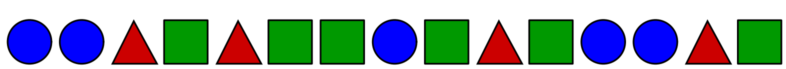
Option 2: ■ → ●●, ▲ → ▲■, ● → ▲■▲

We can check that the rules in *Option 1* produce the correct sequence after three applications, but the rules in *Option 2* do not. After this we can be sure that the three rules Boris used were as follows:



Problem 2: Cleo plays with cards of different shapes. Cleo starts with exactly one shape (a square, a triangle, or a circle) and plays with a set of replacement rules. Two of Cleo's three rules are shown below.

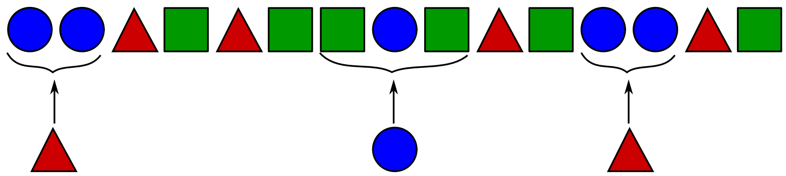
Cleo starts with one shape and applies the replacement rules three times and gets the following result.

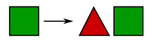


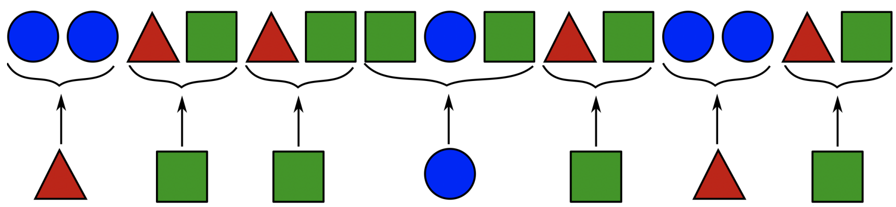
Can you figure out Cleo's replacement rule for the square and which shape Cleo started with?

Solution to 2: Since we do not know what the starting shape is in this problem, let's try and work backwards and see if we can find a solution.

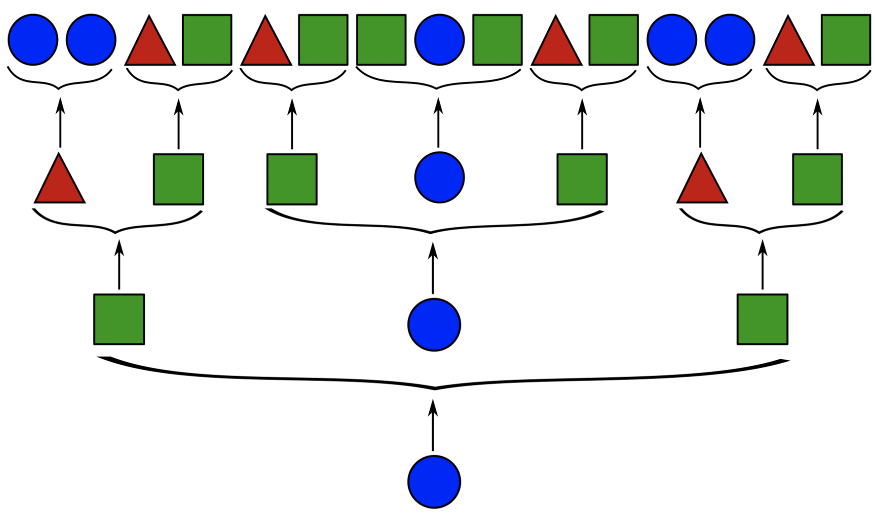
Let's suppose the indicated sections in the final sequence of shapes were obtained by applying the rules for triangle and circle as shown:

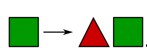


From here we can see that a replacement rule of  could fill in the gaps as shown below.



Working backwards, using the two original rules and the square rule from above, we get the following:



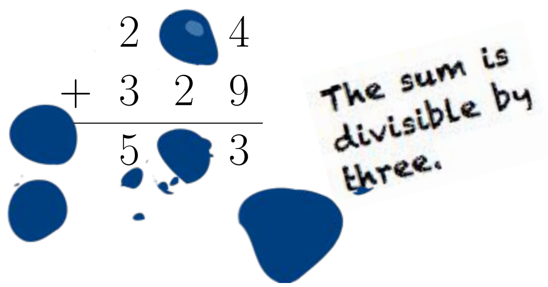
So if Cleo's replacement rule for the square was , and she started with a circle, then Cleo would end up with the sequence given in Problem 2.

It turns out that this is the only possible rule and starting shape that results in the correct final sequence. Can you convince yourself of this? Perhaps you can use reasoning similar to that used in the solution to Problem 1 to determine that there is only one possible answer for Problem 2.

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 16, 2020
What a Mess!

Jing was looking through an old math notebook and found the following mess on one of the pages:



The middle digit of the top and bottom numbers were both smudged and unreadable. Beside the question there was a handwritten note which read: "The sum is divisible by three."

If the missing middle digit in the top number is A and the missing middle digit in the bottom number is B, determine all possible values for A and B.

More Info:

Check the CEMC at Home webpage on Thursday, April 23 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 23.

> This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).
>
> To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
What a Mess!

The sum is divisible by three.

Problem

Jing was looking through an old math notebook and found the following question on one of the pages. The middle digit of the top and bottom numbers were both smudged and unreadable. Beside the question there was a handwritten note which read: “The sum is divisible by three.” If the missing middle digit in the top number is A and the missing middle digit in the bottom number is B, determine all possible values for A and B.

$$\begin{array}{cccc} & 2 & A & 4 \\ + & 3 & 2 & 9 \\ \hline & 5 & B & 3 \end{array}$$

Solution

Solution 1

First we find the possible values of B. For a number to be divisible by 3, the sum of its digits must be divisible by 3. So $5 + B + 3$ must be divisible by 3. The only possible values for B are thus 1, 4, or 7.

We know that $2A4 + 329 = 5B3$ so $2A4 = 5B3 - 329$.

We can try each of the possible values for B in the equation $2A4 = 5B3 - 329$ to find values of A that make the equation true.

1. If $B = 1$, then $513 - 329 = 184$, which cannot equal $2A4$. So when $B = 1$ there is no A to satisfy the problem.
2. If $B = 4$, then $543 - 329 = 214$, which does equal $2A4$ when $A = 1$. So for $A = 1$ and $B = 4$ there is a valid solution.
3. If $B = 7$, then $573 - 329 = 244$, which does equal $2A4$ when $A = 4$. So for $A = 4$ and $B = 7$ there is a valid solution.

Therefore, when $A = 1$ and $B = 4$ or when $A = 4$ and $B = 7$, the given problem has a valid solution.

Solution 2

$$\begin{array}{cccc} & 2 & A & 4 \\ + & 3 & 2 & 9 \\ \hline & 5 & B & 3 \end{array}$$

When the digits in the unit's column are added together, there is one carried to the ten's column. When the digits in the hundred's column are added together we get $2 + 3 = 5$ so there is no carry from the ten's column. Therefore, when the ten's column is added we get $1 + A + 2 = B$ or $A + 3 = B$.

We can now look at all possible values for A that produce a single digit value for B in the number $5B3$. We can then determine whether or not $5B3$ is divisible by 3.

The following table summarizes the results.

A	$B = A + 3$	$5B3$	Divisible by 3 (*yes*/*no*)?
0	3	533	*no*
1	4	543	*yes*
2	5	553	*no*
3	6	563	*no*
4	7	573	*yes*
5	8	583	*no*
6	9	593	*no*

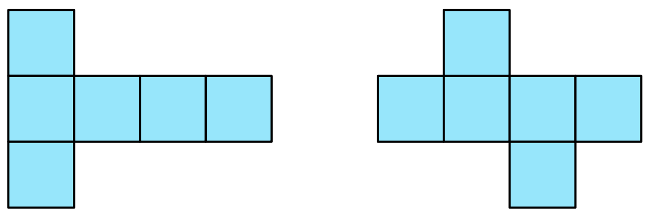
Therefore, when $A = 1$ and $B = 4$ or when $A = 4$ and $B = 7$, the given problem has a valid solution.

CEMC at Home

Grade 7/8 - Friday, April 17, 2020
Cut It Out

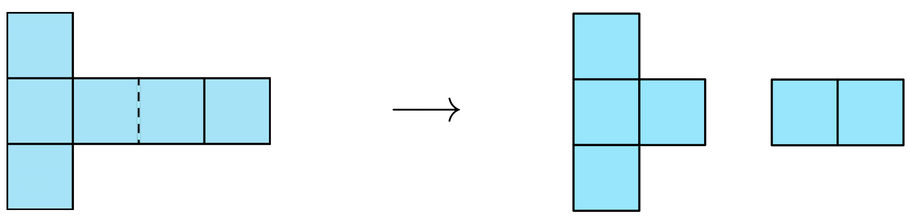
When you look outside your window, many of you will see different buildings or other three-dimensional structures. If you look closely, you can start to break these buildings down into familiar shapes. Understanding how to compose and decompose two-dimensional and three-dimensional shapes is an important skill in mathematics.

In this activity, we are going to explore this idea by looking at nets of three-dimensional objects. A net is a two-dimensional representation of a three-dimensional object, and is what the object would look like if its surface was opened up and laid flat. Look at the two nets below.



Even though these two nets are different, they can both be folded along the lines shown and used to form the surface of a cube. These are two examples of the many different nets of a cube.

Activity 1: Imagine that the first net of a cube given above has been cut in two, as shown below. In how many different ways can you attach these two pieces together in order to form a net of a cube?



To get started, try cutting out the two net pieces and putting them together in different ways.

Follow-up to Activity 1:

- Can you draw a few different nets of a cube that *cannot* be formed by attaching the pieces from Activity 1? (Note that one such net can be found above Activity 1.)
- Can you attach the two pieces from Activity 1 together (matching up edges) in a way that creates a net for an object that is *not* a cube?

Activity 2: Nets of seven different three-dimensional objects have been cut in two and scattered. Find the correct pairs and match them with the appropriate three-dimensional object.

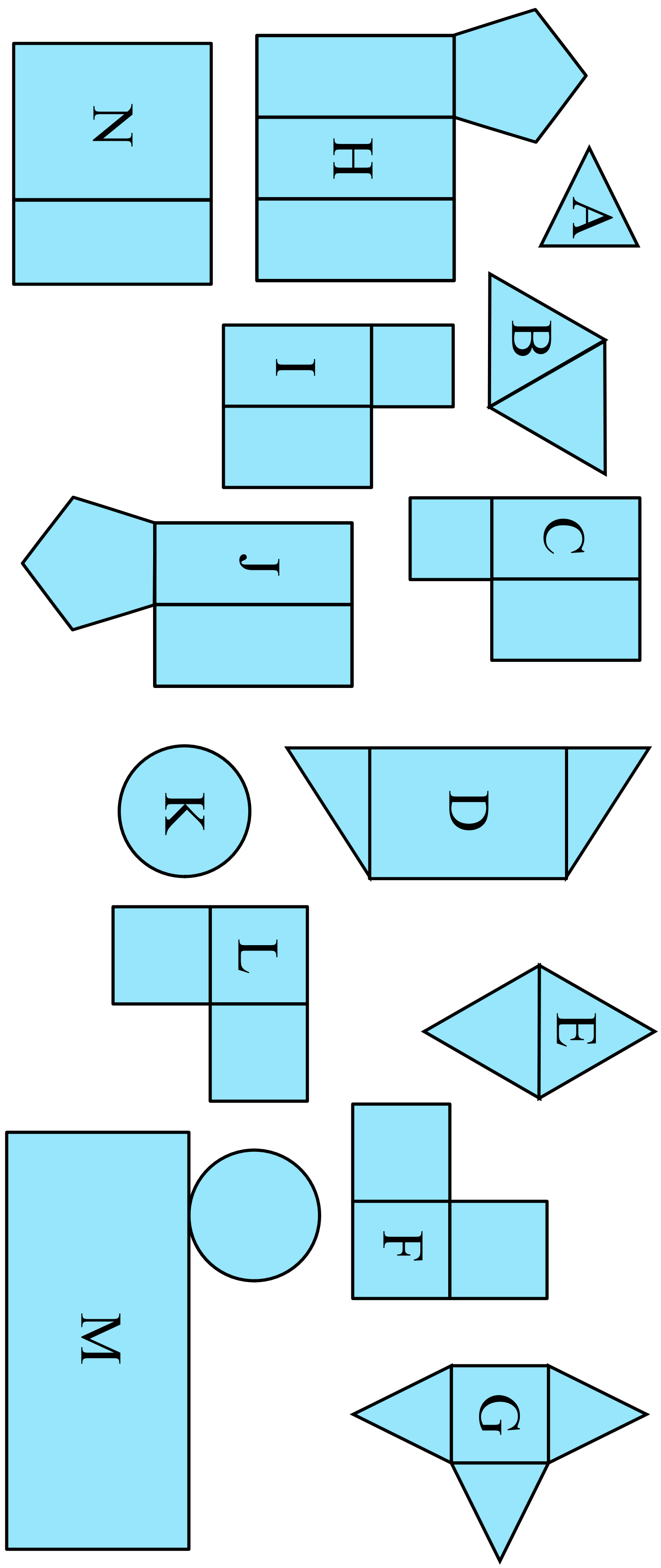
See the next page for the shapes for Activity 2 and a table to record your findings.
To get started, try cutting out the net pieces and putting them together in different ways.

More Info:

Check out the CEMC at Home webpage on Monday, April 20 for a solution to Cut It Out.

This problem was inspired by an activity from NRICH Maths.

For more practice using nets, check out this lesson in the CEMC Courseware.



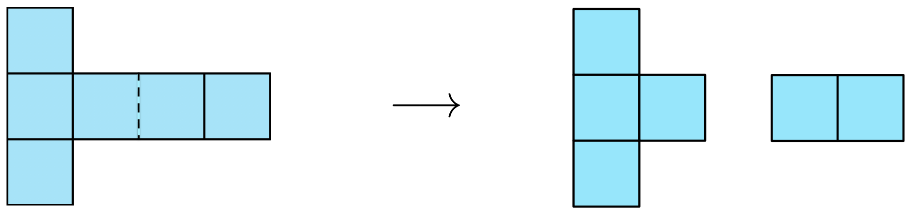
Record your answers to Activity 2 in the table below.

Net Pieces	3D Shape
	cube
	rectangular prism
	triangular prism
	pentagonal prism
	cylinder
	square pyramid
	triangular pyramid

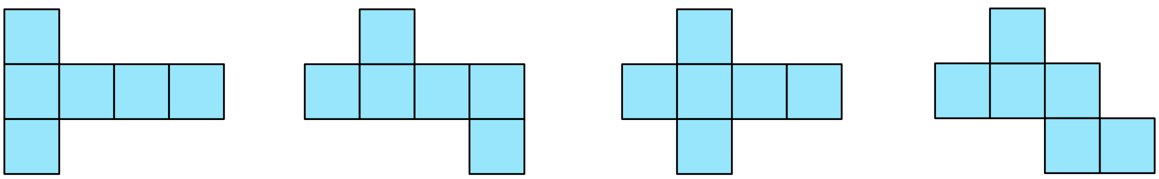
CEMC at Home

Grade 7/8 - Friday, April 17, 2020
Cut It Out - Solutions

Activity 1: Imagine that the following net of a cube given has been cut in two, as shown below. In how many different ways can you attach these two pieces together in order to form a net of a cube?

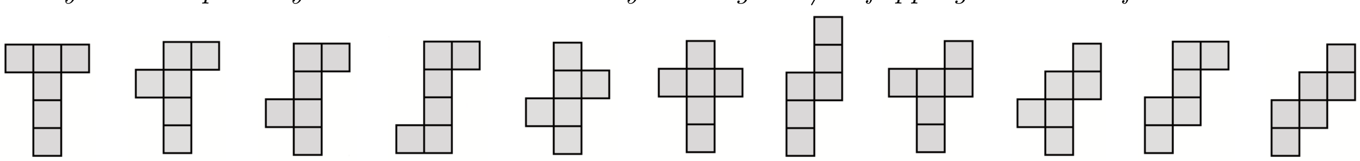


Answer: You can put the pieces together in the following ways to make four different nets of a cube.



The net shown above furthest to the left is the net that we started with (before the cut in Activity 1), and the other three nets are new.

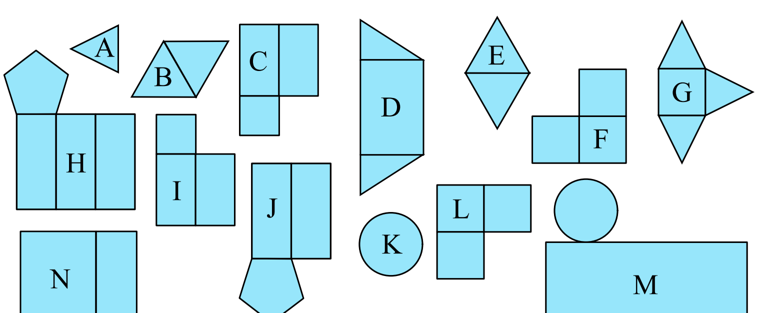
Note that there are eleven different nets of a cube and these are shown below. Every net of a cube that you could possibly draw can be obtained by turning and/or flipping over one of the nets below.



Four of these eleven nets can be made by attaching the pieces from Activity 1 in some way, and the remaining seven cannot. Can you explain why?

Activity 2: The nets of seven different three-dimensional objects have been cut in two and scattered. Find the correct pairs and match them with the appropriate three-dimensional object.

Net pieces:



Answers:

Net Pieces	3D Shape
F and L	cube
C and I	rectangular prism
D and N	triangular prism
H and J	pentagonal prism
K and M	cylinder
A and G	square pyramid
B and E	triangular pyramid

CEMC at Home

Grade 7/8 - Monday, April 20, 2020
Placing Polygons

In today's game, we will take turns drawing different shapes on a playing board.

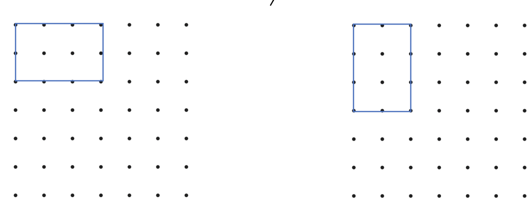
You Will Need:

- Two players
- Many different pieces of dot paper
- A pencil
- An eraser
- A ruler
- A pair of dice

How to Play:

1. Start with a piece of dot paper that is 15 dots by 15 dots (some is provided on the last page).
2. Players alternate turns.
   Decide which player will go first (Player 1) and which player will go second (Player 2).
3. Player 1 starts by rolling the pair of dice. The two positive integers rolled represent the dimensions of a rectangle (width and length, in some order). This player draws a rectangle on the dot paper with the four vertices located at dots, and one vertex located in the top left corner of the grid, that has the appropriate dimensions.

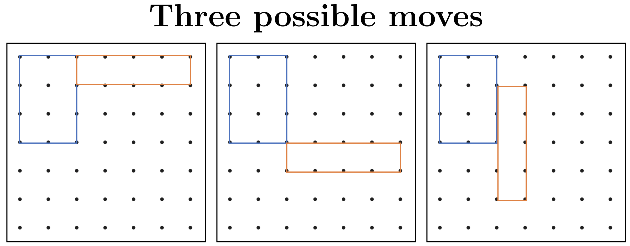
   For example, if a 2 and a 3 are rolled, then there are two options for the rectangle drawn as shown below. Note that 1 unit is the horizontal/vertical distance between two adjacent dots.

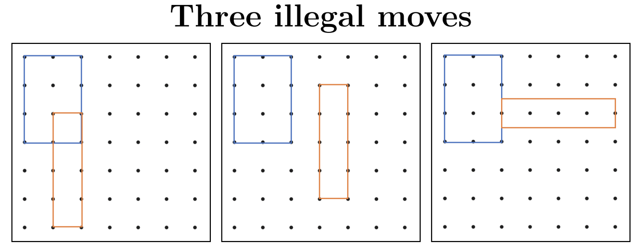
4. Player 2 now rolls the dice and draws a rectangle with length and width based on the numbers rolled, and following these rules:
   - The rectangle drawn must have its four vertices located on dots.
   - The rectangle drawn must touch an existing rectangle.
   - The rectangle drawn must not overlap with any existing rectangle.

   For example, various legal (and illegal) moves are shown below for a roll of 1 and 4.

   Three possible moves

   Three illegal moves

5. The two players alternate rolling the dice and drawing shapes following the rules outlined in 4.
6. If at any time during the game the current player cannot make a legal move based on their roll of the dice, then the other player wins the game.

Play this game a number of times and then think about these questions:

- Is there a minimum number of turns that must be played in this game before it can be won?
- Is there a maximum number of turns that can be played before a game must be won?

Variation 1

Play the same game, but with the following variation at the end: Once the game ends (because the current player cannot make a legal move based on their roll of the dice), each player adds up the total area covered by all of the rectangles they drew on their turns. The player with the largest area wins.

You will need to find a way to clearly indicate which rectangles belong to which player as you play. Will your strategy for playing the game change based on this variation?

Variation 2

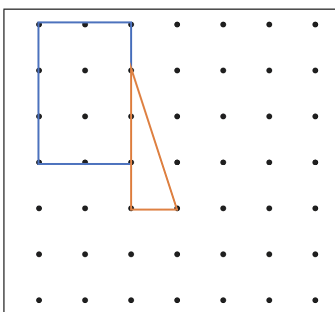
Play another game with the same rules as the original game, but with the following variation:

The two numbers rolled can be interpreted as either

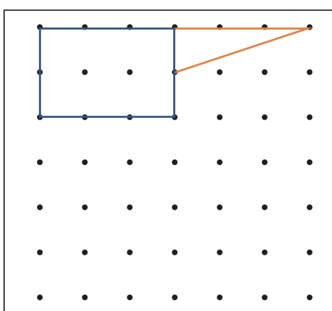
- the base and height of a parallelogram, *or*
- the base and height of a triangle.

On each turn, the current player has the option of drawing any parallelogram or triangle with an appropriate base and height based on their roll of the dice. As before, the shape drawn must have all of its vertices located at dots, and must touch an existing shape on the paper without overlapping with any existing shapes.

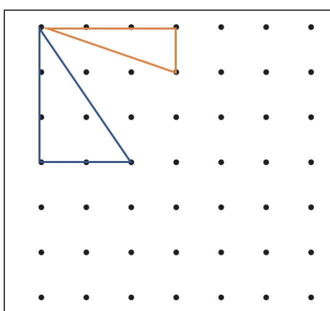
Below are some possibles starts to this variation of the game. Note that you have to be extra careful to draw the shapes accurately in this game, as it will be harder to tell whether or not two shapes are overlapping or just touching. Remember that two triangles (or parallelograms) that have the same base and height can still look very different!



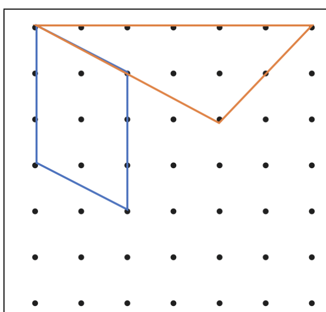
Player 1 rolls 2 and 3
Player 2 rolls 1 and 3



Player 1 rolls 2 and 3
Player 2 rolls 1 and 3



Player 1 rolls 2 and 3
Player 2 rolls 1 and 3

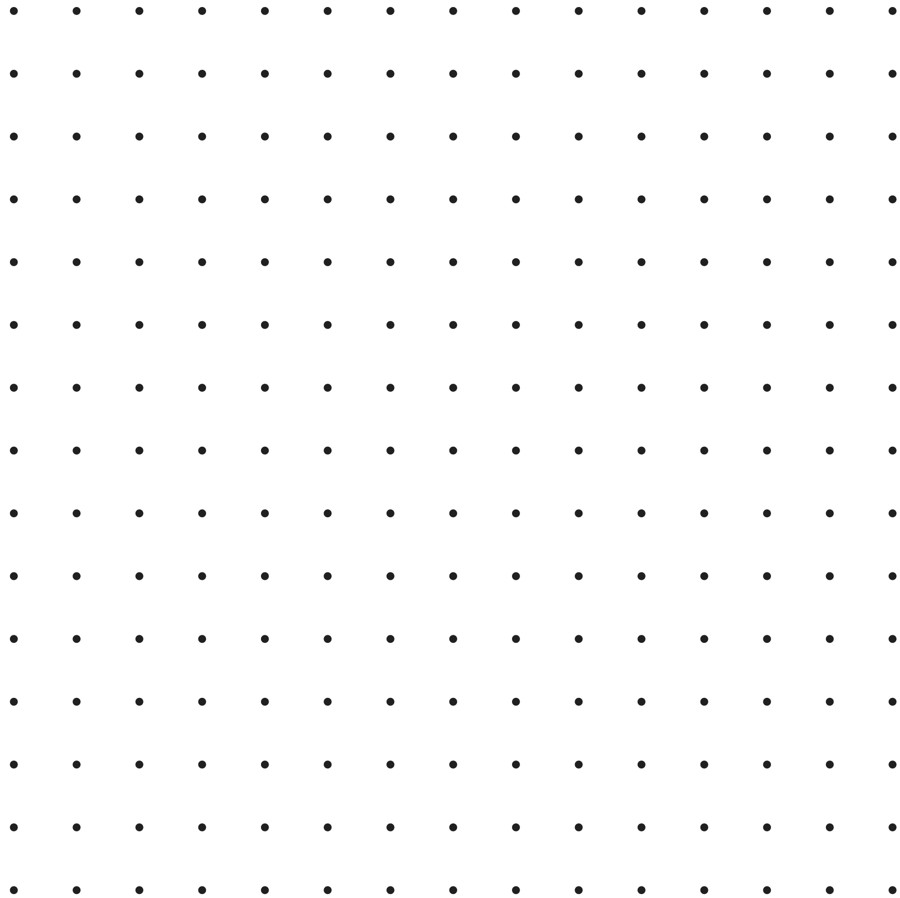


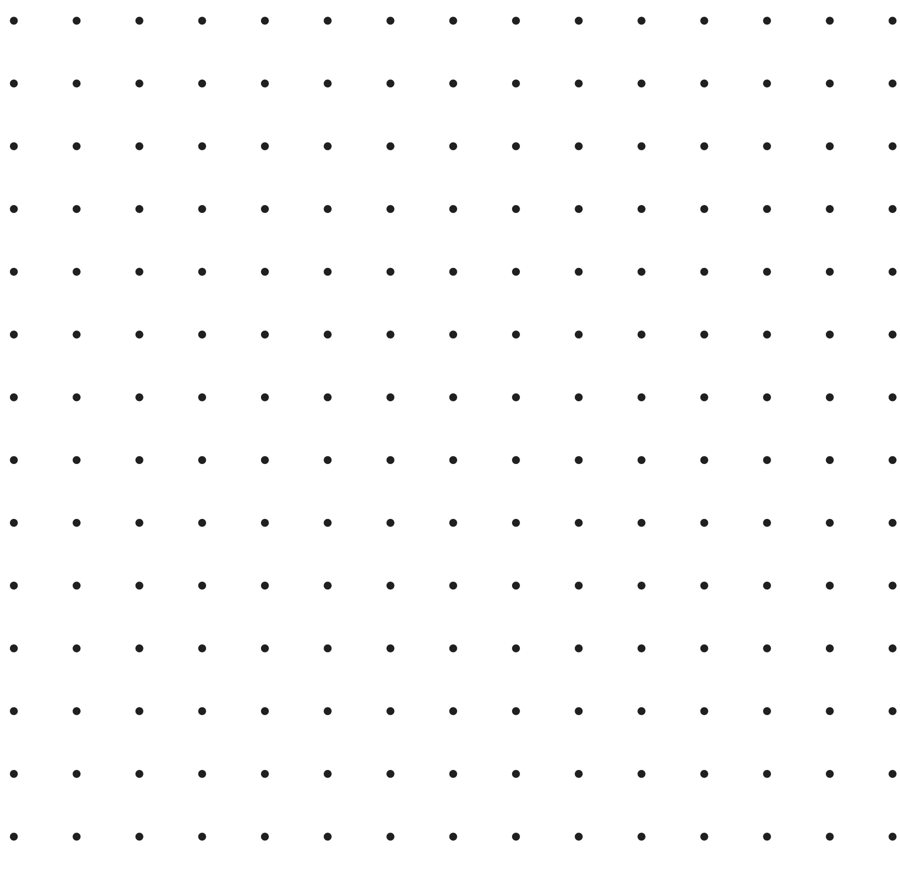
Player 1 rolls 2 and 3
Player 2 rolls 2 and 6

There is a lot of choice involved in this variation of the game. Given a roll of the dice, you get to decide the type of shape (parallelogram or triangle), the type of angles (right angle(s) or not), and the orientation of your shape (the shapes can have up to four different orientations).

More Info:

In these games, we played around with different *polygons*. To learn more about polygons, check out this lesson in the CEMC Courseware. You can also view lessons specifically about triangles, or parallelograms and other quadrilaterals.

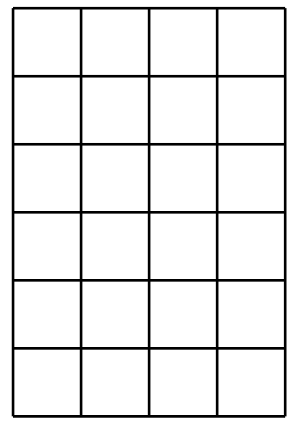




CEMC at Home

Grade 7/8 - Tuesday, April 21, 2020
Square Doodles

Problem 1: The grid below is divided into 24 small squares. Follow the steps to doodle in some of the squares. Read the steps carefully and do them in the order given. No square can have more than one doodle in it.



1. Draw a smiley face in $\frac{1}{4}$ of the squares.
2. Draw a cloud in $\frac{1}{6}$ of the remaining blank squares.
3. Draw a spiral in $\frac{2}{5}$ of the remaining blank squares.
4. Draw wavy lines in $\frac{1}{3}$ of the remaining blank squares.

How many squares have *not* been doodled in after you have finished?

Problem 2: Shreya started with a different blank grid, and was given similar instructions as in Problem 1. Shreya was given one instruction for drawing each of the same four shapes but *the fractions* in Shreya's instructions may have been different and the *order in which Shreya was asked to draw the shapes* may have been different. Shreya ended up with 6 squares with wavy lines, 4 squares with spirals, 5 squares with clouds, 10 squares with smiley faces, and 5 blank squares.

We cannot be sure of exactly what list of instructions Shreya was given. Can you give one possibility for the list of instructions that Shreya could have been working from?

To start, determine the total number of squares in Shreya's grid.

Problem 3: Aryan was given a grid of blank squares and the following instructions, in order.

1. Draw a cloud in $\frac{1}{4}$ of the squares.
2. Draw a spiral in $\frac{1}{6}$ of the remaining blank squares.
3. Draw a smiley face in $\frac{2}{5}$ of the remaining blank squares.

After following the steps, Aryan ended up with exactly 6 blank squares. How many blank squares could Aryan have started with? Is there more than one possibility?

Can you work "backwards"? You know there were 6 blank squares after the third step was completed. Can you figure out how many blank squares there were right before Aryan started the third step?

More Info:

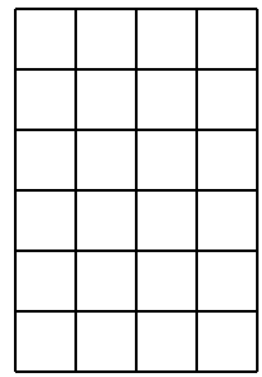
Check out the CEMC at Home webpage on Wednesday, April 22 for a solution to Square Doodles.

For more practice working with fractions, check out this lesson in the CEMC Courseware.

CEMC at Home

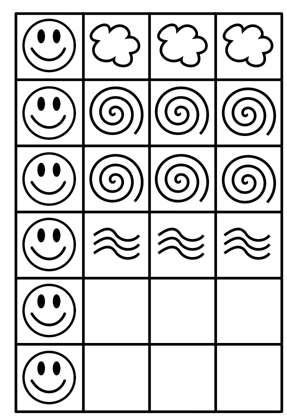
Grade 7/8 - Tuesday, April 21, 2020
Square Doodles - Solution

Problem 1 Summary: Follow the steps below to doodle in some of the squares. How many squares have *not* been doodled in after you have finished?



1. Draw a smiley face in $\frac{1}{4}$ of the squares.
2. Draw a cloud in $\frac{1}{6}$ of the remaining blank squares.
3. Draw a spiral in $\frac{2}{5}$ of the remaining blank squares.
4. Draw wavy lines in $\frac{1}{3}$ of the remaining blank squares.

Solution: We will go through each step below. One possibility for the final grid is also shown.



Step 1. Since $\frac{1}{4}$ of 24 is 6, smiley faces are drawn in 6 blank squares.
There are $24 - 6 = 18$ squares left blank after Step 1.

Step 2. Since $\frac{1}{6}$ of 18 is 3, clouds are drawn in 3 blank squares.
There are $18 - 3 = 15$ squares left blank after Step 2.

Step 3. Since $\frac{2}{5}$ of 15 is 6, spirals are drawn in 6 blank squares.
There are $15 - 6 = 9$ squares left blank after Step 3.

Step 4. Since $\frac{1}{3}$ of 9 is 3, wavy lines are drawn in 3 blank squares.
There are $9 - 3 = 6$ squares left blank after Step 4.

Therefore, 6 squares have not been doodled in after finishing these steps.

Problem 2 Summary: Shreya started with a different blank grid, and was given four instructions as in Problem 1, but possibly with a different order of shapes drawn, and different fractions for the shapes. Shreya ended up with 6 squares with wavy lines, 4 squares with spirals, 5 squares with clouds, 10 squares with smiley faces, and 5 blank squares. Can you give one possibility for the list of instructions that Shreya could have been working from?

Solution: We know each square in Shreya's grid must end up either blank or with one of the four shapes doodled in it. Since $6 + 4 + 5 + 10 + 5 = 30$, we determine that Shreya's grid must have had 30 squares in total.

Let's try and find possible instructions Shreya could have been working from. Let's assume that she was asked to draw the shapes in the order they were listed in the question: 6 wavy lines, 4 spirals, 5 clouds, and then 10 smiley faces, and see if we can find the correct fractions.

- *Shreya drew 6 wavy lines:* Notice that 6 is $\frac{1}{5}$ of 30. If Shreya drew 6 wavy lines during Step 1, then the instruction must have been "Draw wavy lines in $\frac{1}{5}$ of the squares."
  After this step there would be $30 - 6 = 24$ remaining blank squares.
- *Shreya drew 4 spirals:* Notice that 4 is $\frac{1}{6}$ of 24. If Shreya drew 4 spirals during Step 2, then the instruction must have been "Draw spirals in $\frac{1}{6}$ of the remaining blank squares."
  After this step there would be $24 - 4 = 20$ remaining blank squares.
- *Shreya drew 5 clouds:* Notice that 5 is $\frac{1}{4}$ of 20. If Shreya drew 5 clouds during Step 3, then the instruction must have been "Draw clouds in $\frac{1}{4}$ of the remaining blank squares."
  After this step, there would be $20 - 5 = 15$ remaining blank squares.
- *Shreya drew 10 smiley faces:* Notice that 10 is $\frac{2}{3}$ of 15. If Shreya drew 10 smiley faces during Step 4, then the instruction must have been "Draw smiley faces in $\frac{2}{3}$ of the remaining blank squares."
  After this step, there would be $15 - 10 = 5$ remaining blank squares, as expected.

We have found one set of instructions that Shreya could have been working from, however there are many other possibilities. If you assume Shreya drew the shapes in a different order, then you will get a different solution if you follow similar steps above.

There are 24 different correct answers to this problem, because there are 24 different orders in which Shreya could have been asked to draw the shapes. Depending on what order you choose, you may not get fractions that are as nice as the ones in our work above.

Problem 3 Summary: Aryan was given a grid of blank squares and the following instructions and Aryan ended up with exactly 6 blank squares at the end. How many blank squares could Aryan have started with?

1. Draw a cloud in $\frac{1}{4}$ of the squares.
2. Draw a spiral in $\frac{1}{6}$ of the remaining blank squares.
3. Draw a smiley face in $\frac{2}{5}$ of the remaining blank squares.

Solution: It turns out that there is only one possibility for the number of squares in Aryan's grid. To find this number, we will work backwards, starting with what must have happened during Step 3.

During Step 3, Aryan drew smiley faces in $\frac{2}{5}$ of the blank squares and so left the other $\frac{3}{5}$ of these squares blank. We know that exactly 6 squares were still blank at the end of this step. Since 6 is $\frac{3}{5}$ of 10, Aryan's grid must have had 10 blank squares right before Step 3 began.

Using similar reasoning, we can determine that Aryan's grid must have had 12 blank squares right before Step 2 began (since 10 is $\frac{5}{6}$ of 12), and 16 blank squares right before Step 1 began (since 12 is $\frac{3}{4}$ of 16). Try to work out these details for yourself.

Therefore, Aryan must have started with a grid of 16 blank squares.

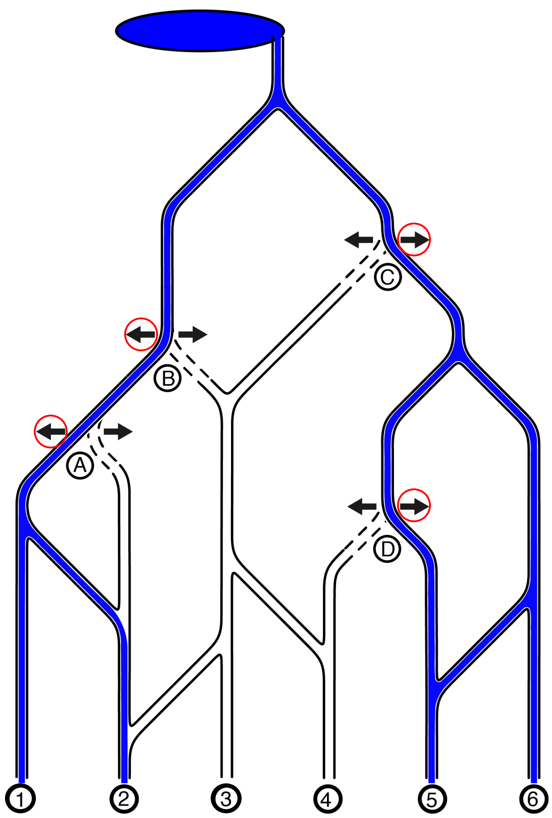
CEMC at Home

Grade 7/8 - Wednesday, April 22, 2020

Go with the Flow

Martin has created an irrigation system to water the fields in his farm. The water flows from a lake at the top of the hill all the way down to six fields numbered 1 to 6 at the bottom. Along the water canals, Martin has installed four water gates (A, B, C, and D), where he can direct the water to flow either to the left or to the right, but not in both directions.

An example showing how these gates can be set to have the water flow to fields 1, 2, 5, and 6 is shown below.



Problem 1: Explain how Martin can set the water gates so that water flows to fields 2, 3, and 4.

Want to check your answer? Use this online exploration to set each gate and see if you are correct.

Problem 2: Martin wants to set the gates so that water flows to fields 2, 3, 5, and 6.

(a) Explain why this is not possible based on how the farm is currently set up.

(b) Explain how the water canals in the farm can be adjusted in order to make this possible.

i. Can you achieve this by removing one existing canal from the irrigation system?

ii. Can you achieve this by adding one new canal to the irrigation system?

Do these changes affect your solution to Problem 1?

More Info:

Check out the CEMC at Home webpage on Thursday, April 24 for solutions to Go with the Flow.

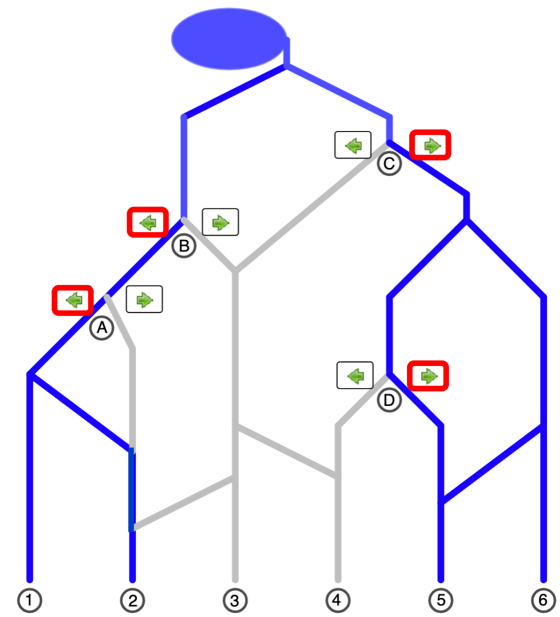
A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

CEMC at Home

Grade 7/8 - Wednesday, April 22, 2020
Go with the Flow - Solutions

Martin has created an irrigation system to water the fields in his farm. The water flows from a lake at the top of the hill all the way down to six fields numbered 1 to 6 at the bottom. Along the water canals, Martin has installed four water gates (A, B, C, and D), where he can direct the water to flow either to the left or to the right, but not in both directions.

An example showing how these gates can be set to have the water flow to fields 1, 2, 5, and 6 is shown to the right.



Problem 1: Explain how Martin can set the water gates so that water flows to fields 2, 3, and 4.

Solution: Notice that we do not want water to flow to field 6. That tells us gate C must be set to the left. Setting gate C to the left means it does not matter which direction we set gate D because no water will be flowing through gate D. Gate B can also be set either to the left or to the right. If gate B is set to the left, then gate A must be set to the right because we do not want water to flow to field 1. If gate B is set to the right, then it does not matter how we set gate A.

The tables below give a summary of all possible ways Martin could set the water gates so that water flows to fields 2, 3, and 4. You should check for yourself that each of these settings achieves the result.

Gate A	Gate B	Gate C	Gate D
Left or Right	Right	Left	Left or Right

Gate A	Gate B	Gate C	Gate D
Right	Left	Left	Left or Right

Problem 2: Martin wants to set the gates so that water flows to fields 2, 3, 5, and 6.

(a) Explain why this is not possible based on how the farm is currently set up.

(b) Explain how the water canals in the farm can be adjusted in order to make this possible.

   i. Can you achieve this by removing one existing canal from the irrigation system?

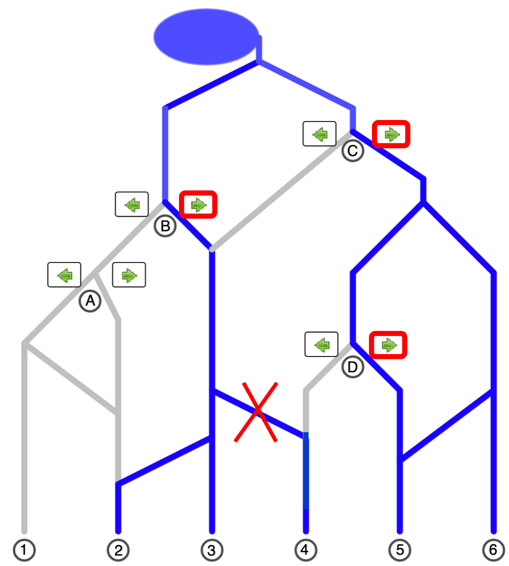
   ii. Can you achieve this by adding one new canal to the irrigation system?

Solution:

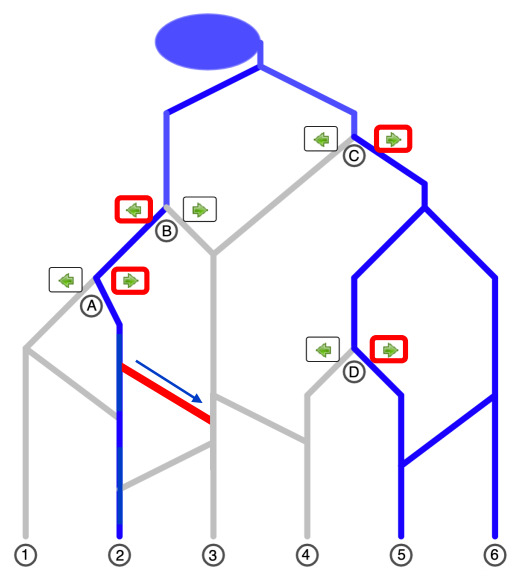
(a) To get water to field 6, gate C must be set to the right. With gate C set to the right, we have no choice but to set gate D to the right, to ensure that water does not flow to field 4. With gates C *and* D set to the right, we have no choice but to set gate B to the right as well to ensure water gets to field 3. However, setting gate B to the right will also result in water flowing into field 4. Therefore, it is not possible to have water flow to only fields 2, 3, 5, and 6.

In fact, you might have noticed from the diagram that whenever water flows into field 3, it must also flow into field 4. This means it is impossible to achieve the goal in Problem 2.

(b) i. By removing the canal shown below, we can have water flow into fields 2, 3, 5, and 6. Removing this canal would allow water to flow into field 3 without also flowing into field 4. This was the problem we had with the original canal setup. Following the logic in the solution to part (a), we set gates B, C, and D to the right, but with this canal removed, we will now get water flowing into field 3 and *not* field 4. (It does not matter which direction we set for gate A because no water will be flowing through it.) After doing this, we will have water flowing to only fields 2, 3, 5, and 6, as desired.



ii. By adding the canal shown below, we can have water flow into fields 2, 3, 5, and 6. Adding this canal would allow water to flow into field 3 without also flowing into field 4. In this case, we should set gate A to the right, gate B to the left, and gates C and D to the right. After doing this, we will have water flowing to only fields 2, 3, 5, and 6, as desired.

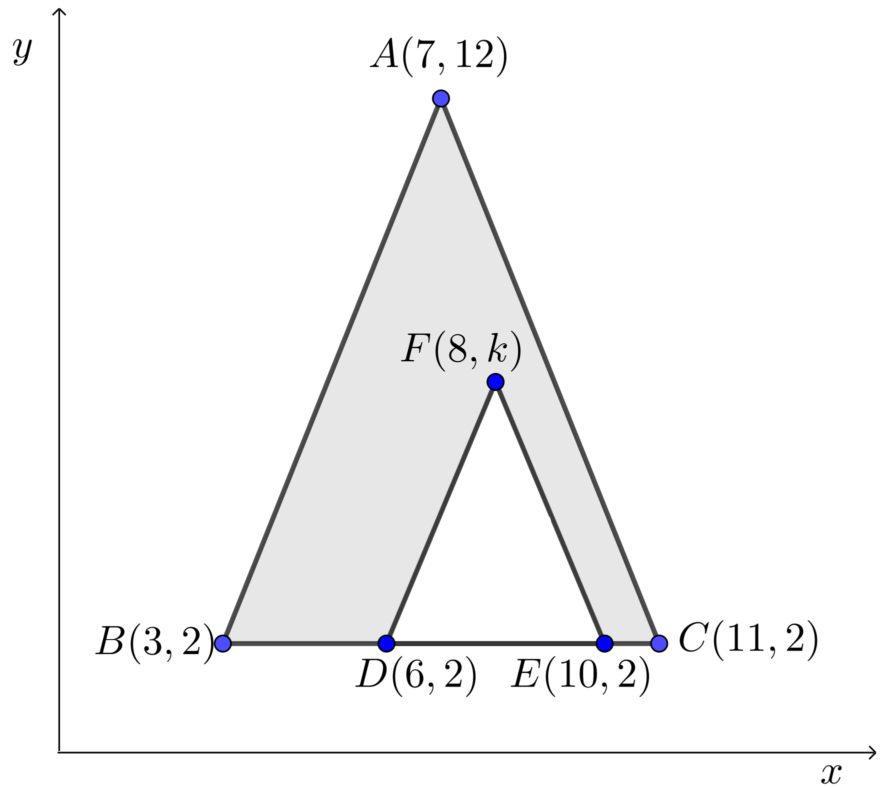


CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 23, 2020
Up to a Certain Point

Points $A(7, 12)$, $B(3, 2)$, $C(11, 2)$, $D(6, 2)$ and $E(10, 2)$ are placed on the Cartesian plane, as shown below. The point F is placed inside $\triangle ABC$ so that the area of the shaded region is 32 units^2.

If the x-coordinate of F is 8, what is the y-coordinate of F?



More Info:

Check the CEMC at Home webpage on Thursday, April 30 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 30.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

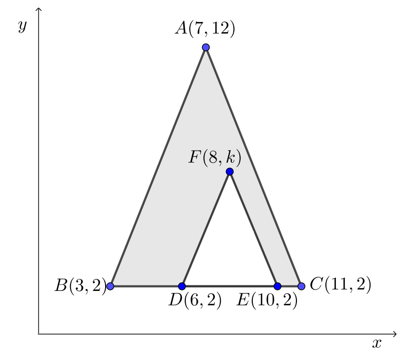
Problem of the Week

Problem C and Solution

Up to a Certain Point

Problem

Points $A(7,12)$, $B(3,2)$, $C(11,2)$, $D(6,2)$ and $E(10,2)$ are placed on the Cartesian plane, as shown to the right. The point F is placed inside $\triangle ABC$ so that the area of the shaded region is 32 units^2. If the x-coordinate of F is 8, what is the y-coordinate of F?

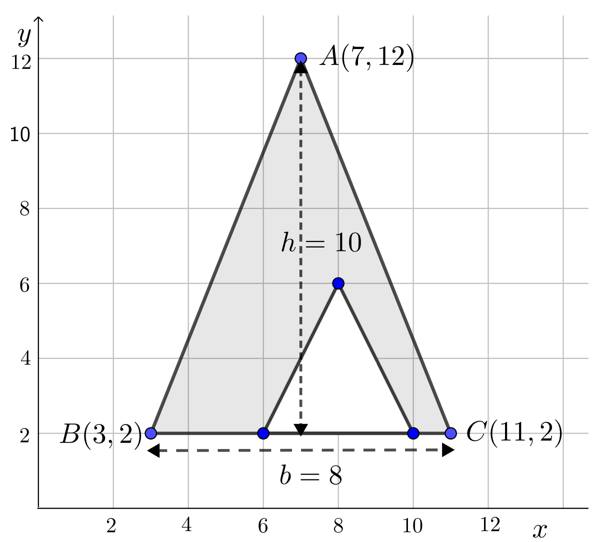


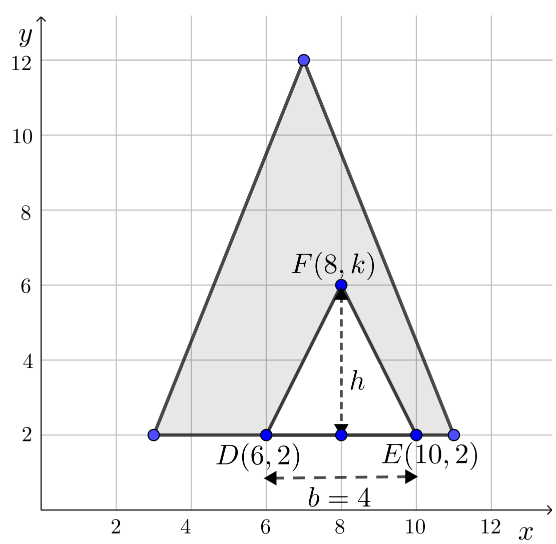
Solution

We will solve this problem in two ways. The first solution uses grid lines, the second solution does not. In both solutions, to find the area of the shaded region we will take the area of the large triangle, $\triangle ABC$, subtract the area of the small triangle, $\triangle DEF$, and then use the given information that this area is equal to 32 units^2.

Solution 1

To determine the height (h) and base (b) of each triangle, we use grid paper:





The area of $\triangle ABC = \frac{b\times h}{2} = \frac{8\times 10}{2} = 40$ (see above left).

The area of $\triangle DEF = \frac{b\times h}{2} = \frac{4\times h}{2} = 2h$ (see above right).

Therefore, the area of the shaded region is $40 - 2h$, which is also equal to 32. So $2h$ must equal 8, and therefore $h = 4$.

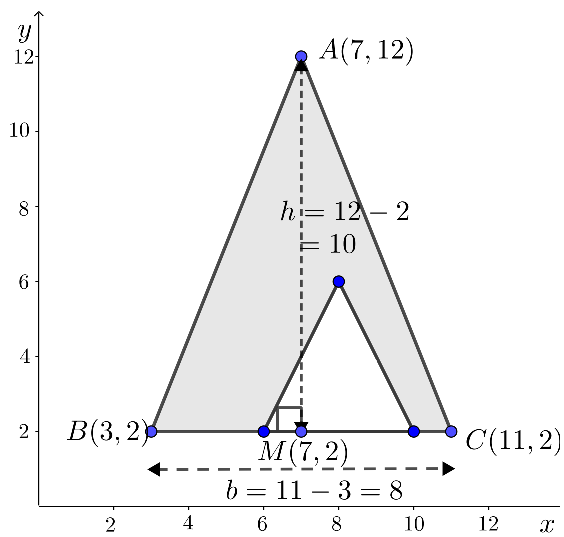
Now point F is $h = 4$ units higher than the base of the triangle, which is 2 units above the x-axis.

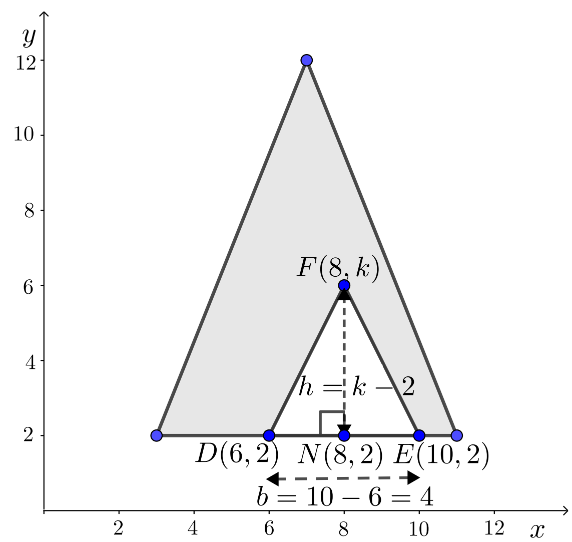
Therefore, the y-coordinate of point F is $2 + 4 = 6$.

Solution 2

In this solution, we use the fact that the distance between two points that have the same x-coordinate is the positive difference between their y-coordinates. We will also use the fact that the distance between two points that have the same y-coordinate is the positive difference between their x-coordinates.

In $\triangle ABC$, drop a perpendicular from vertex A to M on BC. Since BC is horizontal, then AM is vertical. Since every point on a vertical line has the same x-value, M has x-coordinate 7. Similarly, since M is on the horizontal line through $B(3,2)$ and $C(11,2)$, M has y-coordinate 2. Therefore, the base of $\triangle ABC$ is $b = 11 - 3 = 8$ and the height is $h = 12 - 2 = 10$. Now, the area of $\triangle ABC = \frac{b \times h}{2} = \frac{8 \times 10}{2} = 40$ (see below left).





In $\triangle DEF$, drop a perpendicular from vertex F to N on DE. Since DE is horizontal, then FN is vertical. Since every point on a vertical line has the same x-value, N has x-coordinate 8. Similarly, since N is on the horizontal line through $D(6,2)$ and $E(10,2)$, N has y-coordinate 2. Therefore, the base of $\triangle DEF$ is $b = 10 - 6 = 4$ and the height is $h = k - 2$. Now, the area of $\triangle DEF = \frac{b \times h}{2} = \frac{4 \times (k-2)}{2} = 2(k-2)$ (see above right).

We can now solve for k:

$$\begin{array}{rrcl}
 & 40 - 2(k-2) & = & 32 \\
\text{Subtracting 32 from each side:} & 8 - 2(k-2) & = & 0 \\
\text{Adding } 2(k-2) \text{ to each side:} & 8 & = & 2(k-2) \\
 & 4 & = & k - 2 \\
 & 6 & = & k
\end{array}$$

Therefore, the y-coordinate of point F is 6.

CEMC at Home

Grade 7/8 - Friday, April 24, 2020
Order Those Volumes

How can we compare the sizes of three-dimensional objects?

You Will Need:

- A ruler or measuring tape
- Some paper
- A pen or pencil

Introduction

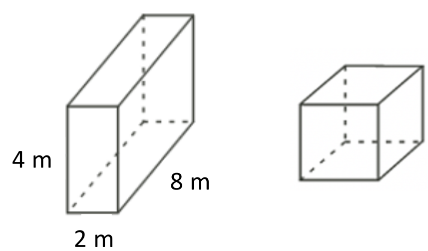
The volume of an object is a measurement of how much space the object takes up. A long time ago, the metric system was proposed and from this system, we gained a basic volume unit: 1 m^3. (Do you know how long ago this system was proposed and first adopted?) This is the space taken up by a cube that has edge lengths of 1 m. If you have a ruler or measuring tape, then you can get a pretty good idea of how much space this is.

Having a standard system of measurement like this allows us to compare volumes of many three-dimensional objects of different shapes.

Comparing rectangular prisms

Since we have a formula for calculating the volume of a rectangular prism, we can compare the volumes of any two rectangular prisms by measuring their dimensions. Remember that a rectangular prism with length ℓ, width w, and height h has volume $V = \ell \times w \times h$.

Problem 1: The rectangular prism and the cube shown below have equal volumes. What is the length of each edge of the cube?



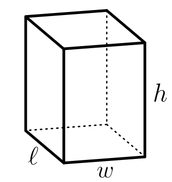
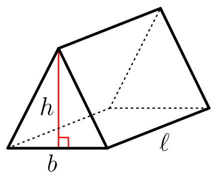
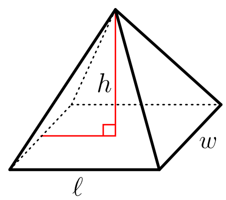
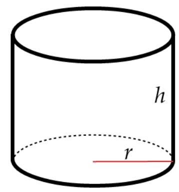
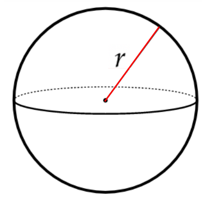
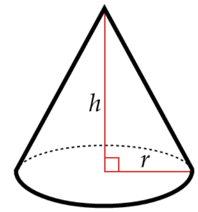
Activity 1: Find five objects that are all rectangular prisms and appear to be reasonably close in size. Order these objects on a table from smallest volume to largest volume, based only on a visual assessment. Once you have done this, get out your ruler or measuring tape and measure the dimensions of each prism. Calculate the approximate volume of each prism based on your measurements and check if your order was correct!

Rectangular Prism	Dimensions	Volume
1		
2		
3		
4		
5		

Comparing other familiar objects

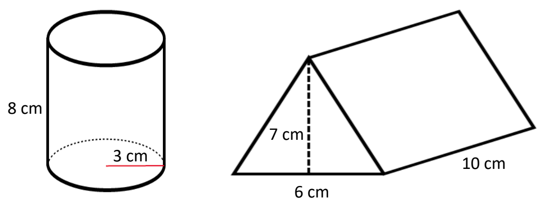
What other volume formulas do you know? Do you know formulas for calculating the volume of some of the three-dimensional objects that we explored last week in the activity Cut It Out? Below are several volume formulas for objects that may or may not be familiar to you.

If you would like to learn more about these objects or formulas, see the links at the bottom of this page.

<table>
<tr><th>Rectangular Prism</th><th>Triangular Prism</th><th>Rectangular Pyramid</th><th>Cylinder</th><th>Sphere</th><th>Cone</th></tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr><td>$V = \ell \times w \times h$</td><td>$V = \dfrac{b \times h \times \ell}{2}$</td><td>$V = \dfrac{\ell \times w \times h}{3}$</td><td>$V = \pi(r \times r \times h)$</td><td>$V = \dfrac{4\pi(r \times r \times r)}{3}$</td><td>$V = \dfrac{\pi(r \times r \times h)}{3}$</td></tr>
</table>

Some of these volume formulas involve the number π (pi). The number π is a decimal number that is non-terminating and non-repeating. If you have not worked with π before, then for the purposes of this activity, you can replace π in each formula with the number 3.14. This is an approximation of the value of π.

Problem 2: A cylinder has radius 3 cm and height 8 cm. A triangular prism has triangular face with base 6 cm and height 7 cm and has a length of 10 cm. Which object has a larger volume?



Activity 2: Try to find a rectangular prism, a triangular prism, a cylinder, a sphere, and one other type of object in the table above (pyramid or cone). Ideally, the five objects are all reasonably close in size. Order these objects on a table from smallest volume to largest volume, based only on a visual assessment. Once you have done this, get out your ruler or measuring tape and try to measure the necessary dimensions of each object. Calculate the approximate volume of each object based on your measurements and check if your order was correct!

Did you gather any objects for which you cannot easily measure the necessary dimensions? Can you approximate the volumes of your five objects well enough to be sure of the correct order of their volumes? Why or why not?

Object	Dimensions	Volume
1		
2		
3		
4		
5		

Comparing volumes without measuring dimensions

Think about an activity where you are asked to gather five objects of *any* shape and compare their volumes. What are some ways that you might accurately compare volumes of objects that have irregular shapes, dimensions that cannot be easily measured, or for which you do not have a volume formula?

More Info: For more on the solids from Activity 2, check out the CEMC Courseware to learn about volumes of prisms, cylinders, pyramids and cones, and spheres.

CEMC at Home

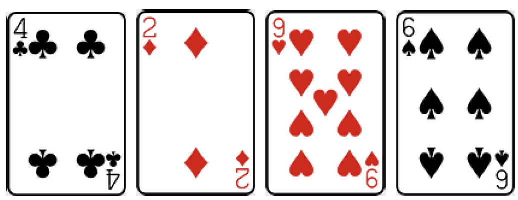
Grade 7/8 - Monday, April 27, 2020
The Game of 24

You Will Need:

- Two or more players
- A standard deck of playing cards, with all Jacks, Queens, and Kings removed (leaving 40 cards in total)
- Some paper and a pencil for score-keeping

How to Play:

1. First, shuffle the deck of 40 cards and put it face down in front of the players.
2. The game has 10 rounds. On the paper, start a column for keeping a running tally of each player's score.
3. Flip the top four cards of the deck face up on the playing surface.
4. Using the numbers on the four cards exactly once each, along with any of the operations of addition, subtraction, multiplication and division, each player privately tries to make a number that is as close to 24 as possible. Aces have a value of 1.
   For example, what numbers can you make with following four cards?

   You can add up the four numbers to get $4 + 2 + 9 + 6 = 21$ *which is pretty close to* 24*, but you can do better! It might be tempting to make exactly* 24 *by calculating* $4 \times 6 = 24$ *but this move is not allowed because this calculation does not use each of the four numbers exactly once. However, we can make exactly* 24 *by introducing parentheses and calculating*

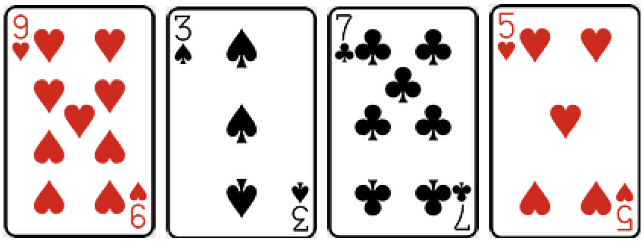
   $$(4 \times 9) - (6 \times 2) = 36 - 12 = 24$$

   This means we first calculate $4 \times 9 = 36$ *and* $6 \times 2 = 12$ *and then subtract these numbers to get* $36 - 12 = 24$.
5. When everyone is finished, all players reveal their calculations and the round is scored as follows: Each player's score is the difference between their calculated number and the number 24. Record each player's score for the round and add this to their running tally for the game.
   For example, if a player finds a way to get 24 *exactly, then their score for the round is* 0*. If a player does not find a way to get* 24*, but manages to make* 23 *or* 25*, then their score is* 1*. If a player makes* 22 *or* 26 *then their score is* 2*, and the scoring continues like this.*
6. Discard the current four cards and then flip the next four cards of the deck face up to play the next round with the same rules.
7. Keep playing until all cards have been used up (a total of 10 rounds). At the end of the game, the winner is the player with the *smallest* tally!

If you would like to practice different ways to combine four whole numbers using operations and parentheses, then think about the examples shown below before you play the game. The calculations follow the order of operations given here:

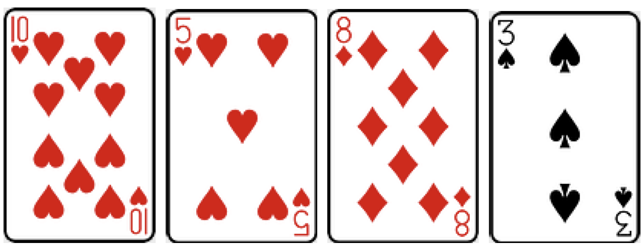
- Multiplication and division are performed before addition and subtraction.
- Any calculations between parentheses $(\cdots)$ are done first, before the calculations outside the parentheses.

Examples:



In this case, we can get 24 exactly by adding:

$$9+3+7+5=24$$

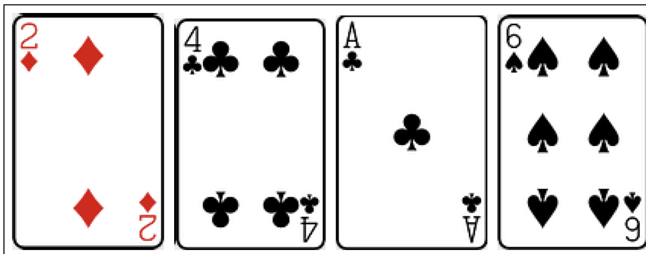


In this case, we can get 26 in at least two different ways:

$$8+3+10+5=26$$

$$8\times 3+10\div 5=24+2=26$$

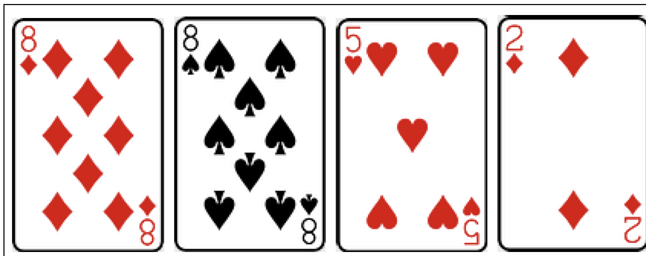
With the help of parentheses, we can get 25 by calculating $(10-5)\times(8-3)=5\times 5=25$.



In this case, we can get 24 exactly with the help of parentheses:

$$6\times 4\times(2-1)=6\times 4\times 1=24$$

(Remember that Aces count as 1 in this game.)



In this case, we can get very close to 24 by adding:

$$8+8+5+2=23$$

With the help of parentheses, we can get 24 exactly:

$$(8\times 5)-(8\times 2)=40-16=24$$

Variation:

Change the rules for how the game is scored to the following:

On each round, the first player to find a way to make 24 with the four cards earns 1 point for the round. No other players can score a point on the round.

If no player can find a way to make 24 with the current set of four cards, then shuffle the four cards back into the deck, and place four new cards face up.

In this variation, the winner is the player who has the most points when the deck has been used up!

CEMC at Home

Grade 7/8 - Tuesday, April 28, 2020

Missing Digits

Each problem below contains a positive integer that has at least one digit missing. A square is used in place of each missing digit.

Problem 1: The two-digit number $5\square$ is divisible by 2. How many possibilities are there for this two-digit number?

Problem 2: The four-digit number $6\square 53$ is divisible by 3. How many possibilities are there for this four-digit number?

Since there are only 10 possibilities for the missing digit, you could solve this problem by substituting each digit, in turn, and checking if the resulting number is divisible by 3. Can you use the following fact to answer this question without actually dividing all 10 resulting numbers by 3?

Did You Know?

A number is divisible by 3 exactly when the sum of its digits is divisible by 3.
For example, 27 is divisible by 3 because $2+7=9$ and 9 is divisible by 3, and 38 is *not* divisible by 3 because $3+8=11$ and 11 is *not* divisible by 3.

Problem 3: The four-digit number $4\square 3\square$ is divisible by 6. How many possibilities are there for this four-digit number?

Note that the two missing digits don't need to be the same.

Did You Know?

A number is divisible by 6 exactly when it is divisible by both 2 and 3.
For example, 48 is divisible by 6 because of the following:

- The last digit is 8, which is an even number. This means 48 is divisible by 2.
- The sum of the digits is $4+8=12$ and 12 is divisible by 3. This means 48 is divisible by 3.

Challenge Problem: The five-digit number $\square 5\square\square 2$ is less than 30 000 and is divisible by 12. How many possibilities are there for this five-digit number?

Did You Know?

A number is divisible by 12 exactly when it is divisible by both 3 and 4.
Also, a number is divisible by 4 exactly when its last two digits (tens and units, in order) form a two-digit number that is divisible by 4.
For example, 236 is divisible by 4 because the two-digit number 36 is divisible by 4, but 534 is not divisible by 4 because the two-digit number 34 is not divisible by 4.

More Info:

Check out the CEMC at Home webpage on Wednesday, April 29 for a solution to Missing Digits.

Try this lesson in the CEMC Courseware for more practice with multiples.

CEMC at Home

Grade 7/8 - Tuesday, April 28, 2020
Missing Digits - Solution

Each problem below contains a positive integer that has at least one digit missing. A square is used in place of each missing digit.

To help us solve these problems, we will use various divisibility tests as outlined in the problem statement:

- A whole number is divisible by 3 exactly when the sum of its digits is divisible by 3.
- A whole number is divisible by 4 exactly when its last two digits (tens and units, in order) form a two-digit number that is divisible by 4.
- A whole number is divisible by 6 exactly when it is divisible by both 2 and 3.
- A whole number is divisible by 12 exactly when it is divisible by both 3 and 4.

Problem 1: The two-digit number 5☐ is divisible by 2. How many possibilities are there for this two-digit number?

Solution: Since the number 5☐ is divisible by 2, its units (ones) digit must be 0, 2, 4, 6, or 8. So there are five possibilities for the given two-digit number. They are 50, 52, 54, 56, and 58.

Problem 2: The four-digit number 6☐53 is divisible by 3. How many possibilities are there for this four-digit number?

Solution: There are 10 possibilities for the digit in the box, and these result in the following four-digit numbers:

$$6053, 6153, 6253, 6353, 6453, 6553, 6653, 6753, 6853, 6953$$

One quick way to check which of these numbers is divisible by 3 is to calculate the sum of their digits and check whether or not this sum is divisible by 3.

Since $6 + 0 + 5 + 3 = 14$ and 14 is *not* divisible by 3, the number 6053 is *not* divisible by 3.

On the other hand, $6 + 1 + 5 + 3 = 15$ and 15 *is* divisible by 3, and so the number 6153 *is* divisible by 3.

Checking the sum of the digits of the other eight numbers, we find that only the numbers 6153, 6453, and 6753 have the sum of their digits divisible by 3.

This means there are three possibilities for the given number. They are 6153, 6453, and 6753.

Problem 3: The four-digit number 4☐3☐ is divisible by 6. How many possibilities are there for this four-digit number? *Note that the two missing digits don't need to be the same.*

Solution: Since the number 4☐3☐ is divisible by 6, it must be divisible by both 2 and 3. The fact that it is divisible by 2 tells us that its units digit must be 0, 2, 4, 6, or 8. Let's look at each of these cases. *See the next page for the case work.*

- Case 1: The units digit is 0.

  In this case the number looks like $4\square 30$. Since the number is also divisible by 3, we know that the sum of its digits must be divisible by 3.

  - If the missing digit is 2, then the sum of the digits is 9, which is divisible by 3.
  - If the missing digit is 5, then the sum of the digits is 12, which is divisible by 3.
  - If the missing digit is 8, then the sum of the digits is 15, which is divisible by 3.

  None of the other possible digits result a sum that is divisible by 3.

- Case 2: The units digit is 2.

  In this case the number looks like $4\square 32$. Since the number is also divisible by 3, we know that the sum of its digits must be divisible by 3.

  - If the missing digit is 0, then the sum of the digits is 9, which is divisible by 3.
  - If the missing digit is 3, then the sum of the digits is 12, which is divisible by 3.
  - If the missing digit is 6, then the sum of the digits is 15, which is divisible by 3.
  - If the missing digit is 9, then the sum of the digits is 18, which is divisible by 3.

  None of the other possible digits result a sum that is divisible by 3.

- Case 3: The units digit is 4.

  In this case the number looks like $4\square 34$. Since the number is also divisible by 3, we know that the sum of its digits must be divisible by 3.

  - If the missing digit is 1, then the sum of the digits is 12, which is divisible by 3.
  - If the missing digit is 4, then the sum of the digits is 15, which is divisible by 3.
  - If the missing digit is 7, then the sum of the digits is 18, which is divisible by 3.

  None of the other possible digits result a sum that is divisible by 3.

- Case 4: The units digit is 6.

  In this case the number looks like $4\square 36$.

  Doing similar work as the other cases, we find that only the digits 2, 5, and 8 produce digit sums that are divisible by 3.

- Case 5: The units digit is 8.

  In this case the number looks like $4\square 38$.

  Doing similar work as the other cases, we find that only the digits 0, 3, 6, 9 produce digit sums that are divisible by 3.

We count that there are $3 + 4 + 3 + 3 + 4 = 17$ possibilities for the four-digit number $4\square 3\square$.

If we would like, we can also list them:

$$4230, 4530, 4830, 4032, 4332, 4632, 4932, 4134, 4434, 4734, 4236, 4536, 4836, 4038, 4338, 4638, 4938$$

Challenge Problem: The five-digit number □5□□2 is less than 30 000 and is divisible by 12. How many possibilities are there for this five-digit number?

Solution: Since the number □5□□2 is less than 30 000, the first digit must be either 1 or 2.

Since the number □5□□2 is divisible by 12, it must be divisible by both 3 and 4.

Since it is divisible by 4, the last two digits must form a two-digit number that is divisible by 4. The two-digit numbers of the form □2 that are divisible by 4 are 12, 32, 52, 72, and 92.

So we know that the number must "start" in one of two ways: with the digits 15 or the digits 25. We also know that the number must "end" in one of five ways: with the digits 12, 32, 52, 72, or 92.

For example, one possibility is that the number is of the following form:

15□12

There are many different numbers that satisfy these properties! Our goal is to find the ones that are *also* divisible by 3.

Notice that all of these integers must have one of the following 10 forms shown in the table below. (Can you see why?) Similar to the previous problem, we determine which values when substituted for the missing digit result in the sum of the five digits in the number being divisible by 3.

Form of the number	**Digits resulting in a number divisible by 3**	**Count**
15□12	0, 3, 6, 9	4
15□32	1, 4, 7	3
15□52	2, 5, 8	3
15□72	0, 3 ,6 ,9	4
15□92	1, 4, 7	3
25□12	2, 5, 8	3
25□32	0, 3, 6, 9	4
25□52	1, 4, 7	3
25□72	2, 5, 8	3
25□92	0, 3, 6, 9	4

Adding up how many numbers we get in each of the 10 cases above, we see that there are

$$4 \times 4 + 6 \times 3 = 34$$

possibilities for the given integer.

This is probably more numbers than you would want to write out!

Note: We solved this challenge problem which involves checking whether five-digit numbers are divisible by 12 without actually attempting to divide a single five-digit number by 12. This shows the power of the divisibility tests! If you work through all of the details it takes to complete the table given above, then you will see that these tests allow us to replace questions like

"Is 25 992 divisible by 3?"

with questions like

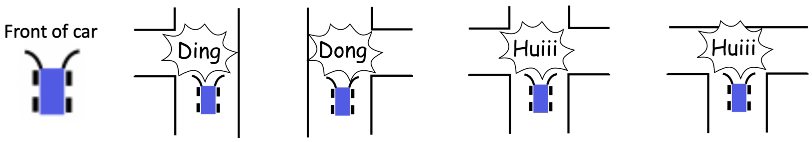
"Is the sum $2 + 5 + 9 + 9 + 2 = 27$ divisible by 3?"

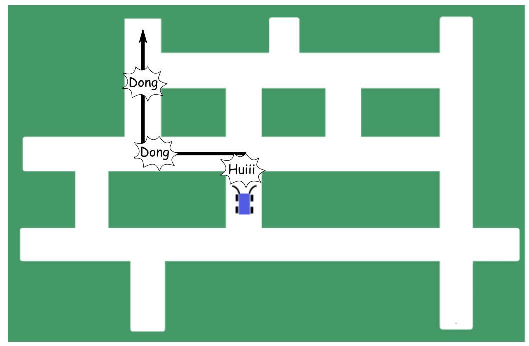
CEMC at Home

Grade 7/8 - Wednesday, April 29, 2020

No Road Blocks

A robotic car has sensors that detect intersections. It produces a different sound depending on whether it is possible to turn left only, turn right only, or turn in both directions at an intersection. The sounds that the car makes at various types of intersections is shown below.



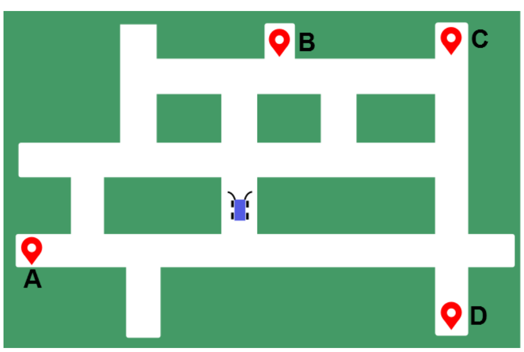


For example, if the car follows the route shown on the map to the right, it would make the sounds Huiii Dong Dong, *in that order.*

Note that the sound the car makes has nothing to do with the direction the car actually turns at the intersection. The sound just indicates the type of intersection the car has encountered.

The robotic car can go straight through an intersection (when possible), turn right (when possible), or turn left (when possible). The robotic car cannot make U-turns and cannot reverse. It automatically stops when it senses an obstacle in front of it (like a dead end).

Problem: The robotic car takes three different trips. On each trip, the car starts off in the position shown in the figure below, and ends up at one of the destinations marked A, B, C, or D. The sequence of sounds produced by the car on each of these trips is shown in the table below. What is the final destination of the robotic car on each trip?



Trip	Sounds Produced	Destination
1	Huiii Ding Huiii Ding	
2	Huiii Dong Dong Dong Ding	
3	Huiii Dong Ding Huiii	

To get started, try drawing a few paths from the starting position to one of the final destinations. Looking at the intersections encountered along one of these paths, determine the list of sounds the car would have made during this trip. Does this match one of the sequences in the table?

Extension: Is it possible for the robotic car to produce exactly the same sequence of sounds on two trips from its starting position (from the Problem above) that have different final destinations (from the list A, B, C, D)? If so, write down this sequence of sounds and the two final destinations. If not, explain why this is not possible.

More Info:

Check out the CEMC at Home webpage on Thursday, April 30 for a solution to No Road Blocks.

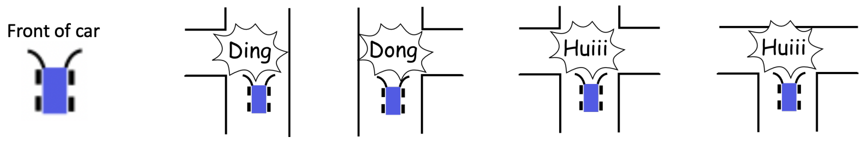
A variation of this problem appeared on a past Beaver Computing Challenge (BCC).

CEMC at Home

Grade 7/8 - Wednesday, April 28, 2020
No Road Blocks - Solution

Problem Summary and Answers

Here are the sounds made by the car at each type of intersection that it encounters. Remember that the car cannot make U-turns and cannot reverse.



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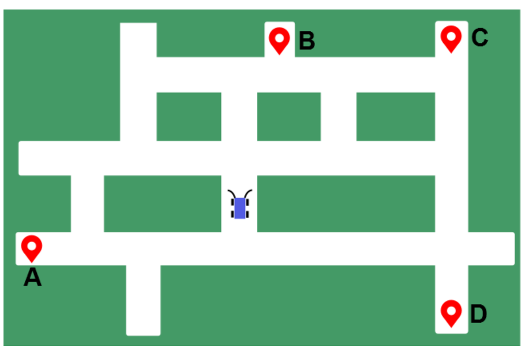


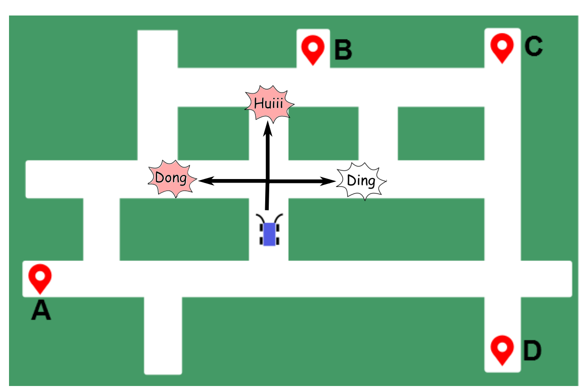
Trip	Sounds Produced	Destination
1	Huiii Ding Huiii Ding	**Answer: C**
2	Huiii Dong Dong Dong Ding	**Answer: B**
3	Huiii Dong Ding Huiii	**Answer: A**

Solution: The final destination of the car on each of the three trips is shown in the table above. Let's explain why these are the only possible final destinations based on the sounds made by the car.

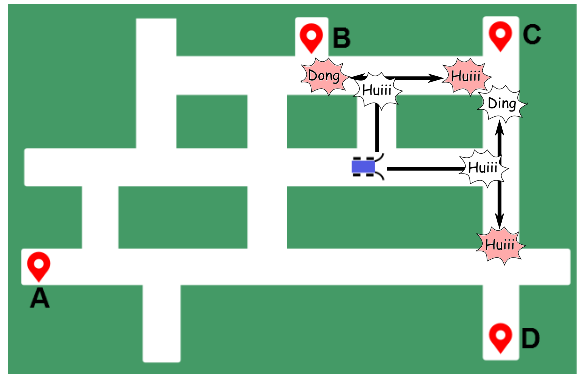
Based on the starting position of the car, just after it starts moving forward it will encounter an intersection where it can turn both left and right and so it will make a Huiii sound first. This will happen no matter what route the car takes on the trip.

During Trip 1, the car made the sounds Huiii Ding Huiii Ding. We can determine which way the car moved at each intersection based on the other three sounds it made before reaching its destination: Ding Huiii Ding.

Once the car reached the first intersection it had the choice of turning left, turning right, or going straight as shown in the map on the right. Notice that the only way the car could have made a Ding sound second is if it turned right at this first intersection. If the car turned left it would have next encountered an intersection with no left turn (Dong) and if it went straight then it would have next encountered an intersection with both a right and a left turn (Huiii). So we know that the car must have made a right turn at this first intersection.



After the car made this first right turn and moved to its second intersection, it had the choice of turning left or going straight as shown in the map on the right. Notice that no matter what choice the car made at this intersection, it would have made the correct third sound (Huiii) at the next intersection. This means we cannot be sure yet which choice it made.

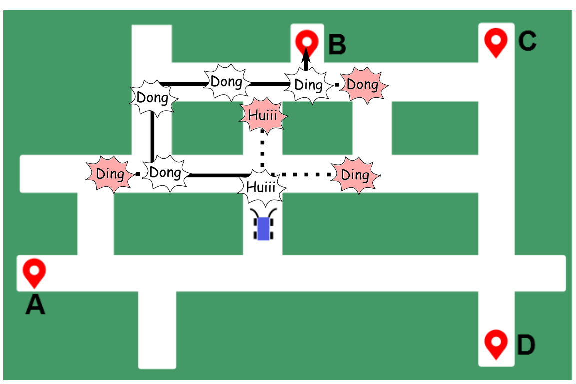


Suppose that the car turned left (travelling upward on the map). Notice that in this case, the only two options for the fourth sound the car could make are Dong (if it turned left next) and Huiii (if it turned right next). Neither of these is the correct fourth sound.

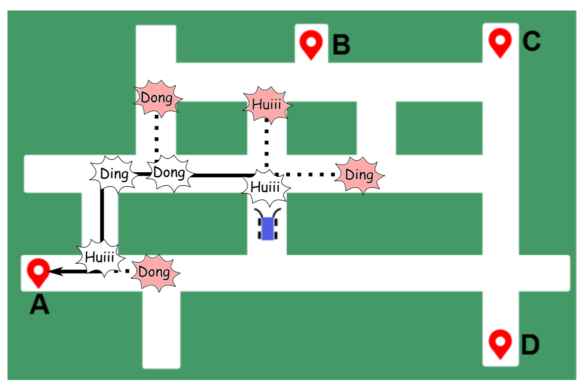
This means the car must have gone straight at this second intersection (travelling instead to the right on the map). Following similar reasoning as earlier, we can see that the car must then have turned left when it reached the next intersection in order to produce a Ding as the fourth sound (rather than a Huiii). Since the car did not make any more sounds during this trip, we know it must not have encountered another intersection, and so it must have travelled straight to destination C.

We can use similar reasoning to find the final destination on the remaining two trips. The two trips are shown in the maps below. Again, we indicate the various choices the car could have made at each intersection and eliminate the possibilities (using the sounds) until we find the correct route.

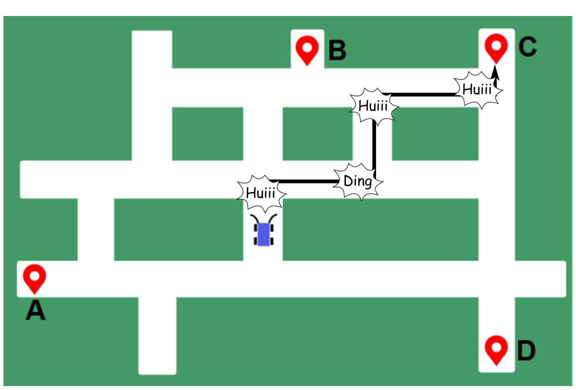
Trip 2: Huiii Dong Dong Dong Ding

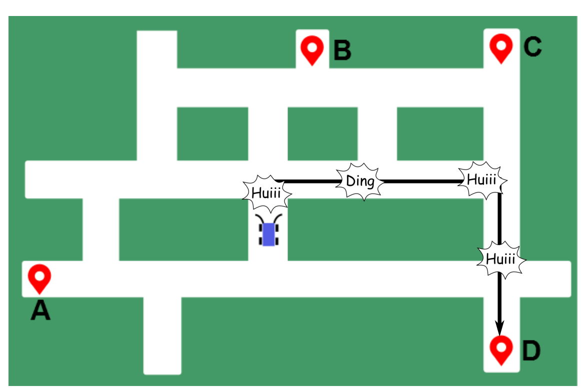


Trip 3: Huiii Dong Ding Huiii



Extension: It *is* possible for the car to produce exactly the same sequence of sounds but arrive at two different destinations on this map. As shown in the maps below, the car would produce the sounds Huiii Ding Huiii Huiii during each of the trips indicated while ending up at C on one trip and D on the other.





CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, April 30, 2020
Digit Swapping

Ali programs three buttons in a machine to swap some digits in a 4-digit number.

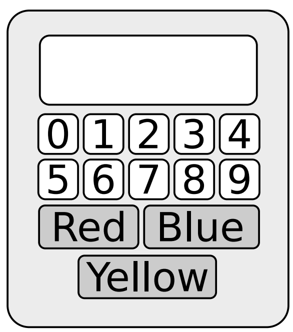
Red button:	swaps the thousands and tens digits
Blue button:	swaps the thousands and hundreds digits
Yellow button:	swaps the hundreds and units (ones) digits

Ali types a 4-digit number into the machine. She then presses the following sequence of buttons to produce 6943 as the output.

$\boxed{\text{Red}}\ \boxed{\text{Yellow}}\ \boxed{\text{Blue}}\ \boxed{\text{Red}}\ \boxed{\text{Yellow}}$

What would the output have been if Ali had instead pressed the following sequence of buttons after typing in her original number?

$\boxed{\text{Blue}}\ \boxed{\text{Red}}\ \boxed{\text{Yellow}}\ \boxed{\text{Blue}}$

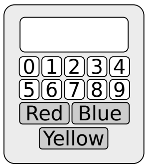


More Info:

Check the CEMC at Home webpage on Thursday, May 7 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, May 7.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php



Problem of the Week
Problem C and Solution
Digit Swapping

Problem

Ali programs three buttons in a machine to swap some digits in a 4-digit number.

Red button:	swaps the thousands and tens digits
Blue button:	swaps the thousands and hundreds digits
Yellow button:	swaps the hundreds and units (ones) digits

Ali types a 4-digit number into the machine. She then presses the following sequence of buttons to produce 6943 as the output.

$$\boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}}$$

What would the output have been if Ali had instead pressed the following sequence of buttons after typing in her original number?

$$\boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}}$$

Solution

First we need to know what number Ali initially typed into the machine in order to produce 6943 as the output.

$$???? \rightarrow \boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}} \rightarrow 6943$$

We can determine this by working backwards. This means we will start with 6943 and go through the sequence of buttons in the opposite order.

$$6943 \rightarrow \boxed{\text{Yellow}} \rightarrow 6349$$
$$6349 \rightarrow \boxed{\text{Red}} \rightarrow 4369$$
$$4369 \rightarrow \boxed{\text{Blue}} \rightarrow 3469$$
$$3469 \rightarrow \boxed{\text{Yellow}} \rightarrow 3964$$
$$3964 \rightarrow \boxed{\text{Red}} \rightarrow 6934$$

So Ali typed 6934 into the machine. Now we want to know the output after pressing the second sequence of buttons.

$$6934 \rightarrow \boxed{\text{Blue}}\,\boxed{\text{Red}}\,\boxed{\text{Yellow}}\,\boxed{\text{Blue}} \rightarrow ????$$

We can go through the second button sequence to determine the new output.

$$6934 \rightarrow \boxed{\text{Blue}} \rightarrow 9634 \rightarrow \boxed{\text{Red}} \rightarrow 3694 \rightarrow \boxed{\text{Yellow}} \rightarrow 3496 \rightarrow \boxed{\text{Blue}} \rightarrow 4396$$

Therefore, the output would have been 4396.

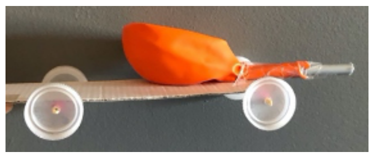
CEMC at Home

Grade 7/8 - Friday, May 1, 2020
RACECAR Backwards is RACECAR

In today's activity we will make a car powered by a balloon, and solve problems involving speeds.

Activity

Make a balloon car using household items! This car needs wheels on axles, a body, and an attached balloon for power. We can get the car to run by filling the balloon with air and letting it go.



See the next page for possible instructions for how to make a balloon car for the activity.

See how far you can make your balloon car go and time how long your car is in motion before it comes to a stop. Once your car comes to a stop, measure the distance travelled. Use the distance and time to calculate the *average speed* of your car on this trip. What might you do to increase the speed of your car? Try some adjustments and keep track of what happens to the speed of the car.

The average speed of a car on a trip is the total distance travelled by the car during the trip divided by the total time it takes the car to complete the trip.

Remember that speed is a unit rate, written as meters per second (m/s), kilometres per hour (km/hr), etc. If your car travelled 45 centimetres in 5 seconds, then to calculate the average speed as a unit rate, you need to figure out how far your car travelled in 1 second, on average. This means you need to figure out what value should go in the box below.

$$\frac{45 \text{ cm}}{5 \text{ sec}} = \frac{\square \text{ cm}}{1 \text{ sec}}$$

Try these problems involving speeds!

1. Tiegan is travelling at a constant speed of 85 km/h. If Tiegan is halfway through a 510 km trip, how much longer will the trip take?

2. A bicycle travels at a constant speed of 15 km/h. A bus starts 195 km behind the bicycle and catches up to the bicycle in 3 hours. What is the average speed of the bus in km/h?

   To get started, determine how far the bicycle has travelled.

3. Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many metres was Greg behind?

   You will likely find this problem more challenging than the first two. Can you use ratios to help you solve this problem?

How to Make a Balloon Car

You Will Need:

- Four plastic bottle caps

  These should have roughly the same diameter.
- One wooden skewer
- One straw
- One pen
- One balloon
- Cardboard
- Tape

Instructions:

1. Cut the wooden skewer in half.
2. Cut the straw into two pieces, each slightly shorter than the wooden skewer pieces.

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3. Cut a piece of cardboard to make the body of the car. The width should be about the same as the length of each straw piece.
4. Poke a hole in the centre of each bottle cap so that the skewer can fit through.

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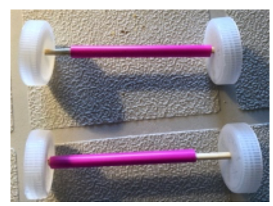
5. Press the bottle cap onto one end of the skewer. Slide the straw around the skewer and then press another bottle cap onto the other end of the skewer so that the straw can rotate. Repeat this process with the other skewer, straw, and bottle caps to form your wheels.
6. Attach the cardboard to the straws using tape.



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7. Put your car down on a flat surface and give it a push. Make sure the car rolls easily and coasts for a bit before stopping. Make any necessary adjustments.
8. Take the pen apart so you are left with just the empty plastic case. Tape the neck of the balloon around one end of the pen case. Wrap the tape very tightly so that no air can escape.

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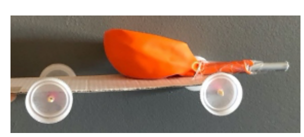
9. Tape the pen case onto the cardboard. Make sure that when the balloon is inflated, it does not touch either wheel.
10. Inflate the balloon and then quickly press your thumb on the end of the pen case. Place the car gently on the ground and then remove your thumb to watch your balloon car go.



More Info:

Check the CEMC at Home webpage on Monday, May 4 for a solution to these problems.

For more practice with speeds and other rates, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Friday, May 1, 2020
RACECAR Backwards is RACECAR - Solution

1. Tiegan is travelling at a constant speed of 85 km/h. If Tiegan is halfway through a 510 km trip, how much longer will the trip take?

   Solution 1: First we will calculate how long it would take Tiegan to complete the entire trip. Since Tiegan is travelling at a constant speed, we know that she will travel 85 km every 1 hr. How long will it take Tiegan to travel 510 km?

   $$\frac{85\text{ km}}{1\text{ h}} \overset{\times 6}{\underset{\times 6}{=}} \frac{510\text{ km}}{\square\text{ h}}$$

   Since 510 km $= 6 \times 85$ km, the entire trip will take 6×1 h $= 6$ h. Since Tiegan is halfway through the trip, the remainder of the trip will take 6 h $\div 2 = 3$ h.

   Solution 2: Since Tiegan is halfway through a 510 km trip, that means she has $510 \div 2 = 255$ km left to travel. Since Tiegan is travelling at a constant speed, we know that she will travel 85 km every 1 hr. How long will it take her to travel 255 km?

   $$\frac{85\text{ km}}{1\text{ h}} \overset{\times 3}{\underset{\times 3}{=}} \frac{255\text{ km}}{\square\text{ h}}$$

   Since 255 km $= 3 \times 85$ km, it will take her 3×1 h $= 3$ h to travel the remaining 255 km.

2. A bicycle travels at a constant speed of 15 km/h. A bus starts 195 km behind the bicycle and catches up to the bicycle in 3 hours. What is the average speed of the bus in km/h?

   Solution: Since the bicycle travels at a constant speed, we know it travels 15 km every 1 h. How far will it travel in 3 h?

   $$\frac{15\text{ km}}{1\text{ h}} \overset{\times 3}{\underset{\times 3}{=}} \frac{\square\text{ km}}{3\text{ h}}$$

   Since 3×15 km $= 45$ km, the bicycle will travel a distance of 45 km in 3 h. At the start, the bicycle was 195 km ahead of the bus. Therefore, in order to catch up to the bicycle, the bus must travel the 195 km plus the additional 45 km that the bicycle travels, or $195 + 45 = 240$ km. So the bus travels 240 km in 3 hours. What is the average speed of the bus as a unit rate?

   $$\frac{240\text{ km}}{3\text{ h}} \overset{\div 3}{\underset{\div 3}{=}} \frac{\square\text{ km}}{1\text{ h}}$$

   Since $240 \div 3 = 80$, the bus must have travelled at an average speed of 80 km/h.

3. Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many metres was Greg behind?

   Solution 1: In the first race, when Azarah crossed the finish line, Charlize was 20 m behind or Charlize had run 80 m.
   Since Azarah and Charlize each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or $100 : 80$.
   Similarly in the second race, when Charlize crossed the finish line, Greg was 10 m behind or Greg had run 90 m.
   Since Charlize and Greg each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or $100 : 90$.
   Let A, C and G represent Azarah's, Charlize's and Greg's speeds, respectively.
   Then, $A : C = 100 : 80 = 25 : 20$ and $C : G = 100 : 90 = 20 : 18$.
   Therefore, $A : C : G = 25 : 20 : 18$ and $A : G = 25 : 18 = 100 : 72$.
   Over equal times, the ratio of their speeds is equal to the ratio of their distances travelled.
   Therefore, when Azarah travels 100 m, Greg travels 72 m.
   When Azarah crossed the finish line, Greg was $100 - 72 = 28$ m behind.

   Solution 2: In the first race, when Azarah crossed the finish line, Charlize was 20 m behind or Charlize had run 80 m.
   Since Azarah and Charlize each travelled these respective distances in the same amount of time, then the ratio of their speeds is equal to the ratio of their distances travelled, or $100 : 80$.
   That is, Charlize's speed is 80% of Azarah's speed.
   Similarly, Greg's speed is 90% of Charlize's speed.
   Therefore, Greg's speed is 90% of Charlize's speed which is 80% of Azarah's speed, or Greg's speed is 90% of 80% of Azarah's speed.
   Since 90% of 80% is equivalent to $0.90 \times 0.80 = 0.72$ or 72%, then Greg's speed is 72% of Azarah's speed.
   When Azarah ran 100 m (crossed the finish line), Greg ran 72% of 100 m or 72 m in the same amount of time.
   When Azarah crossed the finish line, Greg was $100 - 72 = 28$ m behind.

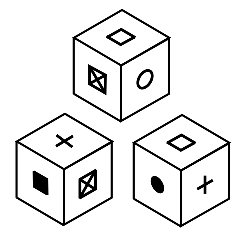
CEMC at Home

Grade 7/8 - Monday, May 4, 2020
Contest Day 1

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #19

Three different views of the same cube are shown. The symbol on the face opposite ● is

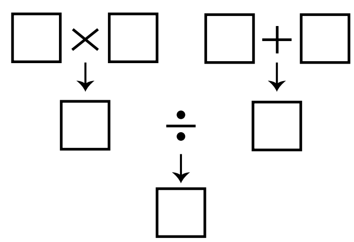


(A) + **(B)** ■ **(C)** ⊠

(D) □ **(E)** ○

2016 Gauss Contest, #20

In the diagram, four different integers from 1 to 9 inclusive are placed in the four boxes in the top row. The integers in the left two boxes are multiplied and the integers in the right two boxes are added and these results are then divided, as shown. The final result is placed in the bottom box. Which of the following integers cannot appear in the bottom box?



(A) 16 **(B)** 24 **(C)** 7

(D) 20 **(E)** 9

More Info:

Check out the CEMC at Home webpage on Monday, May 11 for solutions to the Contest Day 1 problems.

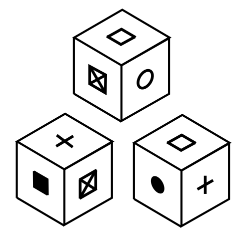
CEMC at Home

Grade 7/8 - Monday, May 4, 2020
Contest Day 1 - Solution

Solutions to the two contest problems are provided below, including a video for the first problem.

2020 Gauss Contest, #19

Three different views of the same cube are shown. The symbol on the face opposite ● is

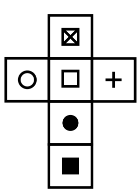


(A) + (B) ■ (C) ⊠

(D) □ (E) ○

Solution:

We begin by recognizing that there are 6 different symbols, and so each face of the cube contains a different symbol.
From left to right, let us number the views of the cube 1, 2 and 3.
Views 1 and 2 each show a face containing the symbol ⊠ .
What symbol is on the face opposite to the face containing ⊠?
In view 1, □ and ○ are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
In view 2, ■ and + are on faces adjacent to the face containing ⊠ , and so neither of these can be the symbol that is on the face opposite ⊠ .
There is only one symbol remaining, and so ● must be the symbol that is on the face opposite ⊠, and vice versa.
A net of the cube is shown below.



ANSWER: (C)

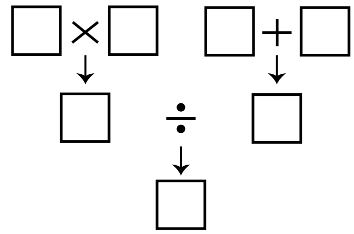
Video

Visit the following link to view another solution to the first contest problem that uses nets:
https://youtu.be/N88l8IXEiHs

See the next page for a solution to the second contest problem.

2016 Gauss Contest, #20

In the diagram, four different integers from 1 to 9 inclusive are placed in the four boxes in the top row. The integers in the left two boxes are multiplied and the integers in the right two boxes are added and these results are then divided, as shown. The final result is placed in the bottom box. Which of the following integers cannot appear in the bottom box?

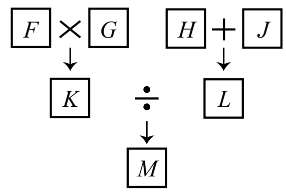


(A) 16 **(B)** 24 **(C)** 7

(D) 20 **(E)** 9

Solution:

We begin by naming the boxes as shown to the right.



Of the five answers given, the integer which cannot appear in box M is 20. Why?

Since boxes F and G contain different integers, the maximum value that can appear in box K is $8 \times 9 = 72$.

Since boxes H and J contain different integers, the minimum value that can appear in box L is $1 + 2 = 3$.

Next, we consider the possibilities if 20 is to appear in box M.

If 3 appears in box L (the minimum possible value for this box), then box K must contain 60, since $60 \div 3 = 20$.

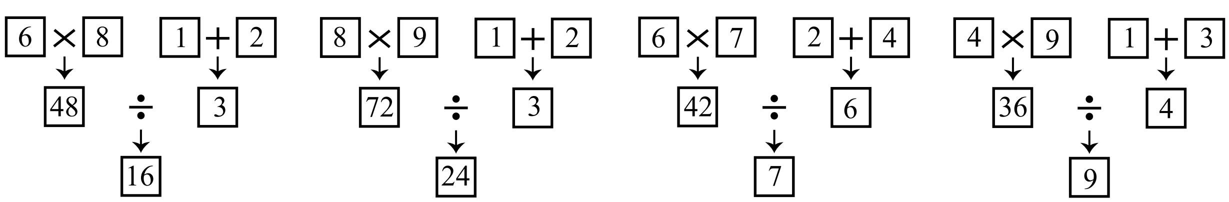
However, there are no two integers from 1 to 9 whose product is 60 and so there are no possible integers which could be placed in boxes F and G so that the product in box K is 60.

If any integer greater than or equal to 4 appears in box L, then box K must contain at least $4 \times 20 = 80$.

However, the maximum value that can appear in box K is 72.

Therefore, there are no possible integers from 1 to 9 which can be placed in boxes F, G, H, and J so that 20 appears in box M.

The diagrams below demonstrate how each of the other four answers can appear in box M.

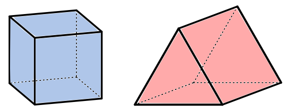


Answer: (D)

CEMC at Home

Grade 7/8 - Tuesday, May 5, 2020
It's in the Bag

Colin has some cubes and some triangular prisms whose rectangular faces are actually squares.



Colin put some number of each of these solids in a bag. In total, the solids in his bag have 21 square faces, 6 triangular faces, and no other types of faces. How many of each solid did Colin put in his bag?

There are several ways to solve this problem. Three different methods are shown below.

Method 1: Make a table

Number of Cubes	Number of Square Faces
1	6
2	12
3	18

Number of Triangular Prisms	Number of Square Faces	Number of Triangular Faces
1	3	2
2	6	4
3	9	6

Notice that if we have 2 cubes and 3 triangular prisms, then we have $12 + 9 = 21$ square faces and 6 triangular faces, as desired. So Colin could have 2 cubes and 3 triangular prisms in his bag. Can you explain why this is the only possibility for the solids in Colin's bag?

Method 2: Use logic

Since cubes do not have triangular faces, the triangular faces must have all come from the triangular prisms. Each triangular prism has 2 triangular faces, so we would need 3 triangular prisms in order to have 6 triangular faces in total. Each triangular prism has 3 square faces, so 3 triangular prisms would have 9 square faces in total. We need 21 square faces. Since $21 - 9 = 12$, that means the remaining 12 square faces must come from cubes. Each cube has 6 square faces, so we would need 2 cubes in order to have 12 square faces in total. This means Colin must have 3 triangular prisms and 2 cubes in his bag.

Method 3: Use variables

Let c represent the number of cubes and t represent the number of triangular prisms in the bag. Since each cube has 0 triangular faces and each triangular prism has 2 triangular faces, the number of triangular faces in the bag must be $2 \times t$. Since there are 6 triangular faces in total, we must have

$$2 \times t = 6$$

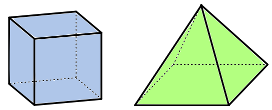
Since each cube has 6 square faces and each triangular prism has 3 square faces, the total number of square faces in the bag must be $6 \times c + 3 \times t$. Since there are 21 square faces in total, we must have

$$6 \times c + 3 \times t = 21$$

Can you use these equations to figure out what the values of t and c must be?

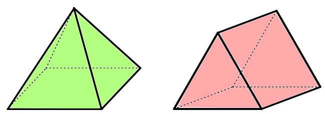
Think about these problem solving methods while you work on the next problems.
Which ones work well for you in these problems, and which ones may not work so well?

Problem 1: Valerie has some cubes and some square-based pyramids.



Valerie put some number of each of these solids in a bag. In total, the solids in her bag have 68 square faces, 56 triangular faces, and no other types of faces. How many of each shape did Valerie put in her bag?

Problem 2: Max has some square-based pyramids and some triangular prisms whose rectangular faces are actually squares.



Max put some number of each of these solids in a bag. In total, the solids in their bag have 13 square faces, some number of triangular faces, and no other types of faces. How many different combinations of solids could Max have put in their bag?

There is no way to know exactly what solids were in Max's bag, but can you find all of the different possibilities? Can you explain how you know you have found them all?

More Info:

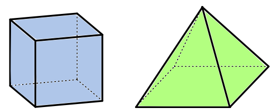
Check out the CEMC at Home webpage on Wednesday, May 6 for a solution to It's in the Bag.

For more practice solving problems using variables, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Tuesday, May 5, 2020
It's in the Bag - Solution

Problem 1: Valerie has some cubes and some square-based pyramids.



Valerie put some number of each of these solids in a bag. In total, the solids in her bag have 68 square faces, 56 triangular faces, and no other types of faces. How many of each shape did Valerie put in her bag?

There are several ways to solve this problem. Three different solutions are shown below.

Method 1: Make a table

Number of Cubes	Number of Square Faces
1	6
2	12
3	18
⋮	⋮
8	48
9	54
10	60
11	66

Number of Pyramids	Number of Square Faces	Number of Triangular Faces
1	1	4
2	2	8
3	3	12
⋮	⋮	⋮
11	11	44
12	12	48
13	13	52
14	14	56

Notice that if we have 9 cubes and 14 pyramids, then we have $54 + 14 = 68$ square faces and 56 triangular faces as needed. So Valerie could have 9 cubes and 14 pyramids in her bag.

Can you use the table to convince yourself that this is the only possibility for the combination of objects in Valerie's bag? We will justify this in the following two solutions.

This method is time consuming because we needed to fill in many rows of the tables before we found a combination that worked. The other methods are more efficient for solving this problem.

Method 2: Use logic

Since cubes do not have triangular faces, the triangular faces must have all come from the pyramids. Each pyramid has 4 triangular faces. Since $56 \div 4 = 14$, there must be 14 pyramids in order to have 56 triangular faces in total.

Each pyramid has 1 square face, so the 14 pyramids have 14 square faces among them. There are 68 square faces in total. Since $68 - 14 = 54$, this means the remaining 54 square faces must come from cubes. Each cube has 6 square faces. Since $54 \div 6 = 9$, there must be 9 cubes in order to have 54 square faces among the cubes.

This means Valerie must have 9 cubes and 14 pyramids in her bag.

Method 3: Use variables

Let c represent the number of cubes and s represent the number of square-based pyramids in Valerie's bag.

Since each cube has 0 triangular faces, and each pyramid has 4 triangular faces, the number of triangular faces in the bag must be $4 \times s$. Since there are 56 triangular faces in total, we must have

$$4 \times s = 56$$

Notice that $4 \times 14 = 56$. The only positive integer that satisfies this equation is $s = 14$ and so we know the number of pyramids must be 14.

Since each cube has 6 square faces and each pyramid has 1 square face, the number of square faces in the bag must be $6 \times c + 1 \times s$. Since there are 68 square faces in total, we must have

$$6 \times c + 1 \times s = 68$$

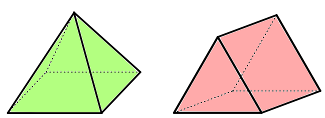
Remember that $s = 14$ so substituting this value we get

$$6 \times c + 1 \times 14 = 68$$

From this we see that $6 \times c$ must equal 54 which means c must equal 9.

Therefore, Valerie must have 9 cubes and 14 pyramids in her bag.

Problem 2: Max has some square-based pyramids and some triangular prisms whose rectangular faces are actually squares.

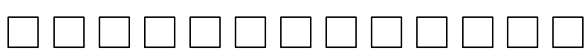


Max put some number of each of these solids in a bag. In total, the solids in their bag have 13 square faces, some number of triangular faces, and no other types of faces. How many different combinations of solids could Max have put in their bag?

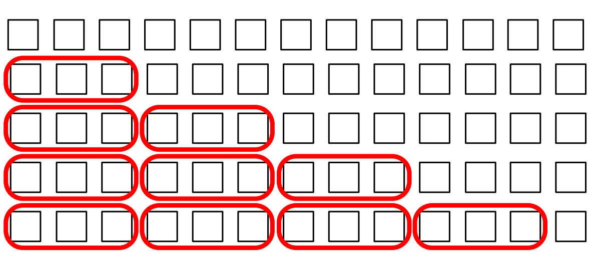
There are several ways to solve this problem. Two different solutions are shown below.

Method 1: Use diagrams

The squares below represent the 13 square faces in Max's bag.



We know each triangular prism has 3 square faces and each pyramid has 1 square face. So we want to figure out how many ways we can put the 13 squares into groups of 3 and 1. The diagram below shows all possible ways to do this, where each group of 3 is circled.



→ 0 triangular prisms and 13 pyramids
→ 1 triangular prism and 10 pyramids
→ 2 triangular prisms and 7 pyramids
→ 3 triangular prisms and 4 pyramids
→ 4 triangular prisms and 1 pyramid

So there are 5 different possible combinations for the solids in Max's bag.

Solution 2: Use variables

Let t represent the number of triangular prisms and s represent the number of square-based pyramids in Max's bag. Each triangular prism has 3 square faces and each pyramid has 1 square face. Since there are 13 square faces in total, we can write the following equation.

$$3 \times t + 1 \times s = 13$$

Notice that there is only one equation and it has two variables. This means we cannot solve this equation in the same way we might solve other equations with only one variable. It also means that the equation may have more than one solution. To find some solutions, we start testing some values.

Suppose there are 0 triangular prisms (and so $t = 0$). Then the equation becomes

$$3 \times 0 + 1 \times s = 13 \quad \text{or} \quad 1 \times s = 13$$

This tells us that in this case we must have $s = 13$.

Suppose there is 1 triangular prism (and so $t = 1$). Then the equation becomes

$$3 \times 1 + 1 \times s = 13 \quad \text{or} \quad 3 + 1 \times s = 13$$

This tells us that in this case we must have $3 + s = 13$ which means $s = 10$.

What are the values of s if there are 2, 3, or 4 triangular prisms (that is, if $t = 2$, $t = 3$, or $t = 4$)? Can there be 5 or more triangular prisms?

It turns out that there are five solutions, and they are shown below:

$$\begin{aligned}
3 \times 0 + 1 \times 13 = 13 &\rightarrow \text{ 0 triangular prisms and 13 pyramids}\\
3 \times 1 + 1 \times 10 = 13 &\rightarrow \text{ 1 triangular prism and 10 pyramids}\\
3 \times 2 + 1 \times 7 = 13 &\rightarrow \text{ 2 triangular prisms and 7 pyramids}\\
3 \times 3 + 1 \times 4 = 13 &\rightarrow \text{ 3 triangular prisms and 4 pyramids}\\
3 \times 4 + 1 \times 1 = 13 &\rightarrow \text{ 4 triangular prisms and 1 pyramid}
\end{aligned}$$

Notice that there cannot be 5 or more triangular prisms is the bag. 5 triangular prisms will contribute $3 \times 5 = 15$ square faces which is more than the total of 13.

So there are 5 different possible combinations for the solids in Max's bag.

Did you use a different approach to solve Problem 2? Is there a way to use a table or other reasoning to solve this problem?

More Info:

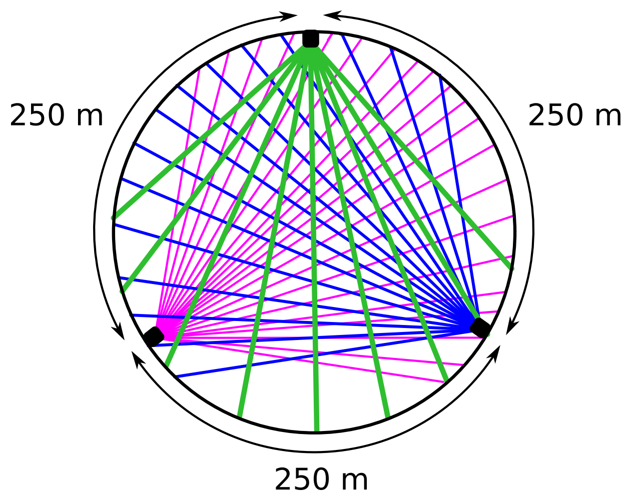
Equations with more than one variable like the one in the solution to Problem 2 are called Diophantine equations. For more practice finding solutions to Diophantine equations, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Wednesday, May 6, 2020

Light Show

William is creating a light show over a circular pond. He has positioned a green light, a pink light, and a blue light so they are equally spaced around the edge of the pond. To get from one light to the next, he must walk 250 metres around the edge of the pond.



Each light has a switch that William can use to turn the light on and off. After positioning the lights, William wants to test each of the different on/off combinations of the three switches to make sure he is happy with how the lights look together.

Problem 1: How many different on/off combinations of the three switches are there?

Need help getting started? Check out the online exploration where you can turn the switches on and off to see all of the different combinations of lights.

Problem 2: William starts at the blue light with all three lights switched off. Explain how William can test four different on/off combinations of the switches (other than "all switches off") by walking a total distance of 750 m around the pond, possibly changing directions during his trip.

Problem 3: William starts at the blue light with all three lights switched off. William wants to test all possible on/off combinations of the switches. What is the shortest possible distance he could walk to complete this task?

Note that William can change directions as many times as he wants during his trip.

More Info:

Check out the CEMC at Home webpage on Thursday, May 7 for a solution to Light Show.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

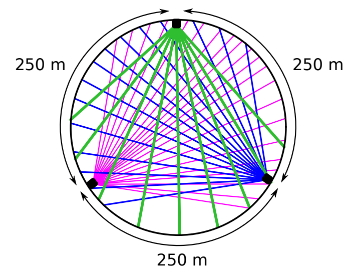
CEMC at Home

Grade 7/8 - Wednesday, May 6, 2020
Light Show - Solution

Summary

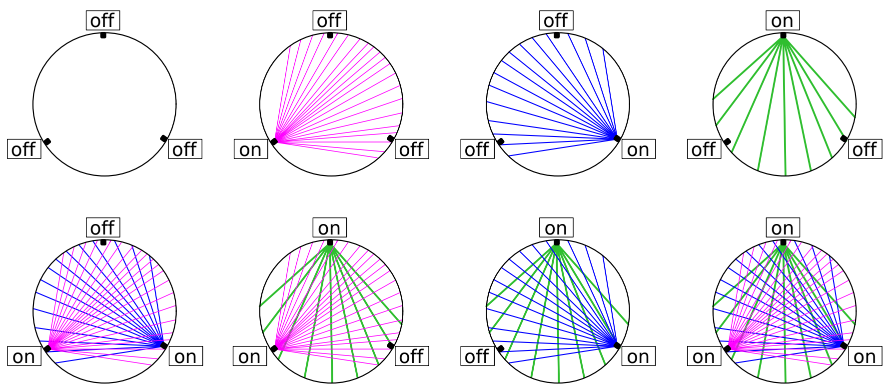
William has positioned a green light, a pink light, and a blue light so they are spaced equally around the pond. To get from one light to another he must walk 250 metres around the edge of the pond.

Each light has a switch that William can use to turn it on and off.



Problem 1: How many different on/off combinations of the three switches are there?

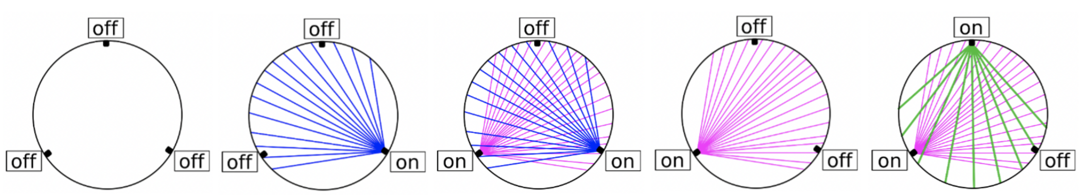
Solution: There are 8 different on/off combinations, as shown below.



Problem 2: William starts at the blue light with all three lights switched off. Explain how William can test four different on/off combinations of the switches (other than "all switches off") by walking a total distance of 750 m around the pond, possibly changing directions during his trip.

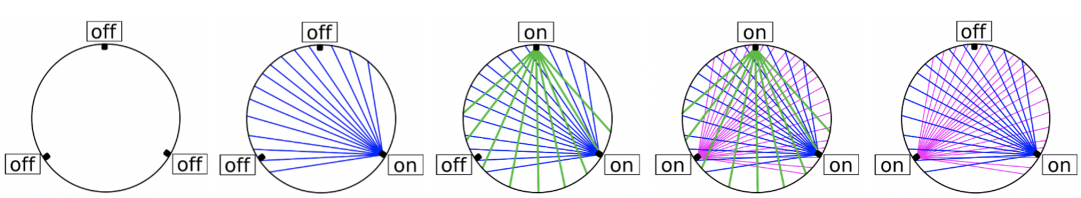
Solution: There are many different ways to achieve this goal.

One option is the following: Start by turning the blue light on. Walk 250 m to the pink light and turn it on. Walk another 250 m *back* to the blue light and turn it off. Walk a final 250 m to the green light and turn it on. The images below show the four different combinations tested on this trip.



Start After 0 m After 250 m After 500 m After 750 m

Another possible option is illustrated using the images below. (Can you describe the trip here?)



Start After 0 m After 250 m After 500 m After 750 m

Problem 3: William starts at the blue light with all three lights switched off. William wants to test all possible on/off combinations of the switches. What is the shortest possible distance he could walk to complete this task?

Solution: William can test all eight of the combinations while walking a total of 1500 m. One way to do this is outlined in the table below.

Action	Blue switch	Pink switch	Green switch	Total distance travelled
Start at the blue switch	off	off	off	0 m
Turn the blue switch on	**on**	off	off	0 m
Turn the pink switch on	on	**on**	off	250 m
Turn the blue switch off	**off**	on	off	500 m
Turn the green switch on	off	on	**on**	750 m
Turn the pink switch off	off	**off**	on	1000 m
Turn the blue switch on	**on**	off	on	1250 m
Turn the pink switch on	on	**on**	on	1500 m

We can check that this table contains a row for each of the eight different combinations, and that this trip will take a total of 1500 m.

This means William can complete this task by walking *at most* 1500 m. Since the question is asking for us to find the *shortest possible* distance, we still need to show that it is not possible for William to walk *less than* 1500 m and still test all eight on/off combinations. Let's see why this cannot be done.

At his starting position, William can test at most 2 different combinations: the initial switch combination ("all switches off") and the combination resulting from flipping this switch (the blue switch). So William has not yet moved and has to test 6 more combinations. Since William can only test one new combination each time he walks to a new switch, every 250 m walk he does will result in testing *at most* one new combination. This means that in order to test all 6 remaining combinations, he has to walk at least 6×250 m $= 1500$ m.

This means the shortest possible distance William can walk to complete the task is 1500 m.

Note that it is not important for this question that William starts at the blue light or that the switches all start off. If William starts at any of the three lights looking at any of the eight combinations, the shortest possible distance he can travel to test all eight combinations is 1500 m.

More Info:

The eight combinations of three switches can be generated in seven steps by flipping exactly one switch at each step, and choosing the order carefully. If we write 0 for "off" and 1 for "on", the list of = on/off combinations can be written as a sequence of bit strings (i.e. using only 0s and 1s). The second, third, and fourth columns in the table in the solution to Problem 3 correspond to the following sequence: 000 100 110 010 011 001 101 111. Such sequences are called Gray codes. Each term in a Gray code is unique, even though it differs by only one bit from the adjacent terms. Gray codes have many applications in electronics and computer science. For example, they are used to minimize the number of actions needed during hardware and software testing.

CEMC at Home features Problem of the Week

Grade 7/8 - Thursday, May 7, 2020
We Took the Cookies

There is a cookie jar that contains a certain number of cookies. Three friends divide the cookies in the following way. First, Harold takes all of the cookies and places them into three equal piles with none left over. He then keeps one of the piles and puts the other two piles back into the jar. Next, Lucie takes the remaining cookies in the jar and places them into three equal piles with none left over. She then keeps one of the piles and puts the other two piles back into the jar. Finally, Livio takes the remaining cookies in the jar and places them into three equal piles, but there is one cookie left over. He then keeps the leftover cookie and one of the piles and puts the other two piles back into the jar.

If there are now 10 cookies in the jar, how many cookies were originally in the cookie jar?

More Info:

Check the CEMC at Home webpage on Thursday, May 14 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution emailed to you on Thursday, May 14.

This CEMC at Home resource is the current grade 7/8 problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week and to find many more past problems and their solutions visit: https://www.cemc.uwaterloo.ca/resources/potw.php

Problem of the Week
Problem C and Solution
We Took the Cookies

Problem
There is a cookie jar that contains a certain number of cookies. Three friends divide the cookies in the following way. First, Harold takes all of the cookies and places them into three equal piles with none left over. He then keeps one of the piles and puts the other two piles back into the jar. Next, Lucie takes the remaining cookies in the jar and places them into three equal piles with none left over. She then keeps one of the piles and puts the other two piles back into the jar. Finally, Livio takes the remaining cookies in the jar and places them into three equal piles, but there is one cookie left over. He then keeps the leftover cookie and one of the piles and puts the other two piles back into the jar. If there are now 10 cookies in the jar, how many cookies were originally in the cookie jar?

Solution

We will present two different solutions. Solution 1 works backwards through the problem. Solution 2 is an algebraic solution.

Solution 1

The last 10 cookies in the cookie jar are also the remaining two of Livio's three piles. Therefore, each pile he made had $10 \div 2 = 5$ cookies. Therefore, there were $5 \times 3 = 15$ cookies in the three piles he made plus 1 more cookie that he kept, for a total of 16 cookies. Therefore, there were 16 cookies in the cookie jar when Livio started dividing the cookies.

These 16 cookies were two of the three piles that Lucie made. Therefore, each pile that she made had $16 \div 2 = 8$ cookies. Therefore, there were $8 \times 3 = 24$ cookies in the three piles that she made. Therefore, there were 24 cookies in the cookie jar when Lucie started dividing the cookies.

These 24 cookies were two of the three piles that Harold made. Therefore, each pile that he made had $24 \div 2 = 12$ cookies. Therefore, there were $12 \times 3 = 36$ cookies in the three piles that he made. Therefore, there were 36 cookies in the cookie jar when Harold started dividing the cookies.

Therefore, there were originally 36 cookies in the cookie jar.

Solution 2

Let the initial number of cookies in the cookie jar be C.

Harold has $\frac{1}{3}C$ cookies in the pile he keeps. Therefore, $\frac{2}{3}C$ cookies are left for Lucie.

Lucie keeps $\frac{1}{3}$ of $\frac{2}{3}C$ cookies. Therefore, $\frac{2}{3}$ of $\frac{2}{3}C$ cookies are left for Livio. That is, $\frac{2}{3} \times \frac{2}{3}C = \frac{4}{9}C$ cookies are left for Livio.

For Livio, the pile he keeps is $\frac{1}{3}$ of one less than what is left. That is $\frac{1}{3} \times \left(\frac{4}{9}C - 1\right)$, and so the remaining number of cookies that he puts back into the cookie jar is equal to $\frac{2}{3} \times \left(\frac{4}{9}C - 1\right)$.

This is also equal to 10. That is,

$$\frac{2}{3} \times \left(\frac{4}{9}C - 1\right) = 10$$

Dividing both sides by $\frac{2}{3}$,

$$\frac{\frac{2}{3} \times \left(\frac{4}{9}C - 1\right)}{\frac{2}{3}} = \frac{10}{\frac{2}{3}}$$

Since $10 \div \frac{2}{3} = 10 \times \frac{3}{2} = \frac{30}{2} = 15$,

$$\frac{4}{9}C - 1 = 15$$

Therefore,

$$\frac{4}{9}C = 16$$

Dividing both sides by $\frac{4}{9}$,

$$\frac{\frac{4}{9}C}{\frac{4}{9}} = \frac{16}{\frac{4}{9}}$$

Since $16 \div \frac{4}{9} = 16 \times \frac{9}{4} = 36$,

$$C = 36$$

Therefore, there were originally 36 cookies in the cookie jar.

CEMC at Home

Grade 7/8 - Friday, May 8, 2020

Reading Minds

In this activity, you will "read minds" using math!

Activity 1: Follow the steps below.

1. Choose a positive integer.
2. Double it.
3. Add 15.
4. Subtract 1.
5. Divide by 2.
6. Subtract your original number.

Did you get 7? You may think I read your mind, but actually all I did was some math! Let's look at each step more closely. We will use a box to represent the starting number. This way we can show that regardless of what number you start with, you will always end up with the number 7.

Step	**Result**
1. Choose a positive integer.	$\square$
2. Double it.	$\square + \square$
3. Add 15.	$\square + \square + 15$
4. Subtract 1.	$\square + \square + 14$ (which is $\square + 7 + \square + 7$)
5. Divide by 2.	$(\square + 7 + \square + 7) \div 2 = \square + 7$
6. Subtract your original number.	$\square + 7 - \square = 7$

As you can see, regardless of what number you start with, when you follow the steps you will always end up with the number 7.

Activity 2: Explain why following the steps below will always result in the number 3.

1. Choose a positive integer.
2. Multiply it by 4.
3. Add 12.
4. Divide by 4.
5. Subtract your original number.

Activity 3: Follow the steps below a few times with different starting numbers.

1. Choose a positive 3-digit number where all the digits are the same.
2. Divide your 3-digit number by the sum of its digits.

What number do you get each time you follow these steps? Can you explain why this happens?

Activity 4: Make up your own math mind reading trick. Try it out on your family and friends and see if they can figure out your trick!

More Info:

Check out the CEMC at Home webpage on Monday, May 11 for a solution to Reading Minds.

For more practice using variables to solve problems, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Friday, May 8, 2020
Reading Minds - Solution

Activity 2: Explain why following the steps below will always result in the number 3.

1. Choose a positive integer.
2. Multiply it by 4.
3. Add 12.
4. Divide by 4.
5. Subtract your original number.

Solution: We will use a box to represent the starting number. We will show that no matter what number you start with, you will always end up with the number 3 at the end.

Step	Result
1. Choose a positive integer.	$\square$
2. Multiply it by 4.	$4 \times \square$ which equals $\square + \square + \square + \square$
3. Add 12.	$\square + \square + \square + \square + 12$ which equals $\square + 3 + \square + 3 + \square + 3 + \square + 3$
4. Divide by 4.	$(\square + 3 + \square + 3 + \square + 3 + \square + 3) \div 4 = \square + 3$
5. Subtract your original number.	$\square + 3 - \square = 3$

The table shows that the result is always 3.

Activity 3: Follow the steps below a few times with different starting numbers.

1. Choose a positive 3-digit number where all the digits are the same.
2. Divide your 3-digit number by the sum of its digits.

What number do you get each time you follow these steps? Can you explain why this happens?

Solution: If you follow these steps, then you will always end up with the number 37.

Since there are only 9 different numbers you could start with in Step 1, you can check for yourself that you get 37 each time. Can you explain why this keeps happening? We check the first few cases in the table below and look for a pattern.

Number chosen	The sum of the digits	Number divided by the sum of its digits
111	3	$\frac{111}{3} = 37$
222	6	$\frac{222}{6} = 37$ since $\frac{222}{6} = \frac{111 \times 2}{3 \times 2} = \frac{111}{3} = 37$
333	9	$\frac{333}{9} = 37$ since $\frac{333}{9} = \frac{111 \times 3}{3 \times 3} = \frac{111}{3} = 37$
444	12	$\frac{444}{12} = 37$ since $\frac{444}{12} = \frac{111 \times 4}{3 \times 4} = \frac{111}{3} = 37$

The table shows that if you start with a number with digits $\square\square\square$ (with the same digit in each box), then the value of the number is $111 \times \square$. This happens because the number $\square\square\square$ has value equal to $100 \times \square + 10 \times \square + 1 \times \square$, which is equal to $(100 + 10 + 1) \times \square$. Also, the sum of the digits in the number is $\square + \square + \square = 3 \times \square$. When you divide $111 \times \square$ by $3 \times \square$ you get the same answer as if you divided 111 by 3, which is 37. This happens no matter what digit is placed in the boxes.

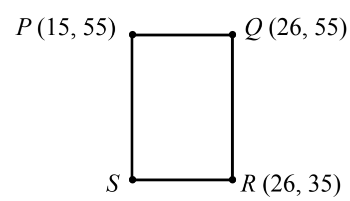
CEMC at Home

Grade 7/8 - Monday, May 11, 2020
Contest Day 2

Today's resource features two questions from the recently released 2020 CEMC Mathematics Contests.

2020 Gauss Contest, #13

Points $P(15,55)$, $Q(26,55)$ and $R(26,35)$ are three vertices of rectangle $PQRS$. The area of this rectangle is



(A) 360 **(B)** 800 **(C)** 220 **(D)** 580 **(E)** 330

2020 Gauss Contest, #23

The list $11, 20, 31, 51, 82$ is an example of an increasing list of five positive integers in which the first and second integers add to the third, the second and third add to the fourth, and the third and fourth add to the fifth. How many such lists of five positive integers have 124 as the fifth integer?

(A) 10 **(B)** 7 **(C)** 9 **(D)** 6 **(E)** 8

More Info:

Check out the CEMC at Home webpage on Thursday, May 21 for solutions to the Contest Day 2 problems.

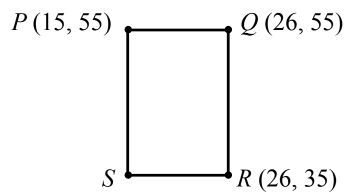
CEMC at Home

Grade 7/8 - Monday, May 11, 2020
Contest Day 2 - Solution

Solutions to the two contest problems are provided below, including a video for the second problem.

2020 Gauss Contest, #13

Points $P(15, 55)$, $Q(26, 55)$ and $R(26, 35)$ are three vertices of rectangle $PQRS$. The area of this rectangle is



(A) 360 **(B)** 800 **(C)** 220 **(D)** 580 **(E)** 330

Solution:

The y-coordinates of points $P(15, 55)$ and $Q(26, 55)$ are equal.
Therefore, the distance between P and Q is equal to the positive difference between their x-coordinates, or $26 - 15 = 11$.
Similary, the x-coordinates of points $R(26, 35)$ and $Q(26, 55)$ are equal.
Therefore, the distance between R and Q is equal to the positive difference between their y-coordinates, or $55 - 35 = 20$.
Since $PQ = 11$ and $RQ = 20$, the area of rectangle $PQRS$ is $11 \times 20 = 220$.

ANSWER: (C)

2020 Gauss Contest, #23

The list $11, 20, 31, 51, 82$ is an example of an increasing list of five positive integers in which the first and second integers add to the third, the second and third add to the fourth, and the third and fourth add to the fifth. How many such lists of five positive integers have 124 as the fifth integer?

(A) 10 **(B)** 7 **(C)** 9 **(D)** 6 **(E)** 8

Solution:

If the first positive integer in the list is a and the second is b, then the third integer is $a + b$, the fourth is $b + (a + b)$ or $a + 2b$, and the fifth is $(a + b) + (a + 2b)$ or $2a + 3b$.
Thus, we are asked to find the number of pairs of positive integers a and b, where a is less than b (since the list is increasing), and for which $2a + 3b = 124$.
What is the largest possible value for b?
If $b = 42$, then $3b = 3 \times 42 = 126$ which is too large since $2a + 3b = 124$. (Note that a larger value of b makes $3b$ even larger.)
If $b = 41$, then $3b = 3 \times 41 = 123$.
However in this case, we get that $2a = 124 - 123 = 1$, which is not possible since a is a positive integer.

Solution continued on the next page.

If $b = 40$, then $3b = 3 \times 40 = 120$ and so $2a = 4$ or $a = 2$.
Thus, the largest possible value for b is 40.
What is the smallest value for b?
If $b = 26$, then $3b = 3 \times 26 = 78$ and so $2a = 124 - 78 = 46$ or $a = 23$.
If $b = 25$, then $3b = 3 \times 25 = 75$.
However in this case, we get that $2a = 124 - 75 = 49$, which is not possible since a is a positive integer.
If $b = 24$, then $3b = 3 \times 24 = 72$ and so $2a = 124 - 72 = 52$ or $a = 26$.
However, if the first integer in the list is 26, then the second integer can not equal 24 since the list is increasing.
Smaller values of b will give larger values of a, and so the smallest possible value of b is 26.
From the values of b attempted thus far, we notice that when b is an odd integer, $3b$ is also odd (since the product of two odd integers is odd), and $124 - 3b$ is odd (since the difference between an even integer and an odd integer is odd).
So when b is odd, $124 - 3b$ is odd, and so $2a$ is odd (since $2a = 124 - 3b$).
However, $2a$ is even for every choice of the integer a and so b cannot be odd.
Conversely, when b is even, $124 - 3b$ is even (as required), and so all even integer values of b from 26 to 40 inclusive will satisfy the requirements.
These values of b are $26, 28, 30, 32, 34, 36, 38, 40$, and so there are 8 such lists of five integers that have 124 as the fifth integer.
Here are the 8 lists:

$$\begin{array}{cccc} 2,40,42,82,124; & 5,38,43,81,124; & 8,36,44,80,124; & 11,34,45,79,124; \\ 14,32,46,78,124; & 17,30,47,77,124; & 20,28,48,76,124; & 23,26,49,75,124. \end{array}$$

ANSWER: (E)

Video

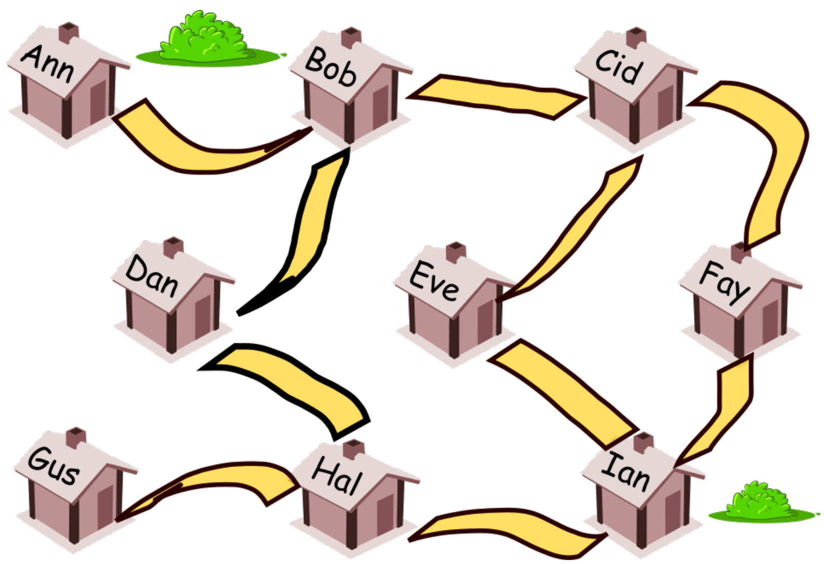
Visit the following link to view another solution to the second contest problem which makes use of a spreadsheet to solve the problem: https://youtu.be/RBM5Q88WvNk.

Don't worry if you have never used spreadsheets before. The video will walk you through the parts that you need to know.

CEMC at Home

Grade 7/8 - Tuesday, May 12, 2020
Volunteer Firefighters

The mayor of a small town is looking for volunteer firefighters. A map showing the possible volunteers' homes and how they are connected by roads is shown below. The mayor's goal is to ensure that every home in the town is either the home of a volunteer or is connected by a single road to the home of a volunteer.



Problem 1: The mayor's goal outlined above can be achieved with only 3 volunteer firefighters. Find a group of 3 volunteer firefighters that will achieve the mayor's goal.

Use the online exploration to test different combinations of firefighters.

Problem 2: Find all possible groups of 3 volunteer firefighters that will achieve the mayor's goal. Explain how you know you have found them all.

Problem 3: Explain why it is not possible for the mayor's goal to be achieved if there are only 2 volunteer firefighters.

Extension: A new road is built between two of the homes in the town and the mayor's goal can now be achieved with only 2 volunteer firefighters. Between which homes in the town could this road have been added?

This new road may be longer and take a less direct route than the other roads on the map.

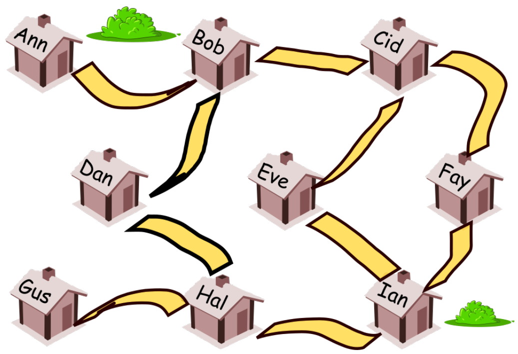
More Info:

Check the CEMC at Home webpage on Wednesday, May 13 for a solution to Volunteer Firefighters.

CEMC at Home

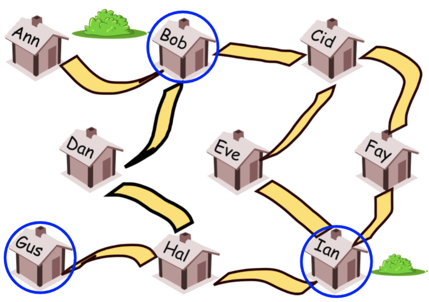
Grade 7/8 - Tuesday, May 12, 2020
Volunteer Firefighters - Solution

Set Up: The mayor's goal is to ensure that every home in the town is either the home of a volunteer or is connected by a single road to the home of a volunteer.



Problem 1: The mayor's goal outlined above can be achieved with only 3 volunteer firefighters. Find a group of 3 volunteer firefighters that will achieve the mayor's goal.

Solution: Here is one possible choice of the group of 3 firefighters.



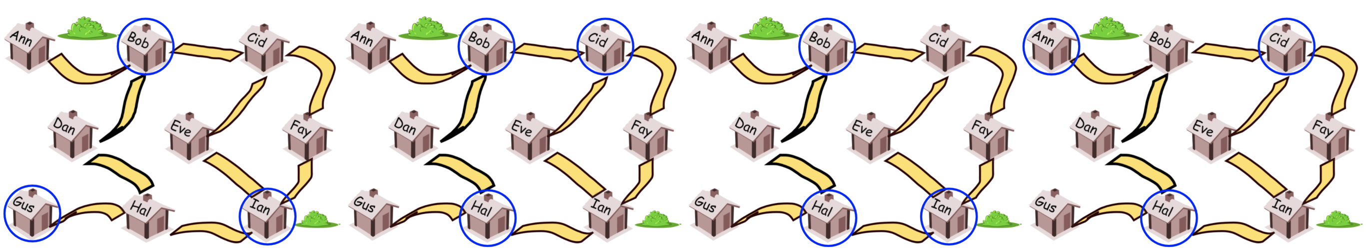
The mayor's goal is achieved with this group because

- Bob, Gus and Ian are firefighters,
- Ann, Dan and Cid are one road from Bob, and
- Eve, Fay and Hal are one road from Ian.

Problem 2: Find all possible groups of 3 volunteer firefighters that will achieve the mayor's goal. Explain how you know you have found them all.

Problem 3: Explain why it is not possible for the mayor's goal to be achieved if there are only 2 volunteer firefighters.

Solution: We will give solutions to Problem 2 and Problem 3 at the same time. First, there are four possible groups of 3 firefighters that will achieve the mayor's goal. They are shown below:



You can check for yourself that each of these four groups achieve the mayor's goal. Let's explain why there cannot be any other groups. (On our way to this, we will also explain why the goal cannot be achieved with only 2 firefighters.)

Ann and Gus have houses that have only one road leading from them. (Ann's road leads to Bob's home and Gus's road leads to Hal's home.) Since each of Ann and Gus needs to either be a firefighter or be next to a firefighter, we need to take one of Ann or Bob as well as one of Gus or Hal in our group of 3 firefighters. (This shows us we need at least 2 firefighters to achieve the goal.)

Notice that no matter what combination of the above is used (Ann and Gus or Ann and Hal or Bob and Gus or Bob and Hal), the 2 firefighters chosen cannot achieve the mayor's goal on their own. For example, Fay will not be a firefighter and will not be one road away from one. This tells us that there is no way to achieve the Mayor's goal with only 2 firefighters (Problem 3).

Now we need to consider all four combinations of Ann, Gus, Bob, and Hal outlined above, and see in how many ways these groups of 2 can be extended to groups of 3 achieving the goal.

Suppose we start with Bob and Hal as firefighters. We see that in this case, the only homes that are not yet connected to either Bob or Hal via a single road are Eve's and Fay's. To fix this using one more firefighter we can add either Cid or Ian. (Notice that adding Ann, Dan, Gus, Eve, or Fay will still leave us with at least one of Eve's and Fay's homes too far from a firefighter.)

This give us two possibilities for the group of 3:

Bob, Hal, and Cid
Bob, Hal, and Ian

Now suppose we start with Bob and Gus. We see that in this case, the only homes that are not yet connected to either Bob or Gus via a single road are Eve's, Fay's, and Ian's. To fix this by adding one more firefighter we *must* add Ian. (Notice that adding Ann, Dan, Hal, Eve, Cid, or Fay will still leave us with at least one home too far from a firefighter.)

This gives us one more possibility for the group of 3:

Bob, Gus, and Ian

If we start with Ann and Hal, then Cid, Eve, and Fay are not yet connected to the home of a volunteer firefighter. The only way to fix this by adding one more firefighter is to add Cid.

This gives us one more possibility for the group of 3:

Ann, Hal, and Cid

Finally, if we start with Ann and Gus, then Dan, Cid, Eve, Fay, and Ian are not connected to the home of a volunteer firefighter. We cannot fix this by adding only one more firefighter. If we add Bob, Dan, or Hal, then Eve and Fay stay disconnected; if we add Cid, Eve, Fay, or Ian, then Dan stays disconnected.

We have now covered all possibilities. Using this reasoning we can be sure that the four groups we have found must be the only groups of 3 that achieve the goal.

Extension: A new road is built between two of the homes in the town and the mayor's goal can now be achieved with only 2 volunteer firefighters. Between which homes in the town could this road have been added?

Solution:

- Having Bob and Ian as the 2 firefighters would leave out only Gus. This could be fixed by adding a road between Gus and Bob or Gus and Ian.
- Having Cid and Hal as the 2 firefighters would leave out only Ann. This could be fixed by adding a road between Ann and Cid or Ann and Hal.

CEMC at Home

Grade 7/8 - Thursday, May 21, 2020
Contest Day 3

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

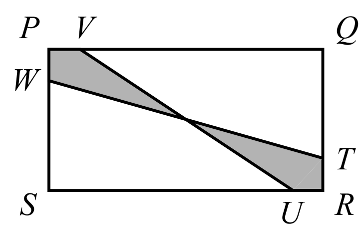
2011 Gauss Contest, #13

Five children had dinner. Chris ate more than Max. Brandon ate less than Kayla. Kayla ate less than Max but more than Tanya. Which child ate the second most?

(A) Brandon **(B)** Chris **(C)** Kayla **(D)** Max **(E)** Tanya

2020 Gauss Contest, #23

In the diagram, rectangle $PQRS$ has $PS = 2$ and $PQ = 4$. Points T, U, V, W are positioned so that $RT = RU = PW = PV = a$. If VU and WT pass through the centre of the rectangle, for what value of a is the shaded region $\frac{1}{8}$ the area of $PQRS$?



(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{5}$

(D) $\frac{1}{3}$ **(E)** $\frac{1}{4}$

More Info:

Check out the CEMC at Home webpage on Monday, May 25 for solutions to the Contest Day 3 problems.

CEMC at Home

Grade 7/8 - Thursday, May 21, 2020
Contest Day 3 - Solution

Solutions to the two contest problems are provided below.

2011 Gauss Contest, #13

Five children had dinner. Chris ate more than Max. Brandon ate less than Kayla. Kayla ate less than Max but more than Tanya. Which child ate the second most?

(A) Brandon **(B)** Chris **(C)** Kayla **(D)** Max **(E)** Tanya

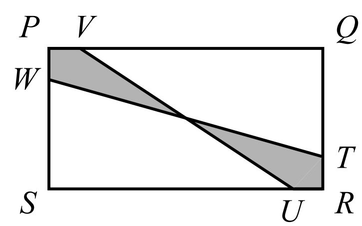
Solution:

Since Kayla ate less than Max and Chris ate more than Max, then Kayla ate less than Max who ate less than Chris.
Brandon and Tanya both ate less than Kayla.
Therefore, Max ate the second most.

ANSWER: (D)

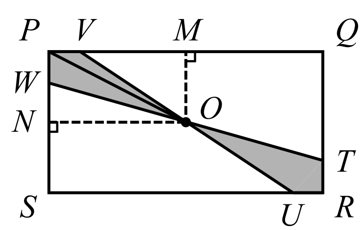
2020 Gauss Contest, #23

In the diagram, rectangle $PQRS$ has $PS = 2$ and $PQ = 4$. Points T, U, V, W are positioned so that $RT = RU = PW = PV = a$. If VU and WT pass through the centre of the rectangle, for what value of a is the shaded region $\frac{1}{8}$ the area of $PQRS$?



(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{5}$
(D) $\frac{1}{3}$ **(E)** $\frac{1}{4}$

Solution:



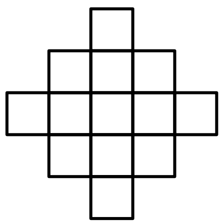
We begin by joining the centre of the rectangle, O, to vertex P.
We also draw OM perpendicular to side PQ and ON perpendicular to side PS.
Since O is the centre of the rectangle, then M is the midpoint of side PQ and so $PM = \frac{1}{2} \times 4 = 2$.
Similarly, N is the midpoint of PS and so $PN = \frac{1}{2} \times 2 = 1$.
$\triangle PVO$ has base $PV = a$ and height $OM = 1$, and so has area $\frac{1}{2} \times a \times 1 = \frac{1}{2}a$.
$\triangle PWO$ has base $PW = a$ and height $ON = 2$, and so has area $\frac{1}{2} \times a \times 2 = a$.
Thus, quadrilateral $PWOV$ has area equal to the sum of the areas of these two triangles, or $\frac{1}{2}a + a = \frac{3}{2}a$.
Similarly, we can show that quadrilateral $RTOU$ also has area $\frac{3}{2}a$ and so the total area of the shaded region is $2 \times \frac{3}{2}a = 3a$.
The area of rectangle $PQRS$ is $4 \times 2 = 8$ and since the area of the shaded region is $\frac{1}{8}$ the area of $PQRS$, then $3a = \frac{1}{8} \times 8$ or $3a = 1$ and so $a = \frac{1}{3}$.

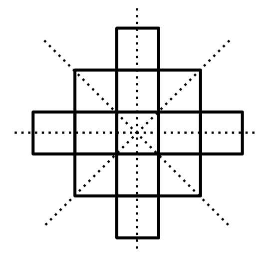
ANSWER: (D)

CEMC at Home

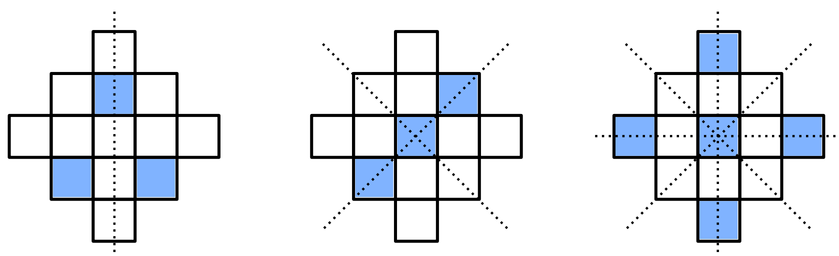
Grade 7/8 - Friday, May 22, 2020
Mirror, Mirror

Thirteen identical squares are arranged as shown in the figure below on the left. In today's activity, we will create symmetrical designs by shading in some of the squares in the figure. Notice that the design with no shaded squares has exactly four lines of symmetry, as shown in the image below on the right.





In each design below, some of the squares in the figure have been shaded. All lines of symmetry for each design are shown. Notice that the rightmost design maintains all four original lines of symmetry, but the other two designs only have one or two lines of symmetry.



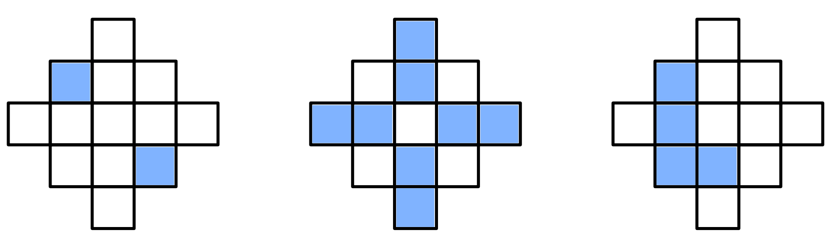
Problem 1: How many different designs are there with exactly two shaded squares that have exactly two lines of symmetry?

You can use the blank squares on the next page to draw some designs.

Problem 2: Is it possible to create a design that has exactly three lines of symmetry? If so, draw one. If not, explain why this is not possible.

You can shade as many squares in the figure as you would like.

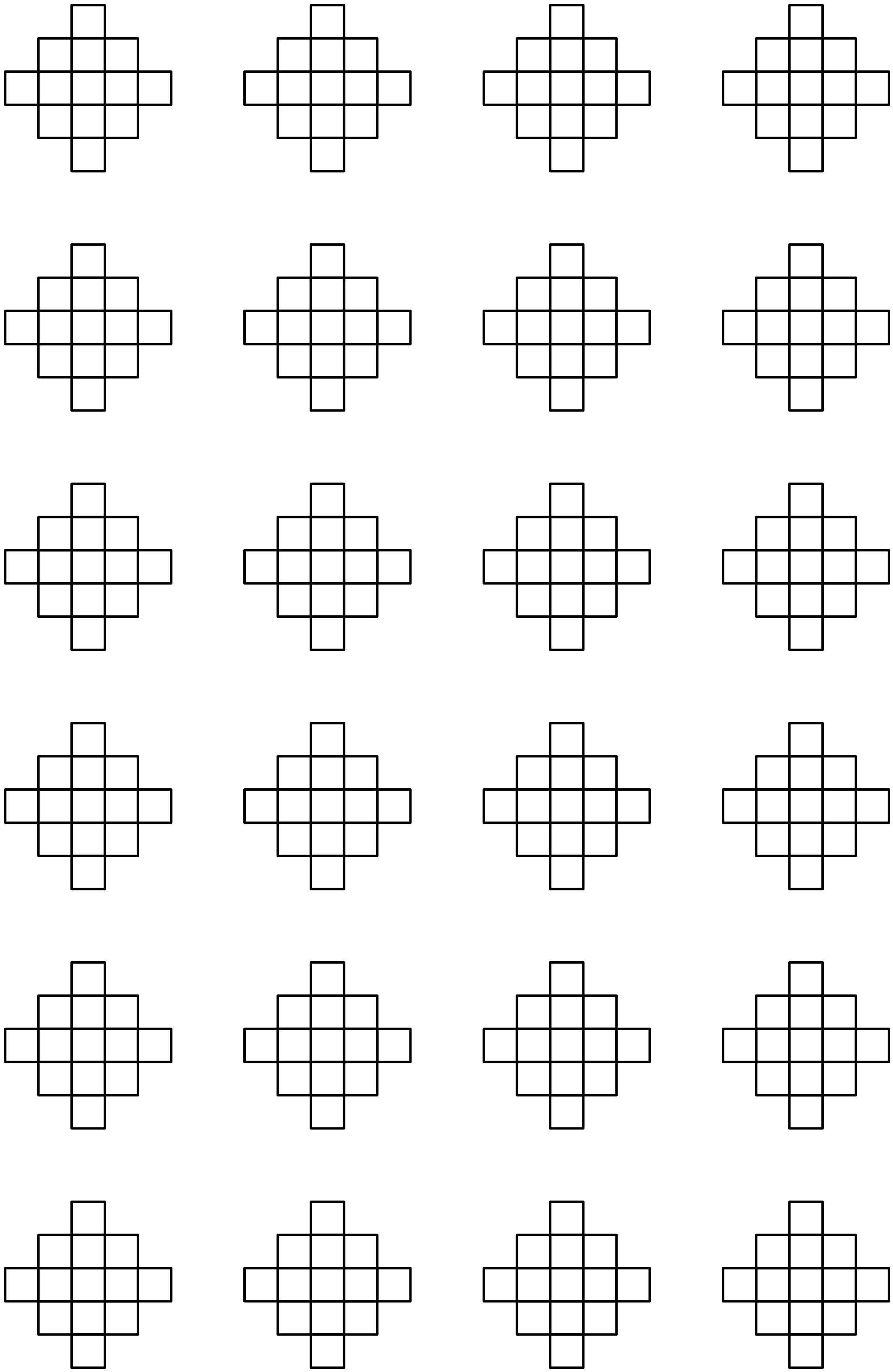
Problem 3: A design has rotational symmetry if we can rotate it about its centre less than a full turn and produce a design that looks identical to the original design. The first two designs below have rotational symmetry but the last design does not.



How many different designs are there with exactly three shaded squares that have rotational symmetry and at least one line of symmetry?

More Info:

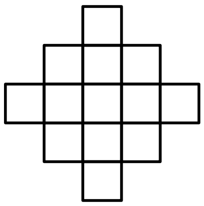
Check out the CEMC at Home webpage on Monday, May 25 for a solution to Mirror, Mirror.

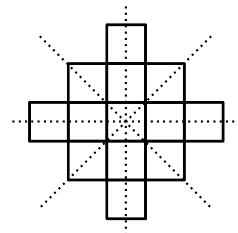


CEMC at Home

Grade 7/8 - Friday, May 22, 2020
Mirror, Mirror - Solution

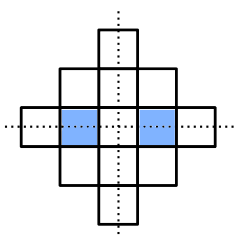
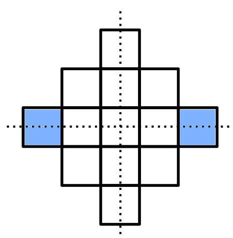
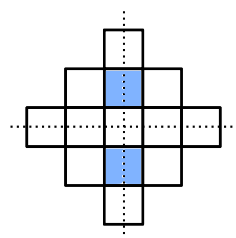
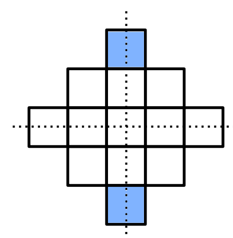
Thirteen identical squares are arranged as shown in the figure below on the left. Notice that the design with no shaded squares has exactly four lines of symmetry, as shown in the image below on the right.

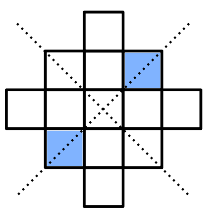
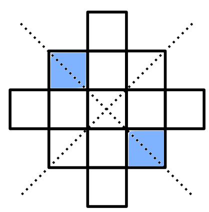




Problem 1: How many different designs are there with exactly two shaded squares that have exactly two lines of symmetry?

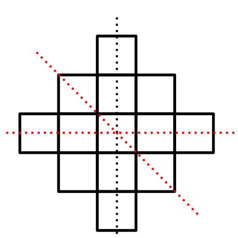
Solution: There are six different designs. They are shown below.

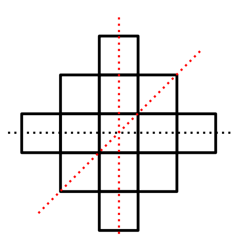



Problem 2: Is it possible to create a design that has exactly three lines of symmetry? If so, draw one. If not, explain why this is not possible.

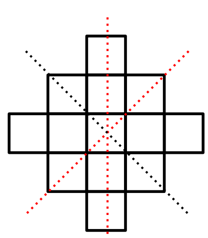
Solution: It is not possible. First, we notice that any group of three of the four lines of symmetry must include a diagonal line *and* a horizontal or vertical line. The four possible combinations of three lines of symmetry are shown below.



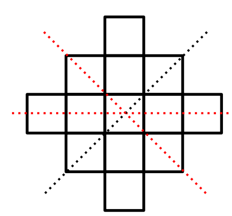
Group 1



Group 2



Group 3

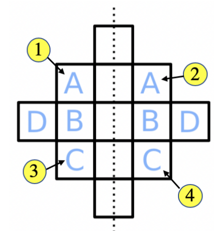


Group 4

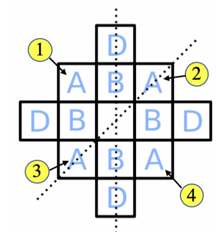
It turns out that any design that has a diagonal line of symmetry *and* a vertical or horizontal line of symmetry must actually have all four lines of symmetry. (This means it cannot be possible to create a design with exactly three lines of symmetry.)

Suppose we have a design that has the three lines of symmetry in Group 2. We will use letters to show the squares that must have the same shading, based on the lines of symmetry. For example, the squares marked with the letter "A" must be either all shaded, or all not shaded.

Since the design has the vertical line of symmetry, the design must have the symmetry indicated in the top image on the right.



Since the design *also* has the diagonal line of symmetry from the lower left corner to the upper right corner, the design must have the additional symmetry shown in the bottom image on the right.

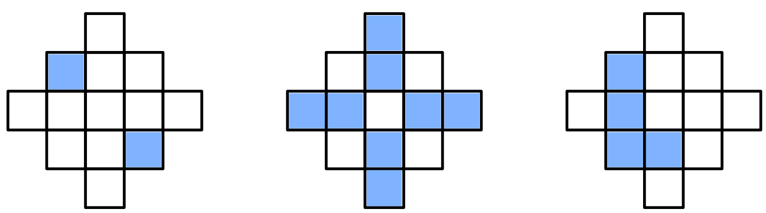


For example, the vertical line of symmetry tells us that the squares marked with 1 and 2 must look the same and that the squares marked with 3 and 4 must look the same. The diagonal line of symmetry tells us that the squares marked with 1 and 4 must look the same. Putting this together, we see that all four of these squares must look the same and so are marked with the same letter, A.

From this, we can see that the design actually has all four lines of symmetry.

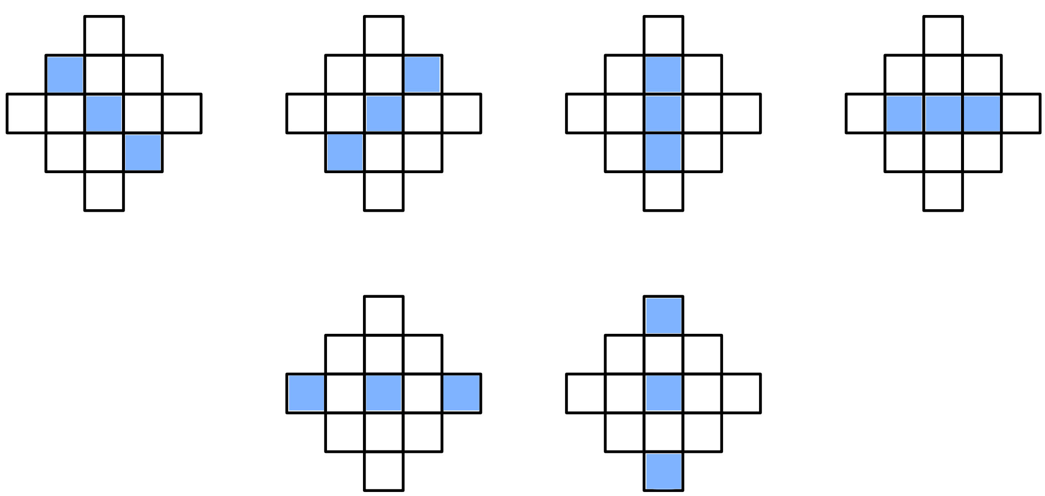
This means any design with these two lines of symmetry will actually have all four lines of symmetry. This argument works for Group 2 and Group 3. The argument for Group 1 and Group 4 is similar.

Problem 3: A design has rotational symmetry if we can rotate it about its centre less than a full turn and produce a design that looks identical to the original design. The first two designs below have rotational symmetry but the last design does not.



How many different designs are there with exactly three shaded squares that have rotational symmetry and at least one line of symmetry?

Solution: There are six different designs. They are shown below.



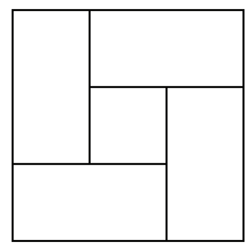
CEMC at Home

Grade 7/8 - Monday, May 25, 2020
Contest Day 4

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2014 Gauss Contest, #18

In the figure shown, the outer square has an area of $9\,\text{cm}^2$, the inner square has an area of $1\,\text{cm}^2$, and the four rectangles are identical. What is the perimeter of one of the four identical rectangles?



(A) 6 cm **(B)** 8 cm **(C)** 10 cm
(D) 9 cm **(E)** 7 cm

2020 Gauss Contest, #22

Celyna bought 300 grams of candy A for \$5.00, and x grams of candy B for \$7.00. She calculated that the average price of all of the candy that she purchased was \$1.50 per 100 grams. What is the value of x?

(A) 525 **(B)** 600 **(C)** 500 **(D)** 450 **(E)** 900

More Info:

Check out the CEMC at Home webpage on Monday, June 1 for solutions to the Contest Day 4 problems.

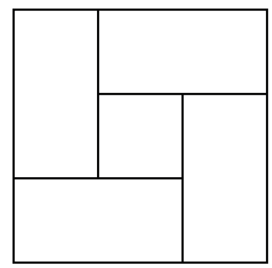
CEMC at Home

Grade 7/8 - Monday, May 25, 2020
Contest Day 4 - Solution

Solutions to the two contest problems are provided below, including a video for the second problem.

2014 Gauss Contest, #18

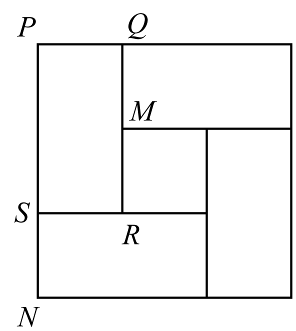
In the figure shown, the outer square has an area of 9 cm^2, the inner square has an area of 1 cm^2, and the four rectangles are identical. What is the perimeter of one of the four identical rectangles?



(A) 6 cm **(B)** 8 cm **(C)** 10 cm

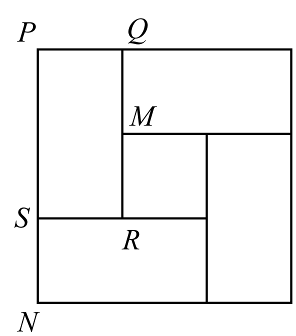
(D) 9 cm **(E)** 7 cm

Solution 1:



The outer square has an area of 9 cm^2, so the sides of this outer square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
The inner square has an area of 1 cm^2, so the sides of this inner square have length 1 cm (since $1 \times 1 = 1$), and thus $MR = 1$ cm.
Since $PN = 3$ cm, then $PS + SN = 3$ cm and so $QR + SN = 3$ cm (since $QR = PS$).
But $QR = QM + MR$, so then $QM + MR + SN = 3$ cm or $QM + 1 + SN = 3$ cm (since $MR = 1$ cm).
From this last equation we get $QM + SN = 2$ cm.
Since each of QM and SN is the width of an identical rectangle, then $QM = SN = 1$ cm.
Using $PS + SN = 3$ cm, we get $PS + 1 = 3$ cm and so $PS = 2$ cm.
Since the rectangles are identical, then $SN = PQ = 1$ cm.
The perimeter of rectangle $PQRS$ is $2 \times (PS + PQ) = 2 \times (2 + 1) = 2 \times 3 = 6$ cm.

Solution 2:



The outer square has an area of 9 cm^2, so the sides of this outer square have length 3 cm (since $3 \times 3 = 9$), and thus $PN = 3$ cm.
Since $PN = 3$ cm, then $PS + SN = 3$ cm.
Since each of PQ and SN is the width of an identical rectangle, then $PQ = SN$ and so $PS + SN = PS + PQ = 3$ cm.
The perimeter of $PQRS$ is $2 \times (PS + PQ) = 2 \times 3 = 6$ cm.

Answer: (A)

See the next page for a solution to the second contest problem.

2020 Gauss Contest, #22

Celyna bought 300 grams of candy A for \$5.00, and x grams of candy B for \$7.00. She calculated that the average price of all of the candy that she purchased was \$1.50 per 100 grams. What is the value of x?

(A) 525 **(B)** 600 **(C)** 500 **(D)** 450 **(E)** 900

Solution:

Celyna spent \$5.00 on candy A and \$7.00 on candy B, or \$12.00 in total.
The average price of all the candy that she purchased was \$1.50 per 100 grams.
This means that if Celyna bought 100 grams of candy, she would have spent \$1.50.
If she bought 200 grams of candy, she would have spent $2 \times \$1.50 = \$3.00$.
How many grams of candy would Celyna need to buy to spend \$12.00?
Since $8 \times \$1.50 = \$12.00$ (or $\$12.00 \div \$1.50 = 8$), then she would need to buy a total of 800 grams of candy.
Celyna bought 300 grams of candy A, and so she must have purchased $800 - 300 = 500$ grams of candy B. The value of x is 500.

Answer: (C)

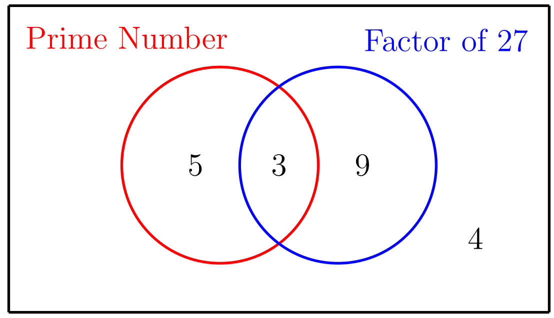
Video

Visit the following link to view three different approaches to solving the second contest problem: https://youtu.be/fWHAtLvCKtA.

CEMC at Home

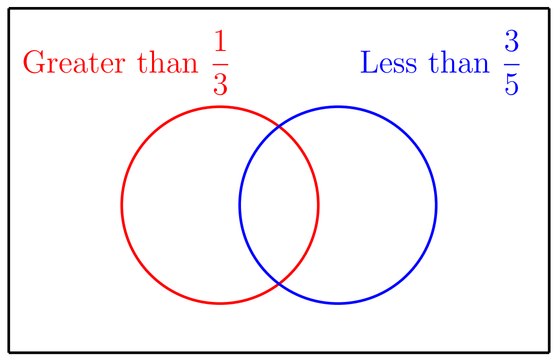
Grade 7/8 - Tuesday, May 26, 2020
Going in Circles

In the Venn diagram shown, the circle on the left contains prime numbers and the circle on the right contains factors of 27. The overlapping area in the middle, contained in both circles, contains prime numbers that are factors of 27. The area outside both circles contains numbers that are neither prime nor factors of 27. We have placed one positive integer in each of the four regions, but they are not the only numbers we could have chosen.



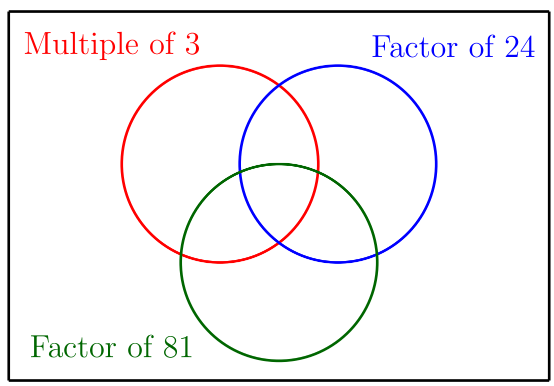
Problem 1

This Venn diagram has four regions. Place a fraction in as many of the regions as you can. Is it possible to find a fraction for each region?



Problem 2

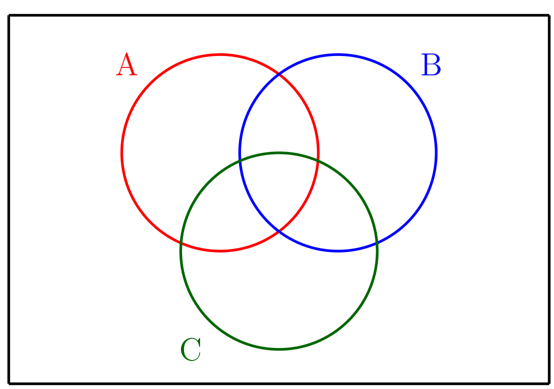
This Venn diagram has eight regions (seven regions "inside" at least one of the circles and one region "outside" all three circles). Place a positive integer in as many of the regions as you can. Is it possible to find a positive integer for each region?



Problem 3

This Venn diagram has eight regions. Place a positive three-digit integer in as many of the regions as you can. Is it possible to find a three-digit integer for each region?

A: 5 is a factor of the sum of the digits
B: The product of the digits is even
C: The mean of the digits is an integer

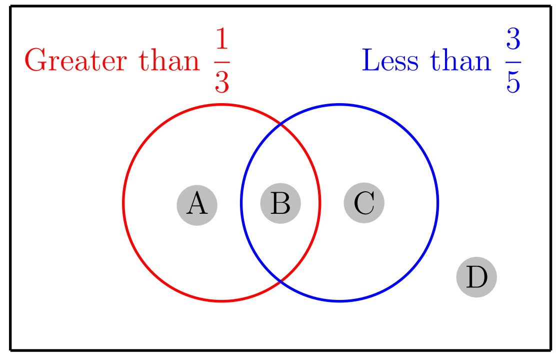


More Info:

Check the CEMC at Home webpage on Wednesday, May 27 for a solution to Going in Circles.

CEMC at Home

Grade 7/8 - Tuesday, May 26, 2020
Going in Circles - Solution

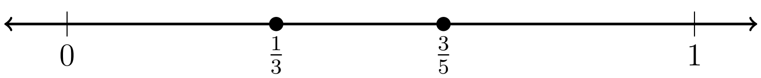


Problem 1

This Venn diagram has four regions. Place a fraction in as many of the regions as you can. Is it possible to find a fraction for each region?

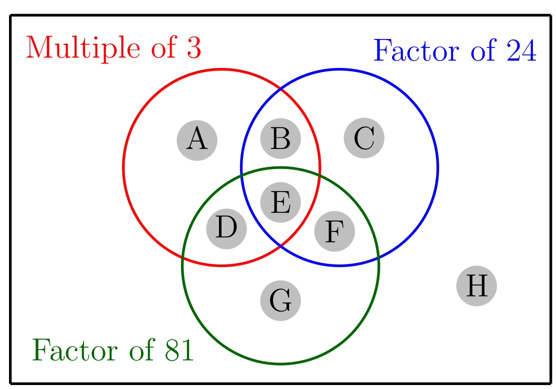
Solution:

We have marked the four regions A, B, C, and D.
We plot the fractions on a number line as a reference:



- Any fraction in Region A must be greater than $\frac{1}{3}$ and *not* less than $\frac{3}{5}$.
  This means the fraction must be greater than $\frac{1}{3}$ and greater than or equal to $\frac{3}{5}$.
  Some examples are $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{3}{4}$. (Any fraction greater than or equal to $\frac{3}{5}$ works.)
- Any fraction in Region B must be greater than $\frac{1}{3}$ and less than $\frac{3}{5}$.
  Some examples are $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{4}{7}$. (Any fraction between $\frac{1}{3}$ and $\frac{3}{5}$ works.)
- Any fraction in Region C must be less than $\frac{3}{5}$ and *not* greater than $\frac{1}{3}$.
  This means the fraction must be less than $\frac{3}{5}$ and less than or equal to $\frac{1}{3}$.
  Some examples are $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$. (Any fraction less than or equal to $\frac{1}{3}$ works.)
- Any fraction in Region D must *not* be greater than $\frac{1}{3}$ and *not* be less than $\frac{3}{5}$.
  This means the fraction must be less than or equal to $\frac{1}{3}$ and greater than or equal to $\frac{3}{5}$.
  It is not possible to find such a fraction and so this region must remain empty.

Therefore, we can place a fraction in three of the four regions.
For example, we could place $\frac{4}{5}$ in region A, $\frac{2}{5}$ in region B, $\frac{1}{5}$ in region C, and no fraction in region D.



Problem 2

This Venn diagram has eight regions (seven regions "inside" at least one of the circles and one region "outside" all three circles). Place a positive integer in as many of the regions as you can. Is it possible to find a positive integer for each region?

Solution:

It is helpful if we first write out the factors of 24 and 81.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 81: 1, 3, 9, 27, 81

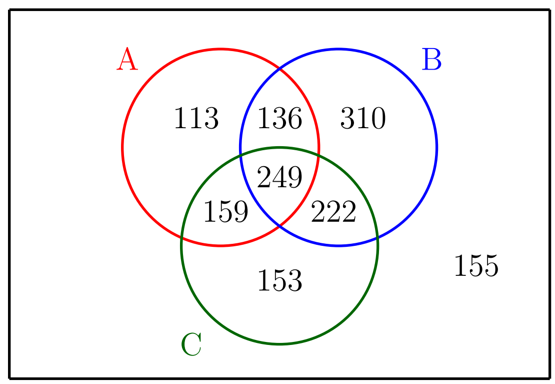
We have marked the eight regions A, B, C, D, E, F, G, and H. We can place a positive integer in each region except for region G.

- Any integer in Region A must be a multiple of 3 but not a factor of 24 and not a factor of 81. Some examples are 18, 21, and 30.
- Any integer in Region B must be a multiple of 3 and a factor of 24 but not a factor of 81. The only options are 6, 12, and 24.
- Any integer in Region C must be a factor of 24 but not a factor of 81 and not a multiple of 3. The only options are 2, 4, and 8.
- Any integer in Region D must be a multiple of 3 and a factor of 81 but not a factor of 24. The only options are 9, 27, and 81.
- Any integer in Region E must be a multiple of 3 and a factor of both 24 and 81. The only option is 3.
- Any integer in Region F must be a factor of both 24 and 81 but not a multiple of 3. The only option is 1.
- Any integer in Region G must be a factor of 81 but not a factor of 24 and not a multiple of 3. It is not possible to find such an integer, so this region must remain empty.
- Any integer in Region H must not be multiple of 3, not be a factor of 24, and not be a factor of 81. Some examples are 5, 7, and 10.

Problem 3

This Venn diagram has eight regions. Place a positive three-digit integer in as many of the regions as you can. Is it possible to find a three-digit integer for each region?

A: 5 is a factor of the sum of the digits
B: The product of the digits is even
C: The mean of the digits is an integer



Solution:

Positive three-digit integers have been placed in the regions in the diagram above. It is possible to place a number in each of the eight regions, and there are other choices you could have made.

There are many ways to go about finding these numbers. You can choose three-digit numbers randomly and then test them to see in which region they belong, hoping to eventually find one for every region, or you can try to reason what digits in each of the regions must look like.

For example, you can note that the product of the digits of an integer is even exactly when the number has at least one even digit. Because of this we know that any number placed within the circle marked B must have at least one even digit, and every number placed outside of this circle must have three odd digits.

We can further note that three odd digits must have an odd sum. So if a number is *outside* circle B but *inside* circle A, then it must have three odd digits that add to an odd multiple of 5. (In fact, they must add to either 5 or 15. Can you see why?) The numbers 113 and 159 both have this property and exactly one of them has the additional property that the mean of its digits is equal to an integer. (The mean of the digits of 113 is $\frac{1+1+3}{3} = \frac{5}{3}$, which is not an integer, and the mean of the digits of 159 is $\frac{1+5+9}{3} = \frac{15}{3} = 5$, which is an integer.)

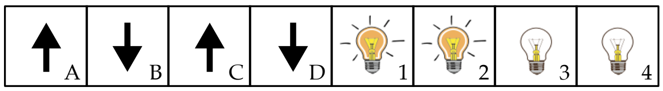
A combination of reasoning and some trial and error is a good approach for this problem!

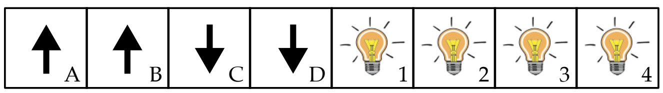
CEMC at Home

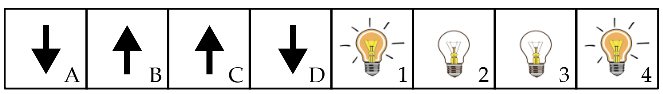
Grade 7/8 - Wednesday, May 27, 2020
Sneaky Switches

As a practical joke, Rachel connected light bulbs to switches so that each switch operates exactly one light bulb but nobody knows which one. Each switch can be either up or down, but we do not know which position corresponds to the connected bulb being on and which position corresponds to the connected bulb being off. To make matters worse, this could be different for different switches.

Problem 1: Rachel connected four switches (marked A, B, C, and D) to four light bulbs (numbered 1, 2, 3, and 4). Three experiments were conducted to determine which switch is connected to which light bulb. The position of each switch and the on/off status of each light bulb in each of the experiments is shown below. Which switch is connected to which light bulb?

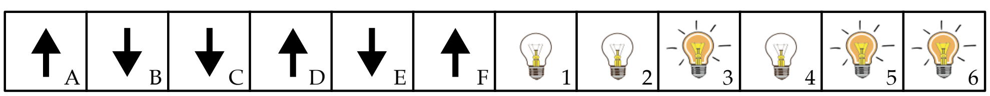
Experiment 1:


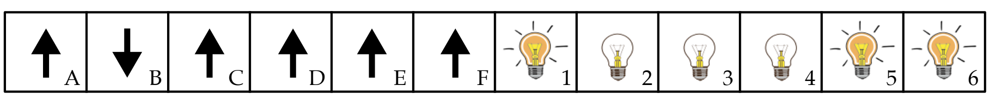
Experiment 2:


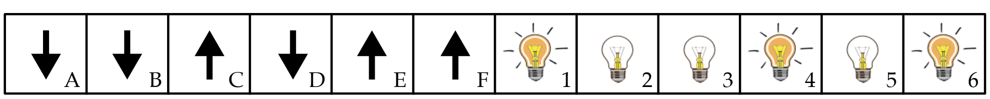
Experiment 3:


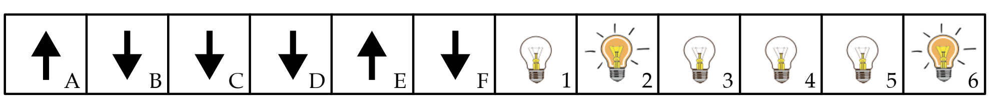
To start, can you determine which switch must be connected to bulb 1?

Problem 2: Rachel connected six switches (marked A, B, C, D, E, and F) to six light bulbs (numbered 1, 2, 3, 4, 5, and 6). Four experiments were conducted to determine which switch is connected to which light bulb. The position of each switch and the on/off status of each light bulb in each of the experiments is shown below. Which switch is connected to which light bulb?

Experiment 1:


Experiment 2:


Experiment 3:


Experiment 4:


Extension: For each problem above, replace the last experiment with a different experiment that could have been used instead of the one given, and would have still allowed you to solve the problem.

More Info:

Check out the CEMC at Home webpage on Thursday, May 28 for a solution to Sneaky Switches.

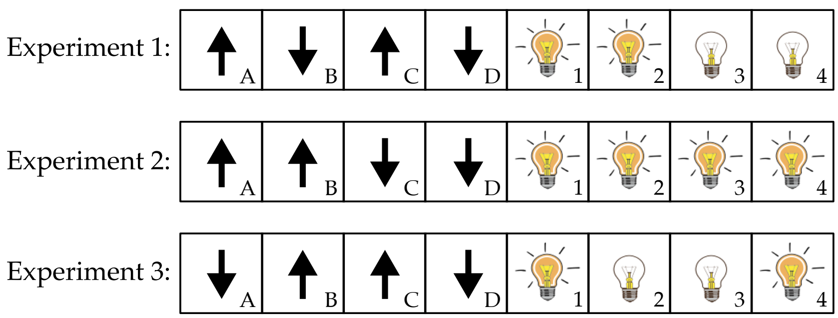
A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

CEMC at Home

Grade 7/8 - Wednesday, May 27, 2020
Sneaky Switches - Solution

Set Up: Rachel connected light bulbs to switches so that each switch operates exactly one light bulb but nobody knows which one. Each switch can be either up or down, but we do not know which position corresponds to the connected bulb being on and which position corresponds to the connected bulb being off. To make matters worse, this could be different for different switches.

Problem 1 Summary: Rachel connected four switches (marked A, B, C, and D) to four light bulbs (numbered 1, 2, 3, and 4). Three experiments were conducted and the results are shown below. Which switch is connected to which light bulb?



Solution:

The only switch that is in the same position in all three experiments is switch D. The only light bulb that has the same on/off status in all three experiments is light bulb 1. This means switch D must be connected to light bulb 1.

Between the first and second experiments, the only switches that change positions are B and C and the only lights that change on/off status are 3 and 4. This means that switches B and C must be connected to light bulbs 3 and 4, in some order.

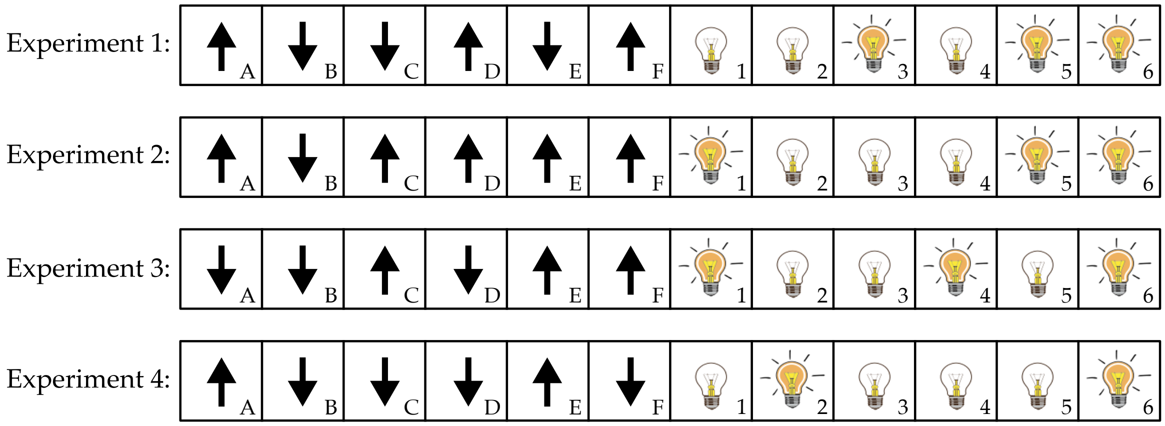
Between the second and third experiments, the only switches that change positions are A and C and the only lights that change on/off status are 2 and 3. This means that switches A and C must be connected to light bulbs 2 and 3, in some order.

Using the two observations above, we can conclude that switch C must be connected to light bulb 3. It follows that switch B is connected to light bulb 4 and switch A is connected to light bulb 2.

In summary, the switches and light bulbs are connected as follows.

Switch	A	B	C	D
Light Bulb	2	4	3	1

Problem 2 Summary: Rachel connected six switches (marked A, B, C, D, E, and F) to six light bulbs (numbered 1, 2, 3, 4, 5, and 6). Four experiments were conducted and the results are shown below. Which switch is connected to which light bulb?



Solution:

Between the first and second experiments, the only switches that change positions are C and E, and the only lights that change on/off status are 1 and 3. This means that switches C and E must be connected to light bulbs 1 and 3, in some order. This is our first observation.

Between the second and third experiments, the only switches that change positions are A and D, and the only lights that change on/off status are 4 and 5. This means that switches A and D must be connected to light bulbs 4 and 5, in some order. This is our second observation.

Between the third and fourth experiments, the only switches that change positions are A, C, and F, and the only lights that change on/off status are 1, 2, and 4. This means that switches A, C, and F must be connected to light bulbs 1, 2, and 4, in some order. This is our third observation.

We know from our first observation that switch C is connected to either light bulb 1 or light bulb 3. Using this along with our third observation, we can conclude that switch C must be connected to light bulb 1. It follows that switch E is connected to light bulb 3. Similarly, using our second observation with our third observation, we can conclude that switch A must be connected to light bulb 4. It follows that switch D is connected to light bulb 5 and switch F is connected to light bulb 2.

We are now left with switch B and light bulb 6, which must be connected. We can see that switch B is in the same position during all four experiments and that light bulb 6 is "on" during all four experiments, and so this is indeed a correct match. (Note that we could have started our solution by observing that switch B must be connected to light bulb 6.)

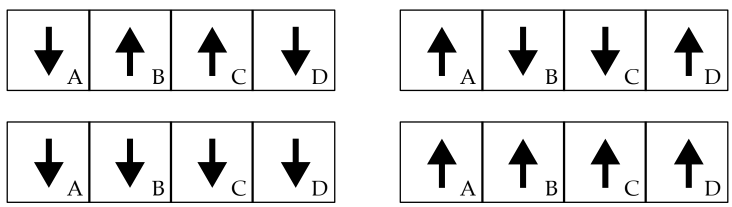
In summary, the switches and light bulbs are connected as follows.

Switch	A	B	C	D	E	F
Light Bulb	4	6	1	5	3	2

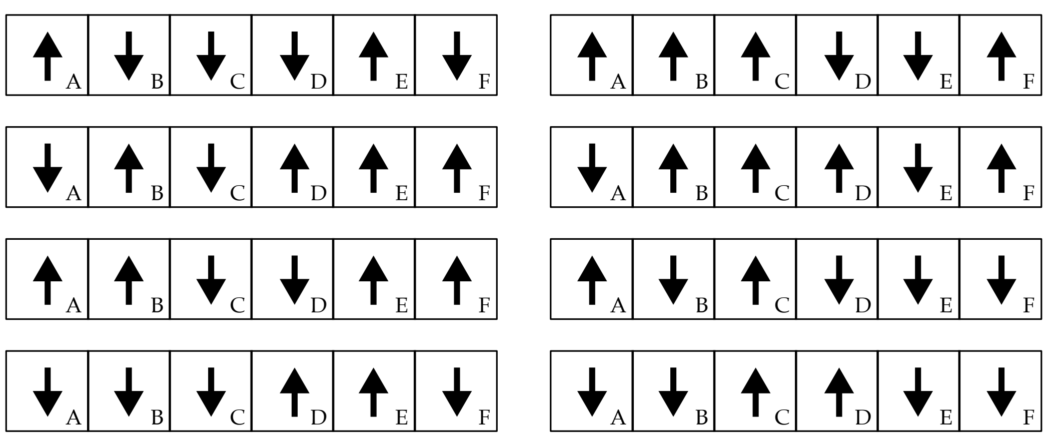
Extension: For each problem above, replace the last experiment with a different experiment that could have been used instead of the one given, and would have still allowed you to solve the problem.

Solution:

In Problem 1, without the final experiment you can determine that switches B and C are connected to light bulbs 3 and 4 (in some order), and thus, switches A and D must be connected to light bulbs 1 and 2 (in some order). So a final experiment needs to distinguish between switches A and D, and switches B and C. This can be achieved by changing two switches from the second experiment: switch A or D and switch B or C. There are four ways to do this, and they are shown below. Notice that the top left experiment shown is actually "Experiment 3" from the original problem. This experiment can be replaced with any of the other three experiments given below.



In Problem 2, without the final experiment you can determine that switches C and E are connected to light bulbs 1 and 3 (in some order), switches A and D are connected to light bulbs 4 and 5 (in some order), and switches B and F are connected to light bulbs 2 and 6 (in some order). So the final experiment needs to distinguish between switches C and E, switches A and D, and switches B and F. This can be achieved by changing three switches from the third experiment: switch C or E, switch A or D, and switch B or F. There are eight ways to do this, and they are shown below. Notice that the top left experiment shown is actually "Experiment 4" from the original problem. This experiment can be replaced with any of the other seven experiments given below.



CEMC at Home

Grade 7/8 - Thursday, May 28, 2020

Just Your Average Sequence

In a sequence of six numbers, every number after the first two is the average of the previous two numbers.

The 4^{th} number in the sequence is 22 and the 6^{th} number in the sequence is 45.

Determine all six numbers in the sequence.

?	?	?	22	?	45

More Info:

Check out the CEMC at Home webpage on Friday, May 29 for two different solutions to Just Your Average Sequence.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 7/8 - Thursday, May 28, 2020
Just Your Average Sequence - Solution

Problem:

In a sequence of six numbers, every number after the first two is the average of the previous two numbers.

The 4^{th} number in the sequence is 22 and the 6^{th} number in the sequence is 45.

Determine all six numbers in the sequence.

?	?	?	22	?	45

Solution:

If x is the average of two numbers y and z, then $\dfrac{y+z}{2} = x$.

It follows that $y + z = 2 \times x$. This idea is used in both Solution 1 and Solution 2 below.

Solution 1

In this solution, we solve the problem by working backwards.

Since the 6^{th} number in the sequence is equal to the average of the two previous numbers, the 6^{th} number must be the average of the 4^{th} and 5^{th} numbers.
So the sum of the 4^{th} and 5^{th} numbers must be 2 times the 6^{th} number, or $2 \times 45 = 90$. Therefore, the 5^{th} number is $90 - 22 = 68$.

We now determine the 3^{rd} number. The 5^{th} number in the sequence is the average of the 3^{rd} and 4^{th} numbers. So the sum of the 3^{rd} and 4^{th} numbers is 2 times the 5^{th} number, or $2 \times 68 = 136$. Therefore, the 3^{rd} number is $136 - 22 = 114$.

We now determine the 2^{nd} number. The 4^{th} number in the sequence is the average of the 2^{nd} and 3^{rd} numbers. So the sum of the 2^{nd} and 3^{rd} numbers is 2 times the 4^{th} number, or $2 \times 22 = 44$. Therefore, the 2^{nd} number is $44 - 114 = -70$.

We now determine the 1^{st} number. The 3^{rd} number in the sequence is the average of the 1^{st} and 2^{nd} numbers. So the sum of the 1^{st} and 2^{nd} numbers is 2 times the 3rd number, or $2 \times 114 = 228$. Therefore, the 1^{st} number is $228 - (-70) = 228 + 70 = 298$.

The sequence of six numbers is $298, -70,\ 114,\ 22,\ 68,\ 45$.

We can indeed check that in this sequence each number after the first two is equal to the average of the previous two numbers.

Solution 2

We will now present a similar, but more algebraic solution.

Let the sequence be

a	b	c	22	d	45

where a represents the 1st number, b represents the 2nd number, c represents the 3rd number and d represents the 5th number in the sequence.

We again solve this problem by working backwards.

Since the 6th number in the sequence is equal to the average of the 4th and 5th numbers, we have $45 = \dfrac{22+d}{2}$.
Multiplying both sides by 2, we obtain $22 + d = 45 \times 2 = 90$. Rearranging, $d = 90 - 22 = 68$.
Therefore, the 5th number in the sequence is 68.

We now determine the 3rd number. Since the 5th number in the sequence is equal to the average of the 3rd and 4th numbers, we have $68 = \dfrac{c+22}{2}$.
Multiplying both sides by 2, we obtain $c + 22 = 68 \times 2 = 136$. Rearranging, $c = 136 - 22 = 114$.
Therefore, the 3rd number in the sequence is 114.

We now determine the 2nd number. Since the 4th number in the sequence is equal to the average of the 2nd and 3rd numbers, we have $22 = \dfrac{b+114}{2}$.
Multiplying both sides by 2, we obtain $b + 114 = 22 \times 2 = 44$. Rearranging, $b = 44 - 114 = -70$.
Therefore, the 2nd number in the sequence is -70.

We now determine the 1st number. Since the 3rd number in the sequence is equal to the average of the 1st and 2nd numbers, we have $114 = \dfrac{a+(-70)}{2}$.
Multiplying both sides by 2, we obtain $a + (-70) = 114 \times 2 = 228$. Rearranging, $a = 228 + 70 = 298$.
Therefore, the 1st number in the sequence is 298.

Therefore, the sequence of six numbers is $298, -70, 114, 22, 68, 45$.

We can indeed check that in this sequence each number after the first two is equal to the average of the previous two numbers.

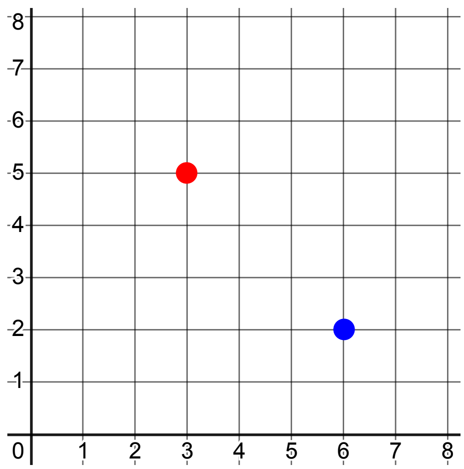
298	-70	114	22	68	45

CEMC at Home

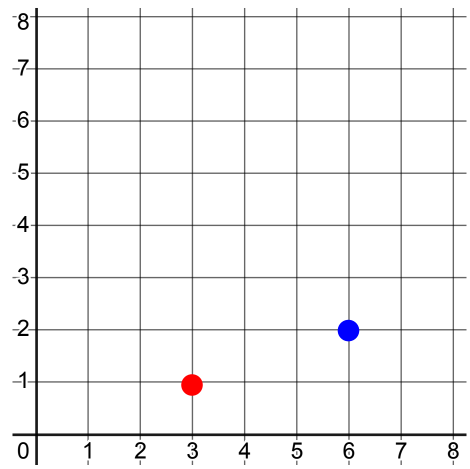
Grade 7/8 - Friday, May 29, 2020
Gridiron Expert

In the activities given below, you will be making different types of shapes by plotting points on a grid.

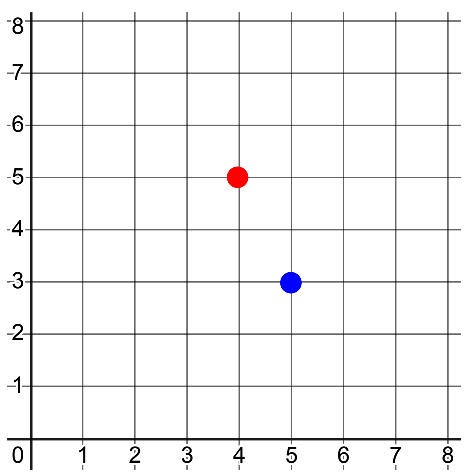
Activity 1: The points $(3, 5)$ and $(6, 2)$ are plotted on the grid below. Plot a third point on the grid so that the three points are the vertices of a right-angled triangle.



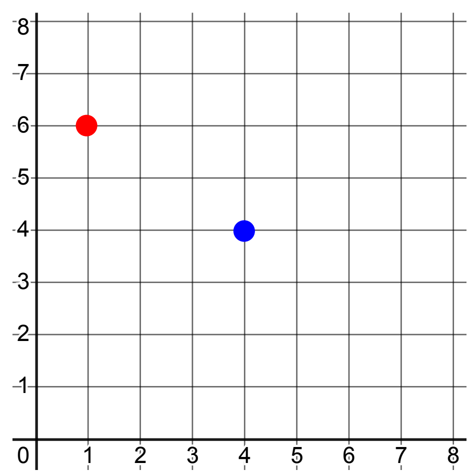
Activity 2: The points $(3, 1)$ and $(6, 2)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a rectangle that is not a square.



Activity 3: The points $(4, 5)$ and $(5, 3)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a quadrilateral with all four sides equal in length.



Activity 4: The points $(1, 6)$ and $(4, 4)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a trapezoid that is not a parallelogram.



There is more than one way to construct the shape described in each of the activities above. Spend some time thinking about how many different ways you could plot the points in each activity. Can you explain why you have the right type of shape each time?

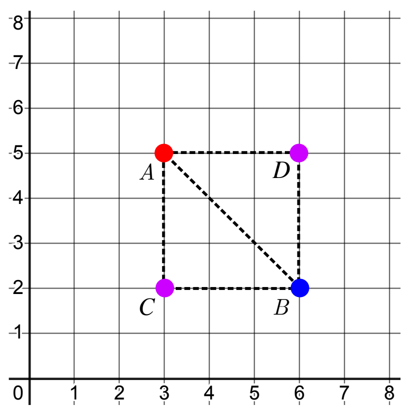
More Info: Check the CEMC at Home webpage on Monday, June 1 for a solution to Gridiron Expert.

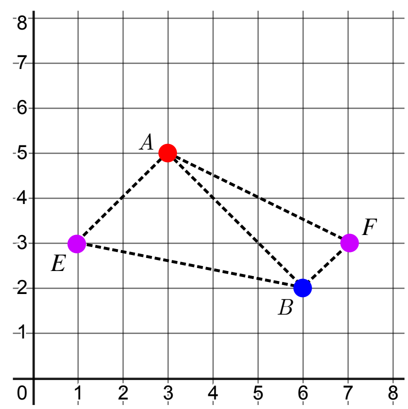
CEMC at Home

Grade 7/8 - Friday, May 29, 2020
Gridiron Expert - Solution

Activity 1: The points $(3, 5)$ and $(6, 2)$ are plotted on the grid. Plot a third point on the grid so that the three points are the vertices of a right-angled triangle.

Solution: We label the original points as A and B as shown. You can plot the third point in several different ways. Four different ways are shown below.





If you plot the third point $C(3, 2)$, then $\triangle ACB$ is a right-angled triangle. Since line segment AC is vertical and line segment CB is horizontal, these line segments are perpendicular and so there is a right angle at C.

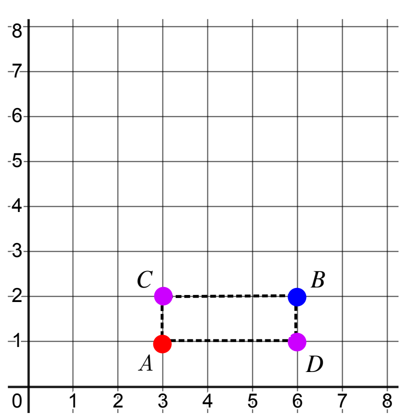
If you instead plot the third point $D(6, 5)$, then you get $\triangle ADB$ with a right angle at D.

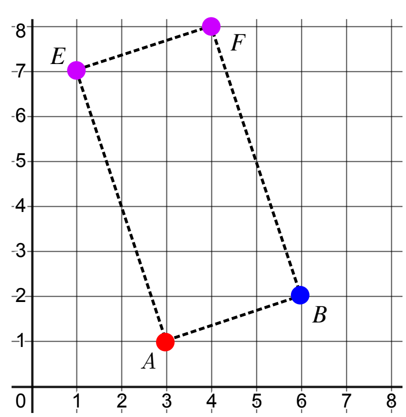
If you plot the third point $E(1, 3)$, then $\triangle EAB$ has right angle $\angle EAB$. If you use a protractor to measure this angle, then you will see that the measure of the angle is around 90°. Do you know how to justify that the line segments EA and AB are indeed perpendicular by just looking at the grid? (Can you explain why $\angle EAB$ is made up of two 45° angles? Think about the diagonals of a square.)

If you plot the third point $F(7, 3)$, then $\triangle ABF$ has right angle $\angle ABF$. Can you explain why?

Activity 2: The points $(3, 1)$ and $(6, 2)$ are plotted on the grid. Plot two more points on the grid so that the four points are the vertices of a rectangle that is not a square.

Solution: We label the original points as A and B as shown. You can plot the two additional points in several different ways. Two different ways are shown below.



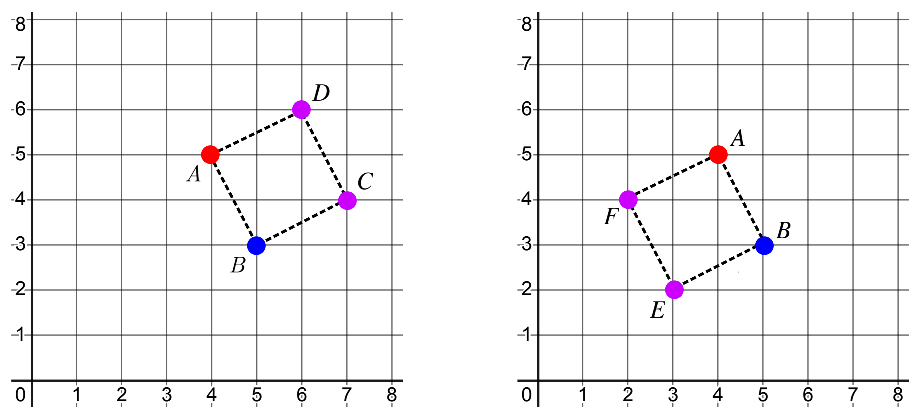


If you plot the two additional points $C(3, 2)$ and $D(6, 1)$, then quadrilateral $ACBD$ is a rectangle, but not a square. Since opposite sides AC and BD are vertical, and opposite sides CB and AD are horizontal, we know that the shape has four right angles. We can also see that the vertical sides have length 1 unit, and the horizontal sides have length 3 units, and so opposite sides are equal in length, but not all sides are equal in length.

If you instead plot the two additional points $E(1, 7)$ and $F(4, 8)$, then quadrilateral $ABFE$ is also a rectangle, but not a square. Since opposite sides AB and EF are diagonals of identical 1×3 rectangles, they must be equal in length. Since opposite sides AE and BF are diagonals of identical 2×6 rectangles, they must be equal in length as well. But, we can see that not all four sides are equal in length. (In fact, side AE is twice the length of side AB. Can you see why?) If you use a protractor to measure the four angles, then you will see that the measure of each angle is around 90°. Do you know how to justify, for example, that the line segments EA and AB are indeed perpendicular by just looking at the grid?

Activity 3: The points $(4, 5)$ and $(5, 3)$ are plotted on the grid. Plot two more points on the grid so that the four points are the vertices of a quadrilateral with all four sides equal in length.

Solution: We label the original points as A and B as shown. You can plot the two additional points in more than one way. Two different ways are shown below.



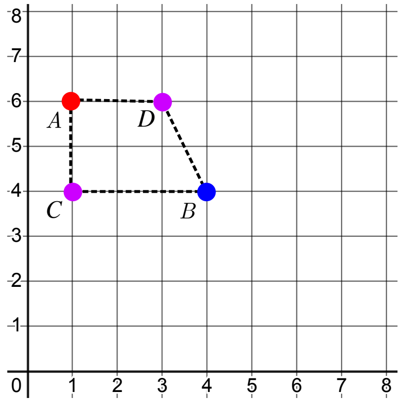
If you plot the two additional points $C(7, 4)$ and $D(6, 6)$, then quadrilateral $ABCD$ has all four sides equal in length. We know that sides AB, BC, CD, and DA are all equal in length because they are all diagonals of identical rectangles. (Each rectangle is 1×2, but they are not all oriented in the same way.)

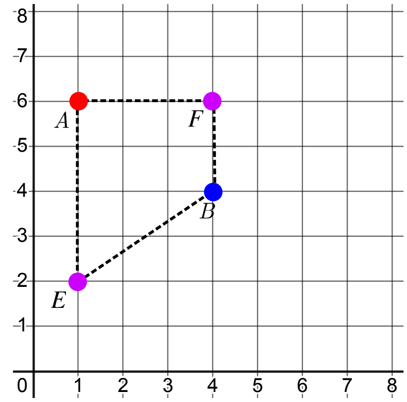
You get a similar situation if you instead plot the two additional points $E(3, 2)$ and $F(2, 4)$ to form quadrilateral $ABEF$.

A quadrilateral with all four sides equal in length is called a rhombus. In addition, quadrilaterals $ABCD$ and $ABEF$ each have four right angles and so they are actually squares. Can you see why?

Activity 4: The points $(1, 6)$ and $(4, 4)$ are plotted on the grid below. Plot two more points on the grid so that the four points are the vertices of a trapezoid that is not a parallelogram.

Solution: We label the original points as A and B as shown. You can plot the two additional points in several different ways. Two different ways are shown below.





If you plot the two additional points $C(1,4)$ and $D(3,6)$, then quadrilateral $ACBD$ is a trapezoid that is not a parallelogram. Since sides AD and CB are horizontal, these opposite sides are parallel, which means $ACBD$ is a trapezoid. Since AC is vertical but DB is not, these opposite sides are not parallel, which means $ACBD$ is not a parallelogram.

You get a similar situation if you plot the two additional points $E(1,2)$ and $F(4,6)$ to form quadrilateral $ABEF$.

Can you find some other ways to plot the two additional points in this activity? In particular, try to plot some trapezoids that have line segment AB as a side.

CEMC at Home

Grade 7/8 - Monday, June 1, 2020
Contest Day 5

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2016 Gauss Contest, #16

Each of $\square$, $\triangle$ and $\blacklozenge$ represents a non-zero number. If $\square = \triangle + \triangle + \triangle$ and $\square = \blacklozenge + \blacklozenge$, then $\square + \blacklozenge + \triangle$ equals

(A) $\square + \triangle$

(B) $\blacklozenge + \triangle + \triangle + \triangle + \triangle$

(C) $\blacklozenge + \blacklozenge + \square$

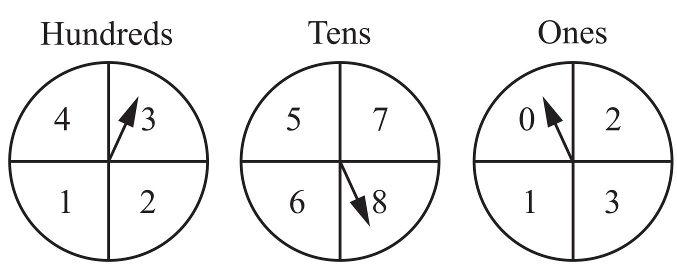
(D) $\triangle + \triangle + \triangle + \blacklozenge + \blacklozenge$

(E) $\blacklozenge + \blacklozenge + \blacklozenge + \triangle + \triangle$

2020 Gauss Contest, #22

Three spinners are shown below. The spinners are used to determine the hundreds, tens and ones digits of a three-digit number. How many possible three-digit numbers that can be formed in this way are divisible by 6?

(A) 11 **(B)** 16 **(C)** 22 **(D)** 12 **(E)** 9



More Info:

Check out the CEMC at Home webpage on Monday, June 8 for solutions to the Contest Day 5 problems.

CEMC at Home

Grade 7/8 - Monday, June 1, 2020
Contest Day 5 - Solution

Solutions to the two contest problems are provided below.

2016 Gauss Contest, #16

Each of $\square$, $\triangle$ and $\blacklozenge$ represents a non-zero number. If $\square = \triangle + \triangle + \triangle$ and $\square = \blacklozenge + \blacklozenge$, then $\square + \blacklozenge + \triangle$ equals

(A) $\square + \triangle$

(B) $\blacklozenge + \triangle + \triangle + \triangle + \triangle$

(C) $\blacklozenge + \blacklozenge + \square$

(D) $\triangle + \triangle + \triangle + \blacklozenge + \blacklozenge$

(E) $\blacklozenge + \blacklozenge + \blacklozenge + \triangle + \triangle$

Solution:

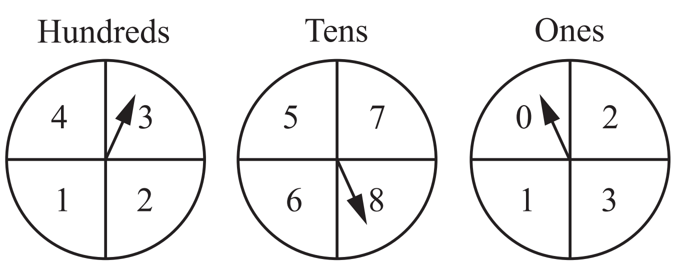
Since $\square = \triangle + \triangle + \triangle$, then by adding a $\blacklozenge$ to each side we get $\square + \blacklozenge = \blacklozenge + \triangle + \triangle + \triangle$.
Since $\square + \blacklozenge = \blacklozenge + \triangle + \triangle + \triangle$, then by adding a $\triangle$ to each side we get that $\square + \blacklozenge + \triangle = \blacklozenge + \triangle + \triangle + \triangle + \triangle$.
(Can you explain why each of the other answers is not equal to $\square + \blacklozenge + \triangle$?)

ANSWER: (B)

2020 Gauss Contest, #22

Three spinners are shown below. The spinners are used to determine the hundreds, tens and ones digits of a three-digit number. How many possible three-digit numbers that can be formed in this way are divisible by 6?

(A) 11 **(B)** 16 **(C)** 22 **(D)** 12 **(E)** 9



See the next page for a solution to the second contest problem.

Solution:

A number is divisible by 6 if it is divisible by both 2 and 3.
To be divisible by 2, the three-digit number that is formed must be even and so the ones digit must be 0 or 2.
To be divisible by 3, the sum of the digits of the number must be a multiple of 3.
Consider the possible tens and hundreds digits when the ones digit is 0.
In this case, the sum of the tens and hundreds digits must be a multiple of 3 (since the ones digit does not add anything to the sum of the digits).
We determine the possible sums of the tens and hundreds digits in the table below.
The sums which are a multiple of 3 are circled.

The Tens Digit

The Hundreds Digit

100s \ 10s	5	6	7	8
1	6	7	8	9
2	7	8	9	10
3	8	9	10	11
4	9	10	11	12

When the ones digit is 0, the possible three-digit numbers are: $150, 180, 270, 360, 450$, and 480.
Consider the possible tens and hundreds digits when the ones digit is 2.
In this case, the sum of the tens and hundreds digits must be 2 less than a multiple of 3 (since the ones digit adds 2 to the sum of the digits).
When the ones digit is 2, the possible three-digit numbers are: $162, 252, 282, 372$, and 462.
The number of three-digit numbers that can be formed that are divisible by 6 is 11.

ANSWER: (A)

CEMC at Home

Grade 7/8 - Tuesday, June 2, 2020
Famous Mathematicians

Throughout human history, many mathematicians have made significant contributions to the subject. These important historical figures often lead fascinating lives filled with interesting stories. Four of these mathematicians are listed below.

Pythagoras	He was a Greek mathematician and philosopher who is best known for discovering the Pythagorean theorem.
Sophie Germain	She was a French mathematician who grew up at a time when women were not allowed to attend universities. Her best known work is in number theory and applied mathematics.
Carl Gauss	At an early age, he added up the numbers from 1 to 100 in only a few seconds. He contributed to many fields in mathematics including algebra, number theory, and geometry.
Ada Lovelace	She contributed significantly to the field of computer science. She is also considered by many to be the first computer scientist to write a computer program.

Choose two of these four mathematicians and for each one you choose:

1. Look up information about the mathematician online. Find a new fact about them that you find interesting and share what you find with friends or family.
2. Are there any mathematical words or ideas connected to this mathematician that sound familiar to you? Try to write down three to five of these words.
3. If you had the chance to go back in time and meet this mathematician, what question would you ask them?

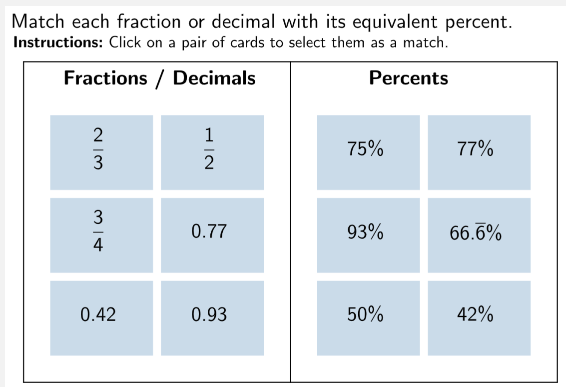
More Info: The CEMC Gauss Math Contest is named in honour of Carl Gauss.

CEMC at Home

Grade 7/8 - Wednesday, June 3, 2020
Interact with Mathematics

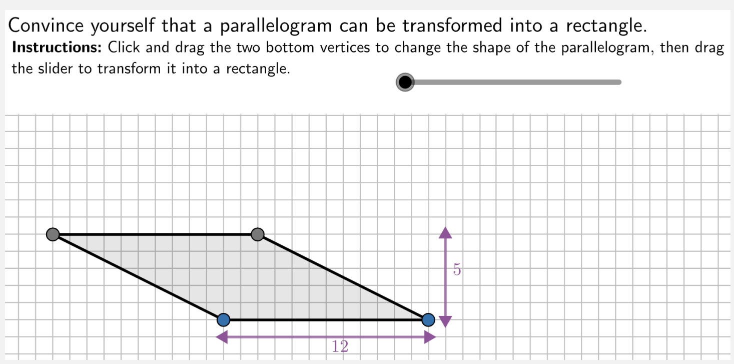
Technology can help us make mathematical discoveries and learn about mathematical objects. Three online examples of this from different areas of mathematics are featured below.

Matching Game: Match decimals and fractions with their equivalent percentages.



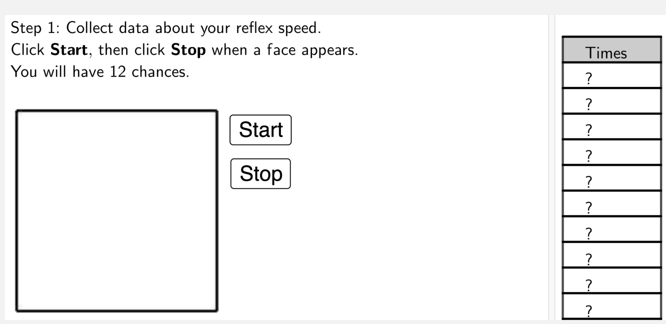
Link to App: https://www.geogebra.org/m/wRkzDXHP

Parallelogram Exploration: Explore how to transform a parallelogram into a rectangle.



Link to App: https://www.geogebra.org/m/x6VYxpHp

Reflex Speed: Test your reflex speed by playing a game. Display the data collected in a histogram.



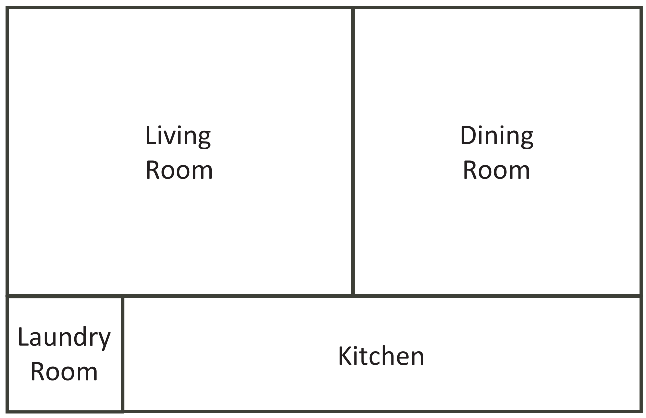
Link to App: https://www.geogebra.org/m/CpQ45m57

More Info: CEMC courseware lessons feature hundreds of interactive mathematics applications.

CEMC at Home

Grade 7/8 - Thursday, June 4, 2020
Kitchen Sized

The rectangular floor plan of the first level of a house is shown in the following diagram.



Both the laundry room and the dining room are square with areas of 4 m^2 and 25 m^2, respectively. The living room is rectangular with an area of 30 m^2.

Determine the area of the kitchen.

More Info:

Check out the CEMC at Home webpage on Friday, June 5 for a solution to Kitchen Sized.

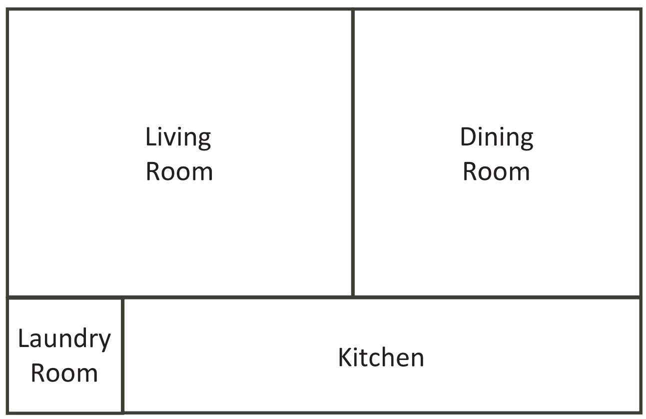
This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 7/8 - Thursday, June 4, 2020
Kitchen Sized - Solution

Problem:

The rectangular floor plan of the first level of a house is shown in the following diagram.



Both the laundry room and the dining room are square with areas of 4 m^2 and 25 m^2, respectively. The living room is rectangular with an area of 30 m^2.

Determine the area of the kitchen.

Solution:

Let the width of a room be the vertical dimension on the diagram. Let the length of a room be the horizontal dimension.

The dining room is a square and has an area of 25 m^2. Its length and width must both be 5 m since Area $= 5 \times 5 = 25$ m^2. The width of the dining room and living room are the same. So the width of the living room is 5 m. But the area of the living room is 30 m^2 so the length of the living room is 6 m since Area $= 5 \times 6 = 30$ m^2.

The laundry room is a square and has an area of 4 m^2. Its length and width must both be 2 m since Area $= 2 \times 2 = 4$ m^2. The width of the laundry room and kitchen are the same. So the width of the kitchen is 2 m.

Now the length of the whole house can be calculated in two ways. We will equate these two expressions to find the length of the kitchen.

$$\begin{aligned}
\text{Length of Laundry Room} + \text{Length of Kitchen} &= \text{Length of Living Room} + \text{Length of Dining Room}\\
2 + \text{Length of Kitchen} &= 6 + 5\\
2 + \text{Length of Kitchen} &= 11\\
\text{Length of Kitchen} &= 9 \text{ m}
\end{aligned}$$

Since the width of the kitchen is 2 m and the length of the kitchen is 9 m, the area of the kitchen is $2 \times 9 = 18$ m^2.

CEMC at Home

Grade 7/8 - Friday, June 5, 2020
Math and CS in the News

Most weeks, our CEMC Homepage provides a link to a story in the media about mathematics and/or computer science. These stories show us how important mathematics and computer science are in today's world. They are a great source for discussions.

Using this article from Phys.org, think about the following questions. (URL also provided below.) Don't worry if you don't understand some of the higher level math discussed in the article.

1. Describe in your own words what it means for a Rubik's Cube to be *scrambled*.
2. What are some situations in life where randomness is important?
3. Research some strategies for solving a Rubik's Cube.
4. Predict the future: According to various online sources, the current record for solving a $3 \times 3 \times 3$ Rubik's Cube is 3.47 seconds for a human and 0.38 seconds for a robot. How do you think technology will help break these records in the future?

URL of the article:
https://phys.org/news/2020-01-hard-scramble-rubik-cube.html

More Info:

A full archive of past posts can be found in our Math and CS in the News Archive. Similar resources for other grades may also be of interest.

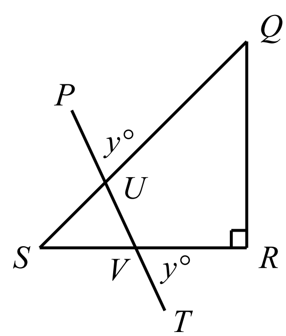
CEMC at Home

Grade 7/8 - Monday, June 8, 2020
Contest Day 6

Today's resource features two questions from the recently released 2020 CEMC Mathematics Contests.

2020 Gauss Contest, #16

In the diagram, $\triangle QRS$ is an isosceles right-angled triangle with $QR = SR$ and $\angle QRS = 90°$. Line segment PT intersects SQ at U and SR at V. If $\angle PUQ = \angle RVT = y°$, the value of y is



(A) 72.5 **(B)** 60 **(C)** 67.5 **(D)** 62.5 **(E)** 70

2020 Gauss Contest, #20

If a and b are positive integers and $\frac{20}{19} = 1 + \cfrac{1}{1 + \frac{a}{b}}$, what is the least possible value of $a + b$?

(A) 16 **(B)** 19 **(C)** 20 **(D)** 38 **(E)** 39

More Info:

Check out the CEMC at Home webpage on Monday, June 15 for solutions to the Contest Day 6 problems.

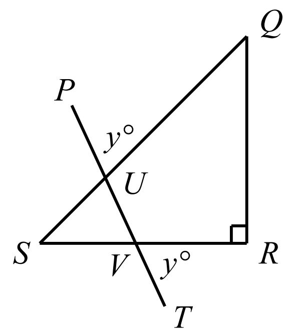
CEMC at Home

Grade 7/8 - Monday, June 8, 2020
Contest Day 6 - Solution

Solutions to the two contest problems are provided below.

2020 Gauss Contest, #16

In the diagram, $\triangle QRS$ is an isosceles right-angled triangle with $QR = SR$ and $\angle QRS = 90^\circ$. Line segment PT intersects SQ at U and SR at V. If $\angle PUQ = \angle RVT = y^\circ$, the value of y is



(A) 72.5 **(B)** 60 **(C)** 67.5

(D) 62.5 **(E)** 70

Solution:

Since $\triangle QRS$ is an isosceles right-angled triangle with $QR = SR$, then $\angle RQS = \angle RSQ = 45^\circ$. Opposite angles are equal in measure, and so $\angle SUV = \angle PUQ = y^\circ$ and $\angle SVU = \angle RVT = y^\circ$. In $\triangle SVU$, $\angle VSU + \angle SUV + \angle SVU = 180^\circ$ or $45^\circ + y^\circ + y^\circ = 180^\circ$ or $2y = 135$ and so $y = 67.5$.

Answer: (C)

2020 Gauss Contest, #20

If a and b are positive integers and $\frac{20}{19} = 1 + \dfrac{1}{1+\frac{a}{b}}$, what is the least possible value of $a + b$?

(A) 16 **(B)** 19 **(C)** 20 **(D)** 38 **(E)** 39

Solution:

We begin by expressing $\frac{20}{19}$ in a form that is similar to the right side of the given equation.
Converting $\frac{20}{19}$ to a mixed fraction we get, $\frac{20}{19} = 1\frac{1}{19} = 1 + \frac{1}{19}$.
Since $\frac{20}{19} = 1 + \dfrac{1}{1+\frac{a}{b}}$ and $\frac{20}{19} = 1 + \frac{1}{19}$, then $1 + \dfrac{1}{1+\frac{a}{b}} = 1 + \frac{1}{19}$ and so $\dfrac{1}{1+\frac{a}{b}} = \frac{1}{19}$.
The numerators of $\dfrac{1}{1+\frac{a}{b}}$ and $\frac{1}{19}$ are each equal to 1, and since these fractions are equal to one another, their denominators must also be equal.
That is, $1 + \frac{a}{b} = 19$ and so $\frac{a}{b} = 18$.
Since a and b are positive integers, then the fractions $\frac{a}{b}$ which are equal to 18 are $\frac{18}{1}$, $\frac{36}{2}$, $\frac{54}{3}$, and so on.
Thus, the least possible value of $a + b$ is $18 + 1 = 19$.

Answer: (B)

CEMC at Home

Grade 7/8 - Tuesday, June 9, 2020
All the Marbles

Problem 1: There are two boxes of marbles. Box A contains _____ red marbles and _____ green marbles. Box B contains _____ red marbles and _____ green marbles. A marble is drawn from each box at random.

(a) Fill in the blanks using four different digits from 1 to 9 so that the probability of drawing a red marble from Box A is higher than the probability of drawing a red marble from Box B.

To get a low probability you want a large number of marbles in the box in total, and a small number of the specific marble in the box. How can this help you choose your numbers?

(b) Fill in the blanks using four different digits from 1 to 9 so that the probability of drawing a red marble is the same for each box.

Need help getting started? Try out your answers on our interactive tool. Note that there are many possible answers for each question. Can you find more than one?

Problem 2: A box contains _____ red, _____ blue, _____ green, and _____ yellow marbles. A marble is drawn from the box at random.

(a) Fill in the blanks using four different digits from 1 to 9 so that the probability of drawing each colour of marble matches the information below.

$$\text{Red: } \frac{5}{24} \qquad \text{Blue: } \frac{1}{8} \qquad \text{Green: } \frac{3}{8} \qquad \text{Yellow: } \frac{7}{24}$$

Is your solution unique? Try to find a second solution, or explain why one does not exist.

(b) Is it possible to fill in the blanks using four different digits from 1 to 9 so that the probability of drawing each colour of marble matches the information below? Explain.

$$\text{Red: } \frac{1}{12} \qquad \text{Blue: } \frac{1}{4} \qquad \text{Green: } \frac{5}{12} \qquad \text{Yellow: } \frac{1}{3}$$

More Info:

Check out the CEMC at Home webpage on Wednesday, June 10 for a solution to All the Marbles.

For more practice with probability, check out this lesson in the CEMC Courseware.

CEMC at Home

Grade 7/8 - Tuesday, June 9, 2020
All the Marbles - Solution

Problem 1: Box A contains ____ red marbles and ____ green marbles. Box B contains ____ red marbles and ____ green marbles. A marble is drawn from each box at random.

(a) Fill in the blanks using four different digits from 1 to 9 so that the probability of drawing a red marble from Box A is higher than the probability of drawing a red marble from Box B.

Solution:

Here it makes sense to have many red marbles and few green marbles in Box A and few red marbles and many green marbles in Box B.

Let's place 9 red marbles and 1 green marble in Box A. This would mean $9 + 1 = 10$ marbles in total in Box A, 9 of which are red. This means that the probability of drawing a red marble from Box A is $\frac{9}{10}$.

Let's place place 2 red marbles and 8 green marbles in Box B. (Remember that we cannot use the digits 1 and 9 again.) This would mean $2 + 8 = 10$ marbles in total in Box B, 2 of which are red. This means that the probability of drawing a red marble from Box B is $\frac{2}{10}$

Since $\frac{9}{10} > \frac{2}{10}$, the probability of drawing a red marble from Box A is higher than the probability of drawing a red marble from Box B.

Note that there are many other strategies and combinations of digits that will also work here. Can you find a few more? Can you find them all?

(b) Fill in the blanks using four different digits from 1 to 9 so that the probability of drawing a red marble from each box is the same.

Solution:

One strategy is to choose, in advance, the probability we want for each of the two boxes, and then find digits that result in this probability.

Suppose we want the probability of drawing a red marble from the boxes to be $\frac{1}{3}$. That means for every red marble in each box, there are 2 green marbles. So the boxes contain twice as many green marbles as red marbles. There are four pairs of digits where one is twice the other:

1 and 2, 2 and 4, 3 and 6, 4 and 8

We want to use these pairs as the numbers of red and green marbles in the boxes. Since we cannot use the same digit more than once, we must pair these in one of the following ways:

- One box: 1 and 2; other box: 3 and 6
- One box: 1 and 2; other box: 4 and 8
- One box: 2 and 4; other box: 3 and 6
- One box: 3 and 6; other box: 4 and 8

Let's choose the first pairing: 1 red marble and 2 green marbles in Box A; 3 red marbles and 6 green marbles in Box B. This means the probability of drawing a red marble from Box A is $\frac{1}{1+2} = \frac{1}{3}$ and the probability of drawing a red marble from Box B is $\frac{3}{3+6} = \frac{3}{9} = \frac{1}{3}$ as well.

It is possible to use this strategy with a different target probability, however some probabilities will not work. For example, can you see why $\frac{1}{2}$ would not work out for this question?

Problem 2: A box contains _____ red, _____ blue, _____ green, and _____ yellow marbles. A marble is drawn from the box at random.

(a) Fill in the blanks using four different digits from 1 to 9 so that the probability of drawing each colour of marble matches the information below.

$$\text{Red: } \frac{5}{24} \qquad \text{Blue: } \frac{1}{8} \qquad \text{Green: } \frac{3}{8} \qquad \text{Yellow: } \frac{7}{24}$$

Solution:

Let's start by writing the probabilities with a common denominator. Since $8 \times 3 = 24$, the lowest common denominator is 24.

$$\text{Red: } \frac{5}{24} \qquad \text{Blue: } \frac{1}{8} = \frac{3}{24} \qquad \text{Green: } \frac{3}{8} = \frac{9}{24} \qquad \text{Yellow: } \frac{7}{24}$$

Notice that the sum of the numerators is $5 + 3 + 9 + 7 = 24$. This mean that if we have 5 red marbles, 3 blue marbles, 9 green marbles, and 7 yellow marbles in the box then we will have 24 marbles in total, and we would get the correct probability for each colour of marble.

There is only one solution to this problem. Can you convince yourself that there is no other way to fill in the blanks according to the rules and get these probabilities?

(b) Is it possible to fill in the blanks using four different digits from 1 to 9 so that the probability of drawing each colour of marble matches the information below? Explain.

$$\text{Red: } \frac{1}{12} \qquad \text{Blue: } \frac{1}{4} \qquad \text{Green: } \frac{5}{12} \qquad \text{Yellow: } \frac{1}{3}$$

Solution:

It is not possible to fill in the blanks in a way that produces the four probabilities given. Here is one way to explain why there is no solution:

We start by writing the probabilities with a common denominator. We can see that the lowest common denominator is 12.

$$\text{Red: } \frac{1}{12} \qquad \text{Blue: } \frac{1}{4} = \frac{3}{12} \qquad \text{Green: } \frac{5}{12} \qquad \text{Yellow: } \frac{1}{3} = \frac{4}{12}$$

Notice that the sum of the numerators is $1+3+5+4 = 13$ and so the sum of these probabilities is

$$\frac{1}{12} + \frac{3}{12} + \frac{5}{12} + \frac{4}{12} = \frac{13}{12}$$

This is a problem! Can you see why? The probabilities of the four possible outcomes should add up to 1, but these four probabilities add up to a value greater than 1.

The probability of drawing each colour of marble represents the fraction of the marbles that are that particular colour. For example, if the probability of drawing a red marble is $\frac{1}{12}$, then it must be the case that $\frac{1}{12}$ of the marbles are red. Similarly, if the other three probabilities are as given, it must be the case that $\frac{3}{12}$ of the marbles are blue, $\frac{5}{12}$ of the marbles are green, and $\frac{4}{12}$ of the marbles are yellow. These fractions cannot possibly all be correct because we cannot have four parts of a whole that add up to more than the whole.

Therefore, we cannot fill in the blanks so that the probability of drawing each marble matches the information given.

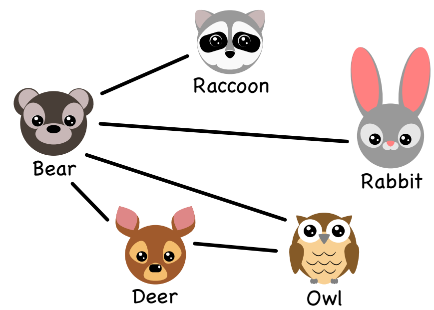
There are other ways to argue that these probabilities cannot be achieved.

CEMC at Home

Grade 7/8 - Wednesday, June 10, 2020
Do I Know You?

A bear, raccoon, rabbit, owl, and deer live in the same forest. Some of the five animals have met before, and some have not. How many of the other animals they have each met before is recorded in the table below. From the information in the table, we can draw a diagram that shows one possibility for which pairs of animals have and have not met. A line is drawn between two animals in the diagram if the two animals have met before, otherwise there is no line drawn.

Animal	Number of Animals Met
Bear	4
Rabbit	1
Raccoon	1
Owl	2
Deer	2



Since the bear has met 4 animals, it must have met all of the other animals. This means there must be a line between the bear and each of the other four animals. Since the rabbit and the racoon have each only met one other animal, it must have been the bear. This means there cannot be any more lines drawn from the rabbit or the racoon. Since the owl has met 2 other animals, it must have also met the deer as shown. Notice that this diagram matches the information in the table.

Problem 1: Five different animals recorded how many of the other animals they had each met before in a table. Which of the following tables are possible? Explain your answers.

Need help getting started? Try drawing a diagram like the one above for each table.

A.

Animal	Number
Elephant	3
Zebra	2
Monkey	4
Tiger	2
Snake	1

B.

Animal	Number
Elephant	1
Zebra	1
Monkey	1
Tiger	1
Snake	1

C.

Animal	Number
Elephant	2
Zebra	3
Monkey	1
Tiger	1
Snake	3

Problem 2: Five different animals recorded how many of the other animals they had each met before in the table shown. Find all possible values for the missing number in the table.

Animals	Number of Animals Met
Cat	3
Dog	3
Hamster	?
Gecko	4
Bird	4

More Info:

Check out the CEMC at Home webpage on Thursday, June 11 for a solution to Do I Know You?

A variation of this problem appeared on a past Beaver Computing Challenge (BCC).

CEMC at Home

Grade 7/8 - Wednesday, June 10, 2020
Do I Know You? - Solution

Problem 1: Five different animals recorded how many of the other animals they had each met before in a table. Which of the following tables are possible? Explain your answers.

A.

Animal	Number
Elephant	3
Zebra	2
Monkey	4
Tiger	2
Snake	1

B.

Animal	Number
Elephant	1
Zebra	1
Monkey	1
Tiger	1
Snake	1

C.

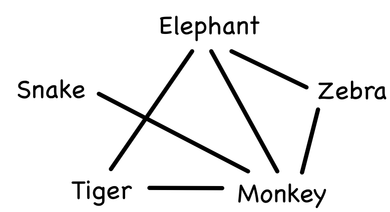
Animal	Number
Elephant	2
Zebra	3
Monkey	1
Tiger	1
Snake	3

Solution:

Let's try drawing a diagram for each table.

A. This table is possible.

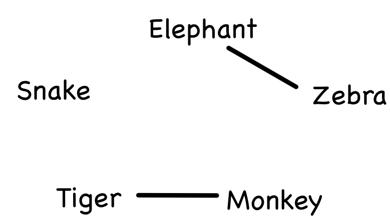
We see that the monkey has met all 4 of the other animals. That means the snake has met only the monkey. The elephant has met 3 animals, so must have also met the zebra and tiger, in addition to the monkey, as it could not have met the snake. The completed diagram given on the right shows a scenario that would result in the numbers in this table. (Is this the only possible diagram?)



B. This table is *not* possible.

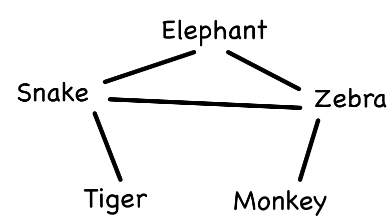
Each animal has met exactly one other animal. This means that each animal must be a part of exactly one line in our diagram and so the animals must be "paired". But there are an odd number of animals so this is impossible.

For example, suppose that we draw a line between the elephant and the zebra and another line between the monkey and the tiger, so that each of these four animals has met exactly one other animal (as in the table). Now, to get the right number for the snake, we need to draw a line from the snake to another animal, but we cannot do so without raising the other animal's number to 2. We will run into a similar problem no matter how we try to pair up the animals.



C. This table is possible.

Suppose that the zebra has met the elephant, the snake, and the monkey. Then the snake could have also met the elephant and the tiger. The completed diagram given on the right shows a scenario that would result in the numbers in this table. (Is this the only possible diagram?)



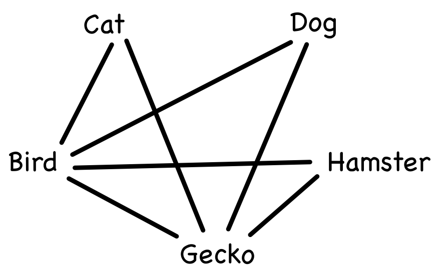
Problem 2: Five different animals recorded how many of the other animals they had each met before in the table shown. Find all possible values for the missing number in the table.

Animals	**Number of Animals Met**
Cat	3
Dog	3
Hamster	?
Gecko	4
Bird	4

Solution:

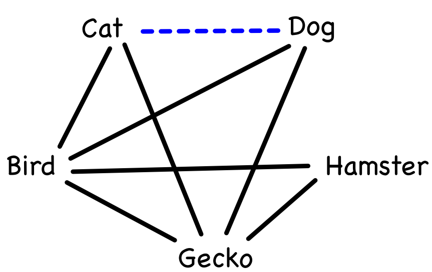
Let's try drawing a diagram to figure out how many other animals the hamster could have met.

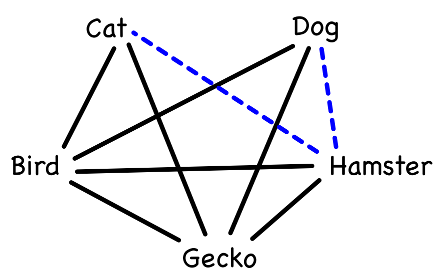
From the table, we can see that the gecko and the bird have each met all 4 of the other animals. So we start off by creating the following diagram that displays this information.



Notice that the above diagram is not complete as the cat and the dog have each met 3 animals, which is 1 more each than is shown in this diagram.

We now have two choices for how to complete the diagram. The cat and the dog could either have met each other and not the hamster (as in the diagram below on the let) or they could have each met the hamster and not each other (as in the diagram below on the right).





In the first case, the hamster has met 2 other animals, and in the second case, the hamster has met all 4 of the other animals.

This means there are two possible values for the missing number in the table: 2 or 4.

CEMC at Home

Grade 7/8 - Thursday, June 11, 2020
Mystery Number

A positive integer has exactly eight positive factors. If two of the factors are 21 and 35, what is the positive integer?

For some integer n, a factor of n is a non-zero integer that divides evenly into n.
For example, 3 is a factor of 18 since $18 \div 3 = 6$, but 4 is not a factor of 18 since $18 \div 4 = 4.5$.

More Info:

Check out the CEMC at Home webpage on Friday, June 12 for a solution to Mystery Number.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 7/8 - Thursday, June 11, 2020
Mystery Number - Solution

Problem:

A positive integer has exactly eight positive factors. If two of the factors are 21 and 35, what is the positive integer?

For some integer n, a factor of n is a non-zero integer that divides evenly into n.
For example, 3 is a factor of 18 since $18 \div 3 = 6$, but 4 is not a factor of 18 since $18 \div 4 = 4.5$.

Solution:

Let n represent the number we are looking for.

We know that four of the positive factors of n are 1, 21, 35 and n. In our solution we will first find the remaining four positive factors and then determine n.

Since 21 is a factor of n and $21 = 3 \times 7$, 3 and 7 must also be factors of n.

Since 35 is a factor of n and $35 = 5 \times 7$, 5 must also be a factor of n.

Since 3 is a factor of n and 5 is a factor of n, and since 3 and 5 have no common factors, $3 \times 5 = 15$ must also be a factor n.

We have found all eight of the positive factors of the unknown number. The positive factors are 1, 3, 5, 7, 15, 21, 35 and n. We now need to determine n.

From the list of factors, we see that the prime factors of n are 3, 5 and 7, and it follows that $n = 3 \times 5 \times 7 = 105$.

Therefore, the positive integer is 105.

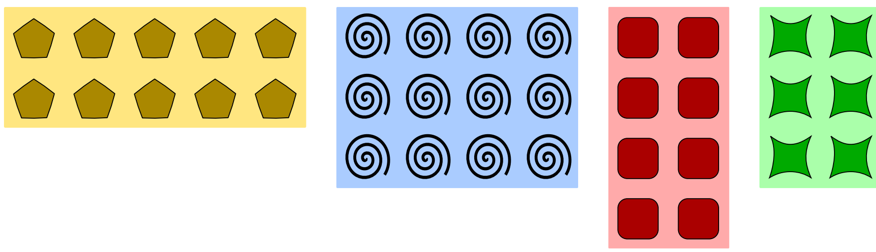
CEMC at Home

Grade 7/8 - Friday, June 12, 2020

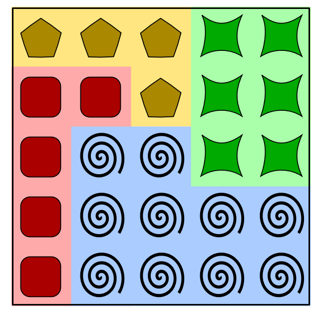
So Many Layers

Problem 1: Kouji covered a wall with four overlapping rectangular sheets of wallpaper as shown. Each sheet of wallpaper is designed using a different image in a repeating pattern. In what order did Kouji place the wallpaper sheets on the wall?

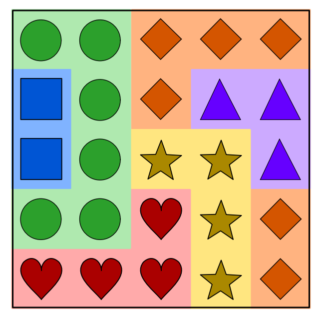
Wallpaper Pieces Used



Result



Problem 2: Tanu covered a wall with six overlapping rectangular sheets of wallpaper as shown. Each sheet of wallpaper is designed using a different image in a repeating pattern.



(a) Give possible dimensions for the six sheets of wallpaper Tanu used.

(b) Determine in what order Tanu placed the wallpaper sheets on the wall.

To help get started, cut out pieces of wallpaper using the images provided on the next page and arrange your pieces to help you solve the problem. Which wallpaper sheets have only one possible set of dimensions and which have more than one?

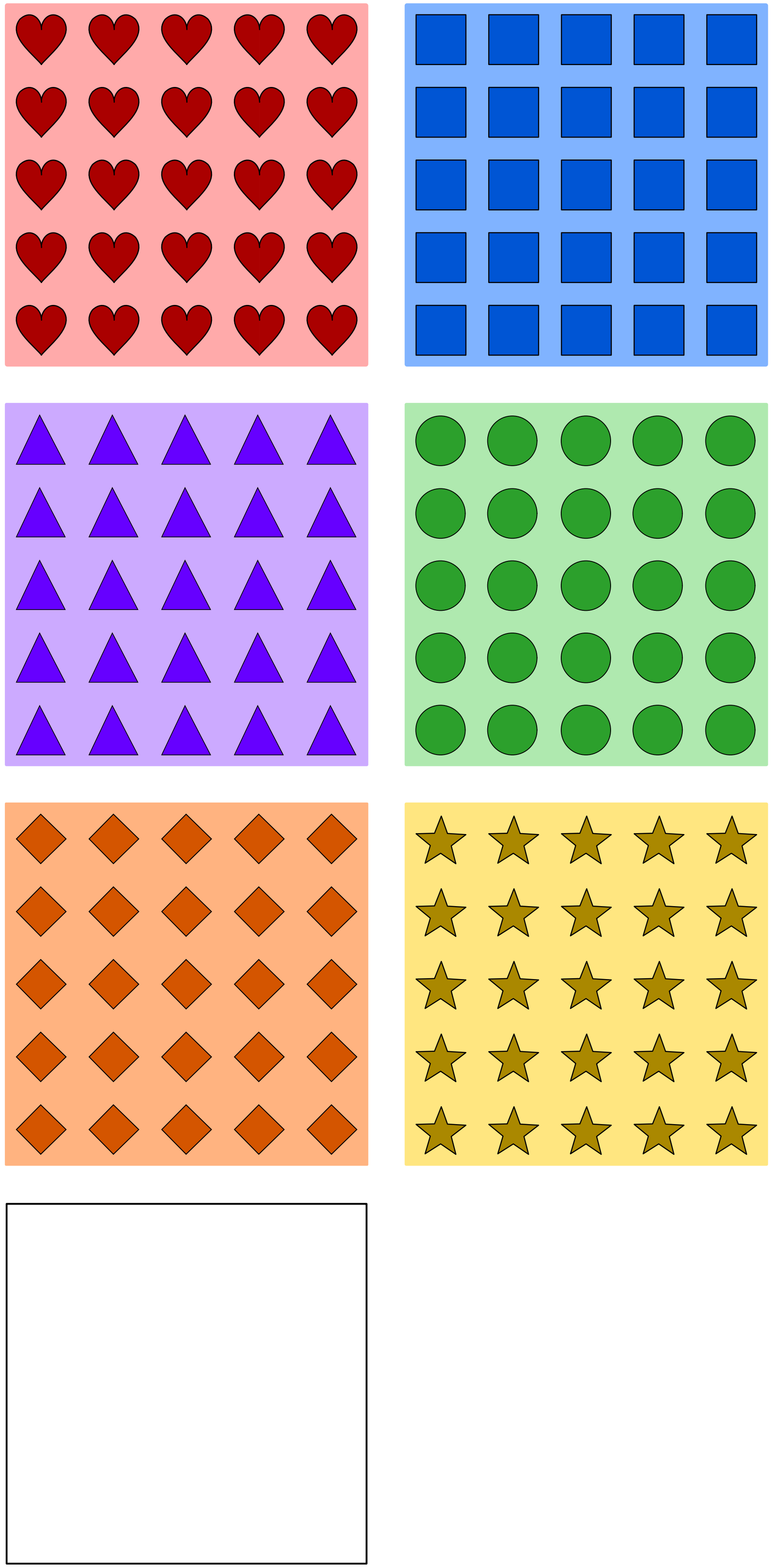
Problem 3: Tanu changes her mind and decides she wants to place the six sheets of wallpaper (from Problem 2) on the wall so that you cannot tell that the wallpaper is overlapping. Each sheet of wallpaper should be visible, but the visible piece should be in the shape of a rectangle so that it looks like the wallpaper pieces were cut to fit right next to each other. Draw one way that Tanu could cover the wall.

There are different possibilities for the dimensions of some of the sheets from Problem 2. You can use the dimensions you found in Problem 2 here, or experiment with different possibilities!

More Info:

Check out the CEMC at Home webpage on Monday, June 15 for a solution to So Many Layers.

A variation of this problem appeared on a past Beaver Computing Challenge (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

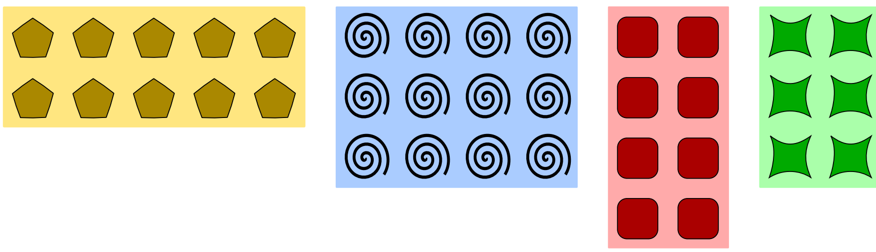


CEMC at Home

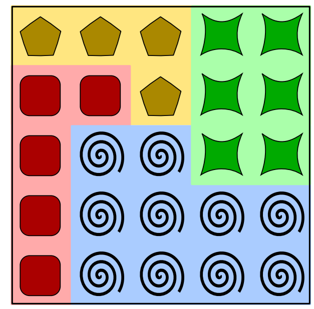
Grade 7/8 - Friday, June 12, 2020
So Many Layers - Solution

Problem 1: Kouji covered a wall with four overlapping rectangular sheets of wallpaper as shown. Each sheet of wallpaper is designed using a different image in a repeating pattern. In what order did Kouji place the wallpaper sheets on the wall?

Wallpaper Pieces Used

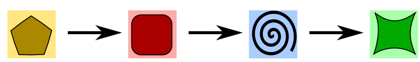


Result



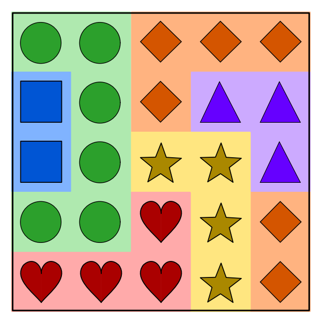
Solution:

The four wallpaper sheets were placed in the following order, from first to last:



To see why this is true, first observe that the wallpaper with the green star-like squares is the only wallpaper that is entirely visible, so it must have been placed last. It remains to justify the order in which the other three sheets were placed. The wallpaper with the red rounded squares is cut off by the blue spirals, so it must have been placed before the blue spirals. The wallpaper with the yellow pentagons is cut off by the red rounded squares, so it must have been placed before the red rounded squares. In order from first to last, the first three sheets to be placed must have been the yellow pentagons, the red rounded squares, and the blue spirals.

Problem 2: Tanu covered a wall with six overlapping rectangular sheets of wallpaper as shown. Each sheet of wallpaper is designed using a different image in a repeating pattern.

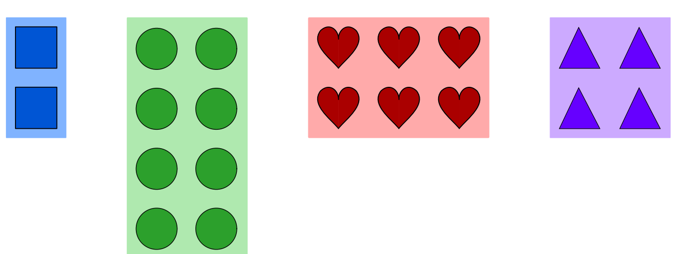


(a) Give possible dimensions for the six sheets of wallpaper Tanu used.

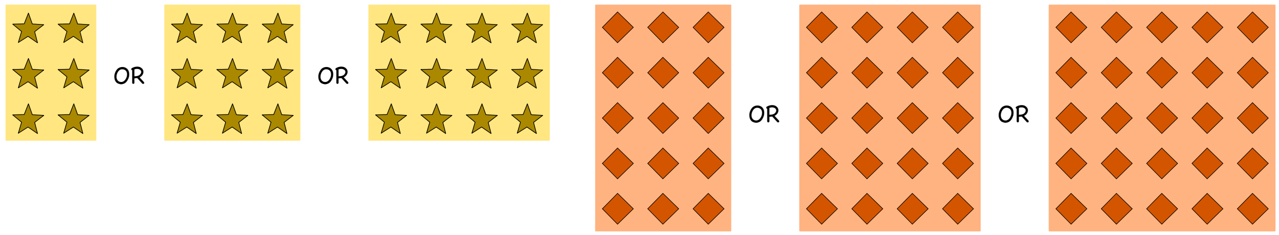
(b) Determine in what order Tanu placed the wallpaper sheets on the wall.

Solution:

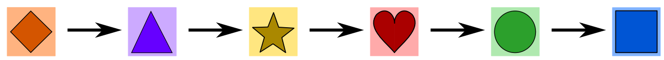
(a) Since each sheet is rectangular, we can see enough to determine that four of the sheets have the dimensions shown below:



We cannot be sure of the dimensions of the remaining two sheets. Here are the different possibilities for these sheets:



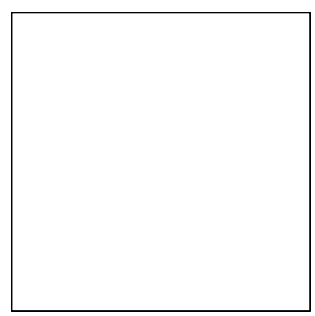
(b) The wallpaper sheets were placed in the following order, from first to last:



The wallpaper with the blue squares is the only wallpaper that is entirely visible, so it must have been placed last. The wallpaper with the green circles is cut off by the blue squares, so it must have been placed before the blue squares. By similar reasoning, the red hearts were placed before the green circles, the yellow stars were placed before the red hearts, the purple triangles were placed before the yellow stars, and the orange diamonds were placed before the purple triangles. Therefore, the sheets must have been placed in the order indicated above.

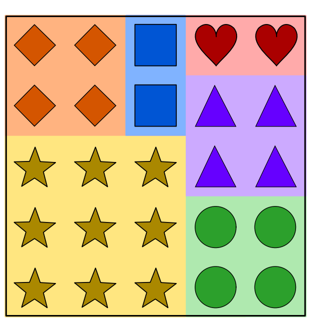
Note that we do not need to know the dimensions of all six wallpaper sheets to determine the order in which they must have been placed. For example, we cannot be sure whether the orange sheet and the green sheet overlap or are placed side-by-side, but this does not stop us from figuring out in which order they were placed.

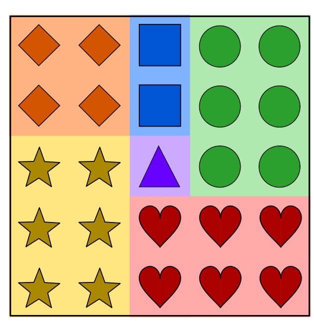
Problem 3: Tanu changes her mind and decides she wants to place the six sheets of wallpaper (from Problem 2) on the wall so that you cannot tell that the wallpaper is overlapping. Each sheet of wallpaper should be visible, but the visible piece should be in the shape of a rectangle so that it looks like the wallpaper pieces were cut to fit right next to each other. Draw one way that Tanu could cover the wall.



Solution:

Your solution may depend on the dimensions you chose for the wallpapers with yellow stars and orange diamonds. Here are two possible solutions. Can you find others?





CEMC at Home

Grade 7/8 - Monday, June 15, 2020
History of Computing

Computers can be found on our desks, in our pockets and even in our refrigerators! This is remarkable because modern computers have been around for less than 100 years. During this time, there has been a constant stream of new discoveries and advances in technology.

Use this online tool to arrange the following list of events in the history of computer science from earliest to most recent.

A. Pong is released and becomes the first arcade game to be commercially successful.

B. Deep Blue is the first computer program to beat a human world chess champion.

C. The Harvard Mark I mechanical computer is built and is used for military purposes during World War II.

D. The first email is sent. It is sent from Ray Tomlinson to Ray Tomlinson.

E. Microsoft introduces the Windows operating system.

F. The Altair 8800 is the first personal computer to sell in large numbers.

G. A robot named Elektro is built which responds to voice commands.

H. Apple announces Siri as a new feature for its products.

I. Tim Berners-Lee posts the first picture on the World Wide Web.

J. The first Microsoft Xbox is available for purchase.

More Info:

Our webpage Computer Science and Learning to Program is the best place to find the CEMC's computer science resources.

CEMC at Home

Grade 7/8 - Tuesday, June 16, 2020
Can You Find the Terms?

Can you find all of the given mathematics and computer science terms in the grid? Good Luck!

A	M	S	T	Q	X	R	S	U	R	V	E	Y
R	H	P	K	L	P	A	I	U	E	F	H	D
E	T	F	R	N	W	T	N	X	R	P	T	J
O	I	J	O	X	V	I	P	V	A	D	P	R
E	R	E	W	H	H	O	O	R	E	A	I	J
L	O	L	T	V	N	L	G	B	R	Z	E	F
B	G	E	E	E	U	O	U	A	N	M	V	I
A	L	U	N	M	T	G	L	O	X	Z	P	H
I	A	T	E	P	G	L	A	T	A	D	U	E
R	T	K	Y	I	E	O	H	I	U	T	X	H
A	D	R	N	L	X	S	F	Z	F	A	E	R
V	C	G	B	V	U	U	M	J	J	O	Z	M
Z	M	A	R	D	I	A	M	E	T	E	R	L

RATIO	SURVEY	ALGORITHM	CRYPTOGRAPHY
VOLUME	EXPONENT	DATA	DEBUGGING
PARALLEL	DIAMETER	NETWORK	VARIABLE

More Info:

Check the CEMC at Home webpage on Wednesday, June 17 for the solution to Can You Find the Terms?

CEMC at Home

Grade 7/8 - Tuesday, June 16, 2020
Can You Find the Terms? - Solution

A	M	S	T	Q	X	R	S	U	R	V	E	Y
R	H	P	K	L	P	A	I	U	E	F	H	D
E	T	F	R	N	W	T	N	X	R	P	T	J
O	I	J	O	X	V	I	P	V	A	D	P	R
E	R	E	W	H	H	O	O	R	E	A	I	J
L	O	L	T	V	N	L	G	B	R	Z	E	F
B	G	E	E	E	U	O	U	A	N	M	V	I
A	L	U	N	M	T	G	L	O	X	Z	P	H
I	A	T	E	P	G	L	A	T	A	D	U	E
R	T	K	Y	I	E	O	H	I	U	T	X	H
A	D	R	N	L	X	S	F	Z	F	A	E	R
V	C	G	B	V	U	U	M	J	J	O	Z	M
Z	M	A	R	D	I	A	M	E	T	E	R	L

RATIO	SURVEY	ALGORITHM	CRYPTOGRAPHY
VOLUME	EXPONENT	DATA	DEBUGGING
PARALLEL	DIAMETER	NETWORK	VARIABLE

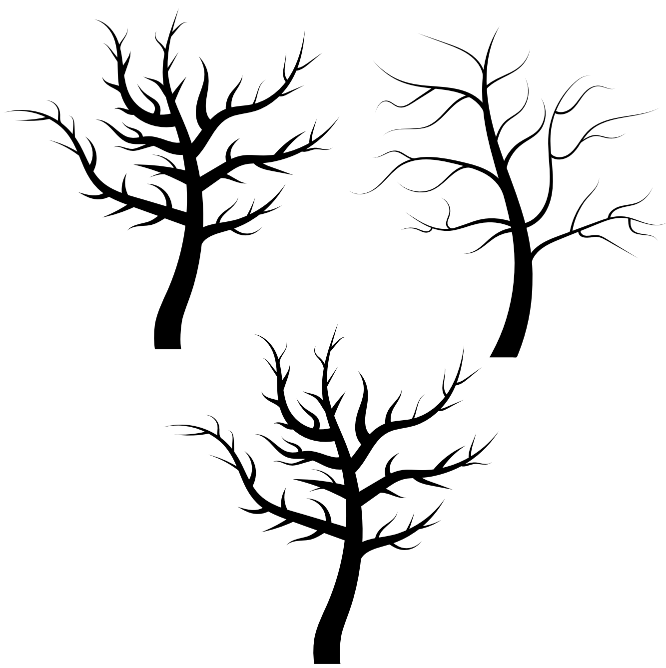
CEMC at Home

Grade 7/8 - Wednesday, June 17, 2020
Out on a Limb

There are 15 Blue Jays and 14 Orioles that wish to rest on the branches of three trees.

Each of the trees will have at least 4 Blue Jays and 2 Orioles in its branches. However, no tree may have more Orioles than Blue Jays in its branches.

Determine the largest number of birds that can be in one tree.



More Info:

Check out the CEMC at Home webpage on Thursday, June 18 for a solution to Out on a Limb.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

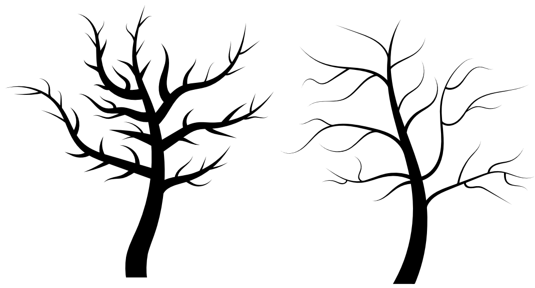
Grade 7/8 - Wednesday, June 17, 2020
Out on a Limb - Solution

Problem:

There are 15 Blue Jays and 14 Orioles that wish to rest on the branches of three trees.

Each of the trees will have at least 4 Blue Jays and 2 Orioles in its branches. However, no tree may have more Orioles than Blue Jays in its branches.

Determine the largest number of birds that can be in one tree.



Solution:

Since each tree contains at least 4 Blue Jays and 2 Orioles, let's start by putting this minimum number of Blue Jays and Orioles in each tree.

Tree 1	Tree 2	Tree 3
4 Blue Jays, 2 Orioles	4 Blue Jays, 2 Orioles	4 Blue Jays, 2 Orioles

The number of Blue Jays not yet in a tree is $15 - 4 - 4 - 4 = 3$.
The number of Orioles not yet in a tree is $14 - 2 - 2 - 2 = 8$.

To produce the greatest number in a tree, as many as possible of the remaining birds should be put in one particular tree. Let's start by putting all of the remaining Blue Jays in Tree 1. Then we have

Tree 1	Tree 2	Tree 3
7 Blue Jays, 2 Orioles	4 Blue Jays, 2 Orioles	4 Blue Jays, 2 Orioles

Let's put as many Orioles in Tree 1 as possible. Since Tree 1 cannot have more Orioles than Blue Jays, we can put at most 5 more Orioles in Tree 1. Now we have

Tree 1	Tree 2	Tree 3
7 Blue Jays, 7 Orioles	4 Blue Jays, 2 Orioles	4 Blue Jays, 2 Orioles

The number of Orioles that are still not in a tree is $14 - 7 - 2 - 2 = 3$.
We cannot place any of these Orioles in Tree 1 because then there will be more Orioles than Blue Jays in that tree. Can we place these in the remaining two trees? We can place 2 in Tree 2 and 1 in Tree 3. So we have

Tree 1	Tree 2	Tree 3
7 Blue Jays, 7 Orioles	4 Blue Jays, 4 Orioles	4 Blue Jays, 3 Orioles

Therefore, the largest number of birds that can be in one tree is 14.

CEMC at Home

Grade 7/8 - Thursday, June 18, 2020
Games and Puzzles

The CEMC has created lots of resources that we hope you have found interesting over the last few months. We also know that there are a lot of online games and puzzles created by other organizations that make use of mathematics and logic. We've highlighted three examples below that you can explore for more mathematical fun!

Deep Sea Math Mystery from Math Playground (https://www.mathplayground.com)

In this puzzle you need to use logic and number sense to figure out how many seashells belong to each sea creature.

Square It from NRICH (https://nrich.maths.org)

Race to be the first to make a square in a grid. Can you develop a strategy to win the game?

Factor Game from NCTM (https://www.nctm.org)

Challenge another person or the computer in this strategy game involving factors of positive integers.

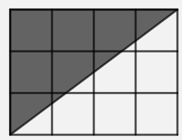
You can find other interesting mathematics related games and puzzles online. Share your favourites using any forum you are comfortable with.

CEMC at Home
Grade 7/8 - Friday, June 19, 2020
Relay Day - Part 1

As part of the CEMC's Canadian Team Mathematics Contest, students participate in Math Relays. Just like a relay in track, you "pass the baton" from teammate to teammate in order to finish the race, but in the case of a Math Relay, the "baton" you pass is actually a number!

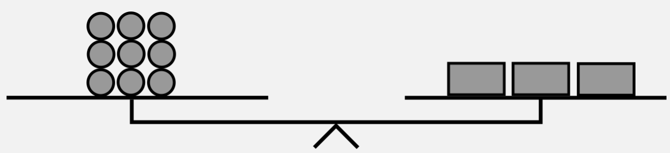
Read the following set of problems carefully.

Problem 1: Twelve 1 unit by 1 unit squares form a rectangle, as shown. What is the area of the shaded region?



Problem 2: *Replace N below with the number you receive.*
In a sequence of numbers, the first term is 3. Each term after the first is determined by multiplying the previous term by 2 and then adding 1. For example, the second term is $2 \times 3 + 1 = 7$ and the third term is $2 \times 7 + 1 = 15$. What is the value of term N?

Problem 3: *Replace N below with the number you receive.*
In the diagram, an equal-armed balance is shown. The mass of each circle is N grams. The rectangles all have the same mass. What is the mass (in grams) of one rectangle?



Notice that you can answer Problem 1 without any additional information.

In order to answer Problem 2, you first need to know the mystery value of N. The value of N used in Problem 2 will be the *answer* to Problem 1. (For example, if the answer you got for Problem 1 was 5 then you would start Problem 2 by replacing N with 5 in the problem statement.)

Similarly, you need the answer to Problem 2 to answer Problem 3. The value of N in Problem 3 is the *answer* that you got in Problem 2.

Now try the relay! You can use this tool to check your answers.

Follow-up Activity: Can you come up with your own Math Relay?
What do you have to think about when making up the three problems in the relay?

In Part 1 of this resource, you were asked to complete a relay on your own. But, of course, relays are meant to be completed in teams! In a team relay, three different people are in charge of answering the problems. Player 1 answers Problem 1 and passes their answer to Player 2; Player 2 takes Player 1's answer and uses it to answer Problem 2; Player 2 passes their answer to Player 3; and so on.

In Part 2 of this resource, you will find instructions on how to run a relay game for your friends and family. We will provide a relay for you to use, but you can also come up with your own!

CEMC at Home

Grade 4 to 12 - Friday, June 19, 2020
Relay Day - Part 2

Relay for Family and Friends

In Part 1 of this resource, you learned how to do a Math Relay. Now, why not try one out with family and friends!

You can put together a relay team and

- play just for fun, not racing any other team, or
- compete against another team in your household (if you have at least 6 people in total), or
- compete with a team from another family or household by
  - timing your team and comparing times with other teams to declare a winner, or
  - competing live using a video chat.

Here are the instructions for how to play.

Relay Instructions:

1. Decide on a team of three players for the relay. The team will be competing together.
2. Find someone to help administer the relay; let's call them the "referee".
3. Each teammate will be assigned a number: 1, 2, or 3. Player 1 will be assigned Problem 1, Player 2 will be assigned Problem 2, and Player 3 will be assigned Problem 3.
4. The three teammates should not see any of the relay problems in advance and should not talk to each other during the relay.
5. Right before the relay starts, the referee should hand out the correct relay problem to each of the players, with the problem statement face down (not visible).
6. The referee will then start the relay. At this time *all three players* can start working on their problems.

   Think about what Player 2 and Player 3 can do before they receive the value of N (the answer from the previous question passed to them by their teammate).
7. When Player 1 thinks they have the correct answer to Problem 1, they record their answer on the answer sheet and pass the sheet to Player 2. When Player 2 thinks they have the correct answer to Problem 2, they record their answer to the answer sheet and pass the sheet to Player 3. When Player 3 thinks they have the correct answer to Problem 3, they record their answer on the answer sheet and pass the sheet to the referee.

8. If all three answers passed to the referee are correct, then the relay is complete! If at least one answer is incorrect, then the referee passes the sheet back to Player 3.

9. At any time during the relay, the players on the team can pass the answer sheet back and forth between them, as long as they write nothing but their current answers on it and do not discuss anything. (For example, if Player 2 is sure that Player 1's answer must be incorrect, then Player 2 can pass the answer sheet back to Player 1, silently. This is a cue for Player 1 to check their work and try again.)

See the next page for a relay for family and friends! This includes instructions for the referee. You can also come up with your own relays to play. You can find many more relays from past CTMC contests on the CEMC's Past Contests webpage.

Sample answer sheets are provided below for you to use for your relays if you wish.

Answer Sheets:

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Relay For Three

Instructions for the Referee:

1. Multiple questions at different levels of difficulty are given for the different relay positions.
   - Assign one of the first three problems (marked "Problem 1") to Player 1.
   - Assign one of the next three problems (marked "Problem 2") to Player 2.
   - Assign one of the last three problems (marked "Problem 3") to Player 3.

   Choose a problem so that each player is comfortable with the level of their question. The level of difficulty of each question is represented using the following symbols:

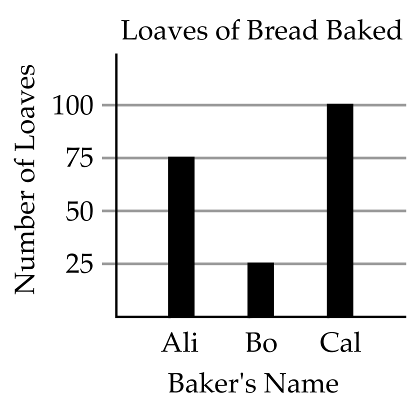
   - These questions should be accessible to most students in grade 4 or higher.
   - These questions should be accessible to most students in grade 7 or higher.
   - These questions should be accessible to most students in grade 9 or higher.

2. Use this tool to find the answers for the relay problems in advance.

Relay Problems (to cut out):

Problem 1

The graph shows the number of loaves of bread that three friends baked. How many loaves did Bo bake?

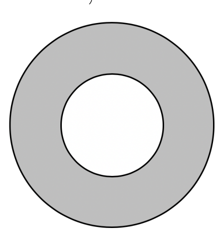


Problem 1

An equilateral triangle has sides of length $x + 4$, $y + 11$, and 20. What is the value of $x + y$?

Problem 1

In the figure shown, two circles are drawn. If the radius of the larger circle is 10 and the area of the shaded region (in between the two circles) is 75π, then what is the *square* of the radius of the smaller circle?



Problem 2 ●

Replace N below with the number you receive.
Kwame writes the whole numbers in order from 1 to N (including 1 and N). How many times does Kwame write the digit '2'?

Problem 2 ■

Replace N below with the number you receive.
The total mass of three dogs is 43 kilograms. The largest dog has a mass of N kilograms, and the other two dogs have the same mass. What is the mass of each of the smaller dogs?

Problem 2 ◆

Replace N below with the number you receive.
The points $(6, 16)$, $(8, 22)$, and (x, N) lie on a straight line. Find the value of x.

Problem 3 ●

Replace N below with the number you receive.
You have some boxes of the same size and shape. If N oranges can fit in one box, how many oranges can fit in two boxes, in total?

Problem 3 ■

Replace N below with the number you receive.
One morning, a small farm sold 10 baskets of tomatoes, 2 baskets of peppers, and N baskets of zucchini. If the prices are as shown below, how much money, in dollars did the farm earn in total from these sales?

Basket of Tomatoes:	\$0.50
Basket of Peppers:	\$2.00
Basket of Zucchini:	\$1.00

Problem 3 ◆

Replace N with the number you receive.
Elise has N boxes, each containing x apples. She gives 12 apples to her sister. She then gives 20% of her remaining apples to her brother. After this, she has 120 apples left. What is the value of x?