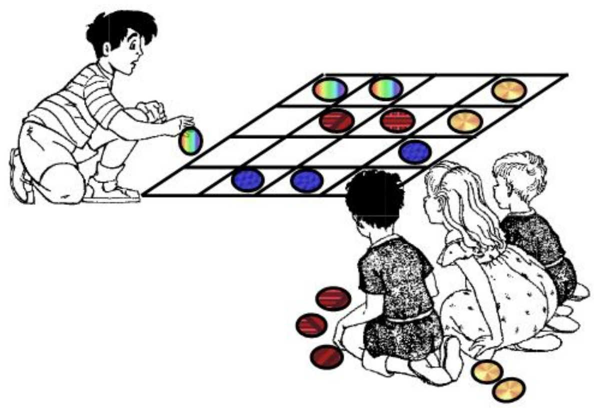
CEMC at Home

Grade 4/5/6 - Monday, March 23, 2020
Sum Bingo

You Will Need:



- Two to four players
- The *Number List* (on the next page)
- The *Game Board* (on the next page)
- 10 to 12 "markers" per player

  (These could be small coins, paperclips, or any other small objects, and should be different for each player.)
- A calculator

How To Play:

1. Players take turns.
2. On your turn, select two numbers from the Number List that you think will have a sum on the Game Board.
3. Say the two numbers out loud and the number on the Game Board that you think is their sum. No calculators yet!
4. Using a calculator, check their sum.
5. If you are correct and the sum is on the Game Board, place one of your markers on the box containing that sum. If you are not correct, do not place one of your markers.
6. The winner is the first to get four markers in a row in any direction (horizontal, vertical, or diagonal).

More Info:

Check out the CEMC at Home webpage on Monday, March 30 for the solution to Sum Bingo.

If you enjoyed this game and would like to try **Product Bingo**, go to

https://cemc.uwaterloo.ca/resources/invitations-to-math/NumberSense-Grade5.pdf

and scroll down to page 50. Have fun!

Number List

23 57 75 43 61 32 95 84 19 115

Game Board

80	127	132	94
159	42	107	93
98	210	84	156
199	100	179	62

CEMC at Home

Grade 4/5/6 - Monday, March 23, 2020
Sum Bingo - Solution

Number List

23 57 75 43 61 32 95 84 19 115

Game Board Solution

$\boxed{80}$ $23 + 57 = 80$ $19 + 61 = 80$	$\boxed{127}$ $32 + 95 = 127$ $43 + 84 = 127$	$\boxed{132}$ $57 + 75 = 132$	$\boxed{94}$ $19 + 75 = 94$
$\boxed{159}$ $75 + 84 = 159$	$\boxed{42}$ $19 + 23 = 42$	$\boxed{107}$ $32 + 75 = 107$ $23 + 84 = 107$	$\boxed{93}$ $32 + 61 = 93$
$\boxed{98}$ $23 + 75 = 98$	$\boxed{210}$ $95 + 115 = 210$	$\boxed{84}$ $23 + 61 = 84$	$\boxed{156}$ $61 + 95 = 156$
$\boxed{199}$ $61 + 115 = 199$	$\boxed{100}$ $43 + 57 = 100$	$\boxed{179}$ $84 + 95 = 179$	$\boxed{62}$ $19 + 43 = 62$

Challenge:

There are 10 numbers in the *Number List* and 16 numbers on the *Game Board*.

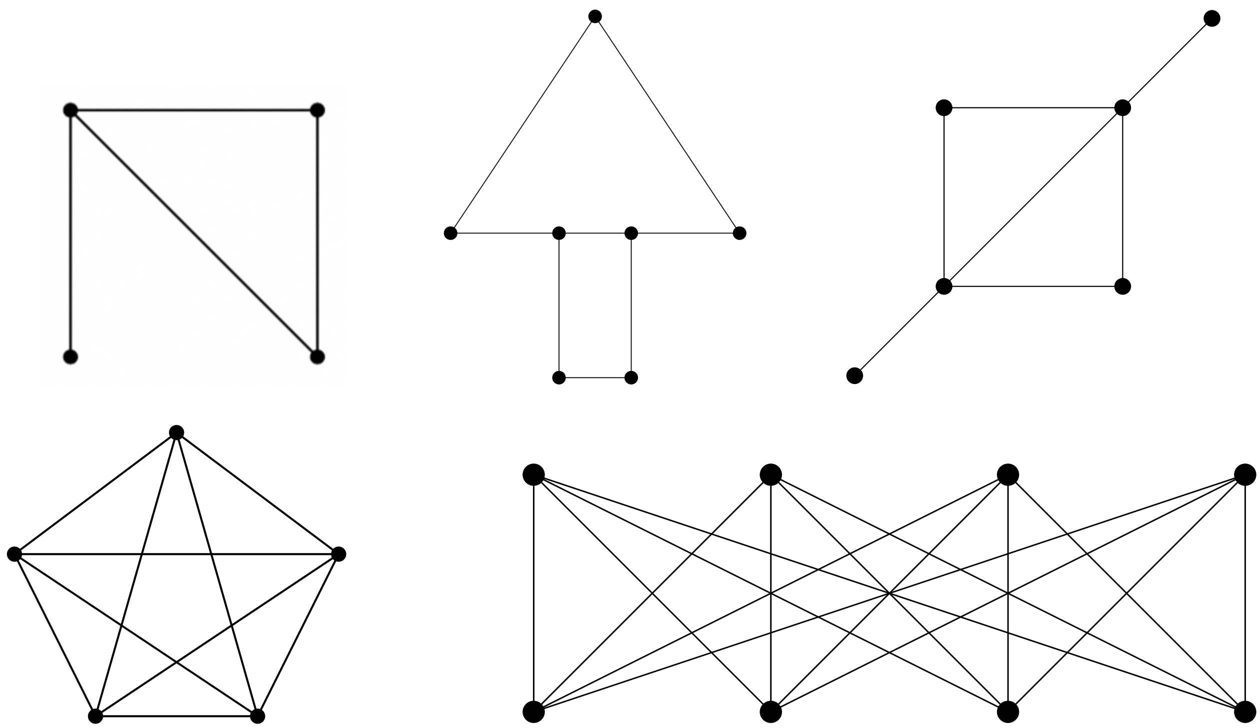
Can you make up an entirely different *Game Board* that uses these same 10 numbers in the *Number List* above?

Can you make up an entirely different game (with a different *Number List*)?

CEMC at Home

Grade 4/5/6 - Tuesday, March 24, 2020
Connect-the-Dots

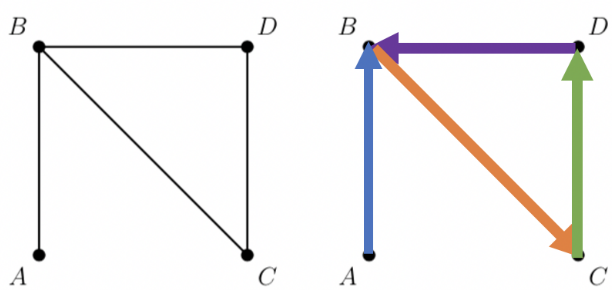
Question 1: In each of the following pictures, find a way to trace over every line exactly once, moving from dot to dot without lifting up your pencil.



Notes: *We are not allowed to trace over the same line twice and we cannot leave any lines out! Take note of all of the dots in the figure. Places where lines meet may or may not correspond to dots. We need to decide at which dot to start our tracing and we do not necessarily need to end our tracing in the same place we started.*

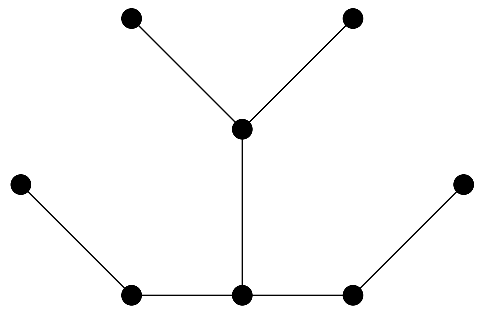
Here is an explanation of one way to complete the task for the top left figure above:

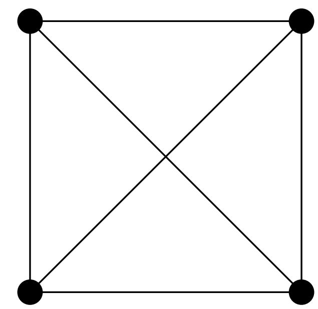
Notice that there are four dots and four lines in the figure. We label each dot with a letter as shown in the figures below. If we start at the letter A, then we can trace all of the lines without lifting the pencil.



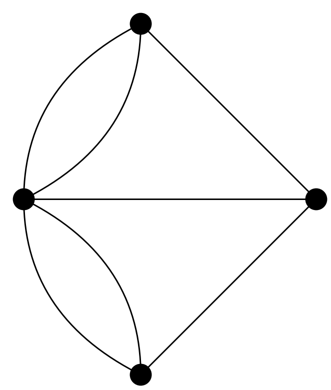
We can trace from A to B, then from B to C, then from C to D, and then from D to B to cover each line exactly once without lifting up our pencil. Can you find different way to trace out the lines?

Question 2: For each of the following pictures, explain why it is *impossible* to trace over every line exactly once, moving from dot to dot without lifting up your pencil.





Question 3: In the picture below, is it possible to trace over every line exactly once, moving from dot to dot without lifting up your pencil? If yes, show how. If no, explain why not.



Question 4: Suppose you are given a picture like the ones in problems 1., 2., 3., with dots connected by a series of lines or curves. Just by looking at the picture, how can you tell whether it is possible to trace over all the lines exactly once without lifting up your pencil?

More Info:

Check out the CEMC at Home webpage on Tuesday, March 31 for the solution to Connect-the-Dots.

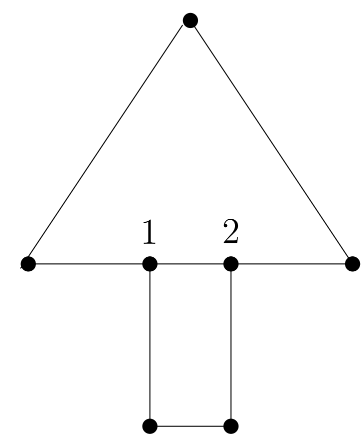
These problems are inspired by the famous *Seven Bridges of Königsberg* problem studied by the mathematician Leonhard Euler. See what you can find out about the Bridges of Königsberg!

CEMC at Home

Grade 4/5/6 - Tuesday, March 24, 2020
Connect-the-Dots - Solution

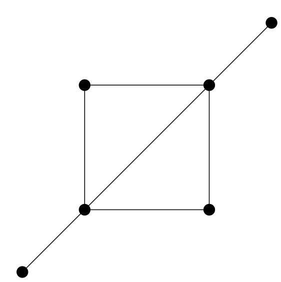
Question 1: In each of the following pictures, find a way to trace over every line exactly once, moving from dot to dot without lifting up your pencil.

Solution: One way to do this is to start from the dot marked "1" and trace over all the outside lines of the picture in order, until returning to dot 1. Then, finish with the line connecting dot 1 to dot 2. Or, you can start from the dot marked "2" and trace over all the outside lines in the picture, returning to dot 2 and finishing with the line from dot 2 to dot 1.

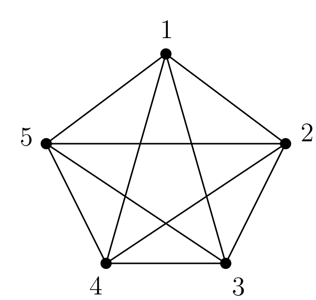


The important thing is that you have to start at either dot 1 or dot 2, since these are the ones with an odd number of lines (or *edges*) coming out of them. Every time you pass through a dot (other than at the beginning or end), you use up an even number of the edges passing through that dot. So, the only way to use up all the edges for a dot (or *vertex*) with an odd number of edges is to make sure that vertex is the first or last vertex!

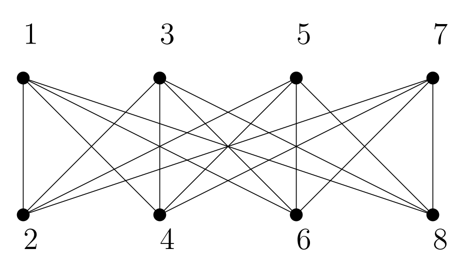
Solution: Here, notice there are exactly two dots (vertices) that have just one edge coming out of them. You must start tracing at one of these vertices, and finish at the other – as soon as you get to one of these vertices, you can't get back to the rest without tracing over the same line twice! So, one way to trace this is to start at the bottom-left vertex, move to the bottom-left corner of the square, trace around the perimeter of the square until you return to where you started, and then trace through the remaining diagonal lines to finish at the top-right vertex.



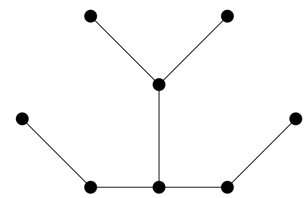
Solution: One way to trace out this diagram is to trace the outside (perimeter) of the pentagon, starting and ending at vertex 1, and then to trace over the inside lines in the pentagon in the following order: $1 \to 3 \to 5 \to 2 \to 4 \to 1$.



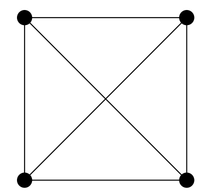
Solution: One possible tracing is $1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 5 \to 2 \to 7 \to 4 \to 1 \to 6 \to 3 \to 8 \to 1$.



Question 2: For each of the following pictures, explain why it is *impossible* to trace over every line exactly once, moving from dot to dot without lifting up your pencil.

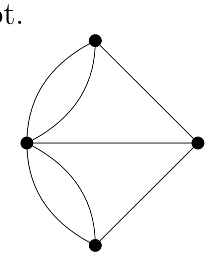


Solution: Notice that in this picture, there are four dots (vertices) that have only one edge coming out of them. Unless one of these vertices is your starting point, as soon as you get to that vertex, you can't get back to the rest of the picture without tracing over the same line twice, so you have to finish there. So, after you start tracing, you can only get to one of these dots before you are forced to stop, making it impossible to reach all four of them.



Solution: As mentioned earlier, every time you pass through a dot (except at the beginning and end), you use up an even number of its edges. So, if you are going to use up all the edges exactly once, at most two of your vertices (dots) can have an odd number of edges touching it. But in this picture, all four of the dots have an odd number of edges, so there's no way to trace over them all exactly once.

Question 3: In the picture below, is it possible to trace over every line exactly once, moving from dot to dot without lifting up your pencil? If yes, show how. If no, explain why not.



Solution: No, it is not possible to trace over all the lines in this picture exactly once, since all four of the dots have an odd number of edges coming out of them (so the same argument works as in the last example).

As the story goes, this particular picture can be used to represent a map of the city of Königsberg back in the 1700s. Each dot represents a landmass, and each edge represents a bridge between the landmasses. The people in this city used to go for walks around town, trying to find a route crossing each bridge exactly once. No one could figure out a way to do this, so they asked the mathematician Leonhard Euler to explain why this was impossible. He came up with the explanation provided here!

Question 4: Suppose you are given a picture like the ones in problems 1., 2., 3., above, with dots connected by a series of lines. Just by looking at the picture, how can you tell whether it is possible to trace over all the lines exactly once without lifting up your pencil?

Solution: As mentioned in the previous problem, if there are more than two dots that have an odd number of lines coming out of them, then there is no way to trace over all the lines in the required way. It turns out that there's also no way to draw such a picture where only one dot has an odd number of lines (why?), so this leaves only the case of zero or two dots with an odd number of edges. In both these cases, it turns out that we can always find a tracing of all the lines. The general idea is to break the problem down into smaller problems, essentially finding a tracing for two smaller parts of the picture and then putting those tracings together. If you are interested, you may want to look into *Fleury's algorithm* for more details!

Mathematicians call pictures like these, with dots and lines (vertices and edges), graphs. *Aside from modelling walks through a city, or connect-the-dots games, graphs can also be used to represent roads between different intersections in a city, or something as complex as the connections between webpages on the Internet (with each vertex representing a webpage, and each edge representing a link between webpages). Mathematicians use graphs to model and help solve many different types of problems.*

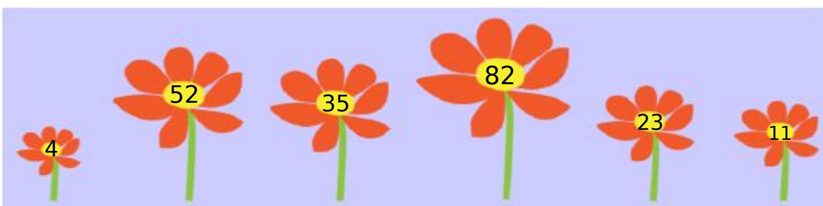
CEMC at Home

Grade 4/5/6 - Wednesday, March 25, 2020
Beever's Choice

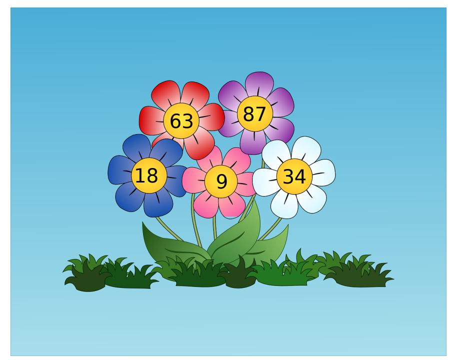
Beever the bee flies to fields of flowers to collect pollen. On each flight, he visits only one flower and can collect up to 10 mg of pollen. He may return to the same flower more than once.

Beever has two favourite flower patches, but visits only one patch daily, making 20 flights per day.

The initial amount of pollen (in mg) in each flower is shown on each flower in the two patches.



Flower Patch 1



Flower Patch 2

Question 1: Suppose that Beever decides to visit Flower Patch 1 today. What is the largest amount of pollen in total that he can collect over 20 flights to this patch?

If Beever chooses a flower for each trip without thinking carefully about each choice, then he is unlikely to gather the largest amount of pollen overall. For example, if Beever takes his first trip to the flower furthest to the left, then he will only be able to gather 4 mg on his first trip. What would happen if Beever decided to visit the flower furthest to the right for his first two trips?

Question 2: Beever's goal is to collect as much pollen as he can today in 20 flights. Do you think Beever should visit Flower Patch 1 or Flower Patch 2? Why?

More info:

Check out the CEMC at Home webpage on Wednesday, April 1 for the solution to Beever's Choice.

Beever's Choice is related to the computing concept of a *greedy algorithm*, a procedure which tries to optimize each step of a process. (Beever will want to take as much pollen as possible on each flight.)

This problem was inspired by a problem from the *Beaver Computing Challenge*. If you enjoyed this problem, try some similar problems here: 2018 BCC for Grade 5/6.

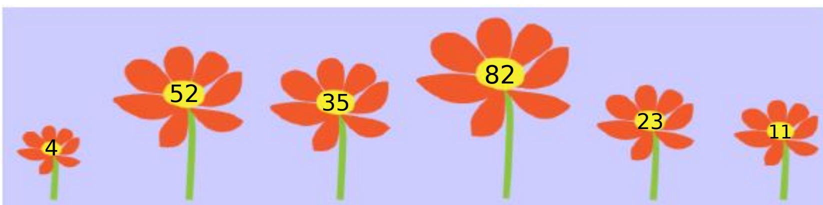
CEMC at Home

Grade 4/5/6 - Wednesday, March 25, 2020
Beever's Choice - Solution

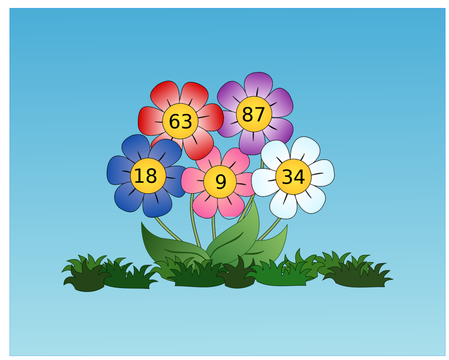
Problem:

Beever the bee flies to fields of flowers to collect pollen. On each flight, he visits only one flower and can collect up to 10 mg of pollen. He may return to the same flower more than once. Beever has two favourite flower patches, but visits only one patch daily, making 20 flights per day.

The initial amount of pollen (in mg) in each flower is shown on each flower in the two patches.



Flower Patch 1



Flower Patch 2

Question 1: Suppose that Beever decides to visit Flower Patch 1 today. What is the largest amount of pollen in total that he can collect over 20 flights to this patch?

Question 2: Beever's goal is to collect as much pollen as he can today in 20 flights. Do you think Beever should visit Flower Patch 1 or Flower Patch 2? Why?

Solution 1: Since Beever can collect up to 10 mg on each flight, he should travel to a flower with at least 10 mg left until it is no longer possible do to so. For example, he can travel five times to the flower with 52 mg, collecting a total of 50 mg, and leaving 2 mg. He can collect the maximum (10 mg) on his first 19 trips totalling

$$50 + 30 + 80 + 20 + 10 = 190 \text{ mg}$$

On his last trip, he will then collect the greatest remaining amount, which is 5 mg from the flower that originally had 35 mg. Thus the largest total amount of pollen he can collect from Flower Patch 1 is $190 + 5 = 195$ mg.

Solution 2: Beever should visit Flower Patch 2. We can use similar reasoning to show that the largest amount of pollen that can be collected from Flower Patch 2 is 197 mg which is more than the maximum amount of 195 mg for Flower Patch 1.

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, March 26, 2020

Chip, Chip, Chooray!

At Biscuit Hill Elementary School, Chip and his sister, Charlene, have decided that they want to make cookies for all of the junior students in their school.

The recipe that they found makes enough chocolate chip cookies of 7 cm diameter for 16 people.

Recipe

1 cup	butter
1 cup	brown sugar
$\frac{1}{2}$ cup	white sugar
2	eggs
2 tsp	vanilla
$2\frac{1}{4}$ cups	flour
1 tsp	baking soda
300 g	chocolate chips

Junior Classes

Mrs. Martin	25 students
Mrs. Laing	26 students
Ms. Richmond	23 students
Mrs. Kelter	24 students
Mr. Hallett	22 students

a) How many batches should Chip and Charlene make so that they make the exact number of cookies needed for all of the students in the junior classes?

b) They decide to make a whole number of batches so that they have some extra cookies to save for later and one cookie for each teacher. What quantity of each ingredient in the recipe will they need?

More Info:

Check the CEMC at Home webpage on Thursday, April 2 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 2.

This CEMC at Home resource is the current grade 5/6 problem from Problem of the Week (POTW). This problem was developed for students in grades 5 and 6, but is also accessible to students in grade 4. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 3/4 problem and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem B and Solution
Chip, Chip, Chooray!

Problem

At Biscuit Hill Elementary School, Chip and his sister, Charlene, have decided that they want to make cookies for all of the junior students in their school.
The recipe that they found makes enough chocolate chip cookies of 7 cm diameter for 16 people.

Recipe

1 cup	butter
1 cup	brown sugar
$\frac{1}{2}$ cup	white sugar
2	eggs
2 tsp	vanilla
$2\frac{1}{4}$ cups	flour
1 tsp	baking soda
300 g	chocolate chips

Junior Classes

Mrs. Martin	25 students
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Ms. Richmond	23 students
Mrs. Kelter	24 students
Mr. Hallett	22 students


a) How many batches should Chip and Charlene make so that they make the exact number of cookies needed for all of the students in the junior classes?

b) They decide to make a whole number of batches so that they have some extra cookies to save for later and one cookie for each teacher. What quantity of each ingredient in the recipe will they need?

Solution

a) There are $25 + 26 + 23 + 24 + 22 = 120$ students in total. Since one recipe makes enough cookies for 16 people, to make exactly enough, Chip and Charlene would need to make $120 \div 16 = 7.5$ batches.

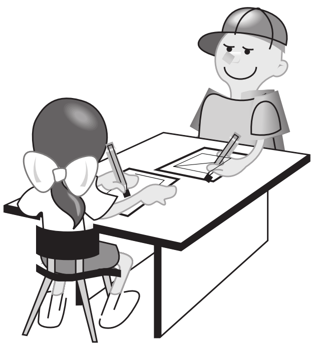
b) Eight batches (128 cookies) will leave 5 for the teachers and 3 to save for later. Thus they will need to multiply all the measurements by eight to get;
$8 \times 1 = 8$ cups butter, $8 \times 1 = 8$ cups brown sugar,
$8 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4$ cups white sugar,
$8 \times 2 = 16$ eggs, $8 \times 2 = 16$ tsp vanilla,
$8 \times (2\frac{1}{4}) = 8 \times 2 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 16 + \frac{8}{4} = 16 + 2 = 18$ cups flour,
$8 \times 1 = 8$ tsp baking soda, and $8 \times 300 = 2400$ g (2.4 kg) of chocolate chips.
(If they want more cookies left over, they will need more batches.)

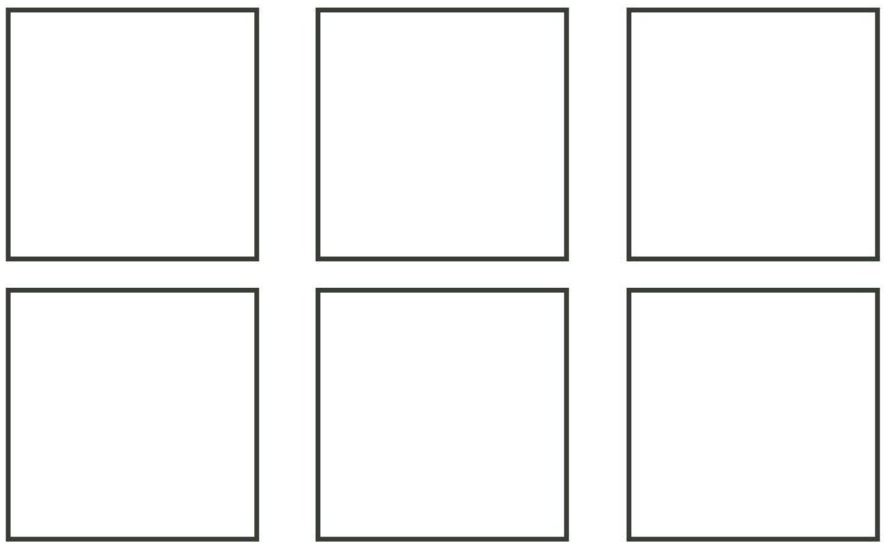
CEMC at Home

Grade 4/5/6 - Friday, March 27, 2020
Quarter Squares

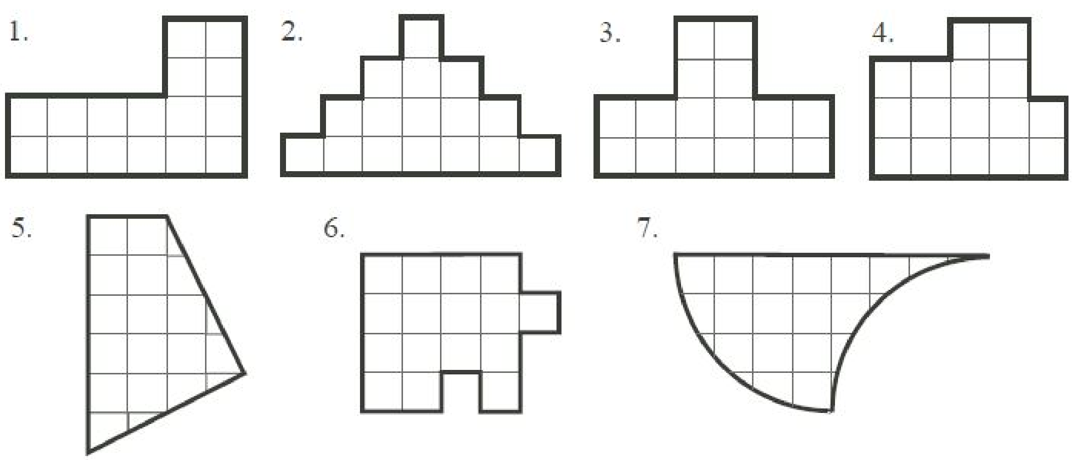
Question 1: On the blank squares below, show six different ways to divide a square into four identical pieces, using only straight line segments.

In math, objects that are identical are called congruent. Congruent shapes have exactly the same shape and size, but may be rotations (turns) and/or reflections (flips) of each other. Can you divide the square into smaller congruent squares or congruent triangles? What other shapes might you use?





Question 2: For each of the templates 1. - 7. given below, discover whether four of these identical shapes could be arranged to create a square. For example, can you arrange four of the shapes from template 1. (of the same size) to form a square? If so, make a sketch on the grid paper on the following page showing how the four identical shapes create a square, and move onto the next template.



Hint: If your visual imagination gets stuck, cut out four copies of the templates to play with. Think about whether you need rotations (turns) and/or reflections (flips) of the pieces.

More info:

Check out the CEMC at Home webpage on Friday, April 3 for the solution to Quarter Squares.

Want another "shape-fitting" activity like this one? Try problem 6 here: 2008 Emmy Noether Circle.

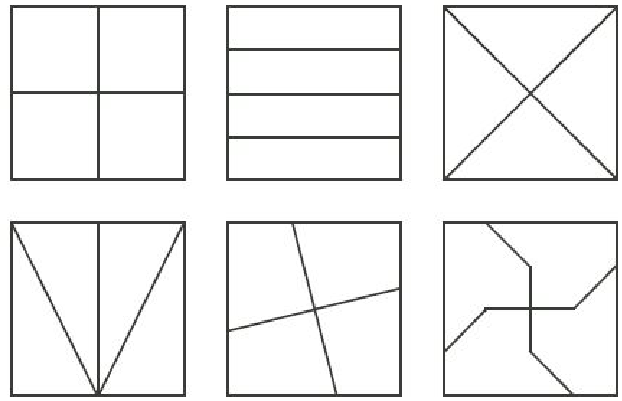
Grid for Quarter Squares

CEMC at Home

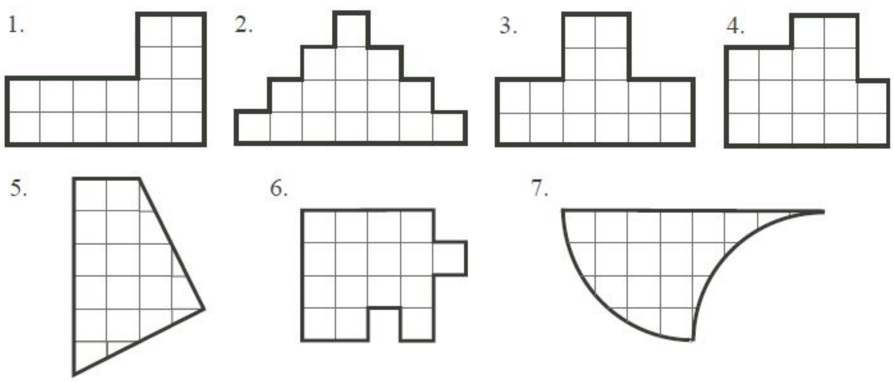
Grade 4/5/6 - Friday, March 27, 2020
Quarter Squares - Solution

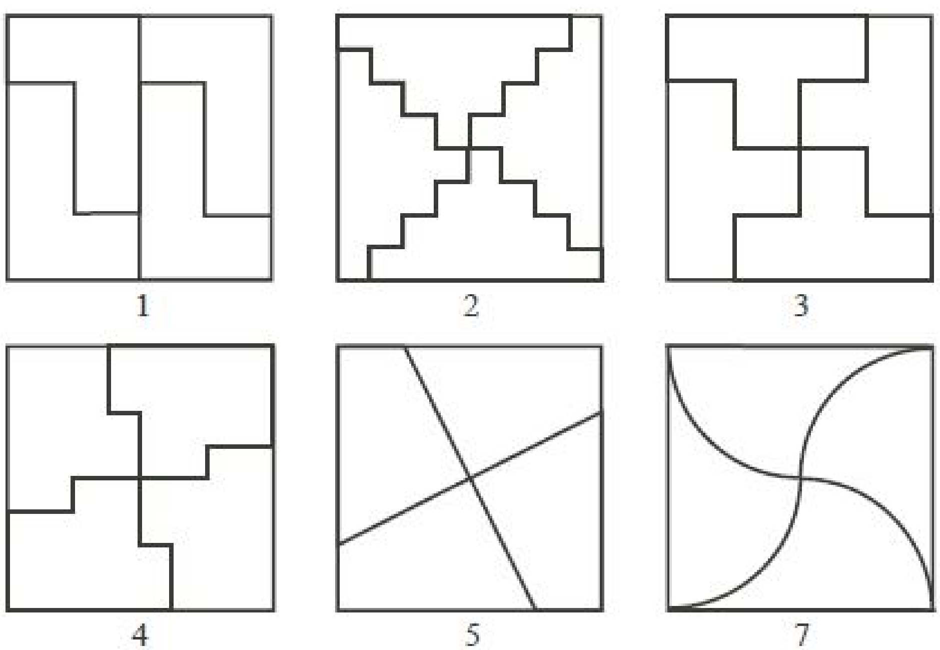
Question 1: Six ways to divide a square into four identical pieces (or four congruent shapes) are shown to the right.

Note that there are countless ways to do subdivisions like the last two, using symmetrical divisions of the square with line segments arranged outward from the centre.



Question 2: The diagrams below reveal how templates 1, 2, 3, 4, 5, and 7 can each be used to form squares with a side length of 8 units.





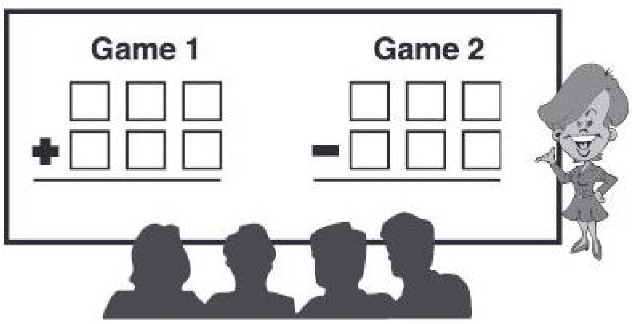
CEMC at Home

Grade 4/5/6 - Monday, March 30, 2020
This Game is Really Sum-thing!

For this activity, you will need to think about *optimizing* sums and differences. This means you will need to think about how to find maximum (greatest) values and minimum (least) values.

Try this example: Let's use the four digits 2, 3, 4, and 5, each exactly once, to form two numbers and then add them. There are many different *sums* you can get by doing this. For example, we could form the numbers 25 and 34 and then add them to get the sum $25 + 34 = 59$. What is the largest possible result that you can get? Think this over and then try out the following games.

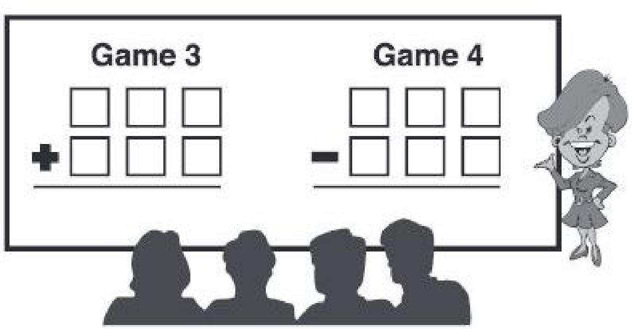
Games 1 and 2: In each game below, you need to place each of the six digits 4, 5, 6, 7, 8, 9, one in each box. In Game 1, the goal is to place the digits in such a way that you obtain the *greatest possible sum*. In Game 2, the goal is to place the digits in such a way that you obtain the *greatest possible difference*.



Digits		
4	5	6
7	8	9

Once you finish Game 1 and Game 2, take a moment to think about the strategies you used to obtain the *greatest* possible sum and difference. (These are strategies for *maximization*!) How will these strategies change if you wish to obtain the *least* possible sum or difference? (These would be strategies for *minimization*!)

Games 3 and 4: In each game below, you need to place each of the six digits 4, 5, 6, 7, 8, 9, one in each box. In Game 3, the goal is to place the digits in such a way that you obtain the *least possible sum*. In Game 4, the goal is to place the digits in such a way that the top number is larger than the bottom number and you obtain the *least possible difference*.



Digits		
4	5	6
7	8	9

More info:

Check out the CEMC at Home webpage on Monday, April 6 for the solutions to these games.

If you'd like more fun with digits and sums, try Problem 1 here: 2007/2008 Emmy Noether Circle 2

CEMC at Home

Grade 4/5/6 - Monday, March 30, 2020
This Game is Really Sum-thing! - Solution

Problem:

In Games 1 and 2, the goal is to maximize the sum and the difference of two three-digit numbers formed from the digits 4, 5, 6, 7, 8, and 9.

In Games 3 and 4, the goal is to minimize the sum and the difference, using the same six digits.

Solution:

GAME 1: The key strategy to obtain the greatest possible sum is to use the greatest digits (8 and 9) in the hundreds column, the next greatest digits (6 and 7) in the tens column, and the other two digits (4 and 5) in the ones column. It does not matter which of the two digits you place in the top or bottom box. Can you see why? The greatest possible sum is 1839, and here are a few ways to place the digits to get this sum.

$$
\begin{array}{cccc} & \boxed{9} & \boxed{7} & \boxed{5} \\ + & \boxed{8} & \boxed{6} & \boxed{4} \\ \hline 1 & 8 & 3 & 9 \end{array}
\qquad
\begin{array}{cccc} & \boxed{9} & \boxed{6} & \boxed{4} \\ + & \boxed{8} & \boxed{7} & \boxed{5} \\ \hline 1 & 8 & 3 & 9 \end{array}
\qquad
\begin{array}{cccc} & \boxed{8} & \boxed{7} & \boxed{4} \\ + & \boxed{9} & \boxed{6} & \boxed{5} \\ \hline 1 & 8 & 3 & 9 \end{array}
\qquad
\begin{array}{cccc} & \boxed{8} & \boxed{6} & \boxed{5} \\ + & \boxed{9} & \boxed{7} & \boxed{4} \\ \hline 1 & 8 & 3 & 9 \end{array}
$$

GAME 2: To obtain a large difference, we want to subtract a small number from a large number. The largest three-digit number we can form is 987 and the smallest three-digit number we can form (with these digits) is 456. To get the largest possible difference we subtract 456 from 987 to get 531.

$$
\begin{array}{cccc} & \boxed{9} & \boxed{8} & \boxed{7} \\ - & \boxed{4} & \boxed{5} & \boxed{6} \\ \hline & 5 & 3 & 1 \end{array}
$$

GAME 3: To obtain the least possible sum, we use the least two digits in the hundreds column, the next least digits in the tens column, and the remaining two digits in the ones column. The least possible sum is 1047, and here are a few ways to place the digits to get this sum.

$$
\begin{array}{cccc} & \boxed{4} & \boxed{6} & \boxed{8} \\ + & \boxed{5} & \boxed{7} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array}
\qquad
\begin{array}{cccc} & \boxed{4} & \boxed{7} & \boxed{8} \\ + & \boxed{5} & \boxed{6} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array}
\qquad
\begin{array}{cccc} & \boxed{5} & \boxed{7} & \boxed{8} \\ + & \boxed{4} & \boxed{6} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array}
\qquad
\begin{array}{cccc} & \boxed{5} & \boxed{6} & \boxed{8} \\ + & \boxed{4} & \boxed{7} & \boxed{9} \\ \hline 1 & 0 & 4 & 7 \end{array}
$$

GAME 4: To obtain the least possible difference we want to make two numbers that are as close as possible in value. To start, we want to make sure the hundreds digits differ by one. This gives us a few possibilities, remembering that we want the top number to be larger than the bottom number:

$$
\begin{array}{cccc} & \boxed{5} & \boxed{} & \boxed{} \\ - & \boxed{4} & \boxed{} & \boxed{} \\ \hline \end{array}
\qquad
\begin{array}{cccc} & \boxed{6} & \boxed{} & \boxed{} \\ - & \boxed{5} & \boxed{} & \boxed{} \\ \hline \end{array}
\qquad
\begin{array}{cccc} & \boxed{7} & \boxed{} & \boxed{} \\ - & \boxed{6} & \boxed{} & \boxed{} \\ \hline \end{array}
\qquad
\begin{array}{cccc} & \boxed{8} & \boxed{} & \boxed{} \\ - & \boxed{7} & \boxed{} & \boxed{} \\ \hline \end{array}
\qquad
\begin{array}{cccc} & \boxed{9} & \boxed{} & \boxed{} \\ - & \boxed{8} & \boxed{} & \boxed{} \\ \hline \end{array}
$$

Some more work, possibly involving some trial and error, will hopefully lead you to find that the least possible difference is 47, which is found by placing the digits as shown below:

$$
\begin{array}{cccc} & \boxed{7} & \boxed{4} & \boxed{5} \\ - & \boxed{6} & \boxed{9} & \boxed{8} \\ \hline & & 4 & 7 \end{array}
$$

CEMC at Home

Grade 4/5/6 - Tuesday, March 31, 2020
Happy Campers

The 48 members of the Junior Division are camped in six tents along Golden Pond. Each tent is a different colour, to help them find their own "home away from home".

Your goal is to discover how many campers reside in each tent. Here are some clues.

1. The tent with the smallest group has 6 campers.
2. The orange tent has the largest group, with 10 campers.
3. The yellow and green tents are the only two tents with the same number of campers.
4. There are a total of 13 campers in the red and blue tents, one of which has the least number of campers.
5. The purple tent has 2 more campers than the blue tent.

Tent	**Number of Campers**
Orange	10
Red	
Blue	
Yellow	
Green	
Purple	

HINT 1: What do clues 1. and 4. tell you if you consider them together?

HINT 2: Don't forget that there are 48 campers in total!

Extension: If you'd like a bit more of this problem, try and figure out whether 11 campers could reside in the orange tent if clues 1., 3., 4, and 5. were still true.

More info:

Check out the CEMC at Home webpage on Tuesday, April 7 for the solution to Happy Campers.

CEMC at Home

Grade 4/5/6 - Tuesday, March 31, 2020
Happy Campers - Solution

Problem:

The 48 members of the Junior Division are camped in six tents along Golden Pond. Each tent is a different colour, to help them find their own "home away from home".

Your goal is to discover how many campers reside in each tent. Here are some clues.

1. The tent with the smallest group has 6 campers.
2. The orange tent has the largest group, with 10 campers.
3. The yellow and green tents are the only two tents with the same number of campers.
4. There are a total of 13 campers in the red and blue tents, one of which has the least number of campers.
5. The purple tent has 2 more campers than the blue tent.

Solution:

Clues 1 and 4 tell us that there are 6 campers in one of the red or blue tents, and 7 in the other. Since there must be at least 6 campers in each tent (clue 1), and 13 in these two tents in total (clue 4), these are the only possibilities. Clues 2 and 4 tell us that there are $10+13=23$ campers in total in the orange, red, and blue tents. Since there are 48 campers over all, we see there must be $48-23=25$ campers in total in the yellow, green, and purple tents.

Since there are either 6 or 7 campers in the blue tent, clue 5 tells us that there are either $6+2=8$ or $7+2=9$ campers in the purple tent. This means that there must be either $25-8=17$ or $25-9=16$ campers in total in the yellow and green tents.

Tent	Number of Campers
Orange	10
Red	6
Blue	7
Yellow	8
Green	8
Purple	9

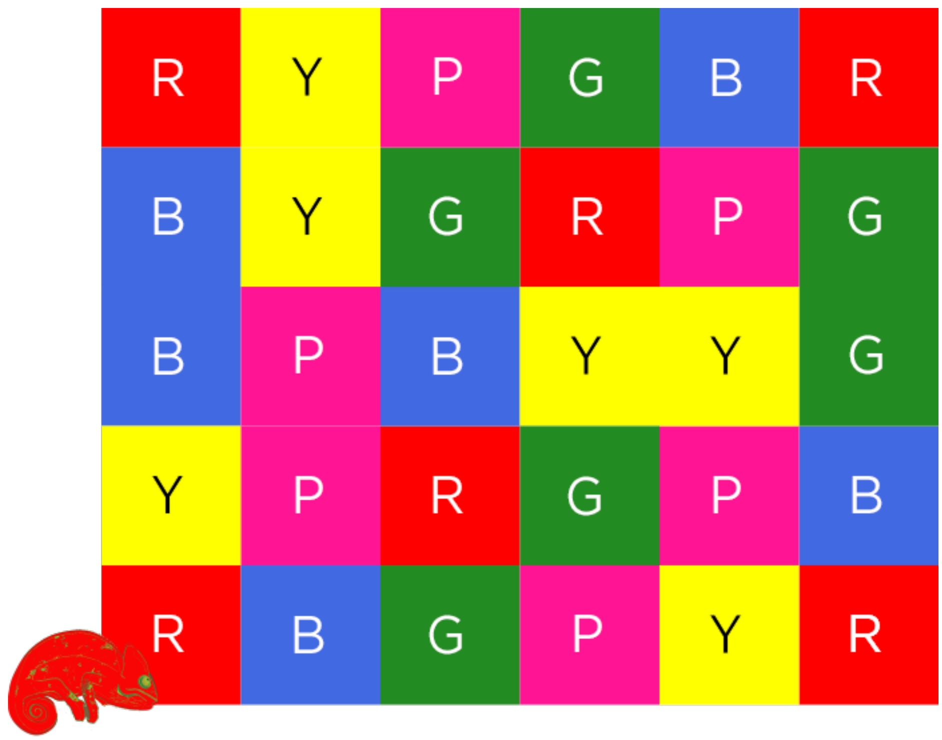
Clue 3 tells us that there are the same number of campers in each of the yellow and green tents which means the total number of campers in these tents must be an even number. This tells us that there must be 16 campers in total in these tents, rather than 17.

We can now be sure that there are 8 campers in each of the yellow and green tents, 9 campers in the purple tent, 7 campers in the blue tent, 6 campers in the red tent, and 10 campers in the orange tent.

CEMC at Home

Grade 4/5/6 - Wednesday, April 1, 2020
Chameleon Gridwalk

Chester the chameleon travels on the grid below, moving from cell to cell. As Chester passes through each cell, his colour changes to match the colour of the cell that he is currently on.



Question 1: Chester travels from the lower left corner of the grid to the upper right corner of the grid, moving between adjacent cells either horizontally or vertically. There are many possible paths he could take, and these will result in him taking on different sequences of colours while he travels. What is the minimum number of different colours that Chester could take on during his trip?

Question 2: Chester travels from the lower left corner of the grid to the upper right corner of the grid, now moving between adjacent cells horizontally, vertically, *or diagonally*. What is the minimum number of different colours that Chester could take on during his trip?

More info:

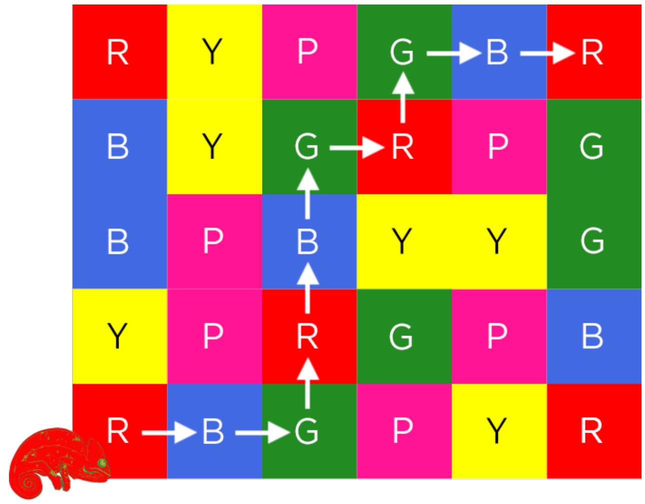
Check the CEMC at Home webpage on Wednesday, April 8 for the solution to Chameleon Gridwalk.

CEMC at Home

Grade 4/5/6 - Wednesday, April 1, 2020
Chameleon Gridwalk - Solution

Solution to Question 1

The minimum number of different colours that Chester could take on while travelling from the lower left corner to the upper right corner, using only horizontal or vertical moves, is 3. The image displayed on the right shows one route that has Chester taking on exactly 3 different colours: red, blue, and green.

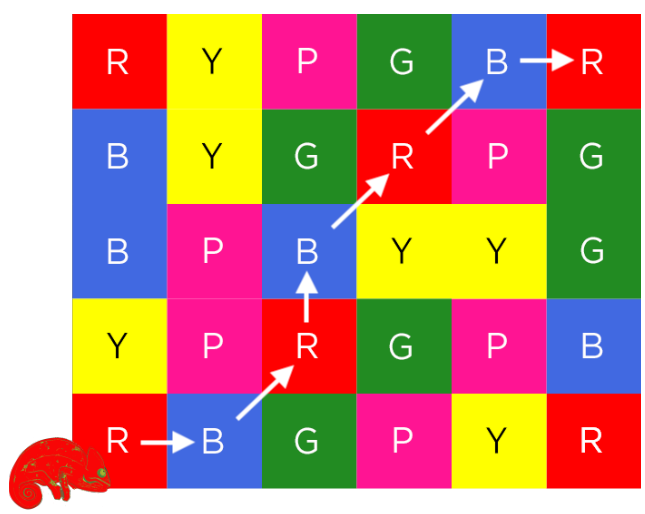


How can we be sure that 3 is the minimum number of different colours that Chester could possibly take on? To show that 3 is the minimum, we have to explain why it is not possible for Chester to walk a path that will have him take on fewer than 3 colours (which means either 1 or 2 colours).

Since Chester's colour is red to begin with, and the next cell he moves to must either be yellow or blue, there is no way that Chester can make it to the right corner without taking on at least 2 colours. If Chester decides to move to the yellow cell above, then we see that there is no path that he can follow from there that only uses red and yellow cells. (The only way to stay on red and yellow cells on his next move would be to move back down to where he started.) So any path will have Chester taking on at least one more colour, making the total number at least 3. If Chester instead decides to move to the blue cell to the right, then we have a similar situation. Again, there is no way to continue from there without picking up a third colour along the way. No matter which path Chester takes from the bottom left corner, he will have to take on at least 3 colours along the the way to the top red corner.

Solution to Question 2

The minimum number of different colours that Chester could take on while travelling from the lower left corner to the upper right corner, using horizontal, vertical, *or diagonal* moves, is 2. The image displayed on the right shows one route that has Chester taking on exactly 2 different colours: red and blue.



It is not possible for Chester to make the trip taking on fewer than 2 colours (which means only 1 colour). Chester's colour is red to begin with, and he will have to take on a second colour (yellow, pink, or blue) as soon as he makes his first move toward the top right corner.

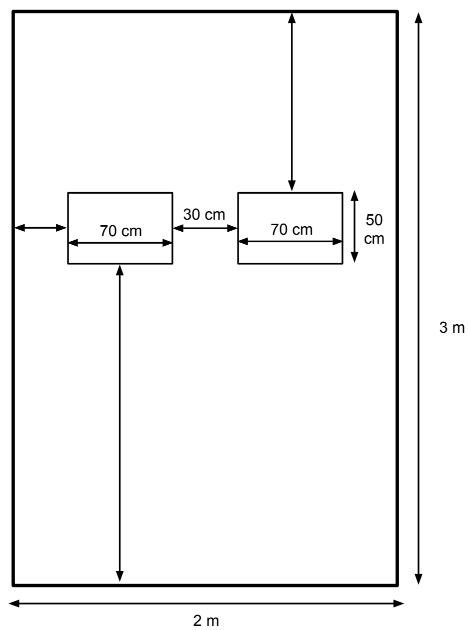
CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, April 2, 2020

Picture Perfect

Two pictures, each 70 cm wide and 50 cm tall, are hung on a rectangular wall so that the top of each picture is 2 m above the floor. The wall is 3 m tall and 2 m wide. The pictures are to be hung so that the horizontal distance from the outside edge of each picture to the nearest wall is the same. Two ways of arranging the pictures are being considered.

a) If the pictures are 30 cm apart on the wall, complete the labels on the following diagram showing where the pictures will be placed on the wall. Note that the diagram is not drawn to scale.



b) If the horizontal distance from the outside edge of each picture to the nearest wall is to be equal to the distance between the two pictures, how far apart should the pictures be?

More Info:

Check the CEMC at Home webpage on Thursday, April 9 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 9.

This CEMC at Home resource is the current grade 3/4 problem from Problem of the Week (POTW). This problem was developed for students in grades 3 and 4, but is also appropriate for students in grades 5 and 6. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 5/6 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem A and Solution
Picture Perfect

Problem

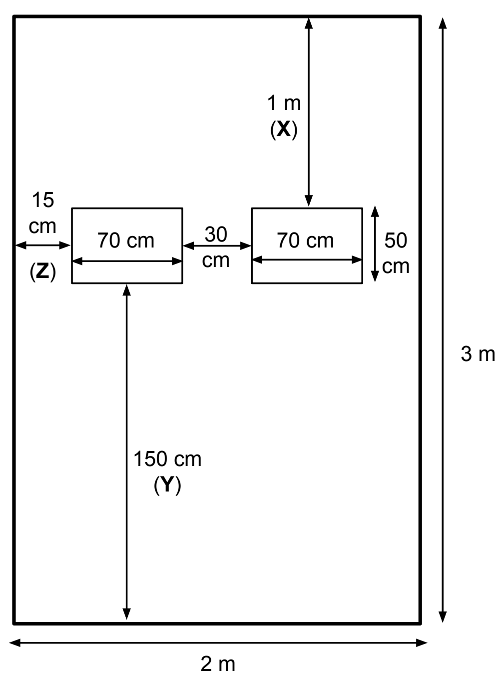
Two pictures, each 70 cm wide and 50 cm tall, are hung on a rectangular wall so that the top of each picture is 2 m above the floor. The wall is 3 m tall and 2 m wide. The pictures are to be hung so that the horizontal distance from the outside edge of each picture to the nearest wall is the same. Two ways of arranging the pictures are being considered.

a) If the pictures are 30 cm apart on the wall, complete the labels on the following diagram showing where the pictures will be placed on the wall. Note that the diagram is not drawn to scale.

b) If the horizontal distance from the outside edge of each picture to the nearest wall is to be equal to the distance between the two pictures, how far apart should the pictures be?

Solution

A) Looking at the original diagram, there are three values missing. Here is the completed diagram:



Here is how we can calculate the missing values:

- (Label **X**) From the problem description we know that the top of each picture is 2 m above the floor. Since the wall is 3 m, then the distance from the ceiling to the top of the picture is $3 - 2 = 1$ m.
- (Label **Y** in the solution) If the distance from the top of the pictures to the floor is 2 m, this is equal to 200 cm. Since the height of each picture is 50 cm, then the distance from the floor to the bottom of the picture is $200 - 50 = 150$ cm.
- (Label **Z**) If the pictures are centred on the wall, then the space on the left side must match the space on the right side. Horizontally, the width of the pictures and the space between them is $70 + 30 + 70 = 170$ cm. Since the wall is 2 m wide and this is equal to 200 cm, the leftover space is $200 - 170 = 30$ cm. If we want the space to be equal on either side, then there must be $30 \div 2 = 15$ cm from the left edge of the picture on the the left to the edge of the wall.

B) The widths of the pictures take up $70 + 70 = 140$ cm of space on the wall. This leaves $200 - 140 = 60$ cm of horizontal space. If we want to equally distribute that space to the left, centre, and right of the pictures we need $60 \div 3 = 20$ cm of space between the two pictures.

Teacher's Notes

Creating and/or reading a diagram properly is a fundamental skill in mathematics. Although a diagram drawn to scale is helpful (or possibly necessary) in some cases, most of the time it is more important that the diagram includes clearly labelled, critical information, and it is unnecessary to have precise measurements. Identifying what is important information is also a useful skill. However, it is a good idea to start a diagram with **any** known information. It is possible you include values that end up being superfluous to the problem, but it is better to have access to extra information than be missing important details.

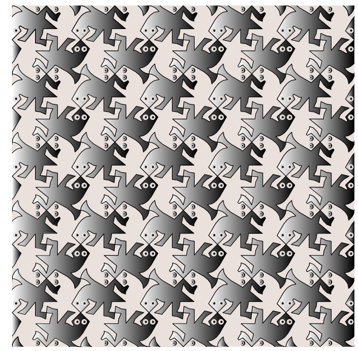
Once you have the initial information labelled, you can infer other values using deductive logic. Logical thinking and *formal logic* are important in the study of mathematics and computer science. In these contexts, we look for precise ways to state our argument that will justify conclusions. A good diagram can be very helpful in this pursuit.

CEMC at Home

Grade 4/5/6 - Friday, April 3, 2020
Dotty Tessellations

A tessellation (or tiling) is an arrangement of one or more shapes in a repeated pattern without overlaps or gaps. Tessellations occur in nature, as illustrated by the honeycomb of bees (a tessellation of hexagons), in masonry (in the walls and floors of buildings), and in artwork.





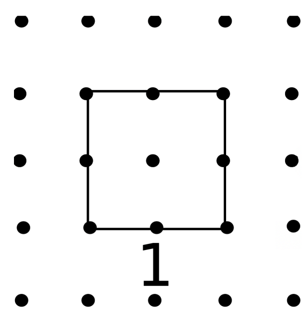
In this activity, we will explore how to make our own tessellations using dot paper.

You Will Need:

- Dot paper (which can be found on the last page)
- A pencil
- An eraser
- A ruler
- Something to colour with (coloured pencils, paints, etc.)

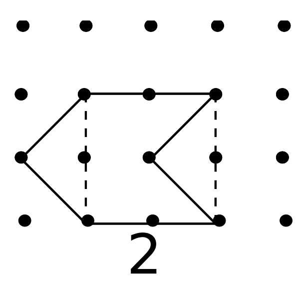
Step 1: Start by drawing a single square, rectangle, or parallelogram on the dot paper using a pencil.

Make sure that it is at least two units long and two units wide, but not so large that it covers too much of the dot paper. An example is shown in image 1.



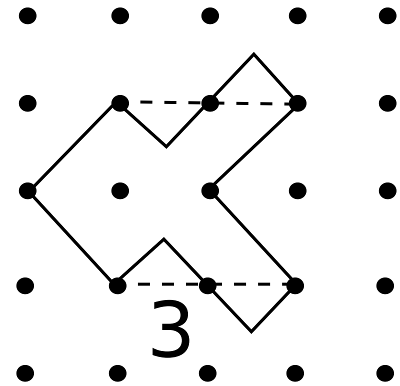
Step 2: “Add” a simple shape to one of the sides of your shape from from Step 1, and then “remove” an identical shape from the opposite side. Erase any lines that are no longer used to form the perimeter of the shape.

In image 2, a triangle is added to the left side of the square by drawing two diagonal lines. An identical triangle is removed from the right side of the square, also by drawing two diagonal lines. The dashed lines in the image show the two vertical edges of the original shape that can now be erased as they are no longer on the perimeter of the shape.



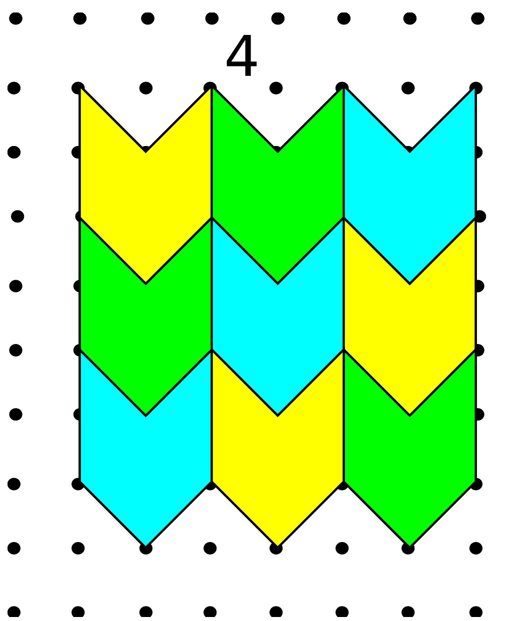
Step 3 (Optional): Go further, if you wish, by now making alterations to the other pair of opposite sides. You can try and use a similar idea as Step 2 (by adding and removing an identical shape) or explore a more complicated alteration.

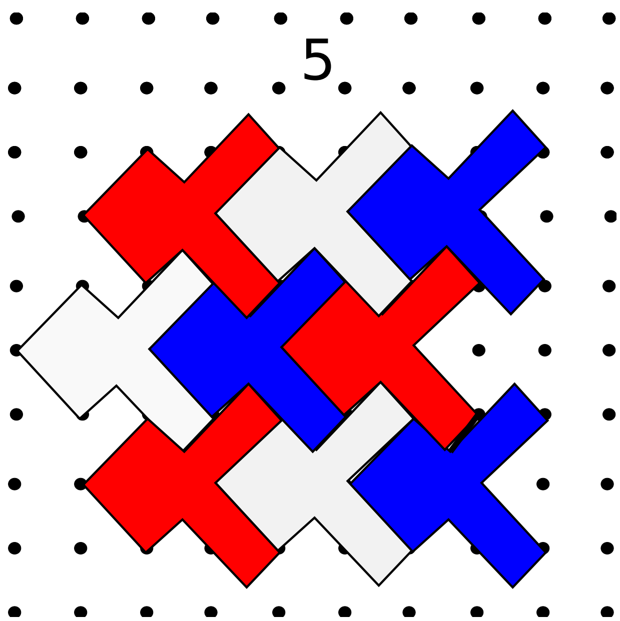
In image 3, we have altered the shape using four identical triangles, with two triangles being "added" and two triangles being "removed". It turns out that this final shape can be used in a tessellation but it might not be immediately clear how to do it! If you choose to do Step 3, and use a more imaginative alteration, then you may want to think about exactly what changes you can and can't make if your goal is to produce a shape that can be used in a tessellation.



Step 4: Draw identical copies of the final shape created in either Step 2 or Step 3 throughout the dot paper. Make sure that your shapes do not overlap and there are no gaps between your shapes. Once you have your tessellation, colour it as imaginatively as you can!

Image 4 shows the final shape from Step 2 tessellated, and image 5 shows the final shape from Step 3 tessellated.





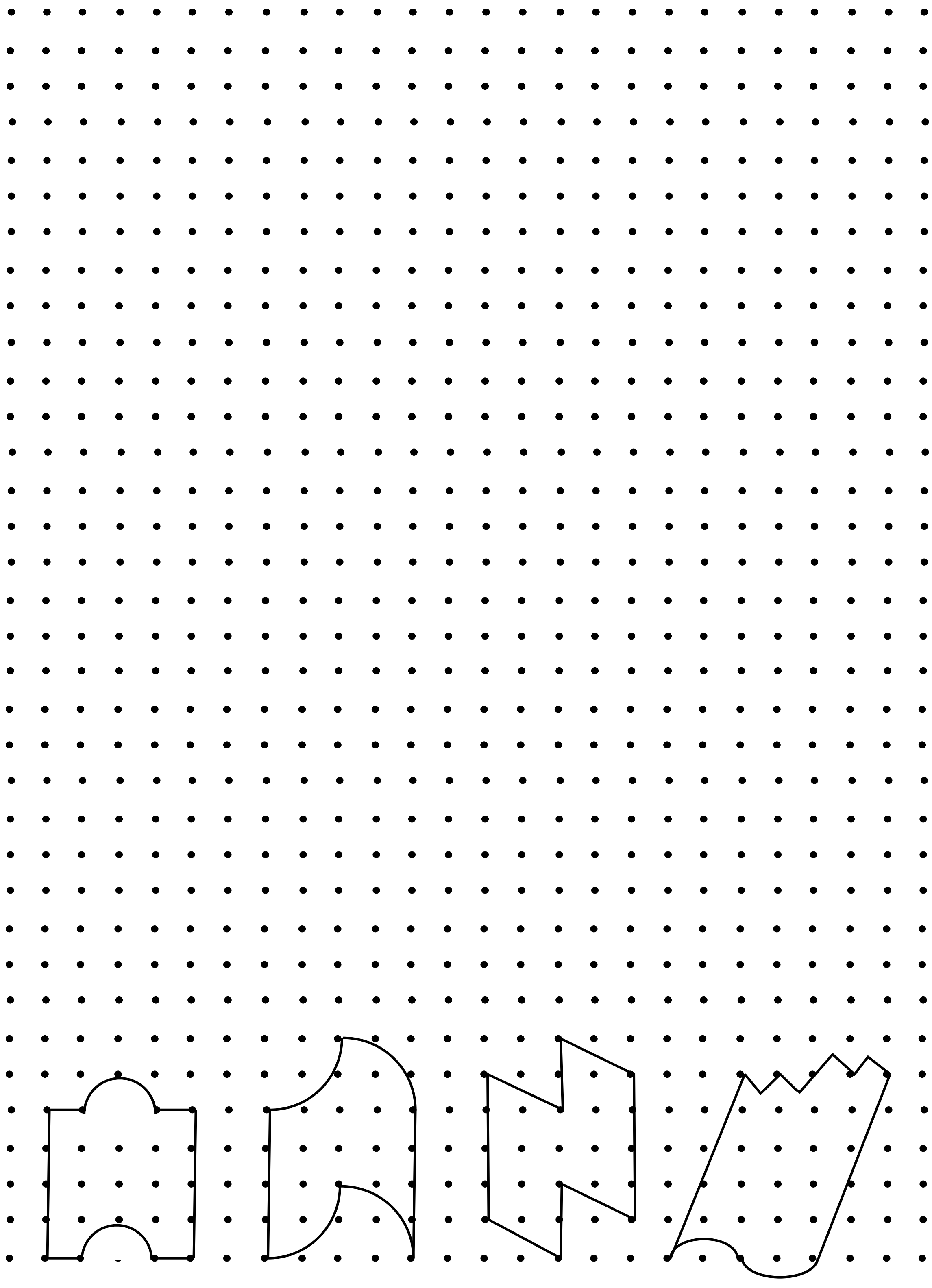
On the next page, you will find some dot paper to work on. You will also find some shapes that have been created for you and can be used in a tessellation. It is easiest if you make your first few shapes by only making alterations that involve straight lines between two dots, but once you get the hang of it, try and make some more complex shapes. Can you make shapes with a mix of straight edges and curved edges that can be used in a tessellation?

More info:

Many works of art are inspired by mathematics! To see some exceptional tessellations, check out the work of the world famous Dutch graphic artist M. C. Escher.

If you would like to explore tessellations further, check out this Math Circles lesson.

Dot Paper for Tessellations



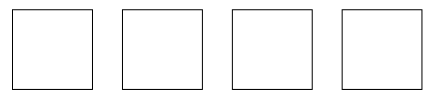
CEMC at Home

Grade 4/5/6 - Monday, April 6, 2020
How Great Is Your Number?

You Will Need:

- At least two players
- A sheet of paper and a pencil for each player
- At least one pair of dice

How To Play:



1. Each player draws four squares on their paper.
2. Players take turns rolling a pair of dice.
3. On your turn, roll the pair of dice and announce the sum of the two numbers rolled.
4. Each player then places the ones digit of this sum in one of the empty squares on their paper.
   For example, if this is the outcome of the roll,

   ⚅ + ⚅ = 12

   then each player places a "2" in one of the empty squares on their paper. If there is more than one empty square remaining, then they have a choice for where to place the "2".
5. Continue taking turns until all squares are filled.
   If there are two players, then each player will roll the dice twice.
6. Each player ends up with a four-digit number. The winner of the game is the player who has formed the greatest four-digit number. (It is possible for the game to end in a tie.)

Play this game a number of times and think about the following questions.

1. What is the least possible ones digit of the sum that you could get on your turn? What is the greatest possible ones digit you could get?
2. If you get a small ones digit, in which square should you place it? Why?
3. If you get a large ones digit, in which square should you place it? Why?
4. Which are the least likely ones digits to occur in this game? Which are the most likely ones digits to occur in this game?

Variations:

A. Try the same game but with three squares, and then with five squares.

B. Try the same game but take the ones digit of the product of the two numbers rolled on the dice, instead of the sum. How does this change the answers to the earlier questions?

More info:

Check out the CEMC at Home webpage on Tuesday, April 14 for answers to these questions.

CEMC at Home

Grade 4/5/6 - Monday, April 6, 2020
How Great Is Your Number? - Solution

Play the game a number of times and think about the following questions.

1. What is the least possible ones digit of the sum that you could get on your turn? What is the greatest possible ones digit you could get?

   Solution: The least possible ones digit is 0. This comes from a sum of 10, which happens if you roll a pair of fives, or a six on one die and a four on the other.

   The greatest possible ones digit is 9. This comes from a sum of 9, which happens if you roll a six on one die and a three on the other, or a five on one die and a four on another.

2. If you get a small ones digit, in which square should you place it? Why?

   Solution: For the four-digit number, the four squares from left to right are the thousands, hundreds, tens, and ones digit, respectively. Since the winner is the player with the greatest four-digit number, small digits should be placed as far as possible to the right. For example, if a digit of 1 comes up, it would make sense to place the digit 1 in an empty square that is farthest to the right.

   How should you you deal with a ones digit of 0 if it comes up? How should you deal with numbers that are smaller than average, but not as small as 1 (for example 3 or 4)?

3. If you get a large ones digit, in which square should you place it? Why?

   Solution: Large digits should be placed as far as possible to the left. For example, if a digit of 9 comes up, it would make sense to place the digit 9 in an empty square that is farthest to the left.

   How should you deal with digits that are larger than average, but not as large as 9? If a ones digit of 6 or 7 comes up first, then where should you place it? You might decide to place the digit 7 in the leftmost square, and then have a ones digit of 9 come up next!

4. Which are the least likely ones digits to occur in this game? Which are the most likely ones digits to occur in this game?

   Solution: Label the two dice "Die #1" and "Die #2". There are 36 possible possible rolls and the chances of getting each of these rolls is the same. Below is a table showing the sum of the two dice in each of the 36 possible rolls.

		Die #1					
		1	2	3	4	5	6
Die #2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Notice that there are two rolls which yield a sum with a units digit of 1: a six on Die #1 and a 5 on Die #2, or a five on Die #1 and a 6 on Die #2, giving $6 + 5 = 5 + 6 = 11$. Similarly, a ones digit of 2 can happen in two ways, $1 + 1 = 2$ or $6 + 6 = 12$, and a ones digit of 3 can happen in two ways, $1 + 2 = 3$ or $2 + 1 = 3$. We can see from the table that the ones digits of 1, 2, and 3 are the least likely to occur in the game (with two ways to achieve each digit).

Similar reasoning shows that there are

- three ways to get a ones digit of 4 ($2 + 2 = 4$, $1 + 3 = 4$, $3 + 1 = 4$),
- three ways to get a ones digits of 0 ($5 + 5 = 10$, $6 + 4 = 10$, $4 + 6 = 10$),
- four ways to get a ones digit of 5 ($1 + 4$, $4 + 1$, $2 + 3$, $3 + 2$),
- four ways to get a ones digit of 9 ($3 + 6$, $6 + 3$, $4 + 5$, $5 + 4$),
- five ways to get a ones digit of 6 ($1 + 5$, $5 + 1$, $2 + 4$, $4 + 2$, $3 + 3$),
- five ways to get a ones digit of 8 ($2 + 6$, $6 + 2$, $3 + 5$, $5 + 3$, $4 + 4$), and
- six ways to get a ones digit of 7 ($1 + 6$, $6 + 1$, $2 + 5$, $5 + 2$, $3 + 4$, $4 + 3$).

Thus the most likely ones digit is 7 (with six ways to achieve this digit).

Variations:

A. Try the same game but with three squares, and then with five squares.

These games will be similar, but will end after a different number of rolls.

B. Try the same game but take the ones digit of the product of the two numbers rolled on the dice, instead of the sum. How does this change the answers to the earlier questions?

The chances of getting certain ones digits change in this variation. For example, the most likely ones digits to occur in this version are 0, 2, and 6, and the least likely ones digit to occur is 7, as it cannot occur at all! Otherwise, the basic strategy of the game remains the same.

CEMC at Home

Grade 4/5/6 - Tuesday, April 7, 2020

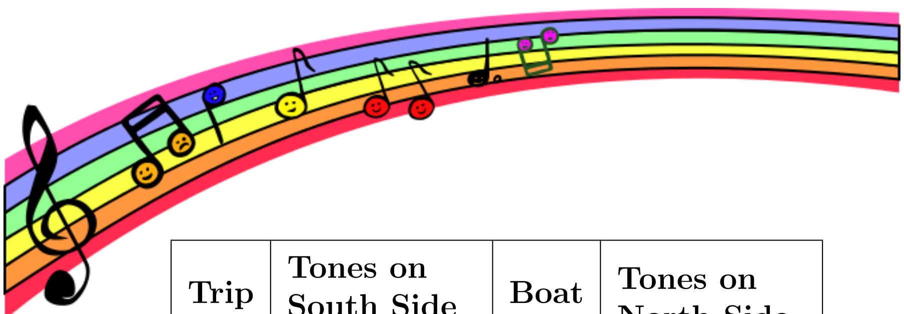
Carrying a Tune

While exploring in the woods, you have found and captured five *Pure Tones*: magical objects that each produce a single, pure musical note. You have put these Tones in glass jars labelled 1, 2, 3, 4, and 5, organized from lowest note to highest note.

In order to take these Tones home, you have to transport them across a river, from the south side to the north side. However, your boat only has storage space for two Tones at a time, plus a seat for you, the driver.

The problem is that these Tones only stay quiet while you are watching them. If they are left alone on one side of the river, they will start making noise. If Tones that are one note apart are left together (like 1 and 2, or 4 and 5), their combined noise will shatter their glass jars, and they will escape.

Design a set of trips back and forth across the river so that you and the five Tones end up on the north side together, without any of them escaping. The table below may help organize your thinking.



Trip	**Tones on South Side**	**Boat**	**Tones on North Side**
	1, 2, 3, 4, 5		None
1		→	
2		←	
3		→	
4		←	
5		→	
6		←	
⋮			

Challenges:

- Can you still solve the puzzle if you can never carry *only one* Tone on your boat?
- This problem is an example of a "river-crossing puzzle". These are puzzles where the goal is to get from one side of a river to another, subject to various constraints. Design your own river-crossing puzzle, solve it, and share it with a friend or family member.

More Info:

Check out the CEMC at Home webpage on Tuesday, April 14 for a solution to Carrying a Tune.

CEMC at Home

Grade 4/5/6 - Tuesday, April 7, 2020
Carrying a Tune - Solution

Problem: While exploring in the woods, you have found and captured five *Pure Tones*: magical objects that each produce a single, pure musical note. You have put these Tones in glass jars labelled 1, 2, 3, 4, and 5, organized from lowest note to highest note.

In order to take these Tones home, you have to transport them across a river, from the south side to the north side. However, your boat only has storage space for two Tones at a time, plus a seat for you, the driver. The problem is that these Tones only stay quiet while you are watching them. If they are left alone on one side of the river, they will start making noise. If Tones that are one note apart are left together (like 1 and 2, or 4 and 5), their combined noise will shatter their glass jars, and they will escape.

Design a set of trips back and forth across the river so that you and the five Tones end up on the north side together, without any of them escaping.

Solution: The first important thing to notice is that you *must* take Tones 2 and 4 across the river on the first trip. If you do not, then the jars on the south side will shatter during your first trip! (Can you see why?) You are safe to leave Tones 2 and 4 on the north side and head back to the south side on your own. Now you have some choice in which Tones you take across the river on your next trip. You can take any two of the three Tones: 1, 3, and 5. (In our example in the table below, we take Tones 1 and 5.) No matter what choice is made, the next important thing to notice is that you *must* take at least one Tone back from the north to the south side of the river once four Tones have made it to the north side. (In our example, we cannot leave tones 1, 2, 4, and 5 together.) You actually must take two Tones back with you. (In our example, we would have to take either 1 or 2 and either 4 or 5 back to the south side on our next trip. We take Tones 2 and 4.)

The table below shows one possible way to get the Tones across successfully in exactly seven trips.

Trip	Tones on South Side	Boat	Tones on North Side
	1, 2, 3, 4, 5		None
1	1, 3, 5	$2, 4 \rightarrow$	2, 4
2	1, 3, 5	$\leftarrow$	2, 4
3	3	$1, 5 \rightarrow$	1, 2, 4, 5
4	2, 3, 4	$\leftarrow 2, 4$	1, 5
5	2, 4	$3 \rightarrow$	1, 3, 5
6	2, 4	$\leftarrow$	1, 3, 5
7	None	$2, 4 \rightarrow$	1, 2, 3, 4, 5

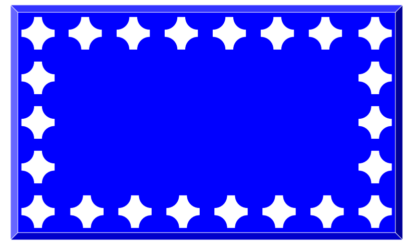
Challenge: Can you still solve the puzzle if you can never carry *only one* Tone on your boat?

Solution: No, it is impossible to get all five Tones across the river under these conditions. If you can only move zero Tones or two Tones across the river on each trip, then the number of Tones on the north side at any given time will always be an even number! Since we need to end up with an odd number of Tones (five) on the north side, we cannot possibly succeed.

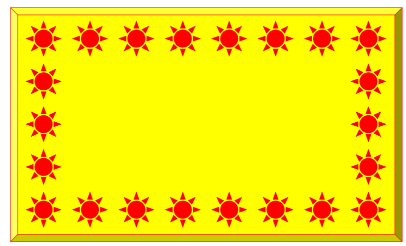
CEMC at Home

Grade 4/5/6 - Wednesday, April 8, 2020
A Stitch in Time...

Friends Nathan and Nia have decided to get together to complete a sewing project. They are making fabric placemats with a fancy pattern on the border.



Nathan's pattern is for a blue placemat with a white border; each one takes 7 minutes to complete.



Nia's pattern is a more complicated design for a yellow placemat with a red border; each one takes 9 minutes to complete.

They set up their sewing machines on opposite ends of a table. As they finish each placemat, they put it on top of a pile in the middle of the table.

They both start sewing at the same time. So Nathan puts the first blue placemat at the bottom of the pile after 7 minutes, and Nia puts a yellow placemat on top 2 minutes later, or 9 minutes after they started.

1. Working upwards, complete the table that shows the order of the placemats in the pile after they have been working for 40 minutes. Enter the time at which each placemat is added to the pile as shown.

2. How many minutes after they start sewing will Nathan try to add a blue placemat to the pile at the same time as Nia tries to add a yellow one?

   We are looking for the first time this happens.

3. Suppose instead that a blue placemat takes 6 minutes to make, and a yellow placemat takes 10 minutes. How many minutes after they start sewing will Nathan and Nia try to add their placemats to the pile at the same time?

   We are looking for the first time this happens.

Time	Table Top
14	blue
9	yellow
7	blue

More Info:

Check out the CEMC at Home webpage on Wednesday, April 15 for a solution to A Stitch in Time.

CEMC at Home

Grade 4/5/6 - Wednesday, April 8, 2020
A Stitch in Time - Solution

Problem Summary:

Nathan's pattern is for a blue placemat; each one takes 7 minutes to complete. Nia's pattern is for a yellow placemat; each one takes 9 minutes to complete. When they finish one of their placemats, they put it on top of a single pile in the middle of the table. They both start sewing at the same time.

1. Working upwards, complete the table that shows the order of the placemats in the pile after they have been working for 40 minutes. Enter the time at which each placemat is added to the pile as shown.
2. How many minutes after they start sewing will Nathan try to add a blue placemat to the pile at the same time as Nia tries to add a yellow one?

   We are looking for the first time this happens.
3. Suppose instead that a blue placemat takes 6 minutes to make, and a yellow placemat takes 10 minutes. How many minutes after they start sewing will Nathan and Nia try to add their placemats to the pile at the same time?

   We are looking for the first time this happens.

Solution:

1. See the completed table at the right, showing the times at which each placemat is completed.
2. Note that Nathan's blue placemats are added to the pile at times that are multiples of 7, whereas Nia's yellow placemats are added to the pile at times that are multiples of 9. This means that they will both try to add a placemat at the same time if that time is a multiple of both 7 and 9. The least number that is a multiple of both 7 and 9 is 63. This tells us that the first time Nathan and Nia will try to add placemats to the pile at the same time is 63 minutes after they start.

   Note: 63 is the *least common multiple* of 7 and 9.
3. With this new timing, the first time Nathan and Nia will try to add placemats to the pile at the same time is 30 minutes after they start. This is because 30 is the least number that is a multiple of both 6 and 10.

   Note: 30 is the *least common multiple* of 6 and 10.

Time	Table Top
49	
45	
42	
36	yellow
35	blue
28	blue
27	yellow
21	blue
18	yellow
14	blue
9	yellow
7	blue

Notice that the answer for 2. is equal to the product of the two times, 7 and 9, but this is not the case for the answer in 3. The product of 6 and 10 is 60, and Nathan and Nia will both try and put placemats on the pile after 60 minutes, but this is not the first time this will happen.

Computer Science Connections

The main work of a computer is handled by its CPU (**C**entral **P**rocessing **U**nit). Over time, CPUs have become smaller and more powerful. Most computers today have *multi-core processors*, which means there are multiple cores (processing units) on a single computer chip in the machine. These cores can work in parallel (i.e., at the same time on different tasks) to improve the speed and performance of the computer.

Although it is not too difficult with today's technology to create these multi-core chips, it turns out that is quite difficult to take full advantage of their parallel processing potential. One issue that limits the effectiveness of parallel processing is that it can be difficult for multiple cores to share resources. In this problem, we can think of Nathan and Nia as separate cores and the pile of placemats in the middle of the table as a shared resource. They will have to coordinate things when both want to add a placemat on the pile at the same time.

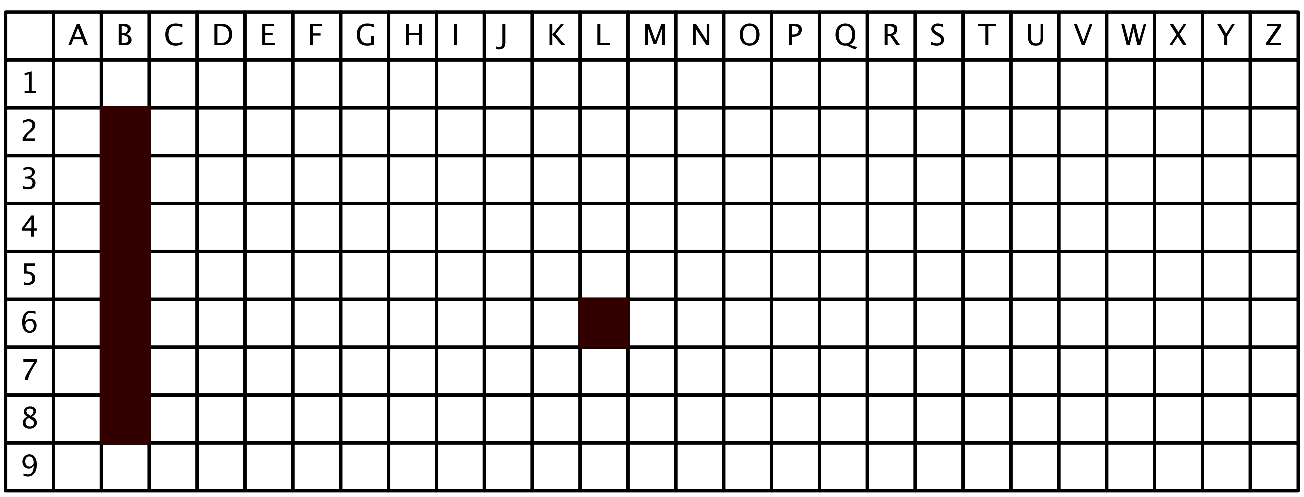
CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, April 9, 2020
Mystery Code

James wants to send an image to a friend. The image is made up of filled in black squares on a grid. The rows of the grid are labelled with numbers from 1 to 9 and the columns of the grid are labelled from A to Z. James encodes the image by providing a series of short codes. Each short code consists of four parts:

- A letter between A and Z followed by a number between 1 and 9 indicating the column and row of the first square to be filled in on the grid,
- followed by an arrow ($\uparrow, \downarrow, \leftarrow, \rightarrow$) indicating the direction to fill,
- followed by a number, indicating the total number of squares to colour.

For example, the short code $B2 \downarrow 7$ tells you to fill in a square at position $B2$ on the grid, and continue filling in 6 more squares directly below that position. And the short code $L6 \rightarrow 1$ tells you to fill in a square at position $L6$ on the grid and no other squares. This code would have you fill in the grid as shown:



Using the blank grid on the next page, determine the image that James sent using the following codes:

$T8 \uparrow 7$ $\quad D4 \leftarrow 1$ $\quad R2 \leftarrow 5$ $\quad I5 \rightarrow 3$ $\quad F2 \downarrow 7$ $\quad L8 \uparrow 6$ $\quad C3 \rightarrow 1$

$P3 \downarrow 6$ $\quad X2 \downarrow 7$ $\quad E3 \uparrow 1$ $\quad W5 \leftarrow 3$ $\quad H8 \uparrow 6$ $\quad K2 \leftarrow 3$ $\quad B2 \downarrow 7$

Use the first blank grid to create the image that James sent using the following codes:

$T8 \uparrow 7$ $\quad D4 \leftarrow 1$ $\quad R2 \leftarrow 5$ $\quad I5 \rightarrow 3$ $\quad F2 \downarrow 7$ $\quad L8 \uparrow 6$ $\quad C3 \rightarrow 1$

$P3 \downarrow 6$ $\quad X2 \downarrow 7$ $\quad E3 \uparrow 1$ $\quad W5 \leftarrow 3$ $\quad H8 \uparrow 6$ $\quad K2 \leftarrow 3$ $\quad B2 \downarrow 7$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1																										
2																										
3																										
4																										
5																										
6																										
7																										
8																										
9																										

An extra blank grid is provided below. Make your own image and provide a classmate with the code that would be used to create your image.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1																										
2																										
3																										
4																										
5																										
6																										
7																										
8																										
9																										

More Info:

Check the CEMC at Home webpage on Thursday, April 16 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 16.

Problem of the Week
Problem A and Solution
Mystery Code

Problem

James wants to send an image to a friend. The image is made up of filled in black squares on a grid. The rows of the grid are labelled with numbers from 1 to 9 and the columns of the grid are labelled from A to Z. James encodes the image by providing a series of short codes. Each short code consists of four parts:

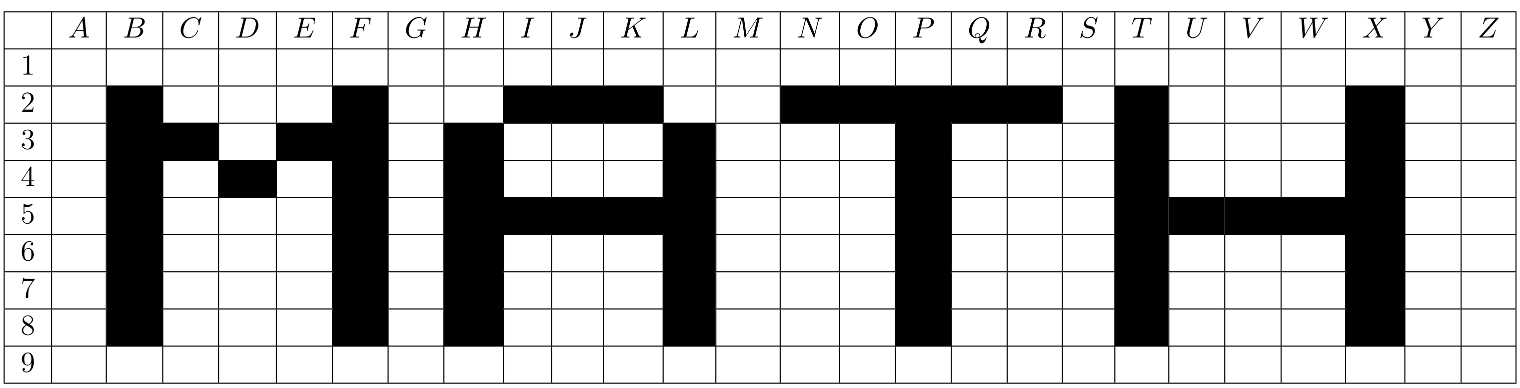
- A letter between A and Z followed by a number between 1 and 9 indicating the column and row of the first square to be filled in on the grid,
- followed by an arrow ($\uparrow, \downarrow, \leftarrow, \rightarrow$) indicating the direction to fill,
- followed by a number, indicating the total number of squares to colour.

Determine the image that James sent using the following codes:

$T8 \uparrow 7$	$D4 \leftarrow 1$	$R2 \leftarrow 5$	$I5 \rightarrow 3$	$F2 \downarrow 7$	$L8 \uparrow 6$	$C3 \rightarrow 1$
$P3 \downarrow 6$	$X2 \downarrow 7$	$E3 \uparrow 1$	$W5 \leftarrow 3$	$H8 \uparrow 6$	$K2 \leftarrow 3$	$B2 \downarrow 7$

Solution

Here is the image:



Teacher's Notes

Digital images are formed by many individual small squares called *pixels*. With images like pictures taken on a phone, the pixels are usually different shades of colour. Before email attachments and scanning became so popular, businesses used fax machines to send scanned documents. A fax was a document scanned by a source machine and then transmitted as data over phone lines to another machine at the receiving end. The transmitted image was normally black and white, like the image in this problem.

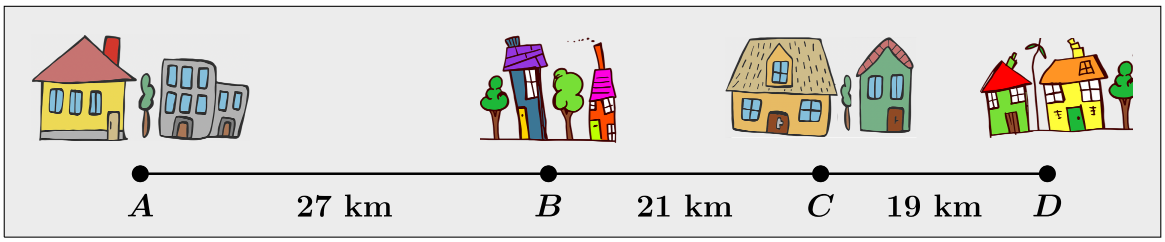
The image in this problem is formed by 63 black pixels and 171 white pixels, for a total of $9 \times 26 = 234$ pixels. We could have described the image by identifying each individual pixel as being either black or white. This would require 234 bits of information. In this problem, we used 14 codes to represent the same information. Although the codes are more complicated to understand, overall they use less data to represent the same information. This concept of using less data to represent information is known as *compression*.

This particular method of compressing data is similar to a technique known as *run-length encoding*. This technique was used by fax machines to reduce the amount of data required to transmit an image over phone lines. Although fax machines are not very popular these days, run-length encoding is still used in other applications, including managing very long gene sequences in DNA research. There are many different techniques for compressing data. You are probably familiar with the results of many of them such as **jpg**, **zip**, and **mp3** files.

CEMC at Home

Grade 4/5/6 - Tuesday, April 14, 2020
Roadmaps to Success

Question 1: Four towns, A, B, C, and D, lie in that order along a straight road. We are told that the distance from A to B is 27 km, the distance from B to C is 21 km, and the distance from C to D is 19 km. This information is shown in the *roadmap* below.

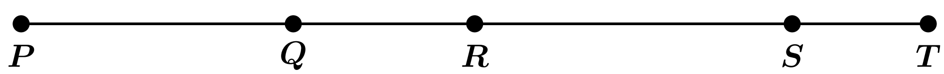


Using these distances, we can see that the distance from A to C equals the sum of the distance from A to B and the distance from B to C, which gives 27 km + 21 km = 48 km. We can keep track of the distances between each pair of towns in the chart shown below. For example, notice that the distance from A to B (27 km) is placed in the same column as A and the same row as B in the chart.

- What is the distance from B to D? Put your answer in the correct place in the chart to the right.
- What is the distance from A to D? Put your answer in the correct place in the chart to the right.

A			
27	***B***		
48	21	***C***	
__	__	19	***D***

Question 2: Five towns, P, Q, R, S, and T, lie in that order along a different straight road.



Since there are five towns, there are 10 different pairs of towns and these are listed below:

$$PQ, PR, PS, PT, QR, QS, QT, RS, RT, ST$$

Look at the chart below, which is meant to give all the distances between pairs of towns, in kilometres. Four of the distances are given: PR, PS, QS, and RT. For example, the distance from Q to S is 31 km. The six remaining distances are missing: PQ, PT, QR, QT, RS, and ST. It may be surprising to you that we can use the just the four pieces of information given here to complete the entire chart!

(a) What is the distance from R to S?

(b) What is the distance from Q to R?

(c) What is the distance from P to T?

(d) Complete the rest of the chart.

P				
__	***Q***			
29	__	***R***		
48	31	__	***S***	
__	__	29	__	***T***

To solve this problem, you might find it helpful to draw the distances given in the chart onto the roadmap above and add new distances when you find them. For example, what does the distance of 31 in the chart above represent on the roadmap? Note that the roadmap is not drawn exactly to scale.

Question 3: Suppose that five towns, U, V, W, X, and Y, lie in that order along a different straight road. Four of the distances between pairs of towns are given in the chart and six distances are missing. Unfortunately, someone made a mistake when measuring and recording the distances. Explain why it is impossible for all four of these distances to be correct.

U				
28	***V***			
102	__	***W***		
__	__	__	***X***	
__	200	125	__	***Y***

More Info:

Check the CEMC at Home webpage on Tuesday, April 21 for a solution to Roadmaps to Success.

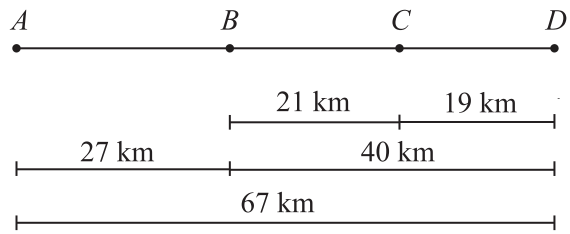
CEMC at Home

Grade 4/5/6 - Tuesday, April 14, 2020

Roadmaps to Success - Solution

1. Since the distance from B to C is 21 km and the distance from C to D is 19 km, then the distance from B to D is 21 km + 19 km = 40 km.
   Since the distance from A to B is 27 km and the distance from B to D is 40 km, then the distance from A to D is 27 km + 40 km = 67 km.

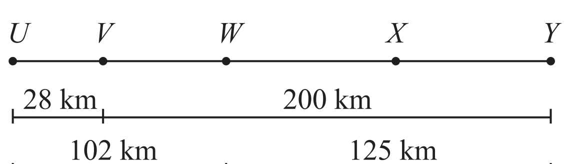
A			
27	B		
48	21	C	
67	40	19	D



2. (a) Since the distance from P to S is 48 km and the distance from P to R is 29 km, then the distance from R to S is 48 km − 29 km = 19 km.
   (b) Since the distance from Q to S is 31 km and the distance from R to S is 19 km, then the distance from Q to R is 31 km − 19 km = 12 km.
   (c) Since the distance from P to R is 29 km and the distance from R to T is 29 km, then the distance from P to T is 29 km + 29 km = 58 km.
   (d) Since the distance from P to R is 29 km and the distance from Q to R is 12 km, then the distance from P to Q is 29 km − 12 km = 17 km.
   Since the distance from P to T is 58 km and the distance from P to Q is 12 km, then the distance from Q to T is 58 km − 17 km = 41 km.
   Finally, the distance from S to T is 10 km. Can you see why?

P				
17	Q			
29	12	R		
48	31	19	S	
58	41	29	10	T

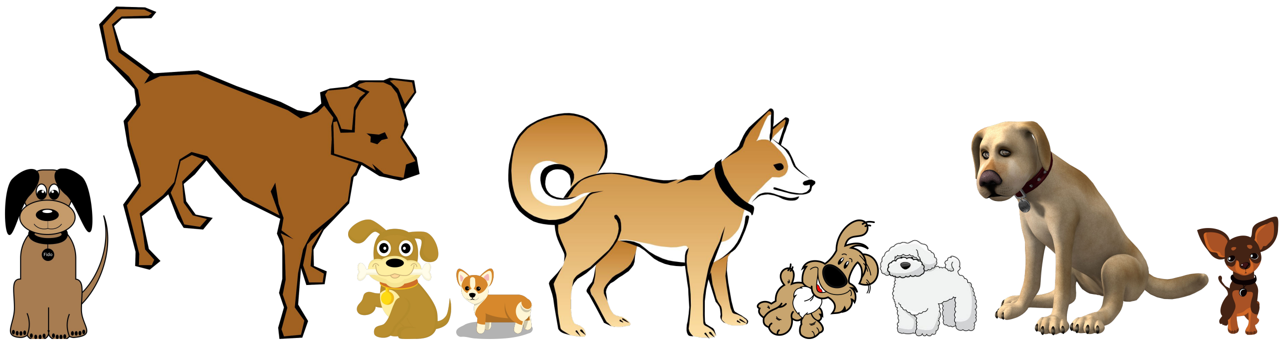
3. The distance from U to Y equals the distance from U to V plus the distance from V to Y.
   This means that the distance from U to Y equals 28 km + 200 km = 228 km.
   The distance from U to Y also equals the distance from U to W plus the distance from W to Y.
   This means that the distance from U to Y equals 102 km + 125 km = 227 km.
   We got two different answers for the distance from U to Y. This cannot be the case, so there must be a mistake in the four distances given.



CEMC at Home

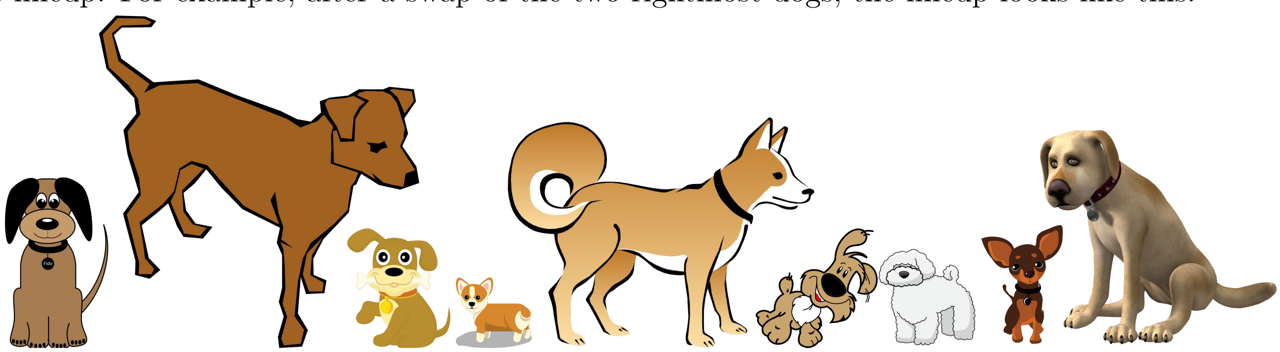
Grade 4/5/6 - Wednesday, April 15, 2020
Doggies Swapped

Consider this lineup of nine dogs of various sizes:



First lineup

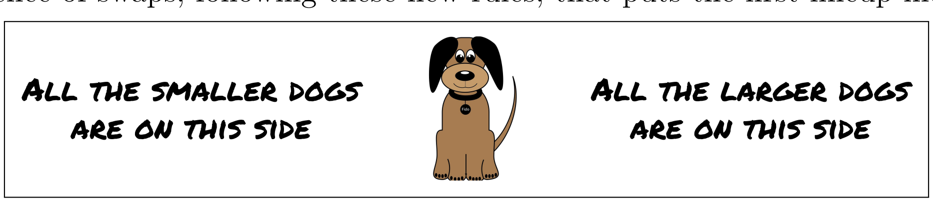
We can move dogs in the lineup by swapping them. A *swap* means two dogs exchange positions in the lineup. For example, after a swap of the two rightmost dogs, the lineup looks like this:



After one possible swap

Problem 1: The goal in this problem is to move the biggest dog (2nd from the left) to the rightmost position, and the smallest dog (4th from the left) to the leftmost position of the lineup, and to do so using the fewest swaps possible. In this problem, we can only swap two dogs that are *right beside each other*, but a dog may get swapped again after it moves into a new position in the lineup. What is the minimum number of swaps required to get the *first lineup* into this form following these rules?

Problem 2: The name of the dog in the leftmost position in the first lineup is **Spot**. The goal in this problem is to rearrange the dogs so that all dogs that are smaller than **Spot** are to **Spot**'s left and all dogs that are larger than **Spot** are to **Spot**'s right. (Otherwise, the dogs can be in any order.) In this problem, we can swap two dogs in *any positions* in the lineup, but each dog can be involved in *at most one swap*. For example, we could choose to swap the dog in the first position with the dog in the last position in the lineup, but then neither dog can be swapped again later. Can you find a sequence of swaps, following these new rules, that puts the first lineup into this form?



More Info:

Check out the CEMC at Home webpage on Wednesday, April 22 for a solution to Doggies Swapped.

CEMC at Home

Grade 4/5/6 - Wednesday, April 15, 2020
Doggies Swapped - Solution

Problem 1: The goal in this problem is to move the biggest dog (2nd from the left) to the rightmost position, and the smallest dog (4th from the left) to the leftmost position of the lineup, and to do so using the fewest swaps possible. In this problem, we can only swap two dogs that are *right beside each other*, but a dog may get swapped again after it moves into a new position in the lineup. What is the minimum number of swaps required to get the *first lineup* into this form following these rules?

Solution: Here is one way get the lineup into this form using exactly 9 swaps that follow the rules:

Remember that we can only swap two dogs that are right beside each other.

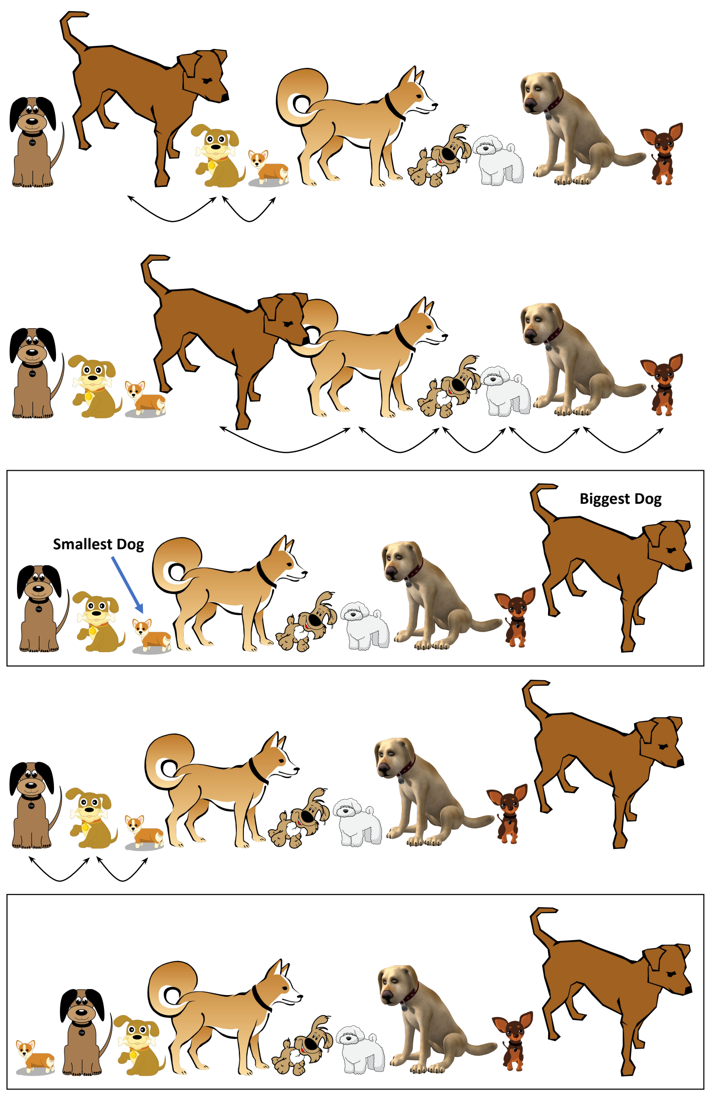
To get the biggest dog to the rightmost position, we could swap the biggest dog, again and again, with the dog immediately to its right, until the biggest dog makes it all the way to the rightmost position of the lineup.

Since the first lineup has 7 dogs to the right of the biggest dog, we can move the biggest dog to the rightmost position using 7 swaps in a row.

Notice that after we have finished moving the biggest dog in the way outlined above, the smallest dog has ended up 3rd from the left in the new lineup. This means we can move the smallest dog to the leftmost position by making 2 more swaps.

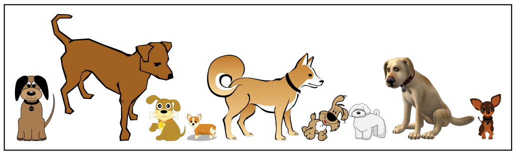
We swap the smallest dog with the dog immediately to its left, and then repeat this once more.

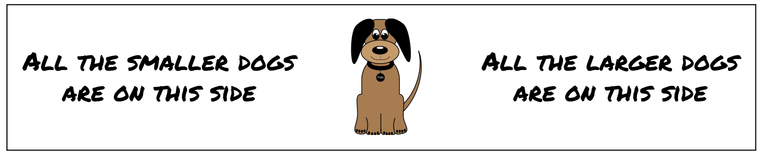
Finally, after these 9 swaps, we end up with the lineup of dogs to the right which is in the correct form.



You may have thought about this question long enough to be convinced that you cannot achieve the goal in fewer than 9 swaps. It turns out that 9 swaps is the best we can do. Can you explain why? Why can it not be done in 8 (or fewer) swaps? Think about how many swaps the biggest dog must take part in (at least 7), the number of swaps the smallest dog must take part in (at least 3), and how many of these swaps could involve *both* the biggest dog and the smallest dog.

Problem 2: The name of the dog in the leftmost position in the first lineup is **Spot**. The goal in this problem is to rearrange the dogs so that all dogs that are smaller than **Spot** are to **Spot**'s left and all dogs that are larger than **Spot** are to **Spot**'s right. In this problem, we can swap two dogs in *any positions* in the lineup, but each dog can be involved in *at most one swap*. Can you find a sequence of swaps, following these new rules, that puts the first lineup into this form?





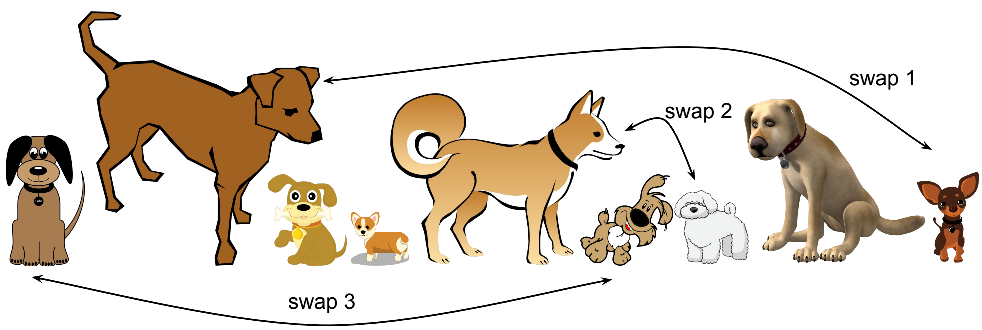
Solution:

Here is one way get the lineup into this form using exactly 3 swaps that follow the rules:

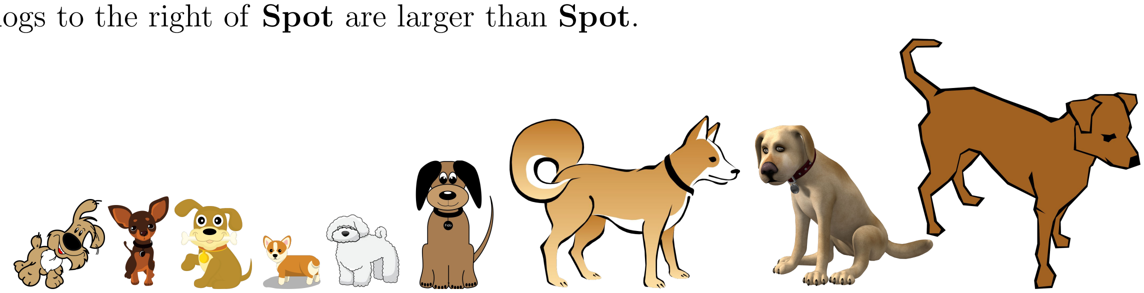
In the starting lineup, find the dog furthest to the right that is smaller than **Spot** and the dog furthest to the left that is bigger than **Spot**. Swap these two dogs (swap 1).

In the new lineup (after swap 1), find the dog furthest to the right that is smaller than **Spot** and the dog furthest to the left that is bigger than **Spot**. Swap these two dogs (swap 2).

In the new lineup (after swap 2), find the dog furthest to the right that is smaller than **Spot**. Swap **Spot** and this dog (swap 3).



Notice that each dog is involved in at most one of these three swaps. After these three swaps, we get the following lineup of dogs where all of the dogs to the left of **Spot** are smaller than **Spot** and all of the dogs to the right of **Spot** are larger than **Spot**.



Computer Science Connections:

In Computer Science, it is important to be able to sort lists of data, that is, arrange lists into a meaningful order. There are many techniques (also known as *algorithms*) for sorting lists. Some algorithms are better than others.

One straightforward algorithm to sort lists is known as *bubble sort*. This technique passes over the list of values multiple times. On each pass, pairs of values that are beside each other are compared. The algorithm starts by comparing the first two values, then moves on to the second and third values, then the third and fourth values, and so on. If the values in a pair are already in the correct order compared to each other, then nothing is done. If the values in the pair are out of order compared to each other, then they are swapped. After the first pass over the list, you are guaranteed to have the largest value at the end of the list. After multiple passes, you end up with a sorted list. This tends to be a very slow way to sort a list.

Another algorithm to sort lists is known as *quicksort*. This technique chooses a value known as a *pivot*, and then swaps values around until the pivot ends up in a spot in the list where all values before the pivot are smaller than the pivot and all values after the pivot are larger than the pivot. Next, the list is split into two parts: the part of the list before the pivot and the part after the pivot. The process is then repeated on each of these smaller lists. The process of moving pivots and splitting lists is repeated until the list is sorted. In most cases, this algorithm is much faster than bubble sort.

There are many other ways you can sort a list. Can you come up with your own algorithm?

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, April 16, 2020

What's My Number?

I am a 3-digit even number.

The sum of my three digits is 20.

I am greater than 40×10.

I am less than $1000 \div 2$.

What number am I?

More Info:

Check the CEMC at Home webpage on Thursday, April 23 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 23.

This CEMC at Home resource is the current grade 3/4 problem from Problem of the Week (POTW). This problem was developed for students in grades 3 and 4, but is also appropriate for students in grades 5 and 6. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 5/6 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem A and Solution
What's My Number?

Problem

I am a 3-digit even number.

The sum of my three digits is 20.

I am greater than 40×10.

I am less than $1000 \div 2$.

What number am I?

Solution

Since $40 \times 10 = 400$, we know the number is greater than 400. Since the number is even, the smallest possible number is 402.

Since $1000 \div 2 = 500$, we know the number is less than 500. Since the number is even, the largest possible number is 498.

So we are looking for an even number between 402 and 498, inclusive. We could check each of the numbers in that range, to see which one has digits that add up to 20. However, that would mean checking 49 numbers. It would be better to reduce the range of numbers to check, if possible.

Here is one way of thinking about the solution:

Since the possible numbers are from 402 to 498, the first digit of the number is 4. Since the sum of the digits is 20, then the middle and last digit must sum to $20 - 4 = 16$. Since the number is even, its last digit is 0, 2, 4, 6, or 8.

If the last digit is 0, then the middle digit must be $16 - 0 = 16$, which is not a digit from 0 to 9.
If the last digit is 2, then the middle digit must be $16 - 2 = 14$, which is not a digit from 0 to 9.
If the last digit is 4, then the middle digit must be $16 - 4 = 12$, which is not a digit from 0 to 9.
If the last digit is 6, then the middle digit must be $16 - 6 = 10$, which is not a digit from 0 to 9.
If the last digit is 8, then the middle digit must be $16 - 8 = 8$, which is a valid digit.

We have examined all possible cases and the only number satisfying all of the conditions is 488.
The number we are looking for is 488.

Teacher's Notes

Here is another way of thinking about the solution:

> Since the numbers from 402 to 498 all start with the digit 4, and the sum of all three digits must be 20, then the sum of the last two digits of the number must be $20 - 4 = 16$.
>
> Since the largest possible digit is 9, then the other digit of the number must be at least $16 - 9 = 7$. So each of the last two digits must be in the range 7 to 9.
>
> Since we are only considering even numbers, then the solution must be a number that ends with an 8.
>
> So we can look at even numbers, that start with a 4, end with an 8, and where the middle digit is at least 7. The possibilities are: 478, 488, and 498. Now we can check the sum of the digits of these numbers:
>
> - $4 + 7 + 8 = 19$
> - $4 + 8 + 8 = 20$
> - $4 + 8 + 9 = 21$

So the only number that satisfies all of the requirements is 488.

CEMC at Home

Grade 4/5/6 - Friday, April 17, 2020
Toothpick Triangles

In this activity, we will explore how many different triangles we can make if our triangles can only have side lengths that are equal to whole number factors of the number 24.

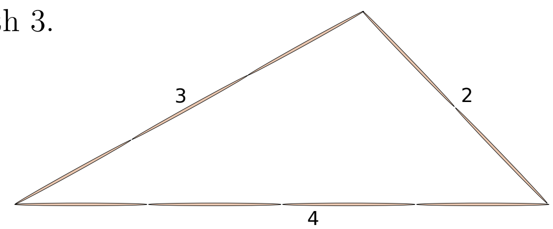
The whole number *factors* of any whole number N are the whole numbers which divide evenly into N. For example, 12 has six whole number factors. They are 1, 2, 3, 4, 6, and 12.

You Will Need:

- A pencil and a piece of paper
- A flat surface to work on
- Around 30 toothpicks

How To Play:

1. Make a list of all whole number factors of 24.
2. Think of each toothpick as having a length of 1 unit. We will line up different numbers of toothpicks in straight lines, end-to-end, to make sides of triangles. If a side is made up of 3 toothpicks, then we will say this side has length 3.
3. Choose three different factors of 24. Using the toothpicks, try and create a triangle that has side lengths equal to these three factors of 24.



   For example, think about the three factors 2, 3 and 4. You will need 2 toothpicks to form a side of length 2, 3 toothpicks to form a side of length 3, and 4 toothpicks to form a side of length 4. These three sides can be arranged on a flat surface to form a triangle as shown above. If you start with the factors 2, 3, and 6 instead, a triangle cannot be formed. Can you figure out why?

4. Form as many different triangles as you can which have side lengths equal to three different factors of 24. Make a list of the side lengths of each of the triangles you form.

   You will have to play with the positions of the toothpick sides to see whether they fit together to form a triangle. You might find it helpful to tape the toothpicks together while you work with a particular side length.

Follow-up Questions:

- There are groups of three factors of 24 that *cannot* be used to form a triangle. Did you discover some of these groups of factors? Why could you not use them to form triangles?
- Think about what you discovered while exploring the factors of 24. Can you find three other numbers between 10 and 50 with factors that can be used to form many different triangles? Are there numbers for which no triangles can be formed?

  If you are allowed to use the same factor more than once to make the side lengths, then you can always make triangles! (Why?) If we want three different factors then it is not always possible.

More info:

Check out the CEMC at Home webpage on Friday, April 24 for a solution to Toothpick Triangles.

CEMC at Home

Grade 4/5/6 - Friday, April 17, 2020
Toothpick Triangles - Solution

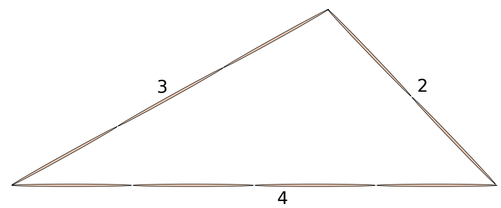
Activity: Using toothpicks, each with a length of 1 unit, form as many different triangles as you can which have side lengths equal to three different factors of 24.

Discussion: The number 24 has eight whole number factors: 1, 2, 3, 4, 6, 8, 12, 24.

If we choose three different factors from this list, then we may or may not be able to form a triangle having these side lengths. Here are some examples:

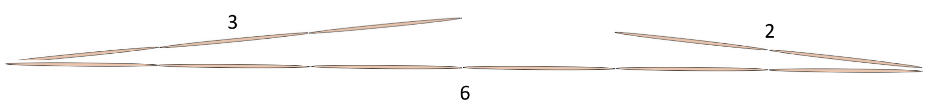
A triangle <u>can</u> be formed using the factors 2, 3, and 4.

If we form sides using 2, 3 and 4 toothpicks, we can arrange them so that each pair of sides meets at a point and a triangle is formed.



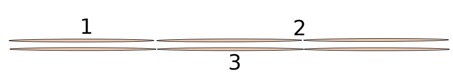
A triangle <u>cannot</u> be formed using the factors 2, 3, and 6.

If we form sides using 2, 3 and 6 toothpicks, then we cannot arrange the sides so that each pair of sides meets at a point. The sides having 2 and 3 toothpicks must touch the ends of the side with 6 toothpicks, but they also must tough each other. As the picture shows, the sides 2 and 3 are not long enough to do so. Since $2 + 3$ is less than 6, we cannot "close" the triangle.



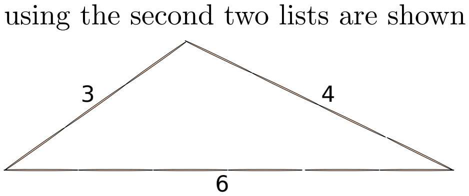
A triangle <u>cannot</u> be formed using the factors 1, 2, and 3.

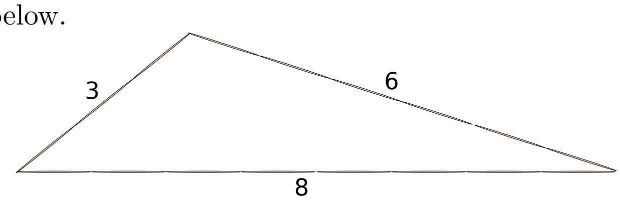
The sides having 1 and 2 toothpicks must touch the ends of the side with 3 toothpicks, but they also must touch each other. As the picture shows, the only way to do this is to lay the toothpicks directly on top of each other, but this does not form a triangle. Since $1 + 2$ is equal to 3 we cannot form a triangle.



Triangles can only be formed if you start with one the five lists of factors in the box shown on the right.

We have already shown that the first list of factors works, and the triangles formed using the second two lists are shown below.





2, 3, 4
3, 4, 6
3, 6, 8
4, 6, 8
6, 8, 12

The triangles that are formed using the last two lists are just scaled versions of two triangles shown earlier, with sides lengths all doubled. *Which ones?*

No other choices of three factors will allow you to form triangles. *Can you figure out why?*

It turns out that the following is true:

Any group of three factors for which one of the numbers is greater than or equal to the sum of the other two numbers cannot be used to form a triangle. Otherwise, a triangle can be formed.

- The list 1, 2, 3 *cannot* be used to form a triangle because 3 is equal to 1 + 2.
- The list 2, 3, 6 *cannot* be used to form a triangle because 6 is greater than 2 + 3.
- The list 2, 3, 4 *can* be used to form a triangle because 2 + 3 is greater than 4, 3 + 4 is greater than 2, and 2 + 4 is greater than 3.

Follow-up Questions:

Here are some facts about the factors of the numbers between 10 and 50:

- If you work with the factors of 12, 20, 24, 30, 36, 40, 42, or 48, then you can make at least one triangle. The factors of all other numbers result in no triangles.

  Since the numbers 12, 36, and 48 also have factors 1, 2, 3, 4, and 6 (among other factors), we know from our earlier work with the factors of 24 that we can form triangles using these factors (and possibly using others as well).

  If you use the factors of a prime number, like 11, then since there are only two factors to work with (1 and 11), you have no chance at making a triangle following the rules. However, if you allow for triangles with equal side lengths, then you can always make triangles! For example, you can make an equilateral triangle with side length 1.

- You can make the most triangles if you work with the factors of 48. The next highest number comes from using the factors of 36, and then 24 (which we worked with).
- You can make only one triangle if you work with the factors of 20.

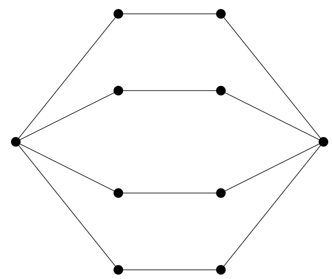
CEMC at Home

Grade 4/5/6 - Monday, April 20, 2020
Tag, and That's It!

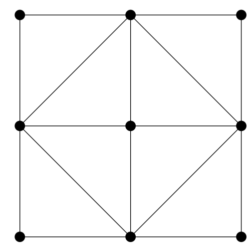
In this activity, we will play a game of tag on a graph!

You Will Need:

- Two players
- A piece of paper and a pencil
- Two counters
  A different small object for each player.



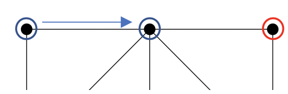
Board 1



Board 2

How to Play:

1. Choose one of the two game boards shown above (Board 1 or Board 2) for the game. Notice that each board consists of dots and line segments drawn between certain pairs of dots.
   Larger versions of these game boards are provided on the next page.
2. Players alternate turns. Decide which player will go first (Player 1) and which player will go second (Player 2). Just like a game of tag, Player 1 is "it", and Player 2 must avoid being caught by Player 1.
3. On the first turn, Player 1 puts their counter on any dot they wish. Next, Player 2 puts their counter on any *other* dot on the game board.
4. Next, Player 1 can move their counter from their current dot to another dot by following a single line segment on the game board. Player 1 can also choose to "pass", and not move their counter at all. Player 2 then moves according to the same rules.
   For example, on Board 2, a player can move from the top left dot to the top middle dot on a single turn, but cannot move from the top left dot to the top right dot, because that means moving across two line segments.

   
5. On all remaining turns, Player 1 and Player 2 take turns moving their counter following the rules outlined in 4. At all times, Player 1 is trying to catch Player 2, and Player 2 is trying to stay away from Player 1.
6. Player 1 can "catch" Player 2 by occupying the same dot as Player 2. If this happens, then Player 1 wins. If Player 1 is unable to catch Player 2 and gives up, then Player 2 wins.

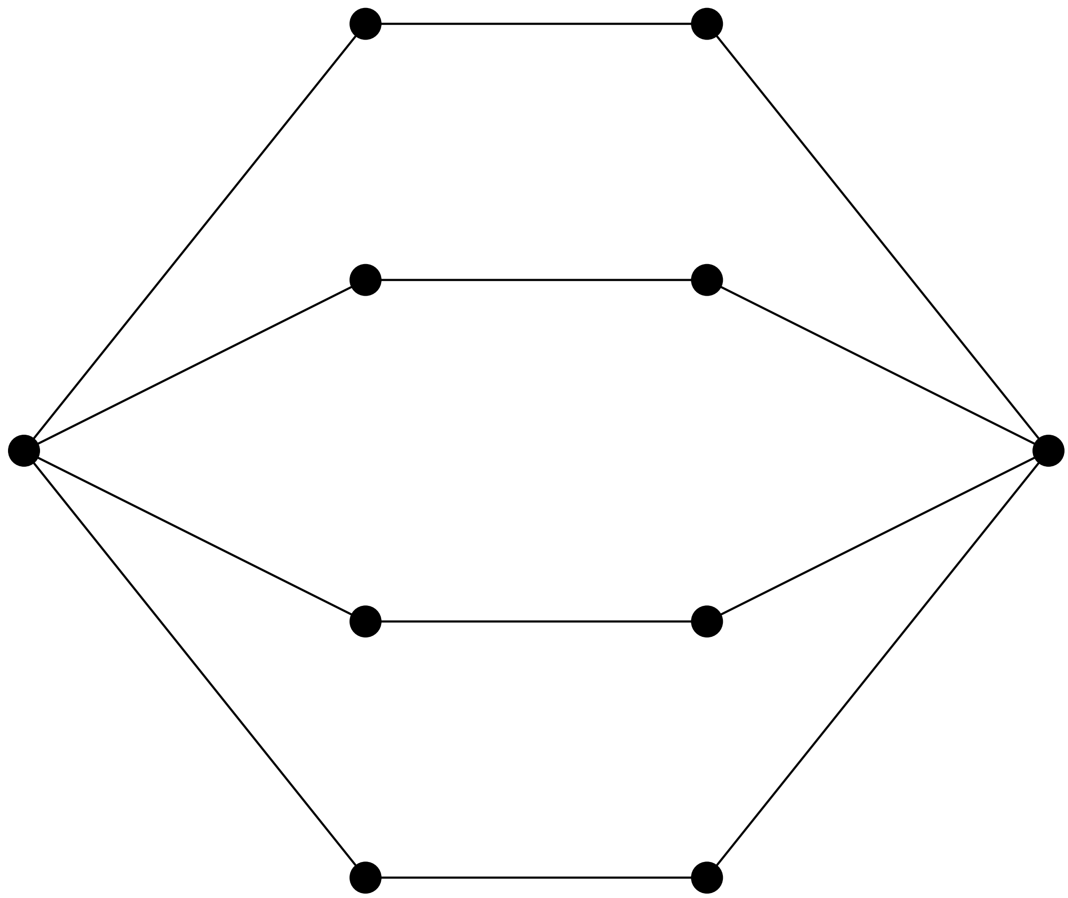
Play this game a number of times using each of the game boards (Board 1 and Board 2).
Alternate who goes first and who goes second. As you play, think about the following questions:

- Who seems to win most often: Player 1 or Player 2? For each of the game boards (Board 1 and Board 2), can you come up with a strategy that will allow you to win every time?
- The game boards for this game are called *graphs*. A graph is made up of dots (called *vertices*), along with lines (called *edges*) that connect certain pairs of vertices. Can you build a new game board (or graph) which gives Player 1 an advantage in the game? What about Player 2?

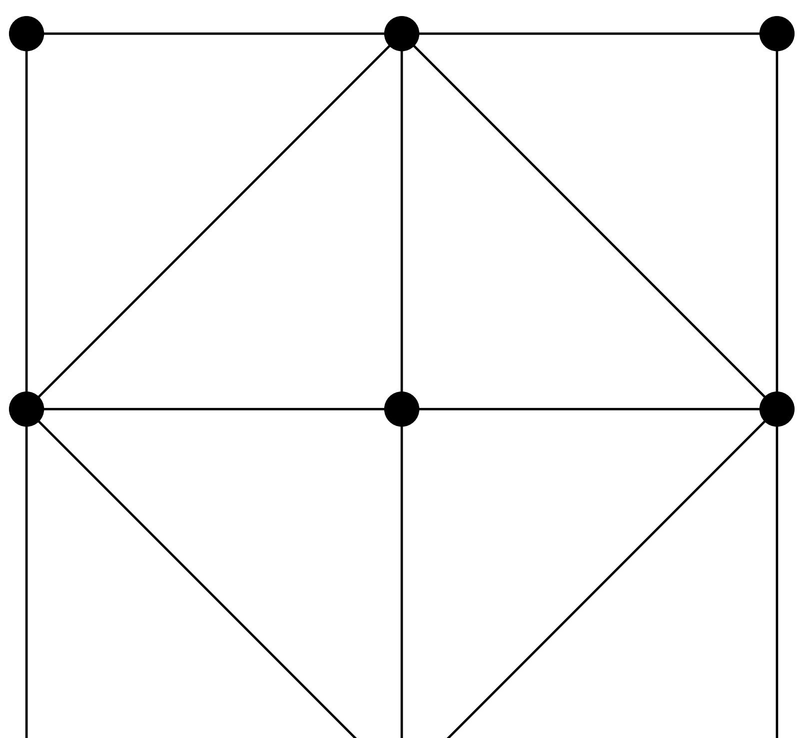
More Info:

Check the CEMC at Home webpage on Monday, April 27 for a discussion of Tag, and That's It!.

Board 1



Board 2



CEMC at Home

Grade 4/5/6 - Monday, April 20, 2020
Tag, and That's It! - Solution

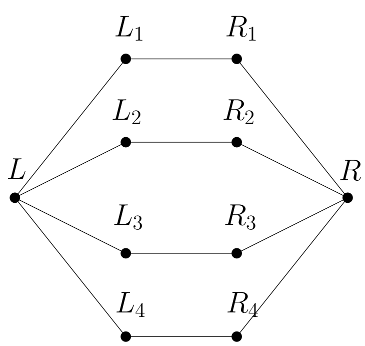
You may have noticed while playing the game that if you play on Board 1, then there is always a way for Player 2 to avoid Player 1 for the entire game. Also, if you play on Board 2, then there is a way for Player 1 to catch Player 2 at some point in the game. We will explain the strategy for each board.

How can Player 2 avoid getting caught by Player 1 on Board 1?

Player 1 has ten different choices for where to place their counter at the start of the game. Player 2 should then place their counter as far as possible from Player 1's counter. This means placing their counter so that it is three line segments away from Player 1's counter.

Can you see why? You can get from one dot to any other dot by crossing at most 3 line segments.

For example, if Player 1 places their counter at R, then Player 2 can place their counter at L. One way for Player 2 to place their counter (based on Player 1's placement) is given in the table below.



Player 1's Vertex	Vertex Player 2 Should Choose
R	L
R_1	L_2
R_2	L_1
R_3	L_4
R_4	L_3
L_1	R_2
L_2	R_1
L_3	R_4
L_4	R_3
L	R

The table above also tells Player 2 how to move in order to stay away from Player 1 throughout the game. For example, if Player 1 moves from R to R_1 on their first move of the game, then Player 2 should move from L to L_2. We see that R_1 and L_2 are paired in the second row in the table above and so this is the right move for Player 2.

Notice that after these two moves, Player 2 is again three line segments away from Player 1.

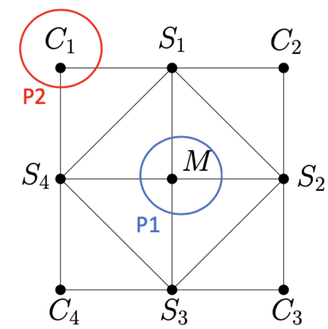
Since Player 2 can always make sure to be three line segments away from Player 1 (at the end of Player 2's turn), Player 1 can never catch Player 2 if Player 2 uses this strategy!

See the next page for a discussion of Board 2.

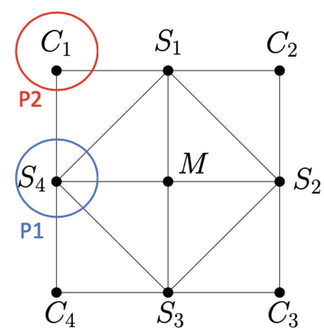
How can Player 1 catch Player 2 on Board 2?

One strategy is for Player 1 to start by placing their counter on the middle dot, marked with M for "middle".

Once Player 1 is occupying M, the only safe move for Player 2 is to occupy one of the corner dots, labelled C_1, C_2, C_3 and C_4. (C is for "corner"). If Player 2 starts anywhere else, then Player 1 can catch Player 2 on the first move. Let's say Player 2 places their counter on C_1 as shown in the diagram on the right.



From M, Player 1 can then move to one of the two dots connected to C_1, labelled S_1 and S_4 in the diagram to the right. Let's say Player 1 moves to S_4.

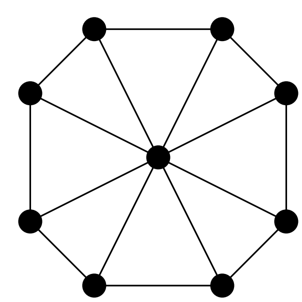


Player 2 now has the choice to pass or to move to S_1. No matter how Player 2 moves, Player 1 can catch Player 2 on the next move and win the game.

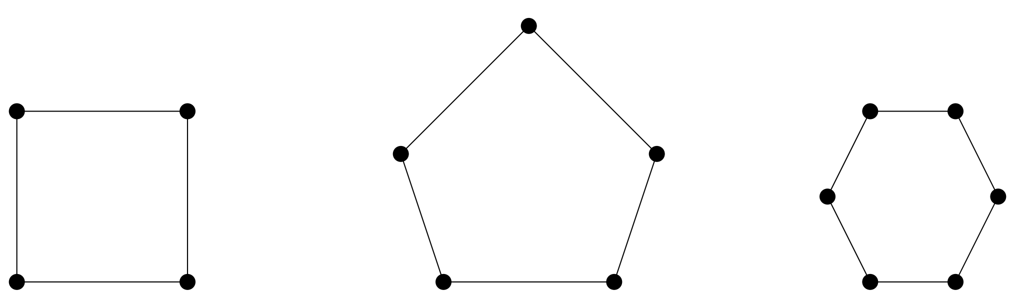
(The strategy is similar if Player 2 moves to S_1 instead, or chooses an entirely different corner to start the game.)

Can you build a new game board (or graph) which gives Player 1 an advantage in the game? What about Player 2?

One easy way to guarantee Player 1 can always win is to include a *universal vertex* in the graph. This is a vertex that is connected to every other vertex by an edge. The "wheel" game board shown on the right gives an example where the vertex in the centre of the wheel is the universal vertex. If Player 1 occupies the universal vertex, they can catch Player 2 on the next turn, no matter where Player 2 goes.



On the other hand, if the game board is a *cycle* of four or more vertices, then Player 2 will always be able to win. Examples of cycles of 4, 5, and 6 vertices are given below to illustrate. The idea is that Player 2 should always stay on the "opposite side" of the cycle from Player 1, keeping as many vertices between the two players as possible. When the cycle has at least 4 vertices, Player 2 will always keep at least two vertices away from Player 1, and so Player 1 will have to give up.



What other examples can you come up with?

CEMC at Home

Grade 4/5/6 - Tuesday, April 21, 2020
Four Square and Ten

Problems:

(a) Using only the four digits 1, 2, 3, and 4, place a digit in each blank square in the grid to the right so that every row, column, *and* diagonal in the grid uses each of the four digits exactly once.

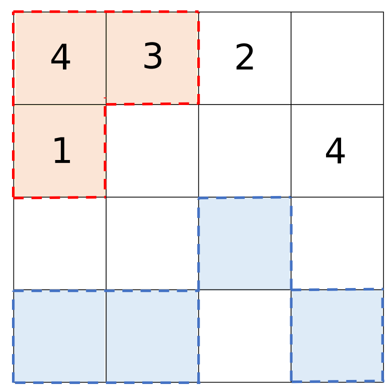
4	3	2	
1			4

(b) Notice that when you add up all 16 digits in your solution grid from part (a), you get a total of 40. In other words, the *sum* of all of the digits in the grid is 40.

Since $40 = 5 \times 8$, it may be possible to divide your solution grid into 5 groups of squares for which the sum of the digits in each group of squares is 8. Can you divide your solution grid from part (a) into 5 groups of squares so that

- the sum of the digits in the squares in each group is 8, *and*
- the squares in each group are *connected*?

A group of squares is connected if each square in the group is touching another square in the group. Squares can touch by sharing an edge or by sharing a vertex as shown in the picture to the right.



Extra Challenges:

1. Can you find more than one solution for part (b)? How many different solutions can you find?

2. Notice that the digits in each row, column, and diagonal in your solution grid from part (a) add to 10. Complete the starting grid from part (a) again, using only the four digits 1, 2, 3, 4, but following these new rules:
   - Each digit can be used more than once in each row, column, or diagonal.
   - The digits in each row, column, and diagonal in your grid must add to 10.

   In how many different ways can you complete the starting grid from part (a) according to these new rules? Are there more possible solutions here than there were for part (a)?

More info:

Check out the CEMC at Home webpage on Tuesday, April 28 for a solution to Four Square and Ten.

CEMC at Home

Grade 4/5/6 - Tuesday, April 21, 2020
Four Square and Ten - Solution

(a) Choosing the correct digits for the blank squares can be done following the steps below. Which squares are filled in during each step is indicated in the diagram by placing the corresponding step numbers in the upper left corner of the squares. *Note that there is only one way to complete the grid following the rules.*

Step 1. Since each row must use each of the digits 1, 2, 3, and 4 exactly once, the last digit in the first row must be 1.

Step 2. The middle two digits of the second row must be 2 and 3, in some order, since 1 and 4 already appear in this row. Since each column also needs to use each of the four digits exactly once, by looking at the first row, we see that 2 must be placed in the second column and 3 must be placed in the third column.

Step 3. The diagonal from lower left to upper right already has a 1 and a 3, so the digit in the third row and second column must be either a 2 or a 4. Since there is already a 2 in the second column, we must place a 4 in this square.

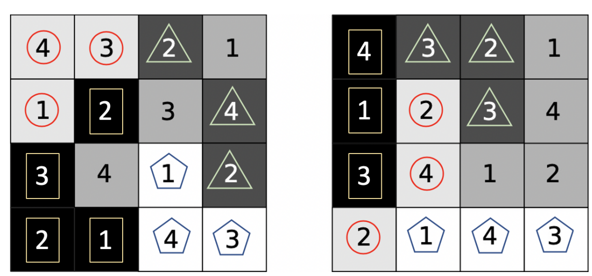
4	3	2	$^{1.}$ 1
1	$^{2.}$ 2	$^{2.}$ 3	4
$^{4.}$ 3	$^{3.}$ 4	$^{5.}$ 1	$^{5.}$ 2
$^{4.}$ 2	$^{4.}$ 1	$^{5.}$ 4	$^{5.}$ 3

Step 4. We now know that the remaining square in the diagonal we worked with in Step 3. must be filled with a 2. This means that the square directly above this 2 must be filled with a 3 (to complete the first column) and the square to the right must be filled with a 1 (to complete the second column).

Step 5. The square in the third row and third column must be filled with a 1 since the third column already has a 2 and a 3, and the third row already has a 4. Now, the only way to complete the diagonal from top left to bottom right is to place a 3 in the square in the bottom right corner. Finally, we see that we have to place a 2 and a 4 as shown to complete the grid.

(b) In the figure to the right, you will see two different ways to divide the solution grid from part (a) into 5 groups of connected squares with the sum of the digits in each group equal to 8.

Are there any other ways to divide the grid?



Extra Challenges:

1. See above for two different ways to divide the grid from part (a) following the rules given in part (b).
2. One possible solution is shown on the right.
   How many different solutions can you find?
   Try to divide these grids as in part (b).

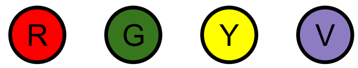
4	3	2	1
1	2	3	4
1	2	3	4
4	3	2	1

CEMC at Home

Grade 4/5/6 - Wednesday, April 22, 2020

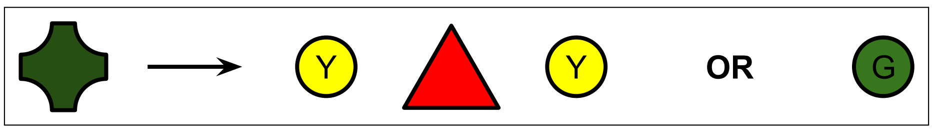
Jessie's Jewelry

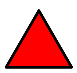
Introduction: Jessie likes to make bracelets and necklaces using four different types of coloured beads.



Although the patterns of the beads in the jewelry may look random, Jessie actually follows strict rules to make the jewelry. Each rule mentions *shapes* and *beads*, and explains how to *replace* a certain shape with a new pattern of shapes and beads.

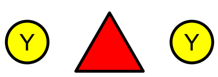
For example, one of Jessie's replacement rules for making jewelry is shown below.

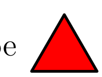


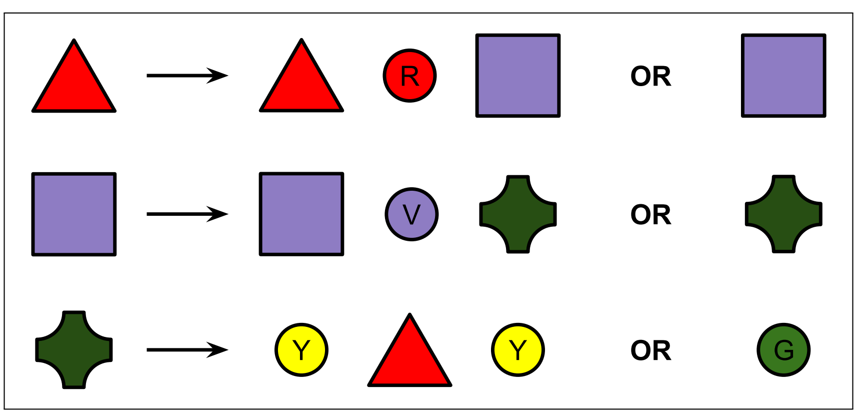
The symbols  and  in the rule represent *placeholder shapes*.

The symbols  and  represent two of the four different types of *beads*.

The rule shown above says that whenever Jessie sees the shape  in a pattern, this shape can be replaced with one of two different things:

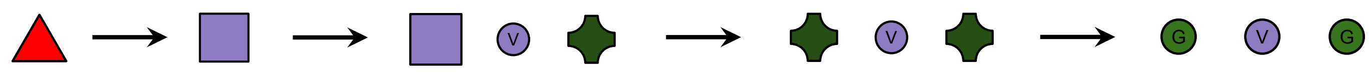
- the pattern , or
- the single bead .

When making a new piece of jewelry, Jessie always starts out with the single placeholder shape  and then uses the three replacement rules shown below.



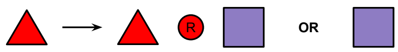
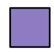
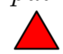
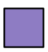
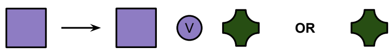
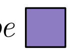
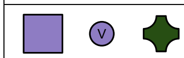
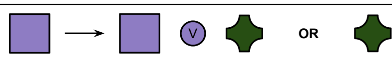
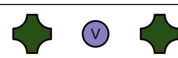
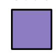
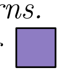
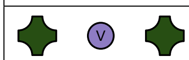
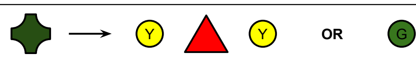
Jessie applies these rules until a final pattern consisting of *only beads* is reached.

Example: Let's see Jessie's rules in action in an example. Here is an outline of one of Jessie's attempts to create a simple piece of jewelry.



Notice how Jessie started with the shape  and ended with a pattern of only beads.

See the table below for an explanation of how this final pattern of beads was created using the rules.

Current Stage of Pattern	Rule Applied	Next Stage of Pattern	Explanation
			*Jessie had two choices for what pattern to substitute for the shape*  *and chose the second option.*
			*Jessie had two choices for what pattern to substitute for the shape*  *and chose the first option.*
			*Jessie had many choices here. Jessie could replace the*  *with one of two patterns and/or replace the*  *with one of two patterns. Jessie chose to apply the rule for*  *only and chose the second pattern option this time.*
			*Jessie had two choices for what pattern to substitute for each of the*  *shapes and chose the second option for each of the two*  *shapes.*

Problems:

1. Start with a  shape and work through the rules yourself. Make different choices than Jessie did in the example above, and create a different pattern of beads than the one shown above.

2. Explain how Jessie can start with a  shape and apply the rules to end up with the following final pattern of beads:

3. No matter how Jessie chooses to apply the rules, there is no way to create the final pattern of beads shown below by starting with a  shape. Can you explain why Jessie cannot create this final pattern?

More Info:

Check out the CEMC at Home webpage on Wednesday, April 29 for a solution to Jessie's Jewelry.

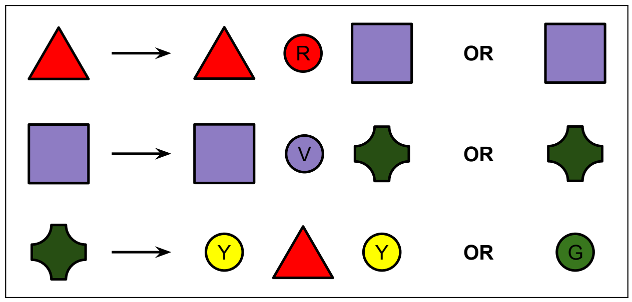
See the next page for an extra challenge problem to try!

Extra Challenge: Which of the following patterns of beads can be made according to Jessie's rules? Explain your answers.

CEMC at Home

Grade 4/5/6 - Wednesday, April 22, 2020
Jessie's Jewelry - Solution

Summary of Jessie's Jewelry: When making a new piece of jewelry, Jessie always starts out with the single placeholder shape and then uses the three replacement rules shown below.



Jessie applies these rules until a final pattern consisting of *only beads* is reached.

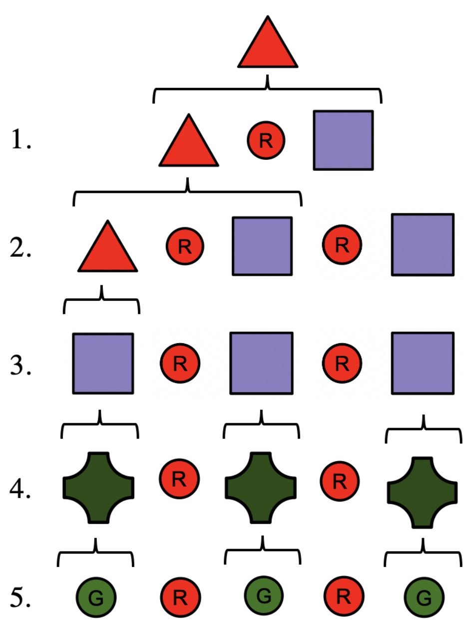
Problems and Solutions:

(a) Start with a shape and work through the rules yourself.

Solution: There are many different possible patterns you can make here. You will see a few patterns that can be made while reading the solutions that follow.

(b) Explain how Jessie can start with a shape and apply the rules to end up with the following final pattern of beads:

Solution: Here is one possible way to end up with the pattern above:



Step 1. The is replaced with the pattern .

Step 2. The is replaced with the pattern .

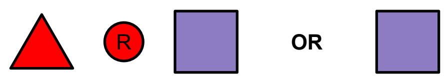
Step 3. The is replaced with a .

Step 4. Each is replaced with a .

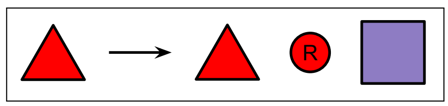
Step 5. Each is replaced with a .

(c) No matter how Jessie chooses to apply the rules, there is no way to create the final pattern of beads shown below by starting with a shape. Can you explain why Jessie cannot create this final pattern?

Solution: Jessie starts with a single . In the first step, Jessie must replace this with one of two things:



Suppose that Jessie replaced the with the first option:

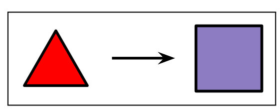


We know that we need to get a V bead immediately to the right of the R bead.

Looking at the rules, the only way this could possibly happen is if a shape appearing to the right of the R bead is replaced with the pattern V at some point. But this would mean there would have to be at least one object between the R bead and the V bead (no matter what replaces the shape in the pattern V).

This tells us that this strategy cannot possibly produce the correct piece of jewelry since we cannot get the R V part of the pattern.

Now suppose that Jessie instead goes with the second option at the beginning:



This means that the only way Jessie could introduce a R bead into the pattern is to reintroduce a shape at some point. Looking at the rules, the only way to introduce a shape would be to replace a shape with the Y Y pattern.

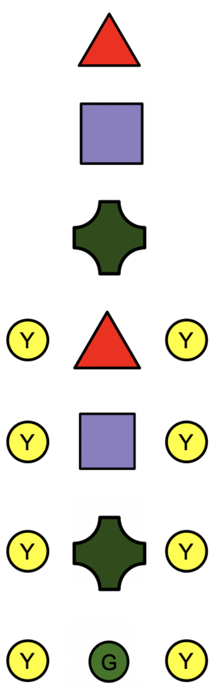
But this means the only way we could get a R bead into the pattern is to have it appear between the two Y beads.

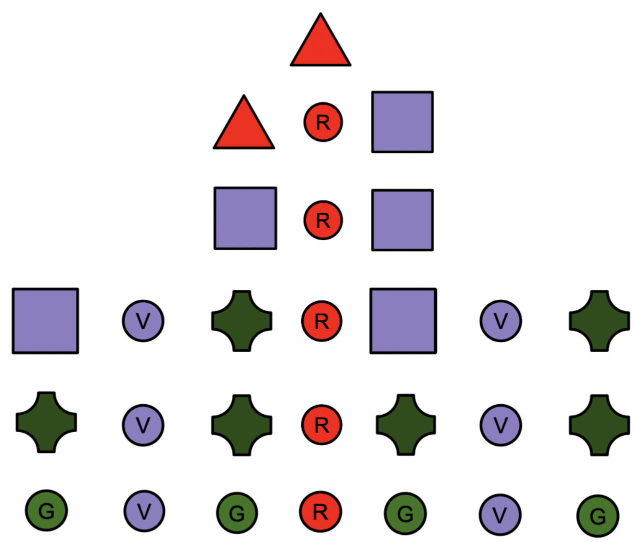
This tells us that this strategy cannot possibly produce the correct piece of jewelry either since the piece of jewelry has the R bead to the left of both Y beads.

Now we have reached a problem: there are no other strategies left to try! This means we can be sure that Jessie cannot make this piece of jewelry following these rules.

Extra Challenge: Which of the following patterns of beads can be made according to Jessie's rules? Explain your answers.

Solution: All three patterns can be made using the rules. We outline the intermediate patterns for the first two pieces of jewelry below, and we leave the third piece of Jewelry for you to think about on your own.





Computer Science Connections:

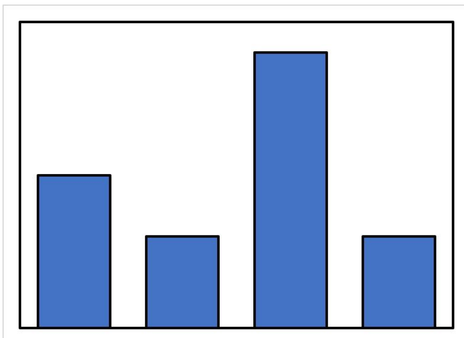
The rules in this problem are similar to rules that are used to describe a *context-free grammar*. In Computer Science we use context-free grammars to define the rules of programming languages.

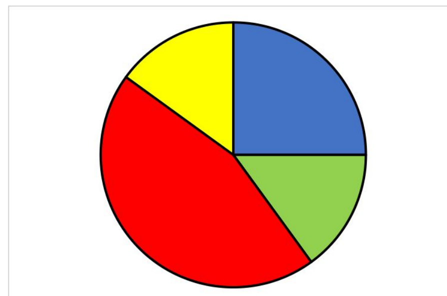
The rules in this problem actually describe simple mathematical expressions. If you substitute $+$ or $-$ operators where you see R, and you substitute $\times$ or $\div$ where you see V, and you substitute matching parenthesis where you see a pair of Y beads surrounding a triangle, then you will see that the sequences that are valid for Jessie's jewelry are also valid mathematical expressions. The sequence that does not follow the rules is not a valid mathematical expression.

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, April 23, 2020
Lost Data

Tanner randomly surveyed 40 students in his school about their ages. The ages given were six, seven, eight, and nine. After gathering the answers, he drew a bar chart and a pie chart to show the results. Unfortunately, before he labelled each chart, he lost the original data. The charts are shown below:





Tanner did remember that 6 students in the survey answered six for their age. He also remembered that $\frac{1}{4}$ of the students surveyed answered eight for their age, and that the most popular answer was age seven. Based on this information, complete the table below:

Age	Number of Students
six	6
seven	
eight	
nine	

More Info:

Check the CEMC at Home webpage on Thursday, April 30 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, April 30.

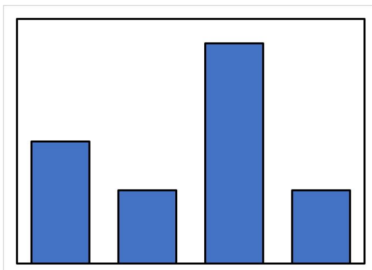
This CEMC at Home resource is the current grade 3/4 problem from Problem of the Week (POTW). This problem was developed for students in grades 3 and 4, but is also appropriate for students in grades 5 and 6. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

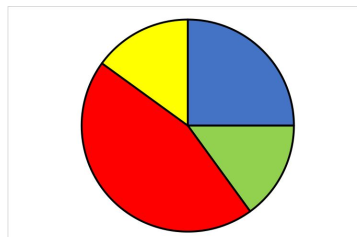
To subscribe to Problem of the Week, to view this week's grade 5/6 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem A and Solution
Lost Data

Problem

Tanner randomly surveyed 40 students in his school about their ages. The ages given were six, seven, eight, and nine. After gathering the answers, he drew a bar chart and a pie chart to show the results. Unfortunately, before he labelled each chart, he lost the original data. The charts are shown below:





Tanner did remember that 6 students in the survey answered six for their age. He also remembered that $\frac{1}{4}$ of the students surveyed answered eight for their age, and that the most popular answer was age seven. Based on this information, complete the table below:

Age	Number of Students
six	6
seven	
eight	
nine	

Solution

We compute $\frac{1}{4}$ of 40 is equal to 10. So 10 of the students who were surveyed are eight.

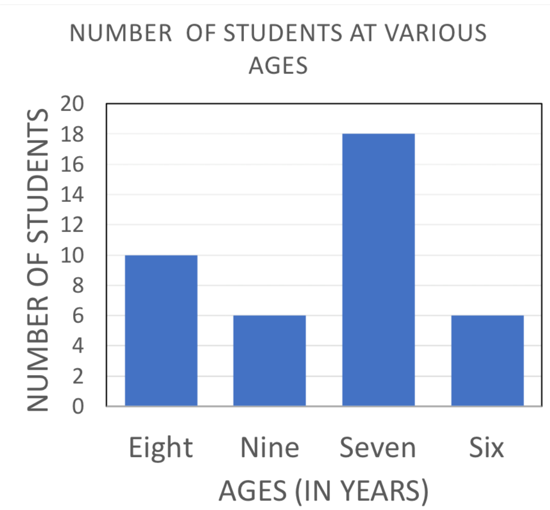
From the pie chart, we can see that there are two data values that are less than $\frac{1}{4}$ of the total number responses, and one data value is greater than $\frac{1}{4}$ of the total number of responses. We can also see from both charts that two of the data values are the same and less than one quarter of the total number of responses. From these two observations, along with the fact that one of the data values is 6, we can conclude that there is a second data value that is also 6.

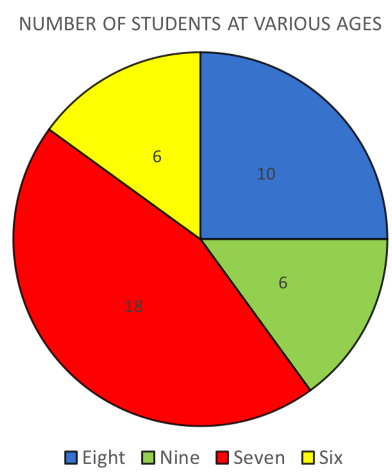
Now we can calculate the sum of three of the data values: $6 + 6 + 10 = 22$. Since 40 students were surveyed, the last data value must be $40 - 22 = 18$. This is the largest data value. Since seven was the most popular answer, there must be 18 students who are seven.

Now we know how many students answered six, seven, and eight for their ages. We also know that there is one data value (6) that we have not assigned to an age. So there must have been 6 students who answered age 9 in the survey. So the completed table is:

Age	Number of Students
six	6
seven	18
eight	10
nine	6

Note: For some reason, Tanner decided to use word labels on the horizontal axis of the bar chart, in alphabetical order. That is, the labels on the horizontal axis were eight, nine, seven and six. His completed graphs are shown below.

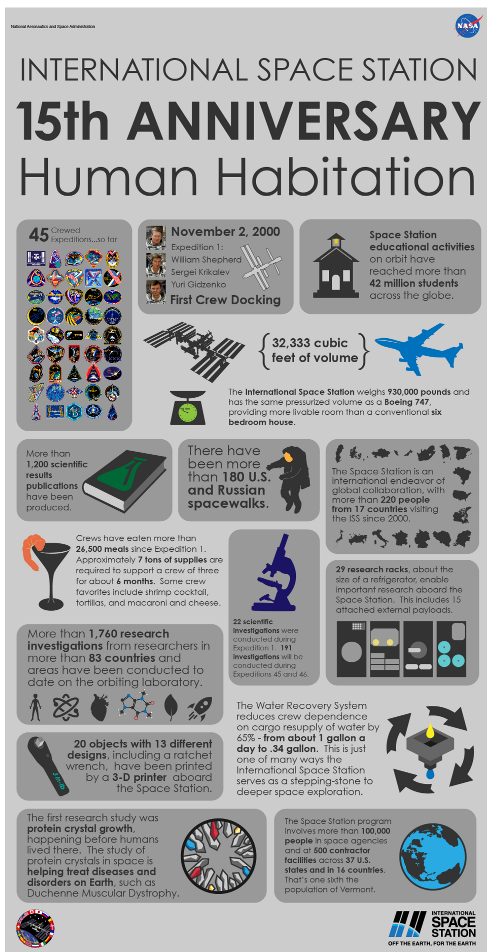




Teacher's Notes

The charts in this problem were generated by an Excel spreadsheet with the same underlying data values: 6, 18, 10, and 6. Different graphical representations of data may make it easier to see relationships among the data values. For example, although it appears that the yellow and green sections of the pie chart are the same size, it is clearer in the bar chart (especially in the solution with grid lines) that these values are the same. However, in the pie chart, it is much easier to see that the blue section is one quarter of the total surveyed.

We see more and more sophisticated examples of using images to represent data. People have been using *infographics* as a way of capturing people's attention - often in an attempt to sell things, but also in an attempt to emphasize important information - for a long time. However with today's technology we see these kinds of images everywhere.



(Image provided by NASA / Public domain)

CEMC at Home

Grade 4/5/6 - Friday, April 24, 2020
Metric Pentathlon

You Will Need:

- Two or more participants
  Enlist your family - this is for all ages! The score cards included are for four participants, but if you have more, you can add extra rows or make your own.
- A metre stick or measuring tape
  If you do not have a metre stick, then can you make one? Only one event uses a metre stick.
- A balloon inflated to about 4 cm across, or a soft, lightweight cloth ball about that size
- Two straws for each participant
  To make a straw, you can use a small strip of paper, roll it into a tube, and tape it into place. The straw should have an opening around 0.5 cm wide and be around 20 cm long. Try to make all of the straws similar in size.
- A cotton ball for each participant (about the size that comes in the top of a pill bottle)
- A paper plate
  If you don't have a paper plate, then you can cut a disk out of cardboard or something similar.
- A watch or clock with a second hand
- Tape
- A pen or pencil

What To Do: For each event, each participant in turn will do the following:

1. Enter your name on the score card for that event.
2. Estimate the distance or time you think you can achieve for the event, and enter that estimate on the score card.
3. Perform the task required by the event.
4. With the help of another participant, measure the actual value you achieve for the time or distance, and enter that measure on the score card.
5. Calculate your score, which is the difference between your estimate and the measured value, and enter it on the score card.

   Subtract the smaller value from the larger value so that the difference is a positive number.

For fairness, vary the order of the participants from event to event. *Think about why!*

The winner of each event is the participant with the *lowest score*, that is the participant with the smallest difference between their estimate and the actual measure.

Event 1: Balloon Toss

For this event you will need:

- The metre stick or measuring tape
- A long piece of tape
- The balloon or cloth ball

Make a line by laying the piece of tape on the floor in a hallway or a room with the most space.

Standing with your toes at the line, estimate how far, in metres, you can toss the balloon or cloth ball and enter your estimate on the score card.

Then toss the balloon underhand, and ask another participant to measure the distance from the line to where the balloon or ball lands on the floor.

Enter their measurement and determine your score.

Score Card for Event 1

Names	Estimate	Measure	Score

NOTE: If a soft cloth ball is used for this event, the distances may be longer.

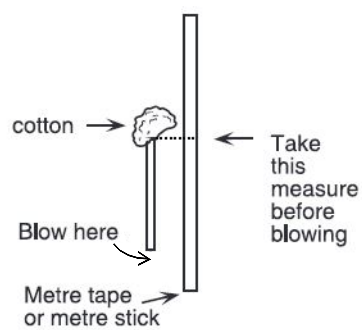
Event 2: Cotton Ball Blow

For this event you will need:

- The metre stick
- One straw for each participant
- The cotton ball(s)

Ask another participant to hold the metre stick vertically against a wall while you complete your turn at the event.

As shown in the diagram to the right, hold a straw vertically and balance your cotton ball on top of the straw. The goal is to blow through the bottom of the straw to move the cotton ball upward.



Estimate how far, in centimetres, you think you can blow the cotton ball vertically, and enter your estimate on the score card.

Ask the other participant to mark the initial and final position of the ball on the metre stick as you blow it upward. Enter the difference in centimetres between those two marks as your measure, and determine your score.

Score Card for Event 2

Names	Estimate	Measure	Score

If you find it hard to point the straw upward and balance the cotton ball while standing, how about trying this event by instead lying on your back!

Event 3: Paper Plate Discus Throw

For this event you will need:

- The metre stick or measuring tape
- A long piece of tape
- The paper plate

Make a line on the floor as you did for Event 1.

Standing with your toes at the line, estimate how far, in metres, you can throw the paper plate, and enter your estimate on the score card.

Then toss the paper plate like a frisbee, and ask another participant to measure the distance from the line to where the plate lands on the floor.

Enter their measurement and determine your score.

Score Card for Event 3

Names	Estimate	Measure	Score

Event 4: Straw Javelin Throw

For this event you will need:

- The metre stick or measuring tape
- A long piece of tape
- One straw for each participant

Make a line on the floor as you did for Event 1.

Standing with your toes at the line, estimate how far, in centimetres, you can throw a second straw, and enter your estimate on the score card.

Then throw the straw, and ask another participant to measure the distance from the line to where the straw lands on the floor.

Enter their measurement and determine your score.

Score Card for Event 4

Names	Estimate	Measure	Score

Event 5: Heel-Toe Walk

For this event you will need:

- The watch or clock
- Two pieces of tape (or other markers)

Place two markers on the floor, approximately 5 metres apart.

Standing with your toes at the first marker, estimate how many seconds it will take you to walk in heel-to-toe fashion to the second marker, and enter your estimate on the score card.

Then ask another participant to use a clock or watch to measure the time in seconds it takes you to do the actual walk.

Enter their measurement and determine your score.

Score Card for Event 5

Names	Estimate	Measure	Score

Once all participants have completed all events, we are ready to declare a winner! The overall winner of the Metric Pentathlon should be the participant who did the best on the events overall, but there are many different ways to measure this. An extra activity might be to decide, as a group, the fairest way to use all of the scores (some in different units) to declare the winner of the Metric Pentathlon.

CEMC at Home

Grade 4/5/6 - Monday, April 27, 2020
Keepin' It Square

In today's game, we will compete to connect the dots to draw squares on dot paper.

You Will Need:

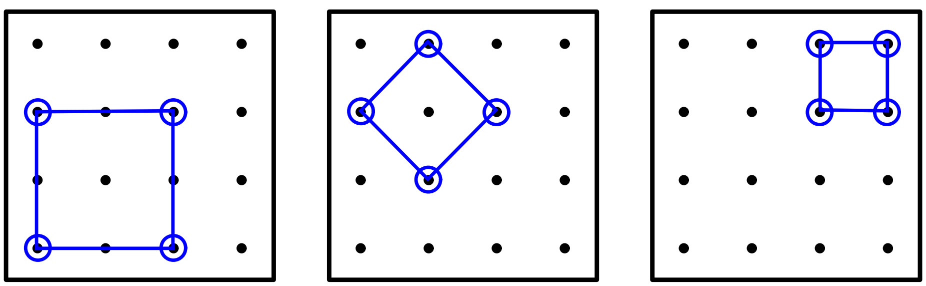
- Two players
- Many different pieces of dot paper

  The game board we will use is 4 dots by 4 dots, but you can play with other boards as well.
- A pencil

How to Play:

1. Start with a piece of dot paper that is 4 dots by 4 dots.
2. The two players take turns circling dots on the dot paper. Decide which player will go first.
3. On your turn, you start by circling one dot that has not yet been circled. You then try to make a square by connecting 4 circled dots on the board.

   Below are some examples of squares. Notice that squares can be different sizes and orientations.



4. The first player to successfully make a square wins.

Variations:

A. Play the same game but keep track (using different coloured pens or pencil crayons) of which dot is circled by which player. In this variation, each player can only make a square using their *own* circled dots.

B. Play the same game but instead of stopping after the first square is drawn, continue until all dots have been circled. The player who made the most squares in total wins the game.

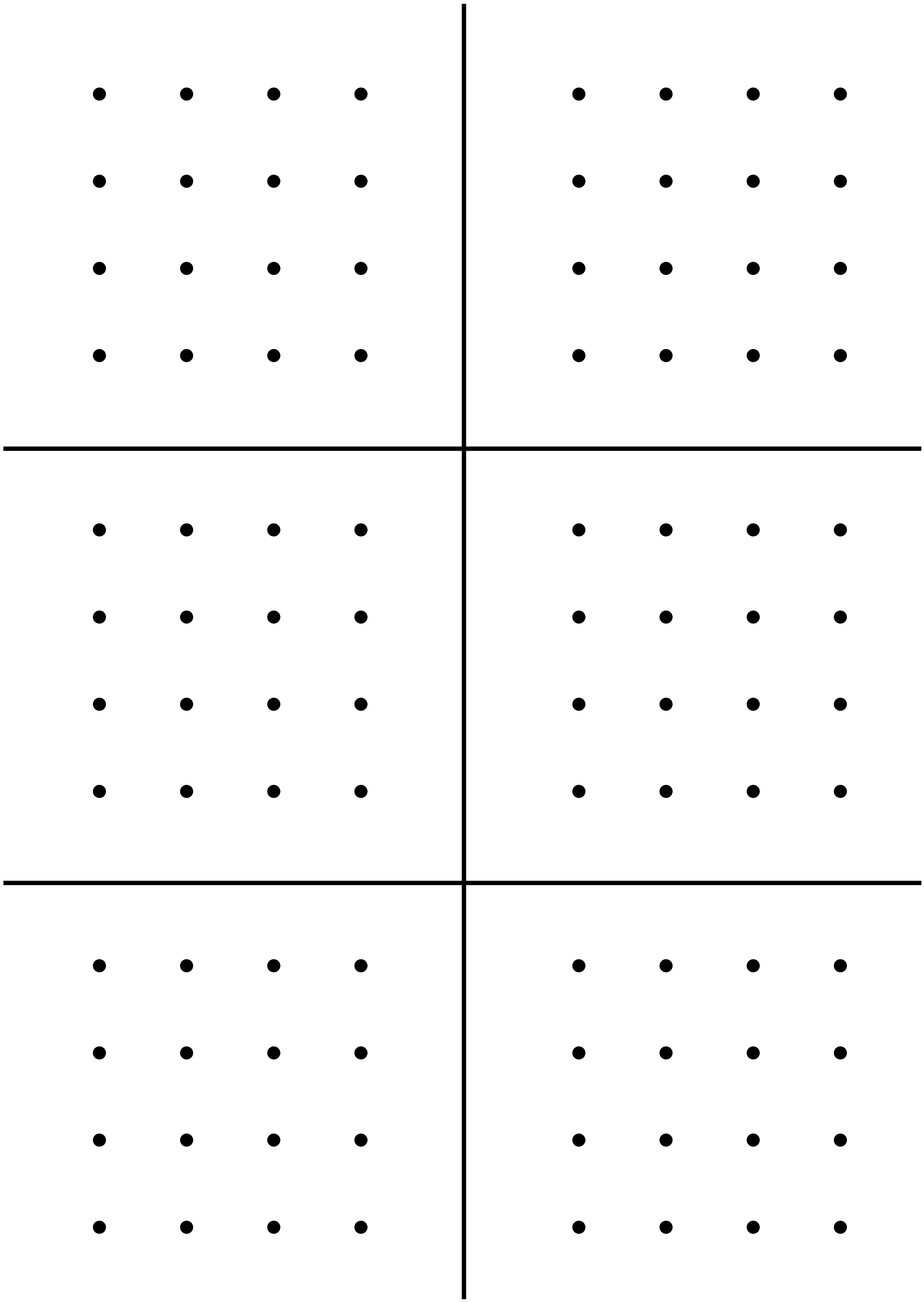
Note that players can make more than one square on a turn. Each square drawn must be a new square, but can share vertices with an existing square and/or overlap with an existing square.

Follow-up Questions:

1. How many different squares are there on a 4 dots by 4 dots game board?
2. How many different squares are there on a 5 dots by 5 dots game board?

More Info:

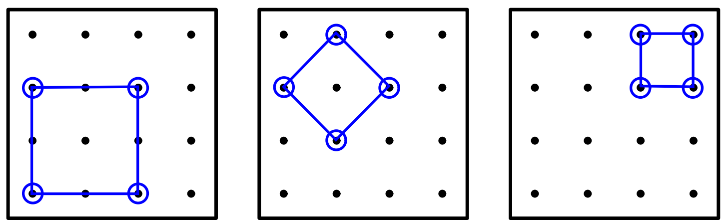
Check out the CEMC at Home webpage on Monday, May 4 for a solution to Keepin' It Square.



CEMC at Home

Grade 4/5/6 - Monday, April 27, 2020
Keepin' It Square - Solution

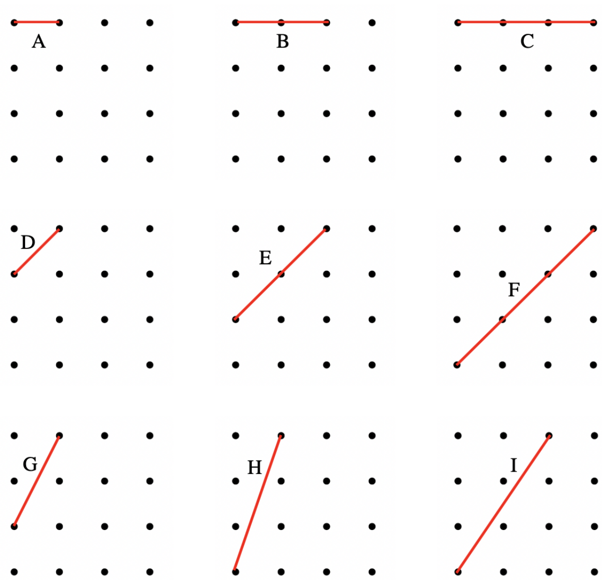
There are many different squares that can be drawn on the following dot paper. A few examples are shown below.



- How many different squares can be drawn that have the same side length as the square shown in the leftmost image?
- How many different squares can be drawn that have the same side length as the square shown in the middle image?
- How many different squares can be drawn that have the same side length as the square shown in the rightmost image?
- What other side lengths are possible for squares drawn on this dot paper?

To make sure that we get a correct count, we need to organize our thinking. We will group the different possible squares based on their side lengths.

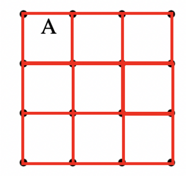
First, convince yourself that there are exactly nine different lengths that a line segment on this board could have. Examples of line segments with each of these lengths are shown below and labelled A - I.



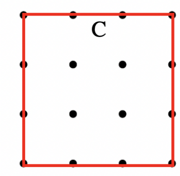
Remember that all vertices of a square drawn must lie on dots. This means the images above show the only possible side lengths of a square on this board.

Now let's count how many different squares of each side length can be drawn.

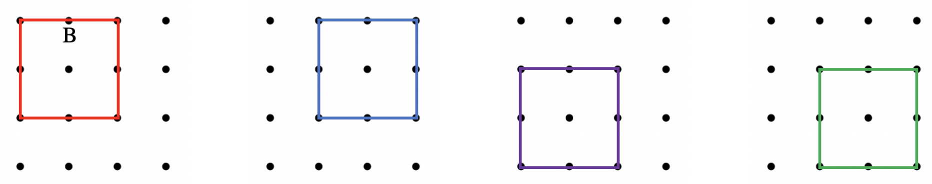
There are 9 squares with side lengths equal to the length of A.



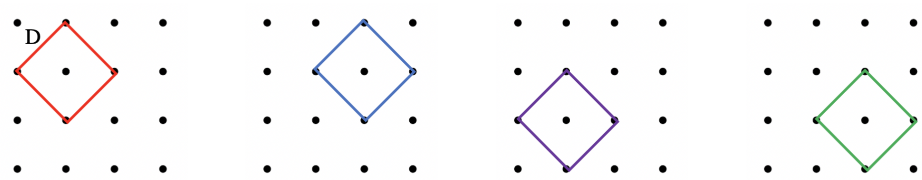
There is 1 square with side lengths equal to the length of C.



There are 4 squares with side lengths equal to the length of B.

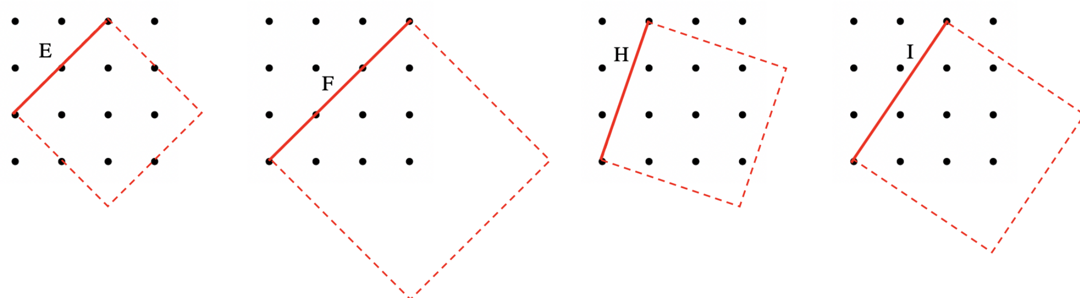


There are 4 squares with side lengths equal to the length of D.

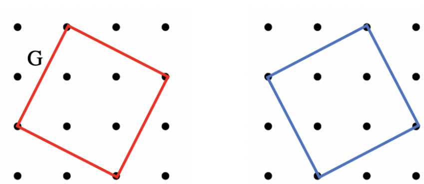


There are 0 squares with side lengths equal to the length of E, F, H, or I.

No matter how you place these line segments on the grid, the squares formed must extend past the edges of the grid. Some examples are shown below.



There are 2 squares with side lengths equal to the length of G.



This means there are $9 + 1 + 4 + 4 + 2 = 20$ different squares that can be drawn on the dot paper.

Can you do a similar count to figure out how many different squares can be drawn on dot paper that is 5 dots by 5 dots (instead of 4 dots by 4 dots)?

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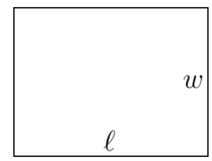
CEMC at Home

Grade 4/5/6 - Tuesday April 28 2020
Figure Out These Rectangles

Problem 1: A rectangle has length ℓ and width w, in centimetres, and a perimeter of 16 cm.

(a) Explain how you know it must be true that $\ell + w = 8$ cm.

(b) Suppose further that the side lengths ℓ and w are positive whole numbers of centimetres with ℓ greater than w. Given this, what are the possible pairs of values for ℓ and w?



Don't use the exact shape of the diagram given to determine the values of ℓ and w. This is just an illustration and does not represent exactly what the rectangle must look like. It can be wider or longer than what is shown here. For example, w could be 1 cm. In this case, what must ℓ be?

Problem 2: Five identical rectangles each with length ℓ and width w are arranged to form the larger rectangle below and to the right. The side lengths ℓ and w are positive whole numbers of centimetres.

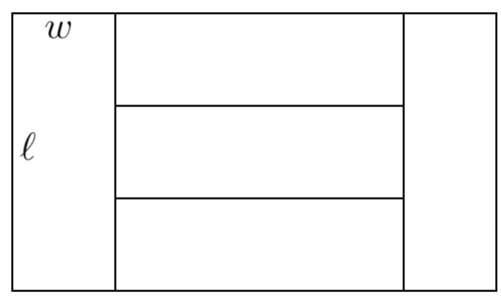
(a) How many widths w make up one length ℓ?

Where in the diagram can we see the relationship between these two values?

(b) Suppose each identical smaller rectangle has perimeter 16 cm and ℓ greater than w. Using your work from Problem 1 and the relationship between ℓ and w from Problem 2 (a), can you figure out the values of ℓ and w?

You found different possibilities for the pair ℓ and w in Problem 1 (b). Which of these pairs of values satisfies the relationship you found in Problem 2 (a)?

(c) If ℓ and w have the values you found in part (b) directly above, what is the total area of the larger rectangle?

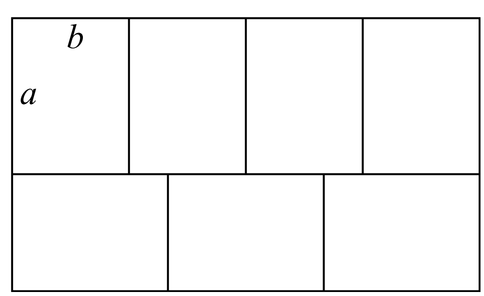


Challenge Problem: Seven identical rectangles each with length a and width b are arranged to form the larger rectangle below and to the right. The side lengths a and b are positive whole numbers of centimetres and the total area of the larger rectangle is $84\,\text{cm}^2$.

(a) What is the area, in square centimetres, of each of the seven smaller identical rectangles?

(b) Explain why 3 times the value of a must be equal to 4 times the value of b. Using this relationship, can you find what the values of a and b must be?

(c) What is the perimeter of the larger rectangle?



More info:

Check the CEMC at Home webpage on Tuesday, May 5 for a solution to Figure Out These Rectangles.

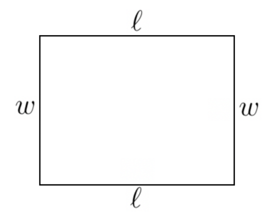
CEMC at Home

Grade 4/5/6 - Tuesday, April 28, 2020
Figure Out These Rectangles - Solution

Problem 1

(a) The perimeter consists of two lengths ℓ and two widths w. Since adding two widths and two lengths together gives 16 cm, adding one width and one length together must give $16 \div 2 = 8$ cm. This means the sum of the length and the width must be 8 cm, or

$$\ell + w = 8 \text{ cm}$$

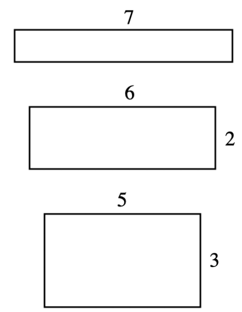


(b) There are only three ways to make 8 by adding two different positive whole numbers:

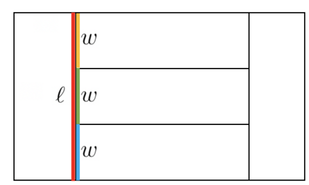
$$1 + 7 = 8,\ 2 + 6 = 8,\ 3 + 5 = 8$$

Since the length is greater than the width, the only possibilities for ℓ and w, in centimetres, are shown in the table below.

width w	length ℓ
1	7
2	6
3	5



Problem 2

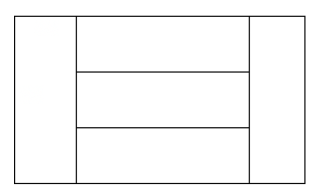


(a) With the sides labelled as shown in the top diagram on the right, we see that the length ℓ is equal to three widths w.

(b) The only pair from Problem 1(b) for which the length is three times the width is the pair $\ell = 6$ cm and $w = 2$ cm.

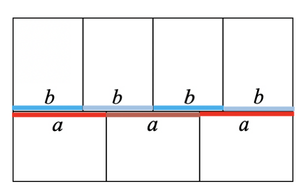
(c) If each smaller identical rectangle has length 6 cm and width 2 cm, then the area of each smaller rectangle is $6 \times 2 = 12$ cm^2.

Since there are 5 smaller rectangles making up the larger rectangle, the area of the larger rectangle must be 5×12 cm^2 $= 60$ cm^2.



Challenge Problem

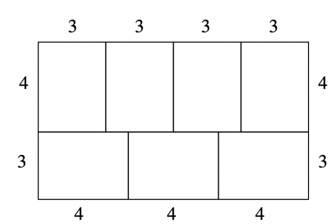
(a) Since the total area of the larger rectangle is 84 cm^2 and it is formed using seven smaller identical rectangles, the area of each smaller rectangle must be $84 \div 7 = 12$ cm^2.



(b) With the sides labelled as shown in the top diagram on the right, we see that three lengths a are equal to four widths b. Since the area of each smaller rectangle is 12 cm^2 we know that a times b must be 12. The factor pairs of 12 are 1 and 12, 2 and 6, and 3 and 4. The only pair that satisfies the correct relationship is 3 and 4. This means $a = 4$ cm and $b = 3$ cm.

(c) Using the labelled diagram, we see that the larger rectangle has length 12 cm and width 7 cm. This means its perimeter is

$$12 \text{ cm} + 7 \text{ cm} + 12 \text{ cm} + 7 \text{ cm} = 38 \text{ cm}$$



CEMC at Home

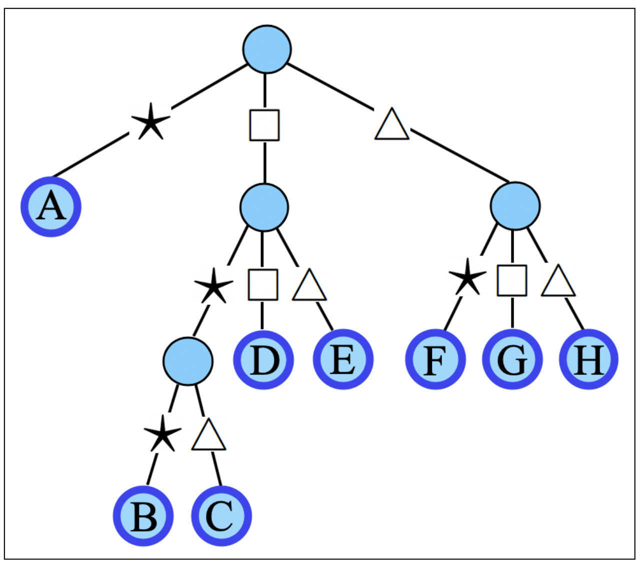
Grade 4/5/6 - Wednesday, April 29, 2020
Cool Codes

Introduction: Charlotte and Jacques want to send secret messages to each other. They will use three different symbols in their messages: $\bigstar$, $\square$, and $\triangle$.

To *encode* their secret messages they use a *key* that only they know. Encoding a message means changing the message from regular text to code involving the symbols $\bigstar$, $\square$, and $\triangle$.

This same key can be used to *decode* their secret messages. Decoding a message means changing the message from code involving the symbols $\bigstar$, $\square$, and $\triangle$ back to regular text.

The key they use to encode and decode messages is the *tree* shown below.



Notice that each of the letters A, B, C, D, E, F, G, and H appear as *leaves* on this tree. This means that this tree can be used to encode any message that contains only these letters.

Each letter has its own code that is determined by the letter's placement on the tree. The code for a letter can be found by following the path on the tree from the top circle to the circle containing the letter, and picking up all of the symbols on the *branches* that you travel along on this path.

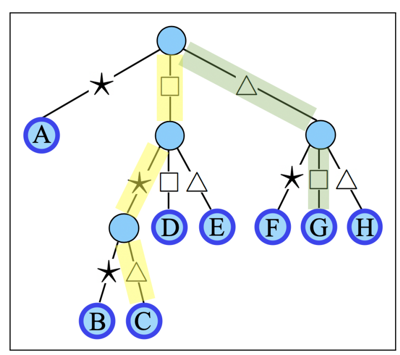
Notice that each *left branch* has the symbol $\bigstar$, each *middle branch* has the symbol $\square$, and each *right branch* has the symbol $\triangle$.

For example, the code for the letter **C** is $\square\bigstar\triangle$.

We find this code by starting at the top circle and taking a middle branch (picking up a $\square$), followed by a left branch (picking up a $\bigstar$), followed by a right branch (picking up a $\triangle$), to end up at **C**.

As another example, the code for the letter **G** is $\triangle\square$.

We find this code by starting at the top circle and taking a right branch (picking up a $\triangle$), followed by a middle branch (picking up a $\square$), to end up at **G**.



To encode a message, we replace each letter in the message with its code from the tree. For example, the encoded version of **BAG** is $\square\bigstar\bigstar\bigstar\triangle\square$ as shown below.

$$\underbrace{\square\bigstar\bigstar}_{\textbf{B}}\underbrace{\bigstar}_{\textbf{A}}\underbrace{\triangle\square}_{\textbf{G}}$$

We have already seen the code for the letter **G**, but you should use the tree to check that these are the correct codes for **B** and **A**. Notice that codes for different letters can be of different lengths.

Example: A message was encoded using the given tree and the resulting code is shown below. Use the tree to decode the message.

□ ★ △ ★ △ □ □ △

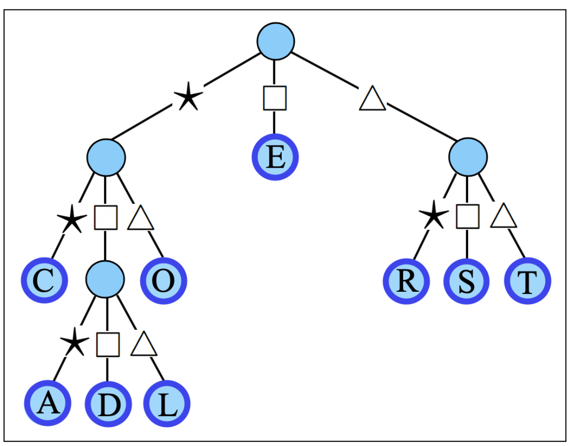
Explanation: You can decode this message letter by letter. Start at the top circle of the tree and follow the path indicated by the symbols until you end up at a letter.

To follow the path indicated by these symbols you must follow a middle branch (□), then a left branch (★), then a right branch (△), and this takes you to the the letter **C**. Once you reach a letter you stop, and restart the process at the top circle of the tree with the next part of the sequence.

To follow the path indicated by the remaining symbols ★ △ □ □ △, we must follow a left branch (★) which leads us straight to the letter **A**, and so we stop and reset again. Continuing in this way until we reach the end of the code, we reveal the original message of **CAGE** as shown below:

Starting at the top circle,	□ → middle, ★ → left, △ → right	⇒	**C**
Going back to the top,	★ → left	⇒	**A**
Going back to the top again,	△ → right, □ → middle	⇒	**G**
Going back to the top again,	□ → middle, △ → right	⇒	**E**

Problem: It is always a good idea to change your key often to keep your messages safe. To send their messages tomorrow, Charlotte and Jacques will use the following different tree.



(a) Using their tree as the key, what is the code for the word **LOST**?

(b) Jacques sent Charlotte the following secret message.

★ □ △ □ △

Explain why Jacques must have made a mistake when encoding the message.

(c) Using their tree as the key, decode the message displayed below.

△ □ □ ★ ★ △ ★ □ △ △ ★ ★ ★ △ ★ □ □ □ △ □ ★ □ ★ △ ★ □ ★ ★ ★ △ ★ △ ★ □ △

Extension:

There are different trees that will encode exactly the same letters, but in a different way. Suppose that the codes for the letters in a tree are as shown in the table on the right.

1. Draw the tree that matches the codes in the table.
2. Charlotte and Jacques think that the tree they used in the problem above is better than the tree that matches the codes that you found in 1. Do you agree with this? Why or why not?

A	★
C	□ ★ ★
D	△ ★
E	□ ★ □
L	△ □
O	□ ★ △
R	□ □
S	□ △
T	△ △

More Info:

Check out the CEMC at Home webpage on Wednesday, May 6 for a solution to Cool Codes.

CEMC at Home

Grade 4/5/6 - Wednesday, April 29, 2020
Cool Codes - Solution

Problem:

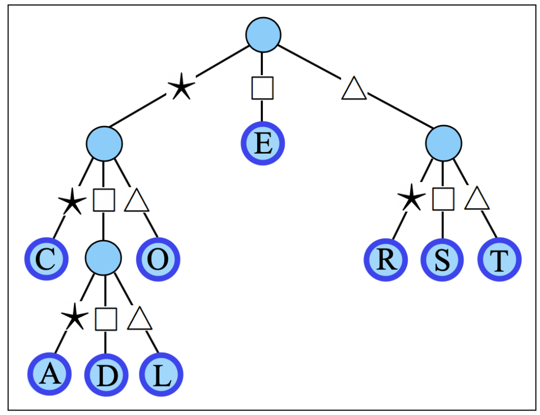
(a) Using their tree as the key, what is the code for the word **LOST**?

(b) Jacques sent Charlotte the following secret message.

★ □ △ □ △

Explain why Jacques must have made a mistake when encoding the message.

(c) Using their tree as the key, decode the message displayed below.



△ □ □ ★ ★ △ ★ □ △ △ ★ ★ ★ △ ★ □ □ □ △ □ ★ □ ★ △ ★ □ ★ ★ ★ △ ★ △ ★ □ △

Solution:

(a) The code for **LOST** is: ★ □ △ ★ △ △ □ △ △

(b) Here is what we get when we try to decode this message:

Starting at the top circle,	★ → left, □ → middle, △ → right	⇒	**L**
Going back to the top,	□ → middle	⇒	**E**
Going back to the top,	△ → right	⇒	?

We have now run out of symbols and are unable to complete the process. Since △ alone is not the code for a letter, this message cannot have been encoded correctly.

(c) The decoded message is: **SECRETCODESARECOOL** (or "secret codes are cool").

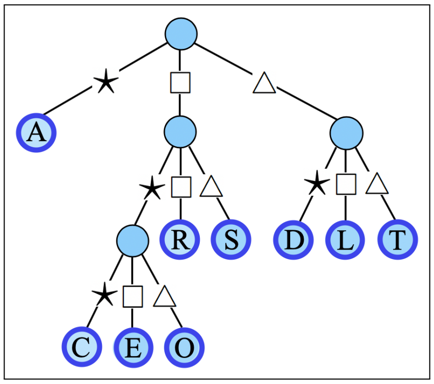
Extension:

1. Draw the tree that matches the codes in the table.
2. Charlotte and Jacques think that the tree they used in the problem above is better than the tree that matches the codes that you found in 1. Do you agree with this? Why or why not?

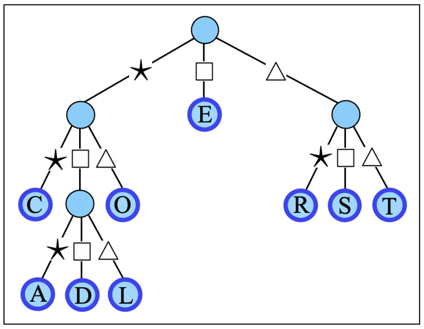
A	★
C	□ ★ ★
D	△ ★
E	□ ★ □
L	△ □
O	□ ★ △
R	□ □
S	□ △
T	△ △

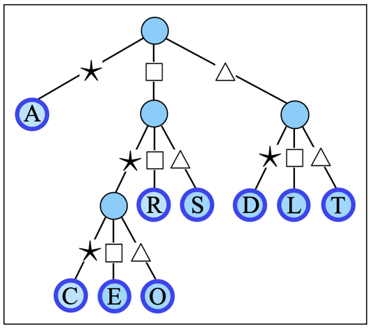
Solution:

1. The tree that matches this encoded table is:



2. Let's compare the two trees:





When using the tree on the left (created by Charlotte and Jacques) to encode the message

SECRETCODESARECOOL

the encoded message has 35 symbols. *Go back and count them for yourself.*

This is already pretty long! However, if we use the new tree on the right to encode the same message, the encoded message will have *even more* symbols, which means we would have to send a longer message. The encoded message in this case would have 45 symbols in total. Can you see why?

We can determine the number of symbols the encoded message will have by counting how many symbols are needed to code each letter, and how many times each letter appears in the original message.

The number of times a letter appears is sometimes called the frequency of the letter.

The rightmost column of the table shows how many symbols are needed to code each letter, in total. When we add up the numbers in this column, we get the total length of the encoded message. The total is 45.

Letter	Code	Frequency of Letter	Total Symbols
A	★	1	1
C	□ ★ ★	3	9
D	△ ★	1	2
E	□ ★ □	4	12
L	△ □	1	2
O	□ ★ △	3	9
R	□ □	2	4
S	□ △	2	4
T	△ △	1	2

The reason it takes more symbols to encode the message with the second tree is that the letters that appear most often in the original message (C, E, and O) happen to be the letters with the longest codes (three symbols instead of one or two).

If you look at the first tree, you will see that the letters C, E, and O have codes of length one or two, rather than three.

So it looks like the first tree (created by Charlotte and Jacques) is a better choice for encoding this particular message since it takes fewer symbols to do so. *What about for other messages?*

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, April 30, 2020

Fishing for Thermoclines

Paige is headed to Lac Nilgault to do some fishing for lake trout and pike. She knows that small pike generally like 18°C water, large pike like 12°C water, and lake trout like 10°C water.
She measures the water temperature in the lake at different depths, and collects the following data.

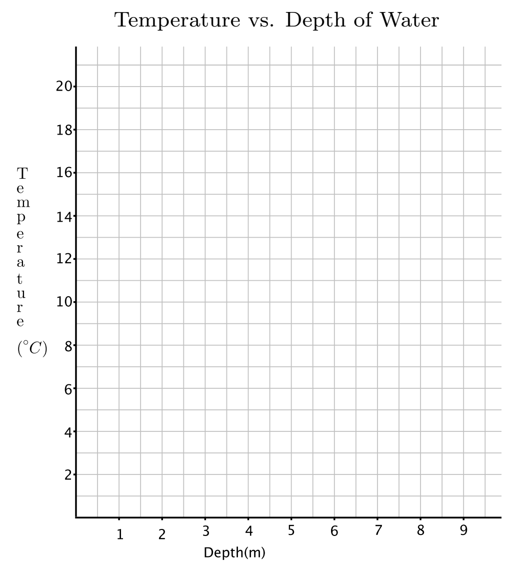
Depth (m)	**Temp** (°C)	**Depth** (m)	**Temp** (°C)
0.5	20	5	15
1	19	5.5	15
1.5	18	6	15
2	18	6.5	12
2.5	18	7	11
3	17	7.5	10
3.5	17	8	10
4	16	8.5	10
4.5	16	9	9

On the grid to the right, plot points to make a broken-line graph to illustrate this data.

a) At what depths should Paige be fishing for small pike?

b) At what depths should Paige be fishing for lake trout?

c) Paige knows that most lakes have a thermocline where the water will rapidly change temperature. At what depth is the top of the thermocline in Lac Nilgault?



More Info:

Check the CEMC at Home webpage on Thursday, May 7 for the solution to this problem.
Alternatively, subscribe to Problem of the Week at the link below and have the solution, along with a new problem, emailed to you on Thursday, May 7.

This CEMC at Home resource is the current grade 5/6 problem from Problem of the Week (POTW). This problem was developed for students in grades 5 and 6, but is also accessible to students in grade 4. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 3/4 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem B and Solution
Fishing for Thermoclines

Problem

Paige is headed to Lac Nilgault to do some fishing for lake trout and pike. She knows that small pike generally like 18°C water, large pike like 12°C water, and lake trout like 10°C water. She measures the water temperature in the lake at different depths, and collects the following data.

Depth m	Temp °C	Depth m	Temp °C
0.5	20	5	15
1	19	5.5	15
1.5	18	6	15
2	18	6.5	12
2.5	18	7	11
3	17	7.5	10
3.5	17	8	10
4	16	8.5	10
4.5	16	9	9

Plot points to make a broken-line graph to illustrate this data.

a) At what depths should Paige be fishing for small pike?

b) At what depths should Paige be fishing for lake trout?

c) Paige knows that most lakes have a thermocline where the water will rapidly change temperature. At what depth is the top of the thermocline in Lac Nilgault?

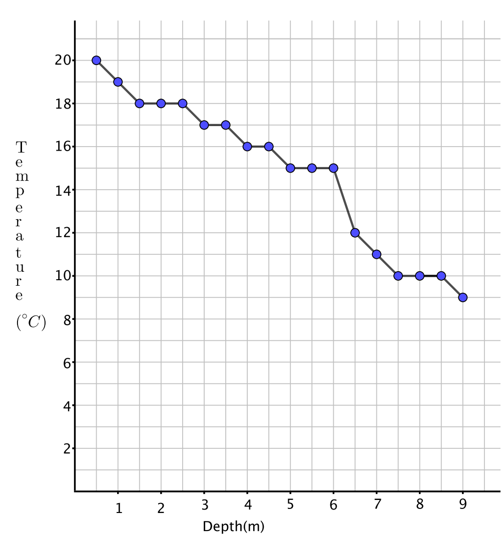
Solution

a) According to the data, Paige should fish for small pike at a depth between 1.5 m and 2.5 m.

b) She should fish for lake trout at a depth between 7.5 m and 8.5 m.

c) We can see from the graph that the temperature starts to quickly drop at a depth of about 6 m. Therefore, the top of the thermocline is at a depth of about 6 m.

Temperature vs. Depth of Water



CEMC at Home

Grade 4/5/6 - Friday, May 1, 2020
This Farmer is No Square

Long ago, there was a farmer whose land was in the shape of a square, each side being exactly 100 metres long.

One day, a woman knocked on the farmer's door and begged for something to eat. Being a kind person, the farmer fed her a nice lunch.

After the woman had eaten, she said "Kind farmer, I am your Queen! As a reward for your kindness, I will grant you enough additional land that you may double the area of your farm. However, your land must remain in the shape of a square."

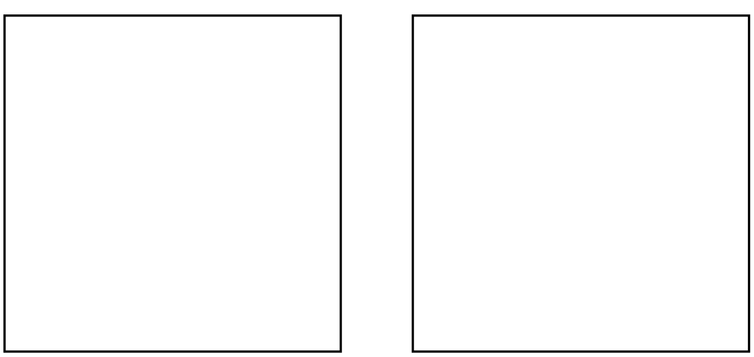
Let's help the farmer figure out how he might determine the side length of his new square of land.

(a) A first thought might be to simply double the length of each side of the original square of land. That is, the new square of land would have sides that are each 200 metres long.

Draw a diagram of his original square of land and of this proposed new one. Explain why this solution does not give the desired result of a new square of land with twice the area of the original one.

(b) Suppose each square below represents a copy of the farmer's original square of land, so their combined area represents the area of the new square of land. Can you divide the given squares in such a way that the pieces can be reassembled in order to form a single larger square? Can you convince yourself that the shape you created is, in fact, a square?

You may cut and arrange these two squares in any way you would like, but your rearrangement must use all of these pieces, be in the shape of a square, and not change the overall area.



Hint:

Young Geo had thoughts analytical
About cutting out shapes most polygonal;
"These boxes," she said,
"Don't get me ahead...
I shall try, instead, some diagonals."

(c) Can you relate the side length of the larger square you created in part (b) to either the side length or the diagonal length of the smaller square?

Challenge: If the farmer's land had been a rectangle, but not necessarily a square, would your technique from part (b) produce a single larger rectangle twice the area of the original rectangle?

More info:

Check out the CEMC at Home webpage on Friday, May 8 for a solution to This Farmer is No Square.

CEMC at Home

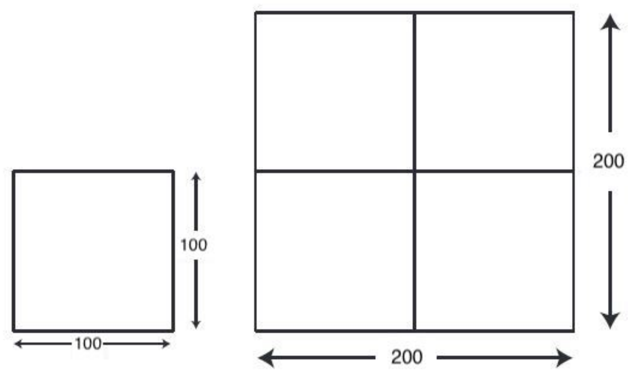
Grade 4/5/6 - Friday, May 1, 2020
This Farmer is No Square - Solution

Problem Summary:

In return for a kind act, the Queen has granted a farmer enough land to double the area of his existing farm, which is square-shaped with each side exactly 100 metres long. However, the new farm must also be in the shape of a square. Let's help the farmer figure out how he might determine the side length of his new square of land.

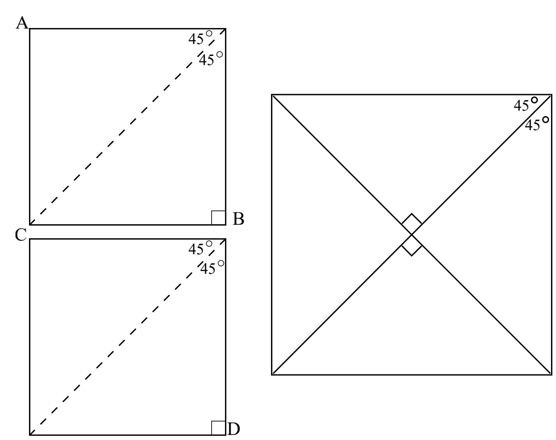
Solution:

(a) The diagram to the right reveals why doubling the side length of the square does not work. The new square that is formed can be divided up into four smaller squares, each with side length 100 metres, as shown. Hence, this plan yields a farm that has four times the original area, not twice the area.



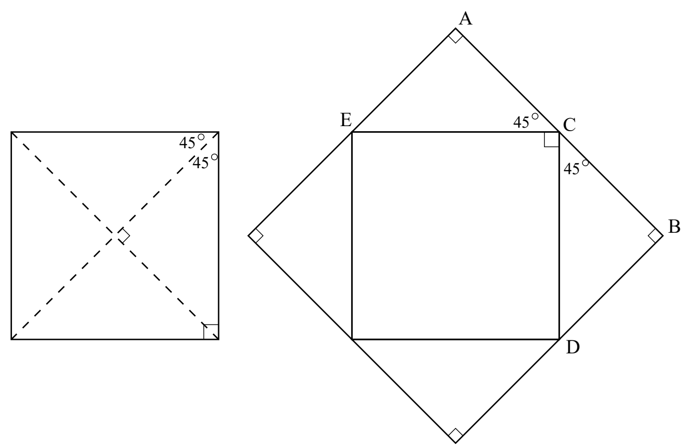
(b) While it is tempting to try to combine the two squares by cutting rectangular pieces and fitting them together, it is not very fruitful. Also, convincing yourself that the figure you end up forming is, in fact, a square, can prove to be quite difficult.

One solution is to cut each square along a diagonal, creating four congruent right-angled triangles. Then line up vertices A, B, C, and D so they become the centre of a new square whose sides are the cut edges.
To justify that this new shape is a square, notice that each side of the new shape corresponds to a diagonal of a smaller square, and so the sides are all equal in length. Also, each new corner angle is formed by two halves of the previous corner angle, and so is a right angle. Thus, the new shape is a square. Also, since it is formed from two smaller squares, it has area that is twice the area of a smaller square.



Another solution is to cut one square along both diagonals, as shown below. This will create four congruent right-angled triangles. Then place each triangle onto one side of the other small square, with the longer side of the triangle (which is also the side of the original square) lined up along the side of the square.

To justify that this new shape is a square, we need to show that the sides of the new shape are all straight lines of the same length. Since ACB is formed by two halves of the previous corner angle and another square's corner angle, can you see why ACB will form a straight line? Also notice that each side of the new shape is formed by two halves of the diagonal of the original smaller square. Therefore, each side length of the new shape is equal to the diagonal length of the original smaller square. So the shape that results from this construction is a square. Also, since it is formed from two smaller squares, it has area that is twice the area of a smaller square.



(c) As established in both solutions to part (b), the side length of the larger square is equal to the diagonal length of the smaller square.

Challenge:

If the farmer's land is a rectangle, but not a square, then both approaches presented in the solutions to part (b) will give a shape that has twice the area, but the shape will not be a square. It will instead be a *rhombus*. Try out both approaches on two identical rectangles and see if you can figure out why!

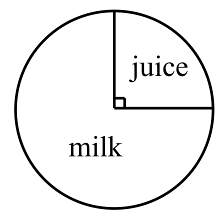
CEMC at Home

Grade 4/5/6 - Monday, May 4, 2020
Contest Day 1

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #6

In the pie chart shown, 80 students chose juice.
How many students chose milk?



(A) 120 **(B)** 160 **(C)** 240
(D) 180 **(E)** 80

2012 Gauss Contest, #15

Yelena chants P, Q, R, S, T, U repeatedly (e.g. $P, Q, R, S, T, U, P, Q, R, \ldots$). Zeno chants $1, 2, 3, 4$ repeatedly (e.g. $1, 2, 3, 4, 1, 2, \ldots$). If Yelena and Zeno begin at the same time and chant at the same rate, which combination will *not* be said?

(A) $T1$ **(B)** $U2$ **(C)** $Q4$ **(D)** $R2$ **(E)** $T3$

More Info:

Check out the CEMC at Home webpage on Monday, May 11 for solutions to the Contest Day 1 problems.

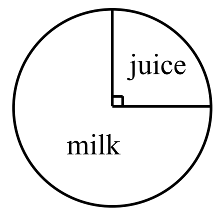
CEMC at Home

Grade 4/5/6 - Monday, May 4, 2020
Contest Day 1 - Solution

Solutions to the two contest problems are provided below, including a video for the second problem.

2020 Gauss Contest, #6

In the pie chart shown, 80 students chose juice.
How many students chose milk?



(A) 120 **(B)** 160 **(C)** 240
(D) 180 **(E)** 80

Solution:

The fraction of the circle which represents students who chose juice is $\frac{1}{4}$.
Therefore, $\frac{1}{4}$ of all students chose juice.
Since the 80 students who chose juice represent $\frac{1}{4}$ of the total number of students, then the total number of students is $4 \times 80 = 320$.
Therefore, $320 - 80 = 240$ students chose milk.

ANSWER: (C)

2012 Gauss Contest, #15

Yelena recites P, Q, R, S, T, U repeatedly (e.g. $P, Q, R, S, T, U, P, Q, R, \ldots$). Zeno recites $1, 2, 3, 4$ repeatedly (e.g. $1, 2, 3, 4, 1, 2, \ldots$). If Yelena and Zeno begin at the same time and recite at the same rate, which combination will *not* be said?

(A) $T1$ **(B)** $U2$ **(C)** $Q4$ **(D)** $R2$ **(E)** $T3$

Solution:

To determine which combination will not be said, we list the letters and numbers chanted by Yelena and Zeno in the table below.

Yelena	P	Q	R	S	T	U	P	Q	R	S	T	U	P	Q	$\cdots$
Zeno	1	2	3	4	1	2	3	4	1	2	3	4	1	2	$\cdots$

Remember that after Yelena chants "U", she will start her pattern all over again beginning from P.
Similarly, after Zeno chants "4", he will start over at 1.
You can see from the table that the pattern for both kids starts to repeat after 12 rounds (it goes back to $P1$, $Q2$, $\ldots$).
To determine which combination will not be said we need only compare the 5 answers with the 12 possibilities given in the table.
The only combination that appears among the 5 answers, but that does not appear in the table, is $R2$.

ANSWER: (D)

Video

Visit the following link for an explanation of the solution to the second contest problem and a discussion of a follow-up problem: https://youtu.be/woHduEnD8M4

CEMC at Home

Grade 4/5/6 - Tuesday, May 5, 2020
Brigitte's Boating Adventure

Brigitte is enjoying rowing her sturdy boat across Blue Lake on a sunny morning. Suddenly, the boat strikes a rock, making a small crack in the hull (the body of the boat).

Water begins to leak into the boat; 3 litres of water leaks in during each minute of time.

But Brigitte's boat is equipped with a small pail and so she can alternate rowing the boat and bailing out the water to keep the boat afloat while she returns to the dock.

(a) If Brigitte does not bail any water out of the boat, how much water would there be in the boat after 1 minute? What about after 30 minutes?

(b) But of course, Brigitte does bail water out of the boat. Using her pail, she removes 1.4 litres during each minute. How many litres of water remain in the bottom of the boat after 1 minute? What about after 30 minutes?

(c) Over each of the next 30 minute periods, the amount that you found in part (b) is added to the water in the bottom of the boat. While alternately bailing and rowing, Brigitte keeps the boat moving toward shore at 2 km per hour.

Alas, despite her efforts, the boat sinks in shallow water just at the end of the dock, at which point it contains a total of 144 litres of water.

Can you use this information to figure out how far away from the end of the dock Brigitte was when the boat struck the rock? Complete the table on the right to help answer this question.

The elapsed time is the time since the boat hit the rock.

Time (hr) Elapsed	**Water (L) in Boat**	**Distance (km) Rowed**
0.5		1 km
1		2 km

Challenge:

Suppose that Brigitte's boat had hit the rock 1 km *further* from the dock than you found in part (c) above. The boat still takes on the same amount of water each minute as before (3 litres) and the boat will still sink once it is holding 144 litres of water.

Brigitte bails faster this time and so she has less time to row. This results in the boat moving at only 1 km per hour. How many litres of water would Brigitte have to bail out of the boat during each minute in order to just reach the dock before the boat sinks?

Hints:

1. How many minutes will the boat now take to reach the dock?
2. How quickly will the boat need to fill to reach 144 litres of water by this time?

More info: Check out the CEMC at Home webpage on Tuesday, May 12 for a solution to Brigitte's Boating Adventure.

CEMC at Home

Grade 4/5/6 - Tuesday, May 5, 2020
Brigitte's Boating Adventure - Solution

Brigitte is enjoying rowing her sturdy boat across Blue Lake on a sunny morning. Suddenly, the boat strikes a rock, making a small crack in the hull (the body of the boat).

Water begins to leak into the boat; 3 litres of water leaks in during each minute of time. But Brigitte's boat is equipped with a small pail and so she can alternate rowing the boat and bailing out the water to keep the boat afloat while she returns to the dock.

(a) If Brigitte does not bail any water out of the boat, how much water would there be in the boat after 1 minute? What about after 30 minutes?

Solution:

If Brigitte does not bail, after 1 minute there would be 3 litres of water in the boat. So after 30 minutes, there would be $3 \times 30 = 90$ litres.

(b) But of course, Brigitte does bail water out of the boat. Using her pail, she removes 1.4 litres during each minute. How many litres of water remain in the bottom of the boat after 1 minute? What about after 30 minutes?

Solution:

Since 3 litres of water leak into the boat every minute, and Brigitte bails 1.4 litres of water out of the boat each minute, the amount of water in the boat after one minute will be $3 - 1.4 = 1.6$ litres.

After 30 minutes, there will be $30 \times 1.6 = 48$ litres in the boat.

(c) Over each of the next 30 minute periods, the amount that you found in part (b) is added to the water in the bottom of the boat. While alternately bailing and rowing, Brigitte keeps the boat moving toward shore at 2 km per hour. Alas, despite her efforts, the boat sinks in shallow water just at the end of the dock, at which point it contains a total of 144 litres of water. How far away from the end of the dock was Brigitte when the boat struck the rock?

Solution:

Every 30 minutes, the boat gains 48 L of water. Using the given speed of the boat, 2 km per hour (or 1 km per half hour), the completed table reveals that 144 litres of water accumulate in the boat after 1.5 hours.

So Brigitte must have been 3 km from the dock when the boat hit the rock.

Time (hr) Elapsed	**Water (L) in Boat**	**Distance (km) Rowed**
0.5 hr	48 L	1 km
1 hr	96 L	2 km
1.5 hr	144 L	3 km

Challenge:

Suppose that Brigitte's boat had hit the rock 1 km *further* from the dock than you found in part (c) above. The boat still takes on the same amount of water each minute as before (3 litres) and the boat will still sink once it is holding 144 litres of water.

Brigitte bails faster this time and so she has less time to row. This results in the boat moving at only 1 km per hour. How many litres of water would Brigitte have to bail out of the boat during each minute in order to just reach the dock before the boat sinks?

Solution:

The distance to be covered is now $3 + 1 = 4$ km.
Thus, at 1 kilometre per hour, it will take 4 hours, or $4 \times 60 = 240$ minutes to reach the dock.
To accumulate 144 litres of water in 240 minutes, the boat must accumulate $144 \div 240 = 0.6$ litres of water each minute. (0.6 litres each minute, over 240 minutes, would mean $240 \times 0.6 = 144$ litres overall.)

So Brigitte would have to bail $3 - 0.6 = 2.4$ litres of water out of the boat during each minute.

CEMC at Home

Grade 4/5/6 - Wednesday, May 6, 2020

Looping Around

Try the following problems. How are the problems and their solutions similar?

Skaters

Seven people are skating in a line on a very long, frozen canal. They begin as shown below.

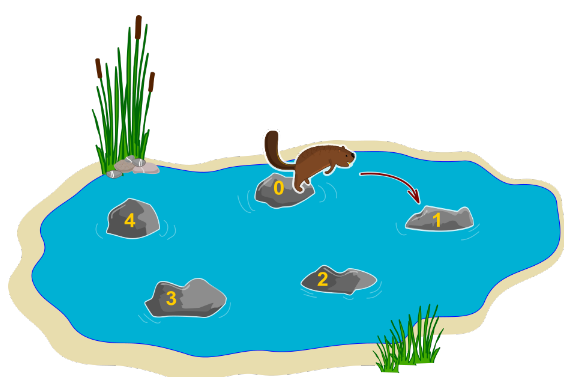
After every minute the person at the front of the line moves to the end of the line. For example, after 1 minute, U will be in front of the line, since V will move behind P.

1. Which skater will be at the front of the line after 3 minutes?
2. Which skater will be at the front of the line after 16 minutes?

Jumping Beaver

An unusual beaver loves to jump. It starts from rock number 0, and jumps clockwise from rock to rock in numerical order. For example, if the beaver jumps 8 times, it ends up on rock number 3:

$$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3$$



1. What rock does the beaver end up on if it jumps 29 times?
2. What rock does the beaver end up on if it jumps 50 times?

More Info:

Check out the CEMC at Home webpage on Wednesday, May 13 for a solution to Looping Around.

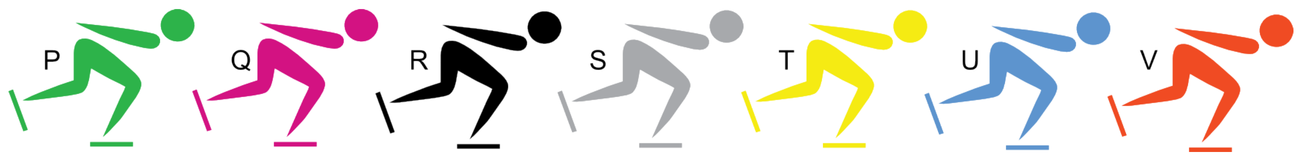
These problems appeared on past Beaver Computing Challenges (BCC). The BCC is a problem solving contest with a focus on computational and logical thinking.

CEMC at Home

Grade 4/5/6 - Wednesday, May 6, 2020
Looping Around - Solution

Skaters

Seven people are skating in a line on a very long, frozen canal. They begin as shown below.

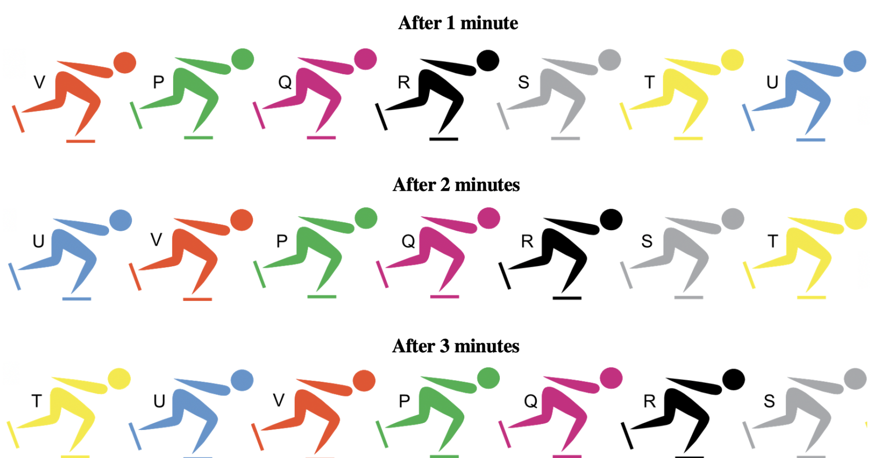


After every minute the person at the front of the line moves to the end of the line. For example, after 1 minute, U will be in front of the line, since V will move behind P.

1. Which skater will be at the front of the line after 3 minutes?
2. Which skater will be at the front of the line after 16 minutes?

Solution:

1. The positions of the skaters after each of the first 3 minutes are shown below.



As shown in the image above, skater S will be at the front of the line after 3 minutes.

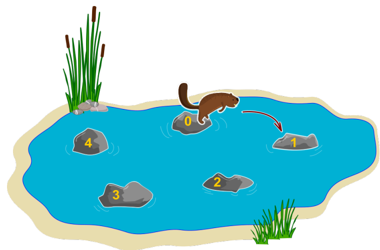
2. We can trace out the positions after each minute.

- at the start, V is in front
- after 1 minute, U is in front
- after 2 minutes, T is in front
- after 3 minutes, S is in front
- after 4 minutes, R is in front
- after 5 minutes, Q is in front
- after 6 minutes, P is in front
- after 7 minutes, V is in front (again)
- after 8 minutes, U is in front (again)
- after 9 minutes, T is in front (again)
- after 10 minutes, S is in front (again)
- after 11 minutes, R is in front (again)
- after 12 minutes, Q is in front (again)
- after 13 minutes, P is in front (again)

Continuing the pattern above, we see that skater T will be in front after 16 minutes.

Jumping Beaver

An unusual beaver loves to jump. It starts from rock number 0, and jumps clockwise from rock to rock in numerical order.



> 1. What rock does the beaver end up on if it jumps 29 times?
> 2. What rock does the beaver end up on if it jumps 50 times?

Solution:

We probably do not want to make a list as we did in the previous question because we would have to keep track of many more steps. We will use a different kind of reasoning here.

If the beaver jumps 5 times, then it will end up where it started (at rock number 0). We can think of 5 jumps as a "lap". After another 5 jumps (or another lap) the beaver will again end up at 0. This pattern will continue.

This means the beaver will be at rock number 0 after 5 jumps, 10 jumps, 15 jumps, 20 jumps, 25 jumps, and so on.

1. The beaver ends up at rock number 4 after 29 jumps. Why is this?

   The beaver is at rock number 0 after 25 jumps. The beaver needs to make 4 more jumps to get to a total of 29 jumps. This will take the beaver to rock number 4.

   $$0 \to 1 \to 2 \to 3 \to 4$$

2. The beaver ends up at rock number 0 after 50 jumps. Why is this?

   Notice that $50 = 10 \times 5$. This means if the beaver jumps 50 times, then the beaver will complete 10 laps exactly, and end up back at rock number 0.

Notice that the two problems in this activity are similar. In the first problem we are studying a repeating pattern. The skaters that are in front of the line follow the pattern

$$\underbrace{V, U, T, S, R, Q, P}, \underbrace{V, U, T, S, R, Q, P}, \underbrace{V, U, T, S, R, Q, P}, \ldots$$

Since this sequence of seven letters keeps repeating, it is possible for us to figure out which skater will be in front after many minutes without having to actually keep listing the skaters. Can you figure out what skater will be in front after 85 minutes?
In the second problem, the rock numbers of the beaver follow a different repeating pattern:

$$\underbrace{0, 1, 2, 3, 4}, \underbrace{0, 1, 2, 3, 4}, \underbrace{0, 1, 2, 3, 4}, \ldots$$

Using the fact that this sequence of five numbers keeps repeating, can you figure out what rock the beaver will be on after 123 jumps?

CEMC at Home features Problem of the Week

Grade 4/5/6 - Thursday, May 7, 2020
How Many Halves?

Robbie won 300 gumballs and he would like to share the winnings with his friends. He decides to list his friends in order based on how long he has known each of them. He wants to give away the most gumballs to the friend he has known the longest. Then he will give the next friend on the list exactly half as many gumballs as the person he has known the longest. He continues to give away exactly half as many to the next friend on his list, until the pattern cannot continue. (He will not give away half a gumball.)

A) If he gives away 100 gumballs to the first friend on the list, how many friends will receive gumballs? Justify your answer.

B) If the last person he gives gumballs to receives 5 gumballs, what is the largest number of gumballs that the first friend on the list can receive? How many gumballs are given away? Justify your answers.

C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.

More Info:

Check the CEMC at Home webpage on Thursday, May 14 for the solution to this problem. Alternatively, subscribe to Problem of the Week at the link below and have the solution emailed to you on Thursday, May 14.

This CEMC at Home resource is the current grade 3/4 problem from Problem of the Week (POTW). This problem was developed for students in grades 3 and 4, but is also appropriate for students in grades 5 and 6. POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students. Each week, problems from various areas of mathematics are posted on our website and e-mailed to our subscribers. Solutions to the problems are e-mailed one week later, along with a new problem. POTW is available in 5 levels: A (grade 3/4), B (grade 5/6), C (grade 7/8), D (grade 9/10), and E (grade 11/12).

To subscribe to Problem of the Week, to view this week's grade 5/6 problem, and to find many more past problems and their solutions, visit the Problem of the Week webpage.

Problem of the Week
Problem A and Solution
How Many Halves?

Problem

Robbie won 300 gumballs and he would like to share the winnings with his friends. He decides to list his friends in order based on how long he has known each of them. He wants to give away the most gumballs to the friend he has known the longest. Then he will give the next friend on the list exactly half as many gumballs as the person he has known the longest. He continues to give away exactly half as many to the next friend on his list, until the pattern cannot continue. (He will not give away half a gumball.)

A) If he gives away 100 gumballs to the first friend on the list, how many friends will receive gumballs? Justify your answer.

B) If the last person he gives gumballs to receives 5 gumballs, what is the largest number of gumballs that the first friend on the list can receive? How many gumballs are given away? Justify your answers.

C) If Robbie wants to maximize the number of friends that receive gumballs, how many gumballs should the first person on the list receive? How many friends receive gumballs? Justify your answers.

Solution

A) If the first friend receives 100 gumballs, then the next friend receives half as many, which is 50 gumballs. Half of 50 is 25 gumballs, which is the amount the next friend on the list receives. Since 25 is an odd number, it cannot be divided in half without ending up with a fraction. So the pattern ends after three friends have received gumballs from Robbie.

B) If we know that the last friend on the list receives 5 gumballs, then as we move up the list, each friend receives twice as many as the previous one. Let's assign the last friend who receives 5 gumballs the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:

Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	5	5
2	10	15
3	20	35
4	40	75
5	80	155
6	160	315

From the information on this table, we can answer the questions.

Since Robbie only has 300 gumballs to give away, if the last person receives 5 gumballs, then the largest amount of gumballs the first person on the list will receive 80 gumballs. There will be a total of 155 gumballs given away.

C) We want to maximize the number of friends to whom Robbie gives gumballs. Using the two solutions above, we notice that the larger the last friend's amount of gumballs the fewer number of friends that can receive gumballs. One way to think about the problem is that we want the last friend to receive the smallest possible amount, which is 1 gumball. So, the second last friend would receive 2 gumballs. As in part (B) let's assign the last friend who receives 1 gumball the number 1. Then we can make a table that shows the pattern of giving gumballs to his friends:

Friend	Number of Gumballs Friend Receives	Total Number of Gumballs Given
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255

From the information on this table, we can answer the questions.

The first person on the list will receive 128 gumballs. A maximum of eight of Robbie's friends can receive gumballs.

Teacher's Notes

In this problem, as we consider one additional friend, the number of gumballs received is reduced by half. For part A), we could write an equation for $g(n)$, the number of gumballs the n^{th} friend gets, as:

$$g(n) = 100 \cdot \left(\frac{1}{2}\right)^{n-1}$$

If we look at the graph of this function, we would see that it is a very steep curve. This is an example of an *exponential function*.

In general, an exponential function is a function of the form:

$$g(n) = m \cdot b^n, \quad \text{where } b > 0 \text{ and } b \neq 1.$$

When $b > 1$, the value of $g(n)$ increases very quickly as n increases. We see exponential growth in real life situations such as compound interest and the spread of viruses.

When $b < 1$ (as in this problem), the value of $g(n)$ decreases very quickly as n increases. We see exponential decay in some chemical reactions and heat transfer.

Exponential functions, exponential growth, and exponential decay are generally seen in high school mathematics and science, and possibly some business studies courses.

CEMC at Home

Grade 4/5/6 - Friday, May 8, 2020
Keeping Your Distance

In this activity, we will play a game with X's and O's on a rectangular grid.

You Will Need:

- Two players
- Some paper
- A pen or pencil

How to Play:

1. Start by drawing a rectangular grid of squares to use as a game board.

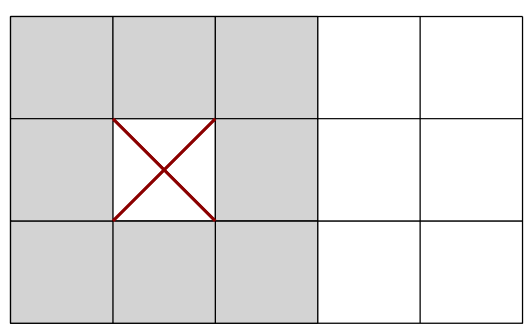
   A 3×5*,* 4×4*, and* 5×5 *game board have been provided for your use on the last page, but you may create game boards of any size.*

2. Players will take turns. Decide which player will go first (Player 1) and which player will go second (Player 2).

3. To begin, Player 1 marks any square in the grid with an X.

4. Next, Player 2 marks any other square in the grid with an O, except that they cannot mark any square that is touching the square with the first X.

   Two squares are touching if they share a side or a vertex. For example, in the game board shown below, Player 2 may not put an O *in any of the squares indicated in grey.*



5. From here, the two players take turns marking the grid with X's (Player 1) and O's (Player 2). Players can never mark a square that touches a square already marked by either player.

6. The first player who is unable to make a move loses, and the other player wins!

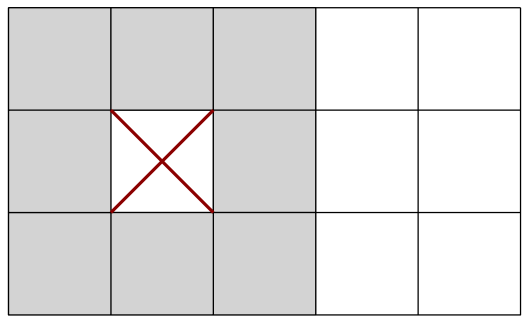
Play this game a number of times.

Try all three of the game boards on the last page, and alternate who goes first and who goes second. Think about the questions on the next page.

Questions:

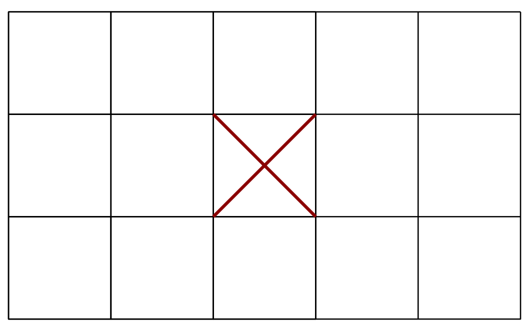
1. Play the game on the 3×5 game board. How many moves are there in the shortest possible game? How many moves are there in the longest possible game?

2. Suppose Player 1 makes the first move shown below.



   How can Player 2 win the game on their first move?

3. Now suppose Player 1 starts the game by claiming the middle square as shown below.



   After making this move, it turns out there is a strategy Player 1 can use to win every time. Can you figure out Player 1's strategy?

4. Can you adjust the strategy you found in 3. so that it works for a 5×5 game board?

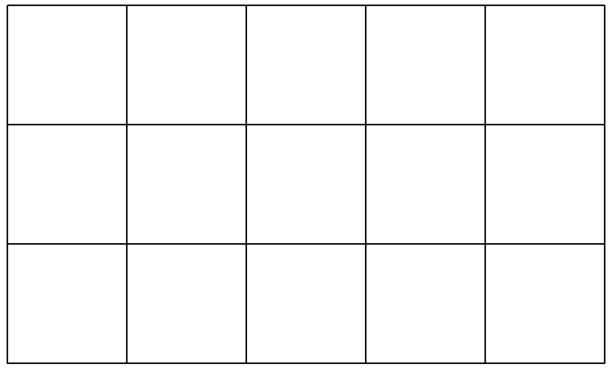
5. Play the game on the 4×4 game board. Which player (Player 1 or Player 2) seems to win most often? Can you come up with a strategy that will make sure this player wins the game every time?

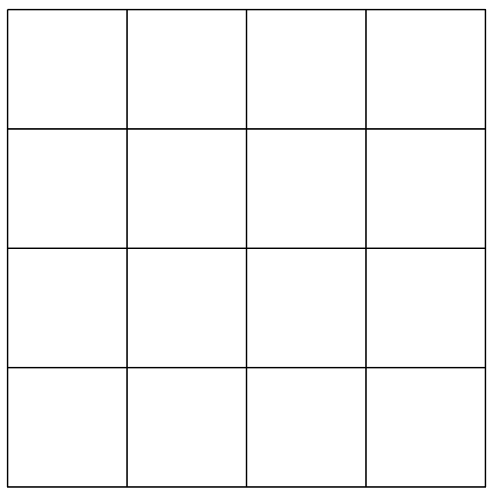
6. Play the game on other larger game boards of your choosing. See if you can find strategies for winning each game!

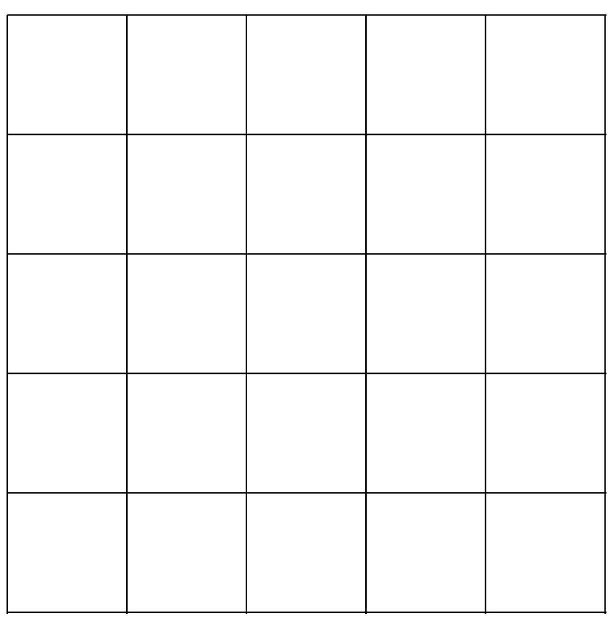
More Info:

Check the CEMC at Home webpage on Friday, May 15 for a discussion of Keeping Your Distance.

Game Boards







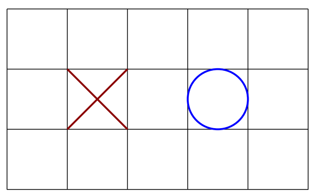
CEMC at Home

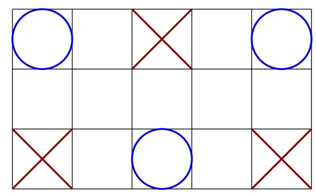
Grade 4/5/6 - Friday, May 8, 2020
Keeping Your Distance - Solution

1. Play the game on the 3×5 game board. How many moves are there in the shortest possible game? How many moves are there in the longest possible game?

   Solution:

   The shortest possible game has 2 moves, and the longest possible game has 6 moves. An example of a finished 2-move game and a finished 6-move game are both given below:

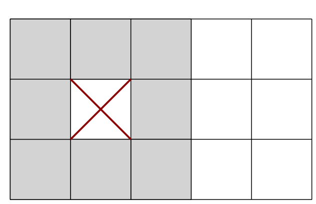
   A game cannot be shorter than 2 moves, because there is no square that Player 1 can mark with an X that will block all of the remaining squares for Player 2.

   To see why a game cannot last longer than 6 moves, think about how many squares could possibly be marked on the game board at the end of a game. On a finished game board, each of the three rows can have at most 3 marked squares. Now, think about the possible number of marked squares in the middle row, and how this affects the other two rows:

   - If there are no marked squares in the middle row, then the top and bottom rows can have up to 3 marked squares each. (This means a maximum of 6 marked squares in total.)
   - If there is 1 marked square in the middle row, then the top and bottom rows can have at most 2 marked squares each. (This means a maximum of 5 marked squares in total.)
   - If there are 2 marked squares in the middle row, then the top and bottom rows can have at most 1 marked square each. (This means a maximum of 4 marked squares in total.)
   - If there are 3 marked squares in the middle row, then there cannot be any other marked squares on the board. (This means a maximum of 3 marked squares in total.)

   This means that a finished game board can have at most 6 marked squares and so a game cannot possibly have more than 6 moves. (A 7th move would mean a 7th marked square which is impossible.)

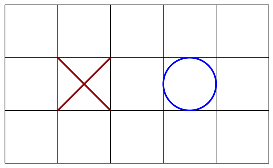
2. Suppose Player 1 makes the first move shown below.

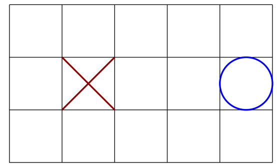
   

   How can Player 2 win the game on their first move?

Solution:

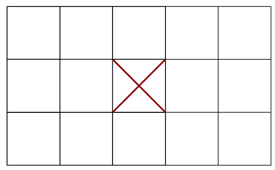
Player 2 actually has two winning moves available. The two moves are shown below.





In either case, every empty square that remains is touching a marked square, and so Player 1 cannot make any legal move on their next turn.

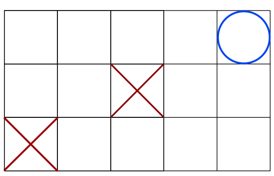
3. Now suppose Player 1 starts the game by claiming the middle square as shown below.

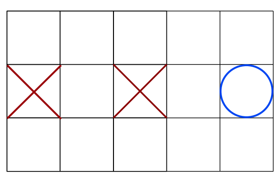


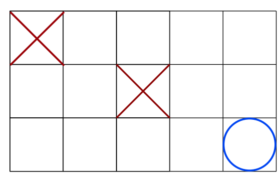
After making this move, it turns out there is a strategy Player 1 can use to win every time. Can you figure out Player 1's strategy?

Solution:

Player 1 can adopt a type of "mirroring strategy", in which they copy everything that Player 2 does, but on the "opposite side of the X" in the centre of the game board. For example, if Player 2 makes any of the three moves shown below (marked with an O) then Player 1 responds by making the corresponding move shown (marked with a second X) on the opposite side.







Player 1's response will be similar if Player 2 chooses to make their first move on the left side of the board.

Because Player 1 is always copying Player 2 by playing on the opposite side of the X in the centre, Player 1 will always be able to make a legal move if Player 2 could. If there's a legal move for Player 2 somewhere on the game board, then there must be a corresponding legal move on the opposite side of the centre X.

Since Player 1 is always one step ahead of Player 2, and will not run out of legal moves until after Player 2 does, Player 1 will win.

4. Can you adjust the strategy you found in 3. so that it works for a 5×5 game board?

Solution:

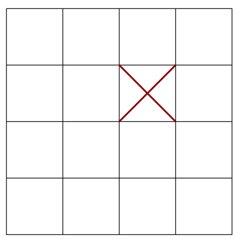
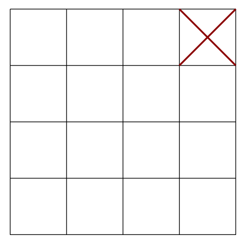
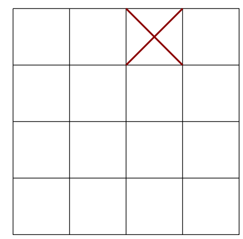
Yes, Player 1's strategy of marking the middle square of the board and then "mirroring" Player 2 will work equally well on a 5×5 game board, for exactly the same reasons as explained above. The key to making this strategy work is that both the rows and the columns on the game board have an odd number of squares. This is necessary in order to make sure there *is* a middle square of the game board (why?). As long as the game board has a middle square, the strategy outlined in in 3. will always work for Player 1.

5. Play the game on the 4×4 game board. Which player (Player 1 or Player 2) seems to win most often? Can you come up with a strategy that will make sure this player wins the game every time?

   Solution:

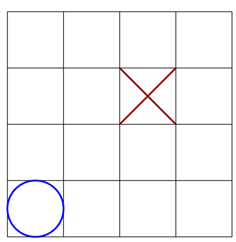
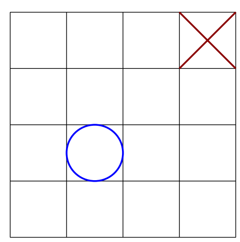
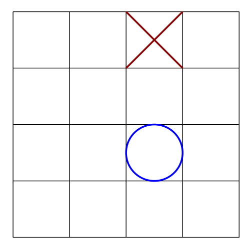
   On the 4×4 game board, the strategy gets more interesting. It turns out that Player 2 can always win, but the strategy is less simple to describe than the 3×5 case.

   Let's suppose that Player 1's first move is one of these three moves:

   Because of the symmetry of the game board, the explanations will be similar if Player 1 chooses a first move other than these three.

   In the three cases above, Player 2 should respond as follows:

   On the first game board, note that Player 1 only has four squares left to play on: the top two squares of the leftmost column, or the rightmost two squares in the bottom row. If Player 1 takes a square in the leftmost column, Player 2 can take either legal square in the bottom row and win. If Player 1 instead goes for one of the squares in the bottom row, then Player 2 can take either legal square in the leftmost column and still win.

   Can you see how to use this explanation above to explain Player 2's strategy on the second game board? Think about reflecting the game board in the diagonal from the upper left corner to the lower right corner. Notice that after this reflection, the marked squares are the same as on the first game board. The strategy here will be similar to the first board.

   On the third game board, Player 1 is only allowed to play in the leftmost column, in which every square is open. But notice that whichever square Player 1 marks, at least one square in that column will remain open for Player 2. Once Player 2 also claims a square in that column, there will be no valid moves left, and so Player 2 wins.

   Think about how to adapt the strategies described above to the other possible game boards after the first two moves.

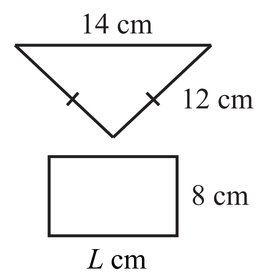
CEMC at Home

Grade 4/5/6 - Monday, May 11, 2020
Contest Day 2

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #9

In the diagram, the perimeter of the triangle is equal to the perimeter of the rectangle. What is the length (L) of the rectangle?



(A) 8 **(B)** 10 **(C)** 11
(D) 14 **(E)** 15

2017 Gauss Contest, #14

When the time in Toronto, ON is 1:00 p.m., the time in Gander, NL is 2:30 p.m. A flight from Toronto to Gander takes 2 hours and 50 minutes. If the flight departs at 3:00 p.m. (Toronto time), what time will the flight land in Gander (Gander time)?

(A) 7:20 p.m. **(B)** 5:00 p.m. **(C)** 6:20 p.m. **(D)** 5:20 p.m. **(E)** 8:50 p.m.

More Info:

Check out the CEMC at Home webpage on Thursday, May 21 for solutions to the Contest Day 2 problems.

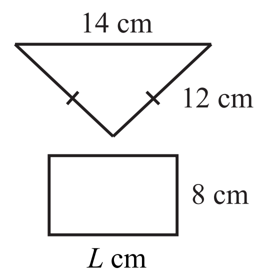
CEMC at Home

Grade 4/5/6 - Monday, May 11, 2020
Contest Day 2 - Solution

Solutions to the two contest problems are provided below, including a video for the first problem.

2020 Gauss Contest, #9

In the diagram, the perimeter of the triangle is equal to the perimeter of the rectangle. What is the length (L) of the rectangle?



(A) 8 **(B)** 10 **(C)** 11 **(D)** 14 **(E)** 15

Solution:

Before you can solve this problem you have to remember that perimeter means the distance around the outside of a shape, and when a shape has a tick on two sides, it indicates that both sides are the same length. This means the triangle has two sides of length 12 cm and one of length 14 cm. Its perimeter is $12 + 12 + 14 = 38$ cm. The rectangle also has the same perimeter. We know that two of the sides of the rectangle are 8 cm long, so together they add to 16 cm. The remaining two sides combined are therefore $38 - 16 = 22$ cm. Since both of these sides are equal in length, the value of L must be $22 \div 2 = 11$.

ANSWER: (C)

Video

Visit the following link for a discussion of the solution to the first contest problem:
https://youtu.be/wmLK4eFEcpo

2017 Gauss Contest, #14

When the time in Toronto, ON is 1:00 p.m., the time in Gander, NL is 2:30 p.m. A flight from Toronto to Gander takes 2 hours and 50 minutes. If the flight departs at 3:00 p.m. (Toronto time), what time will the flight land in Gander (Gander time)?

(A) 7:20 p.m. **(B)** 5:00 p.m. **(C)** 6:20 p.m. **(D)** 5:20 p.m. **(E)** 8:50 p.m.

Solution:

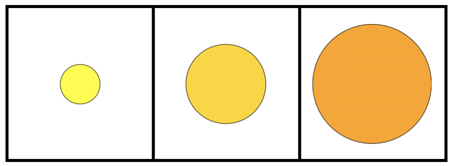
Since the time in Toronto, ON is 1:00 p.m. when the time in Gander, NL is 2:30 p.m., then the time in Gander is 1 hour and 30 minutes ahead of the time in Toronto.
A flight that departs from Toronto at 3:00 p.m. and takes 2 hours and 50 minutes will land in Gander at 5:50 p.m. Toronto time.
When the time in Toronto is 5:50 p.m., the time in Gander is 1 hour and 30 minutes ahead which is 7:20 p.m.

ANSWER: (A)

CEMC at Home

Grade 4/5/6 - Tuesday, May 12, 2020
Shifting Discs

Three discs, each of a different size, are arranged in a grid as shown below. Each disc starts off in its own square with the discs arranged in increasing order of size, so that the smallest disc is in the leftmost square and the largest disc is in the rightmost square.

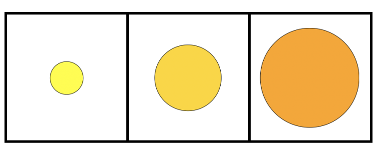


Your goal is to reverse the order of the discs, so that the smallest disc is in the rightmost square and the largest disc is in the leftmost square, however, you must follow certain rules when moving the discs:

- At all times, each square in the grid must contain a single disc, a single stack of discs, or be empty.
- A disc may be moved on top of another disc of larger size, but not of smaller size.
- Any single disc may be moved left or right into any empty square in the grid.
- Discs can only be moved over one square at a time.
  For example, a disc cannot be move directly from the leftmost square to the rightmost square without passing through the middle square.
- Only one disc can be moved at a time. If there is a stack of discs in a square in the grid, then only the top disc in the stack can be moved, not the entire stack at once.

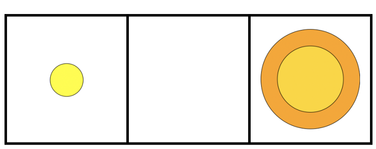
For example, here are three moves performed one after the other that follow the rules:

Start



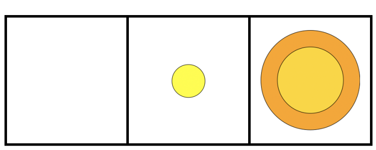
You can either move the small disc on top of the medium disc, or move the medium disc on top of the large disc.

After Move 1



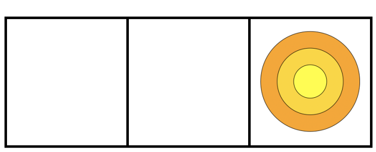
You now cannot move the stack of two discs. You can only move the medium disc back or move the small disc over.

After Move 2



You cannot move the stack of two discs. You cannot move the medium disc as it would have to go on top of the small disc.

After Move 3

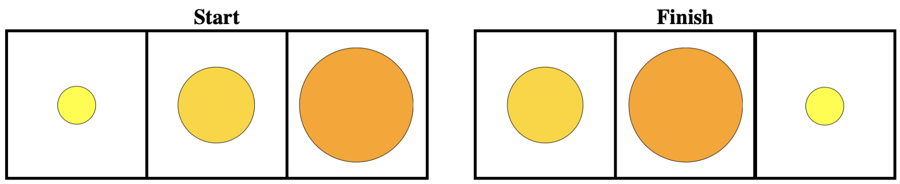


You can only move the small disc from this position.

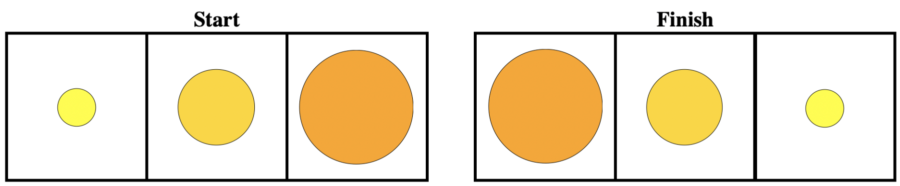
See the next page for some problems to think about while you explore.

Problems:

1. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right.



2. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right, which has the discs in the reverse order.



   Solving this problem is the main goal of the activity!

3. Now suppose you start with four discs instead of three. Just like before, they are all different sizes, and arranged in increasing order of size, with the smallest disc on the left and largest disc on the right. As in 2., you want to reverse the order of the discs, following the same rules. How can you use your solution for moving three discs from 2. to come up with a solution for moving four discs?

4. Building on the previous question, how can you use the solution for four discs to get a solution for the similar puzzle for five discs? In general, if you know how to solve the puzzle for a certain number of discs, how can you use it to solve the similar puzzle with one more disc added?

Extension: Suppose we add one more rule: stacks cannot have more than two discs in them at any time. Do you think there is still a solution to the puzzle with three discs from 2.? Do you think there is still a solution to the similar puzzle with four discs? Explain your answers.

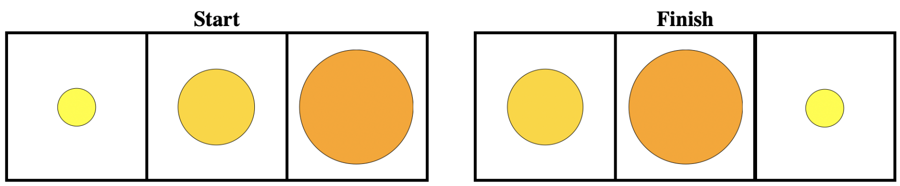
More Info:

Check out the CEMC at Home webpage on Tuesday, May 19 for a solution to Shifting Discs.

CEMC at Home

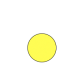
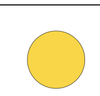
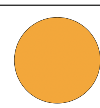
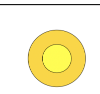
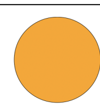
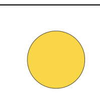
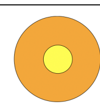
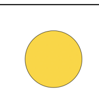
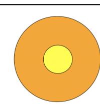
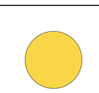
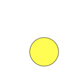
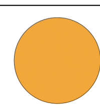
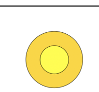
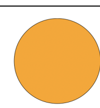
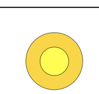
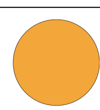
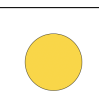
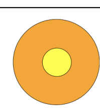
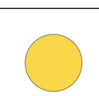
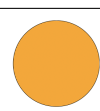
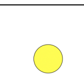
Grade 4/5/6 - Tuesday, May 12, 2020
Shifting Discs - Solution

1. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right.

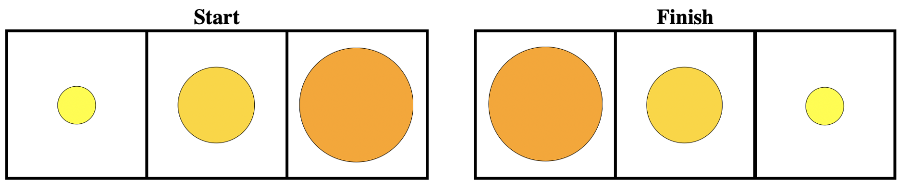


Solution:

The table below shows one possible sequence of moves that produces the correct final arrangement of the discs.

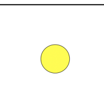
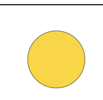
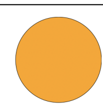
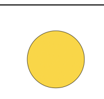
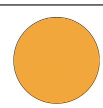
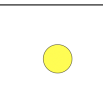
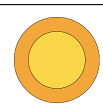
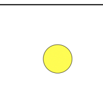
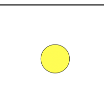
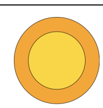
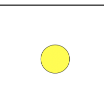
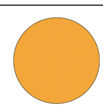
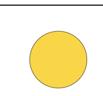
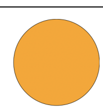
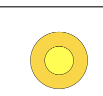
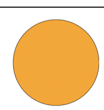
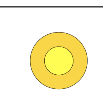
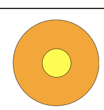
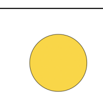
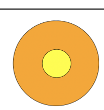
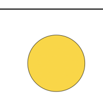
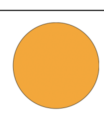
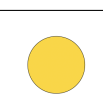
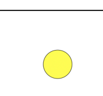
Left Position	Middle Position	Right Position	Explanation
			Starting position
			Move the small disc to the right
			Move the small disc to the right
			Move the medium disc to the left
			Move the small disc to the left
			Move the small disc to the left
			Move the large disc to the left
			Move the small disc to the right
			Move the small disc to the right

2. Describe a sequence of moves that takes the three discs from the starting arrangement shown below on the left to the arrangement shown below on the right, which has the discs in the reverse order.



Solution:

We start by following the moves from 1. to get the discs into the arrangement medium, large, small. Then we continue as shown the table below to get the final arrangement with the discs reversed. Note that we sometimes do more than one move in each row.

Left Position	Middle Position	Right Position	Explanation
			Starting position
			Final position from 1.
			Move the medium disc to the right
			Move the small disc to the left, **twice**
			Move the medium disc to the right
			Move the small disc to the right, **twice**
			Move the large disc to the left
			Move the small disc to the left, **twice**
			Move the medium disc to the left
			Move the small disc to the right, **twice**

3. Now suppose you start with four discs instead of three. Just like before, they are all different sizes, and arranged in increasing order of size, with the smallest disc on the left and largest disc on the right. As in 2., you want to reverse the order of the discs, following the same rules. How can you use your solution for moving three discs from 2. to come up with a solution for moving four discs?

Solution:

Let's label the discs 1, 2, 3, and 4, from smallest to largest. Using this labelling we are tasked with moving the discs from the starting configuration shown below on the left, to the final configuration shown below on the right.

1	2	3	4

4	3	2	1

We illustrate our solution in the table below. Each row of the table shows where each of the four discs (1, 2, 3, and 4) are located in the grid at each stage. If more than one disc is in the same square in the grid at some time, then these discs will be in a stack.

We start our solution by ignoring disc 1 and rearranging discs 2, 3 and 4 using the method from the solution to problem 2 to reverse them: | 1 | 2 | 3 | 4 | becomes | 1 | 4 | 3 | 2 |.

Position A	Position B	Position C	Position D	Explanation
1	2	3	4	Starting position
1	4	3	2	Problem 2. with discs 2, 3, and 4
	4	3	1, 2	Move disc 1 all the way to the right
4	3		1, 2	Move disc 4, then disc 3 to the left
4	1, 3		2	Move disc 1 to the left, twice
4	1, 3	2		Move disc 2 to the left
4	3	2	1	Move disc 1 all the way to the right

4. Building on the previous question, how can you use the solution for four discs to get a solution for the similar puzzle for five discs? In general, if you know how to solve the puzzle for a certain number of discs, how can you use it to solve the similar puzzle with one more disc added?

Solution:

Once we have a set of moves to solve the four-disc puzzle, such as given above, we can solve the five-disc version of the puzzle using the same type of strategy as in the solution to problem 3. We number the discs 1, 2, 3, 4, and 5, from smallest to largest, and illustrate the solution in the table below.

Pos. A	Pos. B	Pos. C	Pos. D	Pos. E	Explanation
1	2	3	4	5	Starting position
1	5	4	3	2	Problem 3. with discs 2, 3, 4, and 5
	5	4	3	1, 2	Move disc 1 all the way to the right
5	4	3		1, 2	Move disc 5, then disc 4, then disc 3 to the left
5	4	1, 3		2	Move disc 1 to the left, twice
5	4	1, 3	2		Move disc 2 to the left
5	4	3	2	1	Move disc 1 all the way to the right

We can use the same strategy to solve the puzzle for any larger number of discs, too! Can you see how to do this? We can always use our solution to the previous puzzle as the first step in our solution to the next puzzle. How can you use the solution to the five-disc puzzle to get a solution to the six-disc puzzle? How can you get from the six-disc puzzle to the seven-disc puzzle?

Extension: Suppose we add one more rule: stacks cannot have more than two discs in them at any time. Do you think there is still a solution to the puzzle with three discs from 2.? Do you think there is still a solution to the similar puzzle with four discs? Explain your answers.

Solution:

It turns out that it is not possible to solve the three-disc puzzle with this new rule in place. Our solution to the three-disc puzzle (shown earlier) used a stack of three discs, but how can we be sure that there isn't some *other* sequence of moves that avoids any stacks of three?

To see why a stack of three discs is unavoidable, we note that we must start with the discs in the order $1, 2, 3$ and need to move disc 3 all the way to the left.

In order to move disc 3 at all, we must first arrange the discs as follows:

1, 2		3

.

Then we can move disc 3 to the left one place:

1, 2	3	

.

Now, in order to move disc 3 to the left again, we need to somehow move discs 1 and 2 past disc 3 to the right. This cannot be done without forming a stack of three at some point. Can you see why? Here is the idea:

In order to have a chance at moving disc 2, we first need to move disc 1 all the way to the right:

2	3	1

. The problem is that there is now no way to move disc 2 past disc 3 without first moving disc 1 off the rightmost square. The only way to have disc 2 occupy the rightmost square after this point is to have disc 2 and disc 1 somehow "pass" each other, over disc 3. The only way to make this happen is to create a stack of all three discs in the middle (which is not allowed).

On the other hand, it *is* possible to solve the four-disc puzzle with this new rule in place. One way to do it is illustrated in the table below:

Pos. A	Pos. B	Pos. C	Pos. D	Explanation
1	2	3	4	Starting position
	2	3	1, 4	Move disc 1 all the way to the right
2	3		1, 4	Move disc 2, then disc 3 to the left
2	1, 3		4	Move disc 1 onto disc 3
2	1, 3	4		Move disc 4 to the left
2	3	4	1	Move disc 1 all the way to the right
	3	2, 4	1	Move disc 2 onto disc 4
3		2, 4	1	Move disc 3 to the left
2, 3		4	1	Move disc 2 all the way to the left
2, 3	4		1	Move disc 4 to the left
3	4	2	1	Move disc 2 two places right
3	1, 4	2		Move disc 1 two places left
3	1, 4		2	Move disc 2 to the right
3	4		1, 2	Move disc 1 all the way to the right
	4	3	1, 2	Move disc 3 two places right
4	3		1, 2	Move disc 4, then disc 3 to the left
4	1, 3		2	Move disc 1 two places left
4	1, 3	2		Move disc 2 to the left
4	3	2	1	Move disc 1 all the way to the right

CEMC at Home

Grade 4/5/6 - Thursday, May 21, 2020
Contest Day 3

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #12

Abdul is 9 years older than Susie, and Binh is 2 years older than Susie. How many years older is Abdul than Binh?

(A) 11 **(B)** 9 **(C)** 14 **(D)** 2 **(E)** 7

2011 Gauss Contest, #10

I bought a new plant for my garden. Anika said it was a red rose, Bill said it was a purple daisy, and Cathy said it was a red dahlia. Each person was correct in stating either the colour or the type of plant. What was the plant that I bought?

(A) purple dahlia **(B)** purple rose **(C)** red dahlia **(D)** yellow rose **(E)** red daisy

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More Info:

Check out the CEMC at Home webpage on Monday, May 25 for solutions to the Contest Day 3 problems.

CEMC at Home

Grade 4/5/6 - Thursday, May 21, 2020
Contest Day 3 - Solution

Solutions to the two contest problems are provided below.

2020 Gauss Contest, #12

Abdul is 9 years older than Susie, and Binh is 2 years older than Susie. How many years older is Abdul than Binh?

(A) 11 **(B)** 9 **(C)** 14 **(D)** 2 **(E)** 7

Solution 1:

Abdul is 9 years older than Susie, and Binh is 2 years older than Susie, and so Abdul is $9 - 2 = 7$ years older than Binh.
For example, if Susie is 10 years old, then Abdul is $9 + 10 = 19$, Binh is $2 + 10 = 12$, and Abdul is $19 - 12 = 7$ years older than Binh.

Solution 2:

We know that Abdul is 9 years older than Susie and Binh is 2 years older than Susie.
Let's say that Susie is 5 years old. That will make Abdul $5 + 9 = 14$ and Binh $5 + 2 = 7$.
This would mean Abdul is $14 - 7 = 7$ years older than Binh.
Let's double-check if this works when Susie is a different age.
This time, let Susie be 8 years old. That will make Abdul $8 + 9 = 17$ and Binh $8 + 2 = 10$.
This would mean Abdul is $17-10 = 7$ years older than Binh, which is the same as our previous answer.

Answer: (E)

2011 Gauss Contest, #10

I bought a new plant for my garden. Anika said it was a red rose, Bill said it was a purple daisy, and Cathy said it was a red dahlia. Each person was correct in stating either the colour or the type of plant. What was the plant that I bought?

(A) purple dahlia **(B)** purple rose **(C)** red dahlia **(D)** yellow rose **(E)** red daisy

Solution:

Anika said that the plant was a *red* rose.
Cathy said that the plant was a *red* dahlia.
If the plant is not red, then both Anika and Cathy are wrong about the colour, so they would have to be right about the plant type. Unfortunately, they each said a different plant type, so that is not possible. This means the plant must be red.
Bill said that the plant was a purple daisy. Bill is wrong about the colour, so he must be right about the flower type. This means the plant is a red daisy.

Answer: (E)

CEMC at Home

Grade 4/5/6 - Friday, May 22, 2020
Toothpick Polyhedrons

You Will Need:

- Toothpicks of the same length
  (at least 12, but preferably more)
- Some miniature marshmallows, licorice bits, or bits of play dough.
- A flat surface on which to work

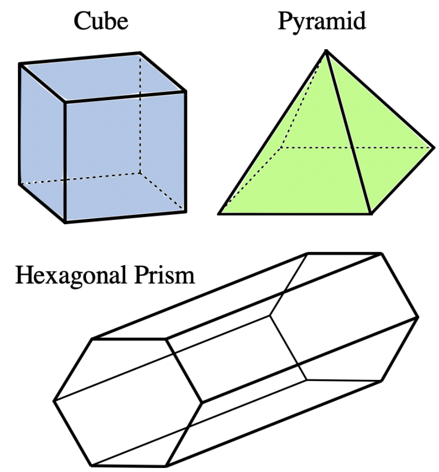
Introduction:

A *polyhedron* is a three-dimensional object with polygons for its faces. Recall that a polygon is a two-dimensional closed shape formed by three or more line segments. Examples of polygons include triangles, quadrilaterals, pentagons, hexagons, and octagons.

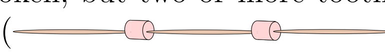
A cube is an example of a polyhedron and has six square faces.

A square-based pyramid is another example of a polyhedron, and has one square face and four triangular faces.

We can build models of polyhedrons by building *skeletons* of the objects. A skeleton includes all of the edges of the polyhedron. For example, the skeleton of a hexagonal prism (another type of polyhedron) is shown on the right.



What to Do:

The goal of this activity is to construct models of polyhedrons using toothpicks joined together with miniature marshmallows. Toothpicks cannot be broken, but two or more toothpicks may be joined together to create edges longer than one toothpick ().

Once you have constructed a new polyhedron, sketch the polyhedron carefully and state its name or make up a fun, suitable name.

Activities and Questions to Explore:

1. Construct a model of a square-based pyramid. How many toothpicks did you use? How many marshmallows did you use? Is there more than one way to do this?
2. See how many different polyhedrons you can construct using exactly 12 toothpicks.
   For example, what could you make with a square base? A triangular base? Other bases?
3. Now see how many different polyhedrons you can construct using *fewer* than 12 toothpicks.
4. Can you construct a polyhedron with a hexagonal base using exactly 12 toothpicks? Why, or why not?

More info:

Check out the CEMC at Home webpage on Friday, May 29 for a solution to Toothpick Polyhedrons.

CEMC at Home

Grade 4/5/6 - Friday, May 22, 2020
Toothpick Polyhedrons - Solution

1. Construct a model of a square-based pyramid. How many toothpicks did you use? How many marshmallows did you use? Is there more than one way to do this?

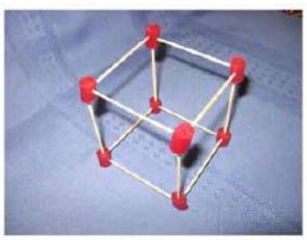
   Solution:

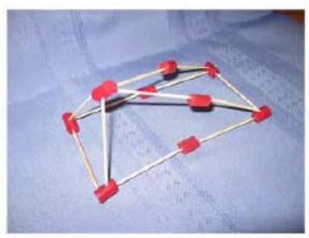
   You can construct a model of a square-based pyramid in more than one way. For example, you can do so using 8 toothpicks and 5 marshmallows, or 12 toothpicks and 9 marshmallows. A diagram of this second way is shown at the end of the solution to 2. below.

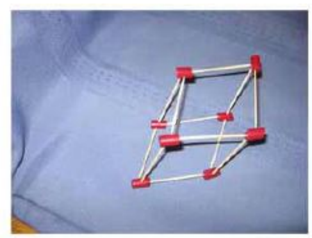
2. See how many different polyhedrons you can construct using exactly 12 toothpicks.

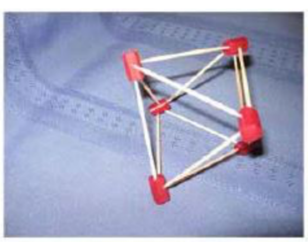
   Solution:

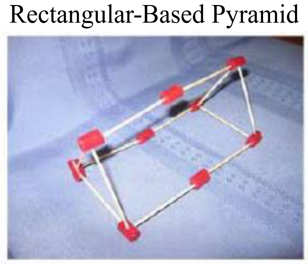
   Here are some photos of models of various polyhedrons which can be built using exactly 12 toothpicks.


Cube


Rectangular-Based Pyramid


Rhombohedron

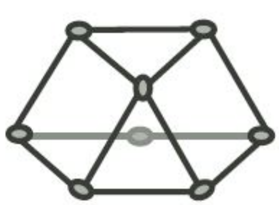

Octahedron

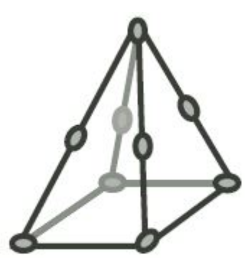

Triangular Prism

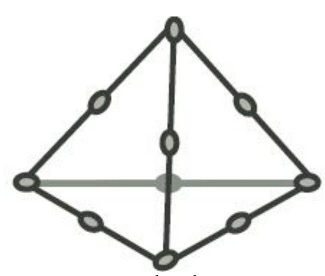

Triangular-Based Pyramid

Notice that the sides of the triangular-based pyramid have sagged a bit under their own weight before the photo was taken. The three toothpicks forming the edges joining at the top vertex should actually be straight.

Here are some sketches of a few more polyhedrons you can model with exactly 12 toothpicks.


You name it!

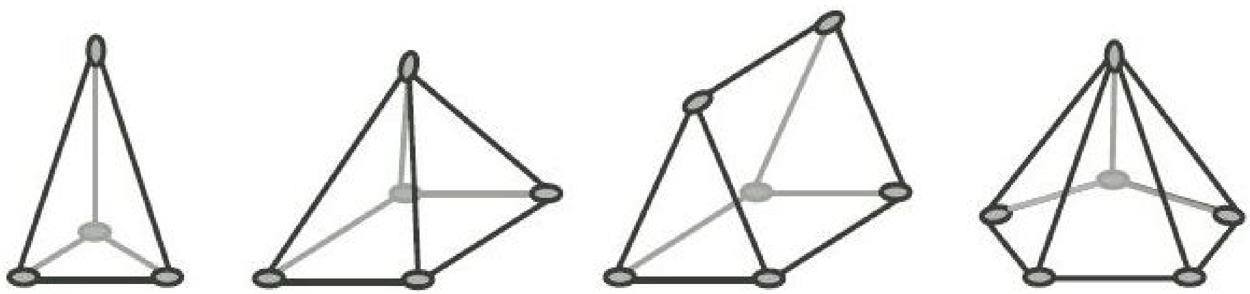

Square-based Pyramid


Tetrahedron

3. Now see how many different polyhedrons you can construct using *fewer* than 12 toothpicks.

   Solution:

   Here are some polyhedrons that you can model using *fewer* than 12 toothpicks. Three of them are pyramids and one is a prism. Can you name each of the polyhedrons?

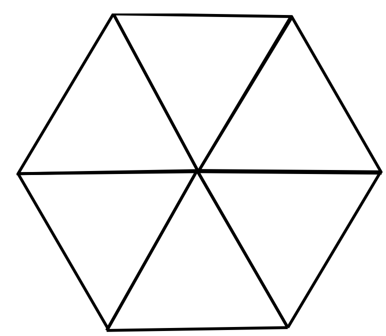


4. Can you construct a polyhedron with a hexagonal base using exactly 12 toothpicks? Why, or why not?

   Solution:

   It is not possible to do this.

   If you try to construct a model of a polyhedron with a hexagonal base, then the base must use at least 6 toothpicks to construct. This would leave you with at most 6 toothpicks to form the other edges. Each of the 6 vertices of the base must have a toothpick coming out, but when you try joining them to form a "peak", you will discover that you actually get a two-dimensional figure!



   This is because a regular hexagon (with equal sides the length of one toothpick) is formed from six equilateral triangles. So your toothpick model would have the shape of the diagram at the right.

CEMC at Home

Grade 4/5/6 - Monday, May 25, 2020
Contest Day 4

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #11

Each of 7 boxes contains exactly 10 cookies. If the cookies are shared equally among 5 people, how many cookies does each person receive?

(A) 14 **(B)** 12 **(C)** 9 **(D)** 11 **(E)** 13

2011 Gauss Contest, #16

A 51 cm rod is built from 5 cm rods and 2 cm rods. All of the 5 cm rods must come first, and are followed by the 2 cm rods. For example, the rod could be made from seven 5 cm rods followed by eight 2 cm rods. How many ways are there to build the 51 cm rod?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

More Info:

Check out the CEMC at Home webpage on Monday, June 1 for solutions to the Contest Day 4 problems.

CEMC at Home

Grade 4/5/6 - Monday, May 25, 2020
Contest Day 4 - Solution

Solutions to the two contest problems are provided below, including a video for the second problem.

2020 Gauss Contest, #11

Each of 7 boxes contains exactly 10 cookies. If the cookies are shared equally among 5 people, how many cookies does each person receive?

(A) 14 **(B)** 12 **(C)** 9 **(D)** 11 **(E)** 13

Solution:

Each of 7 boxes contains exactly 10 cookies, and so the total number of cookies is $7 \times 10 = 70$.
If the cookies are shared equally among 5 people, then each person receives $70 \div 5 = 14$ cookies.

ANSWER: (A)

2011 Gauss Contest, #16

A 51 cm rod is built from 5 cm rods and 2 cm rods. All of the 5 cm rods must come first, and are followed by the 2 cm rods. For example, the rod could be made from seven 5 cm rods followed by eight 2 cm rods. How many ways are there to build the 51 cm rod?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Solution 1:

Using the method of trial and error, we begin by trying one rod with length 5 cm.
If there is one 5 cm rod, then there is $51 - 5 = 46$ cm remaining to be made up of 2 cm rods.
Since $46 \div 2 = 23$, then using one 5 cm rod and twenty-three 2 cm rods gives a possible combination.
Next, let's try two 5 cm rods.
Since $2 \times 5 = 10$ cm, then there is $51 - 10 = 41$ cm remaining to be made up of 2 cm rods.
However, 2 does not divide into 41 evenly, so it is not possible to have two 5 cm rods.
Next, we try three rods with length 5 cm.
If there are three 5 cm rods, then there is $51 - (3 \times 5) = 51 - 15 = 36$ cm remaining to be made up of 2 cm rods.
Since $36 \div 2 = 18$, then using three 5 cm rods and eighteen 2 cm rods gives a possible combination.
Next, let's try four 5 cm rods.
Since $4 \times 5 = 20$ cm, then there is $51 - 20 = 31$ cm remaining to be made up of 2 cm rods.
However, 2 does not divide into 31 evenly, so it is not possible to have four 5 cm rods.
We can see that using an odd number of 5 cm rods allow us to find a solution that works, but using an even number of 5 cm rods does not give a possible combination that works.
(Can you see why this is true? Below, Solution 2 explains this idea further.)
Five 5 cm rods will give a combination that works.
If there are five 5 cm rods, then there is $51 - (5 \times 5) = 51 - 25 = 26$ cm remaining to be made up of 2 cm rods.

See the next page for the rest of Solution 1 and for Solution 2.

Since $26 \div 2 = 13$, then using five 5 cm rods and thirteen 2 cm rods gives a possible combination.
If there are seven 5 cm rods, then there is $51 - (7 \times 5) = 51 - 35 = 16$ cm remaining to be made up of 2 cm rods.
Since $16 \div 2 = 8$, then using seven 5 cm rods and eight 2 cm rods gives a possible combination.
If there are nine 5 cm rods, then there is $51 - (9 \times 5) = 51 - 45 = 6$ cm remaining to be made up of 2 cm rods.
Since $6 \div 2 = 3$, then using nine 5 cm rods and three 2 cm rods gives a possible combination.
However, using eleven 5 cm rods does not work because $11 \times 5 = 55$ cm is greater than 51 cm.
Using any number of 5 cm rods that is greater than 11 will similarly have a total length that is more than 51 cm.
Therefore, there are exactly 5 possible combinations that add to 51 cm using 5 cm rods first followed by 2 cm rods.

Solution 2:

Any number of 2 cm rods add to give a rod having an even length.
Since we need an odd length, 51 cm, then we must combine an odd length from the 5 cm rods with the even length from the 2 cm rods to achieve this.
An odd length using 5 cm rods can only be obtained by taking an odd number of them.
All possible combinations are shown in the table below.

Number of 5 cm rods	Length in 5 cm rods	Length in 2 cm rods	Number of 2 cm rods
1	5	$51 - 5 = 46$	$46 \div 2 = 23$
3	15	$51 - 15 = 36$	$36 \div 2 = 18$
5	25	$51 - 25 = 26$	$26 \div 2 = 13$
7	35	$51 - 35 = 16$	$16 \div 2 = 8$
9	45	$51 - 45 = 6$	$6 \div 2 = 3$

Note that attempting to use 11 (or more) 5 cm rods gives more than the 51 cm length required.
Thus, there are exactly 5 possible combinations that add to 51 cm using 5 cm rods first followed by 2 cm rods.

ANSWER: (A)

Video

Visit the following link to view a discussion of a solution to the second contest problem, and some extensions of the problem: https://youtu.be/Hy-qFPnNatQ

CEMC at Home

Grade 4/5/6 - Tuesday, May 26, 2020
Catching a Code

Try out the following two-player game involving codebreaking!

You Will Need: Two players, paper and a pencil

How to Play:

- The two players in this game alternate between being the "Code Maker" and the "Code Breaker" in each round.
- The players start by choosing an *even* number of rounds to play in the game.
- At the start of each round, the Code Maker chooses a three-digit code, using only the digits 1 through 6 (for example, 146 or 222), and writes the code on a piece of paper, keeping it secret from the Code Breaker.
- The Code Breaker then gets up to ten attempts to guess the Code Maker's code. After each guess, the Code Maker provides two pieces of information about the guess:
  - the number of digits in the guess that appear in the code and are in the correct place, and
  - the number of digits in the guess that appear in the code but are in the wrong place.

  For example, suppose that the Code Maker's code is 263. If the Code Breaker guesses 361, then the Code Maker would give the following information: one of the digits in the guess is in the correct place, and one of the digits in the guess appears in the code but is in the wrong place. If the Code Breaker guesses 336, then the Code Maker would give the following information: no digits are in the correct place, and two digits appear in the code but are in the wrong place. *Note that the two digits here are the 6 and one of the 3s. The Code Maker does not say that all three digits appear in the code because there is only one 3 in the code.*
- The round ends when either the Code Breaker correctly guesses the code or the Code Breaker has made ten incorrect guesses. If the Code Breaker guesses the code in ten or fewer attempts, then the Code Breaker's score for the round is equal to the total number of attempts needed. If the Code Breaker does not guess the code in ten or fewer attempts, then the Code Breaker's score for the round is 11. (The Code Maker does not score.)
- The game ends when the chosen even number of rounds are completed. The winner is the player with the *lowest* total score.

Example: Here is a sample game of Catching a Code, organized in a table.

Code Maker's Code: 234			
Guess #	Guess	Correct Digit – Correct Place	Correct Digit – Wrong Place
1	566	0	0
2	113	0	1
3	423	0	3
4	243	1	2
5	234	3	0

Here, the Code Breaker found the correct code after five attempts. This means the Code Breaker scores 5 points for this round.

Play this game a number of times.

Below, you will find some questions to think about that relate to the sample game provided.

A blank table is also given below for you to use while you play.

Questions:

1. Based on the first guess in the sample game provided, the Code Breaker decides that the correct secret code only contains the digits 1 through 4. Why is this true?
2. After the third guess in the sample game provided, how many possibilities remain for the Code Maker's code? What are they?
3. After the fourth guess in the sample game provided, how many possibilities remain for the Code Maker's code? What are they?

Sample Table:

Guess #	Guess	Correct Digit – Correct Place	Correct Digit – Wrong Place
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

More Info:

Check out the CEMC at Home webpage on Tuesday, June 2 for answers to the questions above.

This game is a pencil-and-paper version of a popular board game. The advantage to the pencil-and-paper version is that you can easily change the rules as you wish. For example, the length of the secret code can be made longer, consisting of four, five, or even more digits. Or, you can increase or decrease the number of possible digits allowed in the code (for example, you can use only the digits 1 through 4, or you can use all of the digits 0 through 9).

CEMC at Home

Grade 4/5/6 - Tuesday, May 26, 2020
Catching a Code – Solution

Code Maker's Code: 234			
Guess #	Guess	Correct Digit – Correct Place	Correct Digit – Wrong Place
1	566	0	0
2	113	0	1
3	423	0	3
4	243	1	2
5	234	3	0

1. Based on the first guess in the sample game provided, the Code Breaker decides that the correct secret code only contains the digits 1 through 4. Why is this true?

   Solution: The feedback given by the Code Maker says that none of the digits in the guess 566 are an exact match with the code, nor are they correct digits in the wrong place. Therefore, the Code Maker's code cannot contain the digits 5 or 6, leaving only 1 through 4 as valid possibilities. Notice that this already cuts down on the number of possible guesses!

2. After the third guess in the sample game provided, how many possibilities remain for the Code Maker's code? What are they?

   Solution: Given the feedback on the third guess above, we know that the Code Maker's code must contain the digits $2, 3, and 4$ in some order, but not the order 423. There are six arrangements of the digits $2, 3, and 4$, and therefore only five arrangements different from 423. These are: 234, 243, 324, 342, and 432. Thus, at first, it looks like there are five remaining possibilities.

   However, we also know that *none* of the digits in the guess 423 are in the same place as the Code Maker's code. This rules out 243, 324, and 432 as well, leaving only two valid possibilities: 234 and 342. In other words, we are looking for a rearrangement of 423 in which *none* of the digits stay in the same place.

3. After the fourth guess in the sample game provided, how many possibilities remain for the Code Maker's code? What are they?

   Solution: Notice we already eliminated 243 as a possible choice for the Code Maker's code at the previous step. In particular, we are already down to two possible codes: 234 and 342. The information we get after the fourth guess is that 243 matches the Code Maker's code in exactly one digit. But 234 matches 243 in only the first digit, and 342 matches 243 in only the second digit. No matter which of the two possibilities is true, the Code Maker will give us the same feedback on the guess 243.

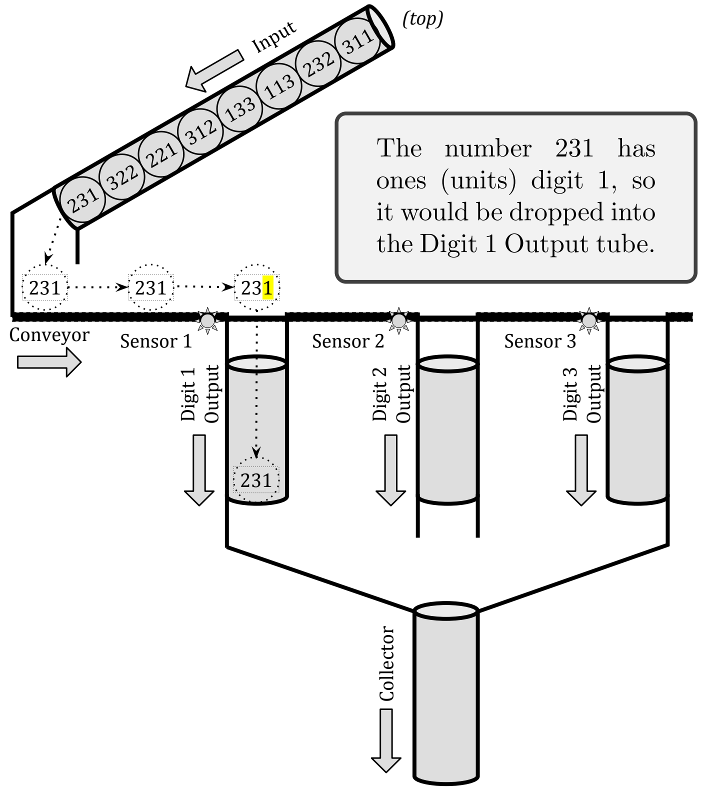
   In other words, the fourth guess is not a useful guess because it adds no new information! It would have been better to guess one of the two remaining possibilities. This would provide a 50% chance of the Code Breaker winning on Guess 4, and would guarantee that the Code Breaker wins by Guess 5.

Note: The famous mathematician and computer scientist Donald Knuth analyzed a version of this game with codes that are four digits long, and formed using six possible digits. Knuth showed that this version of the game can always be won by the Code Breaker in a maximum of five guesses, if they play with the right strategy!

CEMC at Home

Grade 4/5/6 - Wednesday, May 27, 2020
Let's Sort All This Out

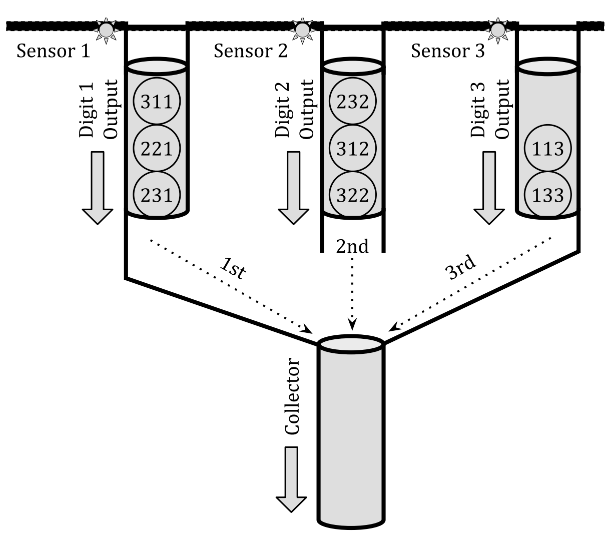
The CEMC has developed a number sorting machine as shown to the right. It sorts three-digit numbers that only contain the digits 1, 2, or 3. Here's how the machine works:



The number 231 has ones (units) digit 1, so it would be dropped into the Digit 1 Output tube.

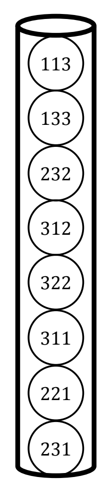
1. The numbers are loaded in the input tube. They drop, one at a time, from the bottom of the input tube and move along a conveyor belt that moves from left to right.
2. There are three sensors along the conveyor belt that can identify the ones, tens, or hundreds digit of a passing number as one of 1, 2, or 3. The sorting machine starts with the sensors set to scan the **ones** (units) digit of each passing number.
3. There are three output tubes, one attached to each sensor. The sensors determine whether a passing number will drop into the tube or pass by the tube.

   If a number with ones digit 1 passes by sensor 1, then a trap door above the Digit 1 Output tube will open and the number drops into the tube. If a number with ones digit 2 or 3 passes by sensor 1, then the trap door stays shut and the number moves on. The other sensors work similarly for the other two digits. The numbers are stored temporarily in the output tubes.
4. Once all of the numbers have been processed from the input tube, they are released from each output tube, one at a time from left to right, and funnelled to the collector tube at the bottom. In other words, all *Digit 1 Output* numbers drop first, followed by the *Digit 2 Output* numbers, and finally the *Digit 3 Output* numbers.



← The diagram on the left shows the result of processing all of the numbers in the input tube when all sensors are set to scan **ones** digits.

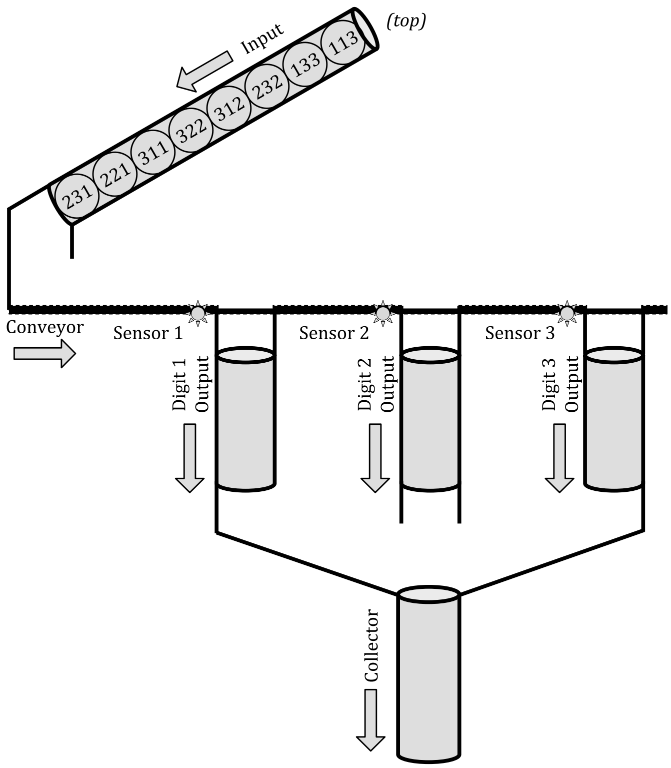
The diagram on the right shows the result after the numbers are released from the output tubes one at a time from left to right. →



Collector Result

5. Next, we take the numbers from the collector tube (the result showing on the first page) and feed them back into the input tube of the machine. We must keep the numbers in the *same order* from bottom to top as shown in the input tube below. We now run the machine again, but this time we set all sensors to scan the **tens** digit of each passing number.

   In the table below, list the numbers as they would appear in the collector tube after the numbers pass through the machine for the second time.



(top)

(bottom)

Collector result when sensing the **tens** digits

6. Finally, we take the numbers from the collector tube after Step 5 is completed and feed them back into the input tube of the machine. Again we must keep the numbers in the *same order* from bottom to top. We now run the machine for a third time, but this time we set all sensors to scan the **hundreds** digit of each passing number.

   In the table to the right, list the numbers as they would appear in the collector tube after the numbers pass through the machine for the third time.

   What do you notice about the order of these numbers?

A diagram of the machine without any numbers is provided on the next page. You may find it helpful to keep track of the numbers as they pass through the machine each time.

(top)

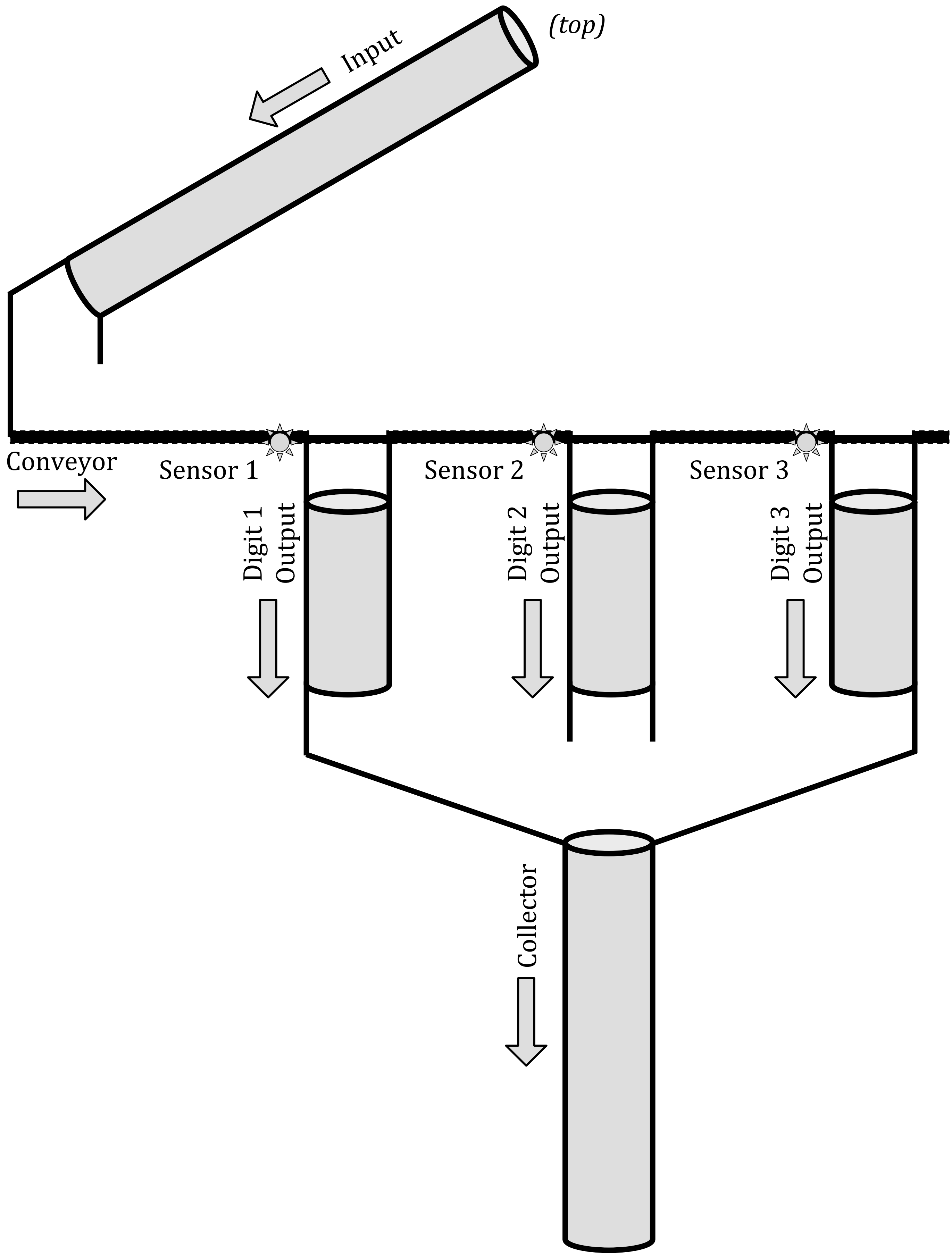
(bottom)

Collector result when sensing the **hundreds** digits

More Info:

Check the CEMC at Home webpage on Wednesday, June 3 for a solution to Let's Sort All This Out.

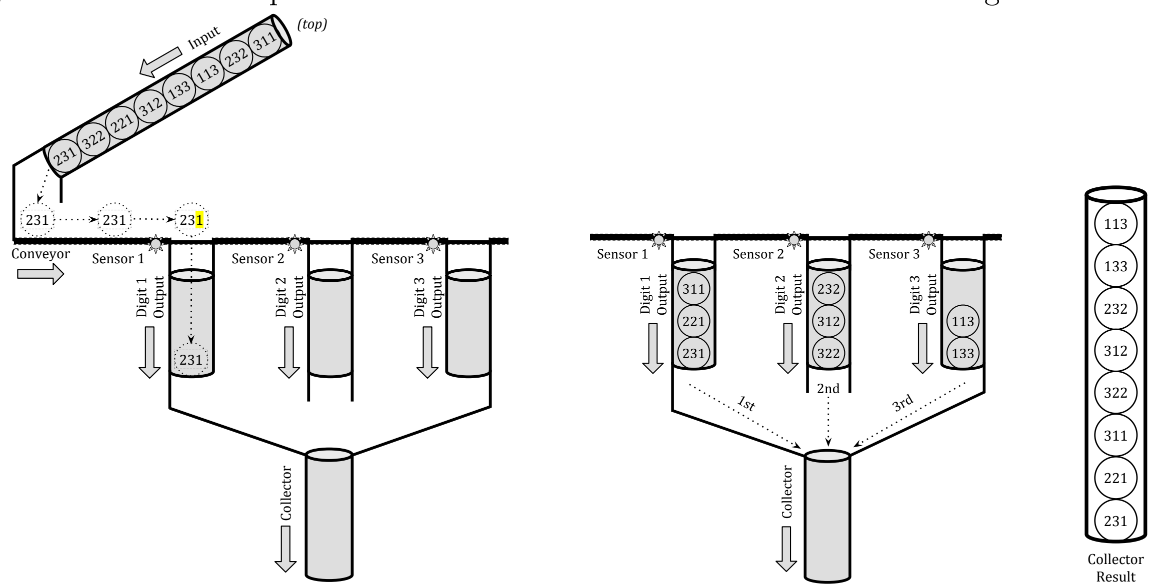
Blank Machine



CEMC at Home

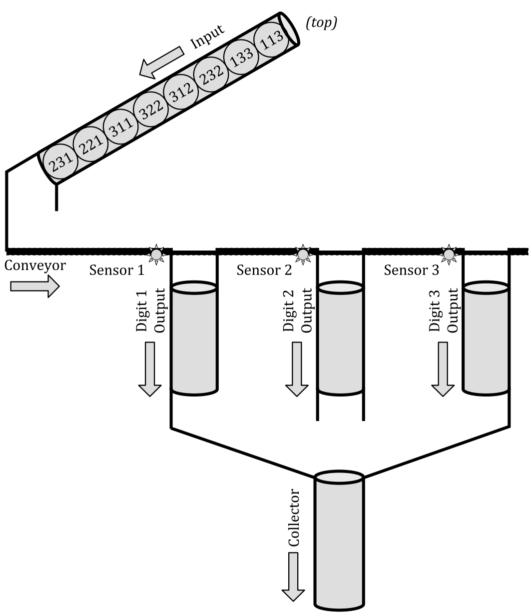
Grade 4/5/6 - Wednesday, May 27, 2020
Let's Sort All This Out - Solution

After the numbers pass through the machine the first time (with the sensors set to scan **ones** digits), the numbers end up in the order shown in the collector below on the right.



The numbers are now put back into the input tube of the machine in the order shown.

After the numbers pass through the machine the second time (with the sensors set this time to scan **tens** digits), the numbers end up in the order shown in the table.



(top)

133
232
231
322
221
113
312
311

(bottom)

After the numbers pass through the machine a third time (with the sensors set this time to scan **hundreds** digits), the numbers end up in the order shown in the collector below.

(top)

322
312
311
232
231
221
133
113

(bottom)

Notice that after the third pass through the machine, the numbers are sorted! The numbers in the collector tube are in order from smallest to largest, if you read them from bottom to top.

Computer Science Connections:

The CEMC Sorting Machine mimics a sorting technique known as *bucket sort*. This sorting technique works with any integers. To do a true bucket sort, you would need output tubes for all ten possible digits (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) rather than just for the digits 1, 2, and 3.

A big part of why the machine works is the way that numbers enter and exit the tubes. The first number that enters a tube (for example, the input tube) will be the first number to eventually leave that same tube. This idea is known as FIFO (First In, First Out). In Computer Science, we use a FIFO data structure called a *queue* which will hold a collection of values. A queue will manage that collection in a way that is similar to the way the tubes in this machine work.

CEMC at Home

Grade 4/5/6 - Thursday, May 28, 2020
Flower Powers

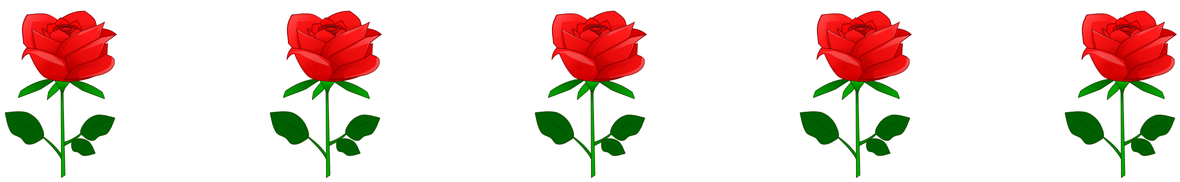
Mr. Digme planted five rose plants in a row along one side of his property.

He then planted one tulip plant in each of the spaces between the roses already in the row.

Next, he planted one daffodil plant in each of the spaces between the plants already in the row.

He then repeated this procedure with daisies, then marigolds, and finally with lilies.

Determine the total number of plants in the row.



To help you get started, think about how many spaces there are between the roses. After planting the tulips, how many roses and tulips would Mr. Digme have planted in total? How many spaces are there between the plants in the row now?
You may not be able to draw out all the plants in the end, but drawing the first few steps and looking for a pattern may be helpful.

More Info:

Check out the CEMC at Home webpage on Friday, May 29 for a solution to Flower Powers.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 4/5/6 - Thursday, May 28, 2020
Flower Powers - Solution

Problem:

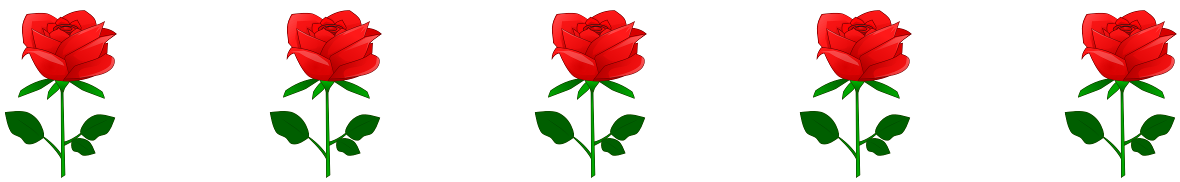
Mr. Digme planted five rose plants in a row along one side of his property.

He then planted one tulip plant in each of the spaces between the roses already in the row.

Next, he planted one daffodil plant in each of the spaces between the plants already in the row.

He then repeated this procedure with daisies, then marigolds, and finally with lilies.

Determine the total number of plants in the row.



Solution:

After planting 5 roses, there were four spaces between the plants. So, Mr. Digme then plants 4 tulips. At this point he has planted $5 + 4 = 9$ plants.

Since there are now 9 plants, there are 8 spaces between plants. So, Mr. Digme plants 8 daffodil plants. At this point he has planted $9 + 8 = 17$ plants.

Since there are now 17 plants, there are 16 spaces between plants. So, Mr. Digme plants 16 daisies. At this point he has planted $17 + 16 = 33$ plants.

Since there are now 33 plants, there are 32 spaces between plants. So, Mr. Digme plants 32 marigold plants. At this point he has planted $33 + 32 = 65$ plants.

Finally, since there are now 65 plants, there are 64 spaces between plants. So, Mr. Digme plants 64 lily plants. At this point he has planted $65 + 64 = 129$ plants.

Therefore, Mr. Digme planted a total of 129 plants in the row.

CEMC at Home

Grade 4/5/6 - Friday, May 29, 2020
The Leaky Tap

An experiment was performed in various households to determine the amount of water that is wasted by a single leaky tap. If you would like to perform this experiment on your own then the instructions are provided below. We have also provided the results of our own experiment for you to work with if you would prefer.

To Perform the Experiment You Will Need:

- A sink with a tap
- A watch or clock that measures seconds
- A clear metric measuring cup (to see the water level)
- A copy of Table 1 below

Instructions for the Experiment:

Turn on the tap, just a little, so that the water drips at a slow but steady rate.

The water should drip slowly enough that you can accurately count the drips, but not so slow that the drips are too irregular.

While the tap drips, complete the following two steps.

1. Count the number of drips in 20 seconds, and record this number in item 1. of Table 1.

   During our experiment, we observed 18 drips in 20 seconds. You can enter the value "18" or the value from your own experiment in the empty cell in the first row of the table.

2. Then place the measuring cup under the tap, and catch all the water that drips out during a 5 minute interval. Record this volume in item 3. of Table 1.

   During our experiment, we accumulated 65 mL of water in 5 minutes.

Table 1: Measurements

	Quantity	Measure	Units
1.	Drips in 20 seconds		drips
2.	Drips in one minute		drips/min
3.	Volume leaked in 5 minutes		mL
4.	Volume that would leak in 1 hour		mL/hr
5.	Volume that would leak in 1 day		L/day
6.	Volume that would leak in 1 year		L/year

After the Experiment:

Complete the table above by calculating the remaining items; use a calculator where needed.

For item 2., think about how many intervals of 20 seconds there are in one minute. For item 4., think about how many intervals of 5 minutes there are in 1 hour. Recall that 1 L is equal to 1000 mL.

See the next page for some questions to think about and an extra activity.

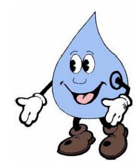
Questions:

1. On average, taking a bath uses about 160 L of water. If a tap leaked at the same rate as yours dripped (or the one from our experiment), about how many days would it take for the tap to leak this amount of water (160 L)?

2. Canada had about 10.2 million households as of 2019. If 1% of those households (about 102 000) had a tap that leaked at the same rate as yours dripped (or the one from our experiment), how many litres of water in total would be leaked in one year? How many inground swimming pools, with capacity about 76 000 L each, could be filled once per year with the water wasted by the leaking taps?

Activity:

Discover your family's weekly water consumption by completing the table below with the help of the members of your household.



My Family's Weekly Water Consumption

Activity	**Average* Amount of Water Used (L)**	**Number per Week**	**Water Used (L)**
Shower (10 minutes)	200 L (standard) 100 L (low flow)		
Tub Bath	160 L		
Washing Machine Load	110 L (top loading) 55 L (front loading)		
Dishwasher Load	20 L		
Cooking and Food Preparation	20 L per day		
Hygiene (teeth, hand washing, etc)	10 L/person/day		
Drinking water	2 L/person/day		

Total Weekly Average Water Used ____________

* These averages were compiled by examining various web sources to determine suitable values.

Comment: While this may give an idea of your family's direct water use, the products and services we all enjoy (such as clothing, food, heating, etc.) involve many indirect uses of water which contribute greatly to the depletion of fresh water. You can learn more about your "water footprint" online.

More info:

Check out the CEMC at Home webpage on Friday, June 5 for a sample solution to The Leaky Tap.

CEMC at Home

Grade 4/5/6 - Friday, May 29, 2020
The Leaky Tap - Solution

Here are the results of one leaky tap experiment, and the follow up calculations.

	Quantity	Measure (with units)
1.	Drips in 20 seconds	18 drips
2.	Drips in 1 minute	54 drips/min
3.	Volume leaked in 5 minutes	65 mL
4.	Volume that would leak in 1 hour	780 mL/hr
5.	Volume that would leak in 1 day	18.72 L/day
6.	Volume that would leak in 1 year	6 832.8 L/year

1. In the experiment, 18 drips were observed in 20 seconds.
2. Since there were 18 drips in 20 seconds and one minute is $60 = 5 \times 20$ seconds, we would expect $5 \times 18 = 54$ drips in 1 minute.
3. In the experiment, 65 mL of water leaked in 5 minutes.
4. Since 65 mL of water leaked in 5 minutes, and 1 hour is $60 = 12 \times 5$ minutes, we would expect $12 \times 65 = 780$ mL of water to leak in 1 hour.
5. Since 780 mL of water would leak in 1 hour, and 1 day is 24 hours, we would expect $24 \times 780 = 18\,720$ mL of water to leak in 1 day. Since 1 L is 1000 mL, this means a rate of 18.72 L in 1 day.
6. Since 18.72 L of water would leak in 1 day, and 1 year has 365 days (as long as it is not a leap year), we would expect $365 \times 18.72 = 6\,832.8$ L of water to leak in 1 year.

Questions:

1. On average, taking a bath uses about 160 L of water. If a tap leaked at the same rate as the tap in the experiment, about how many days would it take for the tap to leak this amount of water?

   Solution: Since 18.72 L of water leak each day, the number of days it would take to leak 160 L is equal to $160 \div 18.72$. Since $160 \div 18.72$ is around 8.55, it would take about $8\frac{1}{2}$ days.

2. Canada had about 10.2 million households as of 2019. If 1% of those households (about 102 000) had a tap that leaked at the same rate as the tap in the experiment, how many litres of water in total would be leaked in one year?

   Solution: Since 6 832.8 L would be leaked from each of the 102 000 households, the total water leaked would be $6\,832.8 \times 102\,000 = 696\,945\,600$ L.

   Note: This wasted water could fill approximately $696\,945\,600 \div 76\,000 \approx 9\,170$ *inground pools (each with capacity about 76 000 L) once per year.*

CEMC at Home

Grade 4/5/6 - Monday, June 1, 2020
Contest Day 5

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2020 Gauss Contest, #11

Each Tuesday, a bus makes its first stop at Gauss Public Library at 1 p.m. It continues to stop at the library every 20 minutes. Its last stop is at 6 p.m. What is the total number of times that the bus stops at Gauss Public Library on a Tuesday?

(A) 16 **(B)** 14 **(C)** 10 **(D)** 20 **(E)** 18

2016 Gauss Contest, #14

One soccer ball and one soccer shirt together cost \$100. Two soccer balls and three soccer shirts together cost \$262. What is the cost of one soccer ball?

(A) \$38 **(B)** \$50 **(C)** \$87.30 **(D)** \$45 **(E)** \$40

More Info:

Check out the CEMC at Home webpage on Monday, June 8 for solutions to the Contest Day 5 problems.

CEMC at Home

Grade 4/5/6 - Monday, June 1, 2020
Contest Day 5 - Solution

Solutions to the two contest problems are provided below.

2020 Gauss Contest, #11

Each Tuesday, a bus makes its first stop at Gauss Public Library at 1 p.m. It continues to stop at the library every 20 minutes. Its last stop is at 6 p.m. What is the total number of times that the bus stops at Gauss Public Library on a Tuesday?

(A) 16 **(B)** 14 **(C)** 10 **(D)** 20 **(E)** 18

Solution:

The bus stops at the library at 1:00 p.m., 1:20 p.m., and 1:40 p.m..
The bus stops at the library at 2:00 p.m., 2:20 p.m., and 2:40 p.m..
Similarly, the bus will stop at those same three times past each hour.
So over the 5 hours, it will stop $5 \times 3 = 15$ times, with an additional final stop at 6:00 p.m., making a total of $15 + 1 = 16$ stops.

Answer: (A)

2016 Gauss Contest, #14

One soccer ball and one soccer shirt together cost \$100. Two soccer balls and three soccer shirts together cost \$262. What is the cost of one soccer ball?

(A) \$38 **(B)** \$50 **(C)** \$87.30 **(D)** \$45 **(E)** \$40

Solution 1:

Since one of the given answers must be the correct cost of one soccer ball, we can test each of the amounts until we find one that works. For example, let's try \$50 for the cost of a ball.
If the cost of a ball is \$50, then a shirt would also cost \$50, because the cost of one ball and one shirt add up to \$100.
In this case, the cost of 2 balls and 3 shirts would be $5 \times \$50 = \$250$ but we are told this total cost is \$262.
This tells us that the cost of a ball is not \$50.

Let's try \$40 for the cost of a ball.
If the cost of a ball is \$40, then the cost of a shirt would be \$60, because $\$40 + \$60 = \$100$.
In this case, the cost of 2 balls is \$80 and the cost of 3 shirts is $3 \times \$60 = \$180$.
The total cost of the 2 balls and 3 shirts is $\$80 + \$180 = \$260$.
We are getting closer, but still just a little off, so let's try \$38 for the cost of a ball.

If the cost of a ball is \$38, then the cost of a shirt would be \$62, because $\$38 + \$62 = \$100$.
In this case, the cost of 2 balls is $2 \times \$38 = \$76$ and the cost of 3 shirts is $3 \times \$62 = \$186$.
The total cost of the 2 balls and 3 shirts is $\$76 + \$186 = \$262$.
Therefore, the cost of one ball is \$38.

Solution 2:

One soccer ball and one soccer shirt together cost \$100.
So then two soccer balls and two soccer shirts together cost $2 \times \$100 = \$200$.
Since we are given that two soccer balls and three soccer shirts together cost \$262, then \$200 added to the cost of one soccer shirt is \$262.
Thus, the cost of one soccer shirt is $\$262 - \$200 = \$62$ and the cost of one soccer ball is $\$100 - \$62 = \$38$.

ANSWER: (A)

CEMC at Home
Grade 4/5/6 - Tuesday, June 2, 2020
Famous Mathematicians

Throughout human history, many mathematicians have made important discoveries. These important historical figures often lead fascinating lives filled with interesting stories. Four of these mathematicians are listed below.

Archimedes	He is considered one of the greatest mathematicians and inventors in history. One thing he is famous for is his study of the surface area and volume of cylinders and spheres.
Leonhard Euler	Two different numbers are named after Euler. Many of the symbols we use when we write mathematics today are based on his writings.
Emmy Noether	She used her genius to figure out important things in mathematics. Her accomplishments were remarkable, but she didn't receive the attention and respect she deserved just because she was a woman.
Jingrun Chen	In the 20th century, he helped us understand some very important problems. Many of these problems involve prime numbers and have still not been fully solved today.

Choose two of these four mathematicians and for each one you choose:

1. Look up information about the mathematician online. Find a new fact about them that you find interesting and share what you find with friends or family.
2. Are there any mathematical words or ideas connected to this mathematician that sound familiar to you? Try to write down three to five of these words.
3. If you had the chance to go back in time and meet this mathematician, what question would you ask them?

More Info:

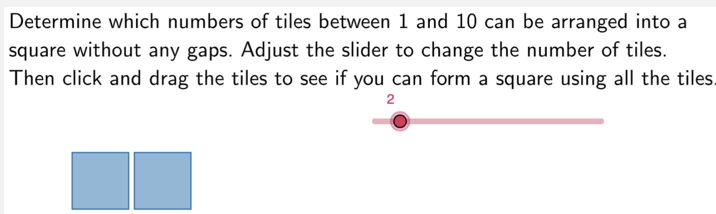
The CEMC Emmy Noether Circles are named in honour of Emmy Noether. This free online project aims to encourage the participation of both teachers and students at the Grade 5 and 6 level in solving problems for enjoyment and satisfaction.

CEMC at Home

Grade 4/5/6 - Wednesday, June 3, 2020
Interact with Mathematics

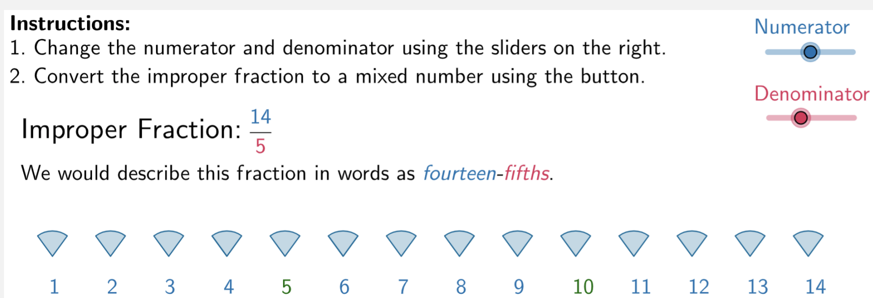
Technology can help us make mathematical discoveries and learn about mathematical objects. Three online examples of this from different areas of mathematics are featured below. Some of these examples show you some of the mathematics you will learn more about in future grades. However, there is still lots you can explore before you get there!

Making Squares: Try arranging different numbers of tiles to make a square.



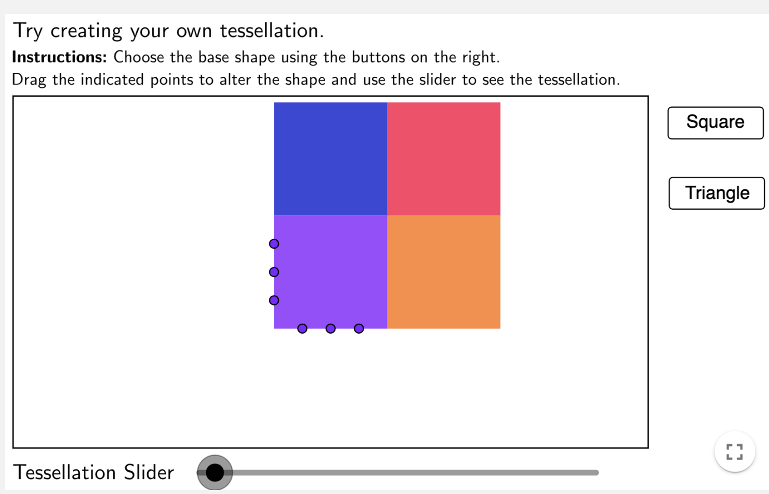
Link to App: https://www.geogebra.org/m/qgKdjyeq

Fraction Exploration: Explore how we represent improper fractions as mixed numbers.



Link to App: https://www.geogebra.org/m/gNZKP9G6

Tessellations: Create pictures using tessellations made from squares and triangles.



Link to App: https://www.geogebra.org/m/VZagPTjQ

More Info: CEMC courseware lessons feature hundreds of interactive mathematics applications.

CEMC at Home

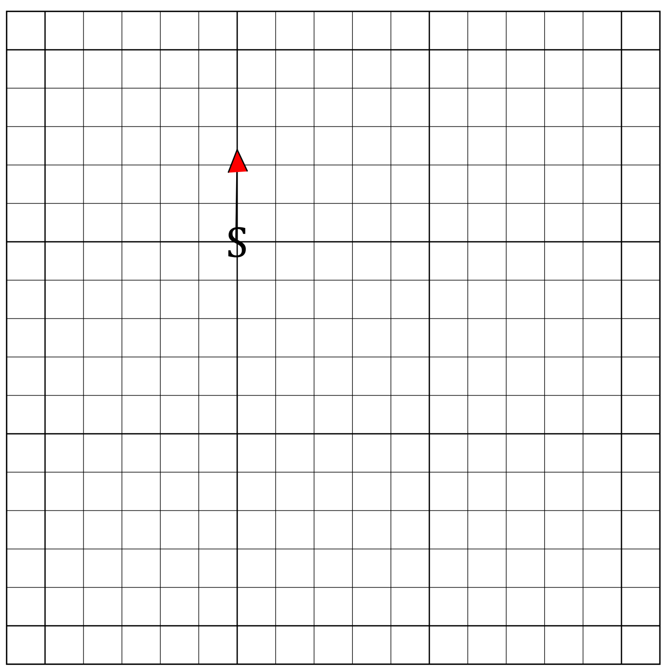
Grade 4/5/6 - Thursday, June 4, 2020
Crossing Paths

Red and Justin are planning a countryside ride on their ATV. They will follow this pattern:

- drive five kilometres and turn right;
- drive five kilometres and turn right;
- drive five kilometres and turn left;
- keep repeating these three steps until they return to their starting point.

(a) On the grid below, the side of each square is one kilometre. Map out their route, starting from the point **S** in the direction shown. Then determine how far they will have travelled when they get back to **S**.

(b) What is the name of the shape enclosed by their route? What is the area of this shape?



More Info:

Check out the CEMC at Home webpage on Friday, June 5 for a solution to Crossing Paths.

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 4/5/6 - Thursday, June 4, 2020
Crossing Paths - Solution

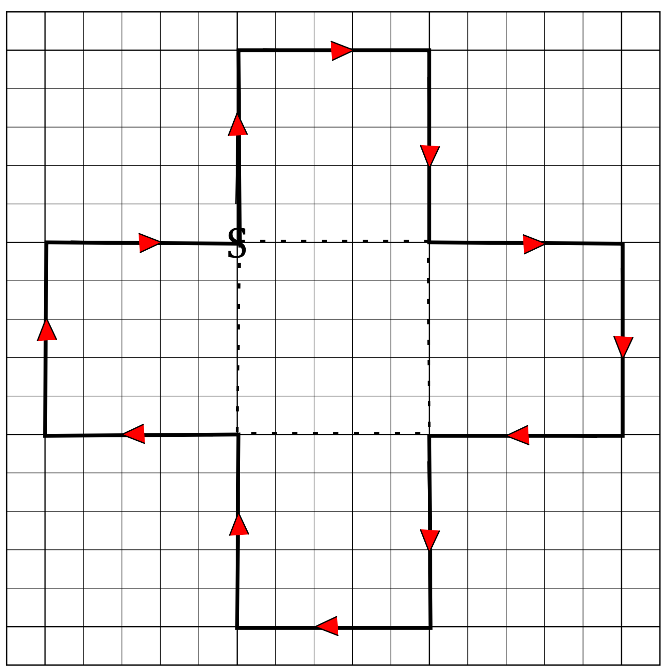
Problem:

Red and Justin are planning a countryside ride on their ATV. They will follow this pattern:

- drive five kilometres and turn right;
- drive five kilometres and turn right;
- drive five kilometres and turn left;
- keep repeating these three steps until they return to their starting point.

(a) On the grid below, the side of each square is one kilometre. Map out their route, starting from the point **S** in the direction shown. Then determine how far they will have travelled when they get back to **S**.

(b) What is the name of the shape enclosed by their route? What is the area of this shape?



Solution:

(a) Their route is shown on the above grid as a solid line, with arrows indicating the directions travelled. Since each line segment has length 5 km, the total distance they travelled is $12 \times 5 = 60\,\text{km}$.

(b) The 12-sided geometric shape enclosed by their route is called an *irregular dodecagon*.
The dashed lines on the grid show that this shape consists of 5 squares, each with side length 5 km. Thus the total area enclosed is $5 \times (5 \times 5) = 125\,\text{km}^2$.

CEMC at Home

Grade 4/5/6 - Friday, June 5, 2020
Math and CS in the News

Most weeks, our CEMC Homepage provides a link to a story in the media about mathematics and/or computer science. These stories show us how important mathematics and computer science are in today's world. They are a great source for discussions.

Using this article from Gizmodo, think about the following questions. (URL also provided below.) You may want to read this article and watch the video with an adult. If there is a word that you don't understand, see if you can figure out what it means by yourself or with the help of an adult. Don't worry if you don't understand some of the higher level math discussed.

1. Who is your favourite Pixar character? Find an image of them online. What geometric shapes do you think were used to model the character?
2. Which Pixar character do you think was easier to model than Geri? Why?
3. Design your own character using any of these geometric shapes: sphere, ellipsoid, rectangular prism, cylinder or cone.
4. Predict the future: What will animated movies look like in 20 years thanks to new mathematics and computer science?

URL of the article:
https://gizmodo.com/how-pixar-uses-math-to-make-characters-look-perfect-1657339566

More Info:

A full archive of past posts can be found in our Math and CS in the News Archive. Similar resources for other grades may also be of interest.

CEMC at Home

Grade 4/5/6 - Monday, June 8, 2020
Contest Day 6

Today's resource features a question from one of the recently released 2020 CEMC Mathematics Contests, along with a question from one of our past contests.

2013 Gauss Contest, #13

Jack, Kelly, Lan, Mihai, and Nate are sitting in the 5 chairs around a circular table. Lan and Mihai are sitting beside each other. Jack and Kelly are not sitting beside each other. The 2 people who are seated on either side of Nate are

(A) Jack and Lan **(B)** Jack and Kelly **(C)** Kelly and Mihai

(D) Lan and Mihai **(E)** Mihai and Jack

2020 Gauss Contest, #15

Emil and Olivia ran a race. Their race times totalled 1 hour 52 minutes. If Emil's time was 4 minutes less than Olivia's time, how many minutes did it take Olivia to run the race?

(A) 78 **(B)** 56 **(C)** 58 **(D)** 74 **(E)** 55

More Info:

Check out the CEMC at Home webpage on Monday, June 15 for solutions to the Contest Day 6 problems.

CEMC at Home

Grade 4/5/6 - Monday, June 8, 2020
Contest Day 6 - Solution

Solutions to the two contest problems are provided below.

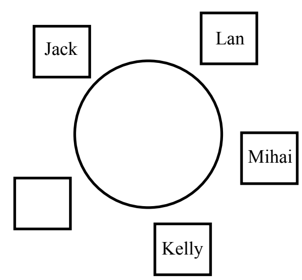
2013 Gauss Contest, #13

Jack, Kelly, Lan, Mihai, and Nate are sitting in the 5 chairs around a circular table. Lan and Mihai are sitting beside each other. Jack and Kelly are not sitting beside each other. The 2 people who are seated on either side of Nate are

(A) Jack and Lan **(B)** Jack and Kelly **(C)** Kelly and Mihai
(D) Lan and Mihai **(E)** Mihai and Jack

Solution:

Since Lan and Mihai are seated beside each other, while Jack and Kelly are not, the only possible location for the remaining chair (Nate's chair) is between Jack and Kelly. Therefore, the 2 people who are seated on either side of Nate are Jack and Kelly.



ANSWER: (B)

2020 Gauss Contest, #15

Emil and Olivia ran a race. Their race times totalled 1 hour 52 minutes. If Emil's time was 4 minutes less than Olivia's time, how many minutes did it take Olivia to run the race?

(A) 78 **(B)** 56 **(C)** 58 **(D)** 74 **(E)** 55

Solution:

There are 60 minutes in 1 hour, and so there are $60 + 52 = 112$ minutes in 1 hour 52 minutes.
If Emil's race time was 54 minutes, then Olivia's race time was 4 minutes more, or 58 minutes.
In this case, their race times total $54 + 58 = 112$ minutes, as required.
Therefore, it took Olivia 58 minutes to run the race.

ANSWER: (C)

CEMC at Home

Grade 4/5/6 - Tuesday, June 9, 2020
Doggy Description

An unknown dog wanders the neighbourhood every night, scavenging in people's garbage pails and making a mess. Four of the kids in the neighbourhood (Mathias, Li Jing, Rajiv, and Olivia) think they have each seen the guilty dog, but the culprit is hard to see clearly in the dark.

Below are their descriptions, each giving exactly four details about the dog: the dog's colour, hair type, collar colour, and tail length.

Each witness has exactly one of the four details correct in their description. Each detail is described correctly by exactly one of the four witnesses.

1. Mathias says the dog is white, fluffy, wears a red collar, and has a long tail.
2. Li Jing says the dog is black, has short hair, wears a red collar, and has a long tail.
3. Rajiv says the dog is brown, has long hair, wears a blue collar, and has a long tail.
4. Olivia says the dog is spotted, fluffy, wears a red collar, and has a short tail.

Determine the correct description of the guilty dog.

Completing the table below may help to sort out which details are correct in the descriptions.

Witness	Colour	Hair Type	Collar Colour	Tail Length
Mathias				
Li Jing				
Rajiv				
Olivia				

HINTS:

1. Start by thinking about the length of the dog's tail.
2. If more than one description says the dog is fluffy, could that be correct?

Challenge:

Work with a friend or family member to make up a different set of four witness descriptions that could also be used to determine the correct description of the dog. (Remember that each witness should have exactly one detail correct and each detail should be described correctly by exactly one witness.)

More info:

Check out the CEMC at Home webpage on Tuesday, June 16 for a solution to Doggy Description.

CEMC at Home

Grade 4/5/6 - Tuesday, June 9, 2020
Doggy Description - Solution

Problem: An unknown dog wanders the neighbourhood every night, scavenging in people's garbage pails and making a mess. Four of the kids in the neighbourhood (Mathias, Li Jing, Rajiv, and Olivia) think they have each seen the guilty dog, but the culprit is hard to see clearly in the dark.

Below are their descriptions, each giving exactly four details about the dog: the dog's colour, hair type, collar colour, and tail length.

Each witness has exactly one of the four details correct in their description. Each detail is described correctly by exactly one of the four witnesses.

1. Mathias says the dog is white, fluffy, wears a red collar, and has a long tail.
2. Li Jing says the dog is black, has short hair, wears a red collar, and has a long tail.
3. Rajiv says the dog is brown, has long hair, wears a blue collar, and has a long tail.
4. Olivia says the dog is spotted, fluffy, wears a red collar, and has a short tail.

Determine the correct description of the guilty dog.

Solution:

Here is the completed table of details.

Child	Colour	Hair Type	Collar Colour	Tail Length
Mathias	white [4]	fluffy	red	long
Li Jing	black	short [3]	red	long
Rajiv	brown	long	blue [2]	long
Olivia	spotted	fluffy	red	short [1]

Noting that each witness can only be right about one detail, and each detail is described correctly by only one witness, we reason as follows (in the order indicated by the numbered items in the table).

1. Since exactly one of the witnesses has the tail length correct, it must be Olivia's description of the tail length that is correct. This means the dog must have a short tail.
2. Since exactly one of the witnesses has the collar colour correct, it must be Rajiv's description of the collar colour that is correct. This means the dog must have a blue collar.
3. Since Rajiv and Olivia can only be correct about one detail each, based on the work above we know they most both be wrong about the hair type of the dog. This means that the dog's hair cannot be long and cannot be fluffy. Since one of the witnesses has the hair type correct, it must be Li Jiang who has it correct, and so the dog's hair must be short.
4. Based on the work above, we know Matias is wrong about tail length, collar colour, and hair type. Since Matias must have one detail correct, it must be the colour. This means the dog must be white.

Thus we conclude that the dog is a white, short-haired dog with a short tail, wearing a blue collar.

CEMC at Home

Grade 4/5/6 - Wednesday, June 10, 2020
Treasure Hunt

You found a treasure map that contains information to find hidden treasures. The map is a grid with rows labelled with letters and columns labelled with numbers. Each square of the grid is identified by a unique name such as **B3** or **D5** and contains either a number or a ◇.

Under some of the ◇ spots on the grid, there is hidden treasure. To find the treasure, you must know the rules of the map and be given a correct starting position.

Here are the rules:

- You will be given a starting position including: a starting square that contains a number and a starting direction which indicates how you start moving in the map (e.g., **A3** ↓).
- From the starting position, move through the grid in the given direction, accumulating the sum of the numbers in the squares that you pass through (including the one on the starting square). Let's call this accumulated sum **S**.

Partial Map:

	1	2	3	4	5	6
A	◇	15	13	3	6	◇
B	2	5	15	16	10	7
C	7	9	◇	1	4	◇
D	◇	11	5	3	17	4

- When you reach a square containing a ◇, there are four possibilities:
  - If **S** is even and its leftmost digit is even (e.g., **S** $= 24$), then you make a quarter turn ($90°$) counterclockwise and continue accumulating sums along this new path.
  - If **S** is even and its leftmost digit is odd (e.g., **S** $= 34$), then you make a quarter turn clockwise and continue accumulating sums along this new path.
  - If **S** is odd and its leftmost digit is even (e.g., **S** $= 45$), then you will keep moving in the same direction and continue accumulating sums along the same path.
  - If **S** is odd and its leftmost digit is odd (e.g., **S** $= 125$), then you have found a treasure!
- While you continue searching for treasure, the accumulated sum **S** continues to grow. Keep moving through the map, changing directions at the squares containing a ◇ symbol when necessary, until you find a treasure.

Example: Suppose you have the *Partial Map* above and you are given the starting position **A3** ↓. This is the path that you would take through the map:

Starting position

	1	2	3	4	5	6
A	◇	15	13	3	6	◇
B	2	5	15	16	10	7
C	7	9	◇	1	4	◇
D	◇	11	5	3	17	4

You start at square **A3** and move downwards in the map. You move from **A3** to **B3** to **C3** and accumulate a sum of **S** $= 13 + 15 = 28$ when you reach your first ◇ spot. Since 28 is even and its leftmost digit is even, you make a quarter turn counterclockwise, making your new direction →.

	1	2	3	4	5	6
A	◇	15	13	3	6	◇
B	2	5	15	16	10	7
C	7	9	◇	1	4	◇
D	◇	11	5	3	17	4

From square **C3** you move to the right in the map. You move from **C3** to **C4** to **C5** to **C6** and accumulate a sum of **S** $= 28 + 1 + 4 = 33$ when you reach your second ◇ spot. Since 33 is odd and its leftmost digit is odd, you have found a treasure at square **C6**!

Here is the complete treasure map.

	1	2	3	4	5	6	7	8	9	10
A	◇	15	13	3	6	◇	12	4	◇	9
B	2	5	15	16	10	7	6	14	3	8
C	7	9	◇	1	4	◇	7	2	◇	18
D	◇	11	5	3	17	4	6	4	9	19
E	2	7	15	13	◇	8	7	◇	10	◇
F	13	14	10	17	1	24	6	12	3	12
G	◇	5	◇	21	1	◇	11	◇	12	◇
H	7	3	10	4	◇	3	11	12	1	5
I	17	6	◇	5	9	5	◇	14	◇	8
J	1	7	15	2	16	6	3	9	10	11

Problem 1

The starting position **J6** ↑ will lead you to a second treasure. Find where this treasure is located.

Problem 2

The starting position **C1** → will lead you to a third treasure. Find where this treasure is located.

Let the treasure hunt begin!

Feel free to explore other starting positions on the grid. If from some starting position you happen to reach the end of the grid without an instruction to turn (and without finding a treasure), then stop and start again with another starting position.

More Info:

Check out the CEMC at Home webpage on Wednesday, June 17 for a solution to Treasure Hunt.

CEMC at Home

Grade 4/5/6 - Wednesday, June 10, 2020
Treasure Hunt - Solution

The starting position **J6** $\uparrow$ will lead you to a treasure at **G10**.

	1	2	3	4	5	6	7	8	9	10
A	◇	15	13	3	6	◇	12	4	◇	9
B	2	5	15	16	10	7	6	14	3	8
C	7	9	◇	1	4	◇	7	2	◇	18
D	◇	11	5	3	17	4	6	4	9	19
E	2	7	15	13	◇	8	7	◇	10	◇
F	13	14	10	17	1	24	6	12	3	12
G	◇	5	◇	21	1	◇	11	◇	12	◇
H	7	3	10	4	◇	3	11	12	1	5
I	17	6	◇	5	9	5	◇	14	◇	8
J	1	7	15	2	16	6	3	9	10	11

- **J6** to **I6** to **H6** to **G6**:
  S is $6 + 5 + 3 = 14$ which is even with leftmost digit odd (turn clockwise).
- **G6** to **G7** to **G8**:
  S is $14 + 11 = 25$ which is odd with leftmost digit even (continue straight).
- **G8** to **G9** to **G10**:
  S is $25 + 12 = 37$ which is odd with leftmost digit odd. There is treasure at **G10**!

The starting position **C1** $\rightarrow$ will lead you to a treasure at **C9**.

	1	2	3	4	5	6	7	8	9	10
A	◇	15	13	3	6	◇	12	4	◇	9
B	2	5	15	16	10	7	6	14	3	8
C	7	9	◇	1	4	◇	7	2	◇	18
D	◇	11	5	3	17	4	6	4	9	19
E	2	7	15	13	◇	8	7	◇	10	◇
F	13	14	10	17	1	24	6	12	3	12
G	◇	5	◇	21	1	◇	11	◇	12	◇
H	7	3	10	4	◇	3	11	12	1	5
I	17	6	◇	5	9	5	◇	14	◇	8
J	1	7	15	2	16	6	3	9	10	11

- **C1** to **C2** to **C3**:
  S is $7 + 9 = 16$ which is even with leftmost digit odd (turn clockwise).
- **C3** to **D3** to **E3** to **F3** to **G3**:
  S is $16+5+15+10 = 46$ which is even with leftmost digit even (turn counterclockwise).
- **G3** to **G4** to **G5** to **G6**:
  S is $46 + 21 + 1 = 68$ which is even with leftmost digit even (turn counterclockwise).
- **G6** to **F6** to **E6** to **D6** to **C6**:
  S is $68+24+8+4 = 104$ which is even with leftmost digit odd (turn clockwise).
- **C6** to **C7** to **C8** to **C9**:
  S is $104+7+2 = 113$ which is odd with leftmost digit odd. There is treasure at **C9**!

CEMC at Home

Grade 4/5/6 - Thursday, June 11, 2020
Amazing Grids

If possible, for each maze find your way from the top left square to the bottom right square, moving only horizontally or vertically to adjacent squares that satisfy the property given in the title.

Multiples of 4, in increasing order

4	10	14	18	22	26	30	34	38	42
8	14	24	28	32	36	40	44	46	50
12	16	20	28	30	38	42	46	50	54
16	18	22	26	34	42	70	66	62	58
20	24	26	28	38	46	50	62	66	62
22	28	30	32	42	50	54	58	70	66
36	32	34	36	46	54	58	62	74	70
40	34	36	38	50	54	58	66	70	74
44	48	52	56	60	62	66	70	74	78
46	50	54	56	64	68	72	76	80	84

Multiples of 7, in increasing order

7	14	21	27	34	41	48	55	62	69
14	20	28	33	40	47	54	61	70	76
21	27	35	42	49	54	61	68	77	83
28	35	40	47	56	61	63	75	84	90
34	41	47	50	63	68	75	82	90	97
41	48	54	58	70	77	84	89	97	104
49	55	61	65	72	79	91	96	104	111
56	62	68	72	79	86	98	103	111	118
63	69	75	82	89	96	105	112	119	125
70	76	83	89	96	103	110	117	126	133

Factors of 48, in any order

48	2	4	12	24	48	3	8	6	10
1	9	13	15	20	40	13	22	14	12
24	12	6	4	1	2	16	18	22	24
5	8	10	14	18	22	26	20	28	16
6	3	14	7	1	4	3	15	20	10
10	16	22	5	20	2	28	18	16	14
8	12	26	9	13	6	3	8	22	24
14	2	4	13	20	1	17	24	3	5
16	7	6	1	4	12	14	26	22	1
8	4	20	5	18	24	2	6	8	3

Composite numbers, in any order

45	1	41	16	15	27	33	18	9	11
22	15	7	11	25	33	37	20	23	32
3	18	13	41	13	40	7	17	8	12
17	9	7	43	2	12	18	21	28	35
23	12	33	27	7	35	11	23	42	40
32	2	40	48	13	19	21	13	16	25
37	16	19	11	12	43	23	24	32	40
9	25	18	14	8	25	41	11	46	50
16	31	4	1	2	30	51	28	24	17
25	43	6	22	27	29	38	19	63	42

Factors

The whole number *factors* of any whole number N are the whole numbers which divide evenly into N. For example, 12 has six whole number factors. They are 1, 2, 3, 4, 6, and 12.

Composite numbers

A *composite number* is a whole number that has whole number factors in addition to 1 and itself. For example, 6 is a composite number because it has whole number factors 1, 2, 3, and 6. The factors 2 and 3 are in addition to 1 and 6 (the number itself). However, the number 1 is not composite because its only whole number factor is 1, and the number 7 is not composite because its only whole number factors are 1 and 7 (the number itself).

More Info:

Check out the CEMC at Home webpage on Friday, June 12 for a solution to Amazing Grids.

CEMC at Home

Grade 4/5/6 - Thursday, June 11, 2020
Amazing Grids - Solution

Problem:

If possible, for each maze find your way from the top left square to the bottom right square, moving only horizontally or vertically to adjacent squares that satisfy the property given in the title.

Multiples of 4, in increasing order

4	10	14	18	22	26	30	34	38	42
8	14	24	28	32	36	40	44	46	50
12	16	20	28	30	38	42	46	50	54
16	18	22	26	34	42	70	66	62	58
20	24	26	28	38	46	50	62	66	62
22	28	30	32	42	50	54	58	70	66
36	32	34	36	46	54	58	62	74	70
40	34	36	38	50	54	58	66	70	74
44	48	52	56	60	62	66	70	74	78
46	50	54	56	64	68	72	76	80	84

Multiples of 7, in increasing order

7	14	21	27	34	41	48	55	62	69
14	20	28	33	40	47	54	61	70	76
21	27	35	42	49	54	61	68	77	83
28	35	40	47	56	61	63	75	84	90
34	41	47	50	63	68	75	82	90	97
41	48	54	58	70	77	84	89	97	104
49	55	61	65	72	79	91	96	104	111
56	62	68	72	79	86	98	103	111	118
63	69	75	82	89	96	105	112	119	125
70	76	83	89	96	103	110	117	126	133

Factors of 48, in any order

48	2	4	12	24	48	3	8	6	10
1	9	13	15	20	40	13	22	14	12
24	12	6	4	1	2	16	18	22	24
5	8	10	14	18	22	26	20	28	16
6	3	14	7	1	4	3	15	20	10
10	16	22	5	20	2	28	18	16	14
8	12	26	9	13	6	3	8	22	24
14	2	4	13	20	1	17	24	3	5
16	7	6	1	4	12	14	26	22	1
8	4	20	5	18	24	2	6	8	3

Composite numbers, in any order

45	1	41	16	15	27	33	18	9	11
22	15	7	11	25	33	37	20	23	32
3	18	13	41	13	40	7	17	8	12
17	9	7	43	2	12	18	21	28	35
23	12	33	27	7	35	11	23	42	40
32	2	40	48	13	19	21	13	16	25
37	16	19	11	12	43	23	24	32	40
9	25	18	14	8	25	41	11	46	50
16	31	4	1	2	30	51	28	24	17
25	43	6	22	27	29	38	19	63	42

Solution:
Solutions are given on the first three mazes. The final maze has no solution; two (of many possible) attempts are shown with dotted lines, both working forward and backward, but both dead-end.

CEMC at Home

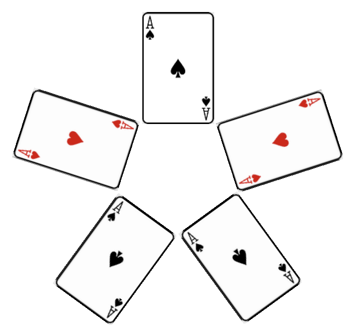
Grade 4/5/6 - Friday, June 12, 2020
It's in the Cards

Good friends Bahaa and Helena are each working on some top-secret government projects. They would like to discuss the projects they both work on, but due to secrecy rules, must not reveal that they are working on a particular project unless their friend is also working on that same project.

Together, the two friends have devised a cooperative game using playing cards to tell them what they need to know. Here is how the game works:

- Bahaa and Helena identify a particular project; let's call it Project X.
- Each of Bahaa and Helena takes one Ace of Hearts and one Ace of Spades. An additional Ace of Spades is placed face down in front of them. (All cards of the same type should be identical.)
- Bahaa goes first. Bahaa will place his two cards face down on top of the card already on the table. If Bahaa is working on the project, he first puts down the Ace of Spades and then the Ace of Hearts. If Bahaa is not working on the project, then he puts down the Ace of Hearts, then the Ace of Spades.
- Next, it's Helena's turn. Helena will place her two cards face down on top of the three cards already on the table. If Helena is working on the project, she first puts down the Ace of Hearts and then the Ace of Spaces. If Helena is not working on the project, she first puts down the Ace of Spades and then the Ace of Hearts.

  Notice that she does the opposite of what Bahaa did.
- Now Bahaa and Helena ask a mutual friend to pick up the deck. While Bahaa and Helena close their eyes, the friend places the cards from the deck face up, in order, so that they form a circle.

  The friend is not aware of the rules of the game and must place the cards in order, but so that Bahaa and Helena cannot tell which card was on the top of the pile by looking at the circle of cards.
- Looking at the circle of cards, Bahaa and Helena now know what information about the project it is safe to share with one another.

Activity: Let's explore how Bahaa's and Helena's game works!

Decide which person will play the role of Bahaa and which player will play the role of Helena. You can enlist a friend to secretly place the cards in a circle for you, but you do not need to. You can also use homemade cards if you do not have three identical decks of playing cards.

Play four different rounds of this game, and complete the table below:

Round	**1**	**2**	**3**	**4**
Bahaa on Project X?	Yes	No	No	Yes
Helena on Project X?	No	Yes	No	Yes
Resulting card circle				

To learn how Bahaa and Helena can use this game to find out which projects they both work on, think about the questions:

1. The four rounds from the activity show what the result of the game would be in all four possible cases for Project X. Three of the four cases result in card circles that can be rotated so that they all look the same. The remaining case results in a card circle that looks different from the other three, no matter how the circle is rotated. Which three cases result in card circles that look the same (when rotated)?

2. If Bahaa is working on Project X, then he can use the card circle to determine whether or not Helena is also working on Project X. Can you explain why?

3. If Bahaa is *not* working on Project X, then he *cannot* use the card circle to determine whether or not Helena is working on Project X. Can you explain why?

Try this!

Make up your own table for four different projects.

Player's Name	Project 1	Project 2	Project 3	Project 4

Player's Name	Project 1	Project 2	Project 3	Project 4

Each player should fill in their name, and place a "Yes" or "No" for each project, but keep their answers hidden from the other player. Play the game again and see if you can use what you have learned about the game to determine which projects you are both working on.

Note that the role of the mutual friend can be replaced by the players taking turns "cutting the cards" a few times after they have all been placed in the pile. To cut a pile of cards, you split the pile in two, lifting the upper part of the pile from the top and placing the lower part of the pile on top of it.

Remember the goal of the game is to achieve the following:

- *If you are working on a project, then you can use the card circle to determine whether or not the other person is also working on the project, and*
- *if you are not working on a project, then you cannot use the card circle to determine whether or not the other person is working on the project.*

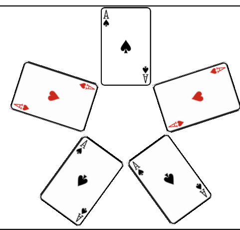
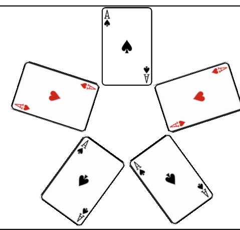
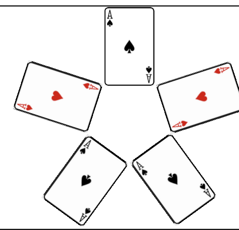
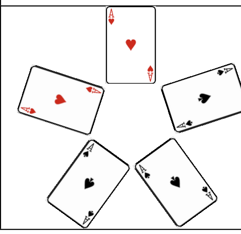
More Info:

Check out the CEMC at Home webpage on Friday, June 19 for a solution to It's in the Cards.

CEMC at Home

Grade 4/5/6 - Friday, June 12, 2020
It's in the Cards - Solution

Activity Solution:

Round	**1**	**2**	**3**	**4**
Bahaa on Project X?	Yes	No	No	Yes
Helena on Project X?	No	Yes	No	Yes
Resulting card circle				

Your answers should be the card circles shown above or some rotation of these card circles.

Questions

1. The four rounds from the activity show what the result of the game would be in all four possible cases for Project X. Three of the four cases result in card circles that can be rotated so that they all look the same. The remaining case results in a card circle that looks different from the other three, no matter how the circle is rotated. Which three cases result in card circles that look the same (when rotated)?

   Solution: The first three rounds all have card circles that look the same (when rotated). These all have the Aces of Hearts separated by an Ace of Spades. The final round has a card circle that is different. This one has the Aces of Hearts beside each other in the circle.

   Can you explain why the cards end up like this? Think about the rules for how Bahaa and Helena place their cards in the pile.

2. If Bahaa is working on Project X, then he can use the card circle to determine whether or not Helena is also working on Project X. Can you explain why?

   Solution: If Bahaa is working on Project X, then we know we are in the situation of either "Round 1" or "Round 4" of the Activity. If Bahaa observes a final card circle that has the two Aces of Hearts separated, then he can be sure that Helena is *not* working on Project X. If Bahaa observes a final card circle that has the two Aces of Hearts beside each other in the circle, then he can be sure that Helena *is* working on Project X (and so knows it is safe to reveal that he is too).

3. If Bahaa is *not* working on Project X, then he *cannot* use the card circle to determine whether or not Helena is working on Project X. Can you explain why?

   Solution: If Bahaa is *not* working on Project X, then we know we are in the situation of either "Round 2" or "Round 3" of the Activity. Bahaa would observe a card circle with the Aces of Hearts separated, regardless of whether Helena is working on Project X or not. This means he cannot tell which is the case. (If Helena is indeed working on Project X, then the game will not give this away.)

CEMC at Home

Grade 4/5/6 - Monday, June 15, 2020
History of Computing

Computers can be found on our desks, in our pockets and even in our refrigerators! This is remarkable because modern computers have been around for less than 100 years. During this time, there has been a constant stream of new discoveries and advances in technology.

Use this online tool to arrange the following list of events in the history of computer science from earliest to most recent.

A. Tim Berners-Lee posts the first picture on the World Wide Web.

B. The top selling mobile game is Angry Birds.

C. The first email is sent. It is sent from Ray Tomlinson to Ray Tomlinson.

D. The Apple Computer Company is created.

E. The first Microsoft Xbox is available for purchase.

F. Keyboard input is introduced as a way of entering data into a computer.

G. Deep Blue is the first computer program to beat a human world chess champion.

More Info:

Our webpage Computer Science and Learning to Program is the best place to find the CEMC's computer science resources.

CEMC at Home

Grade 4/5/6 - Tuesday, June 16, 2020
Can You Find the Terms?

Can you find all of the given mathematics and computer science terms in the grid? Good Luck!

C	P	R	O	G	R	A	M	Y	A	V
A	L	G	O	R	I	T	H	M	E	K
V	S	O	F	T	W	A	R	E	E	E
S	Z	E	U	M	I	D	N	L	U	O
Y	G	U	G	W	E	N	G	G	Q	U
M	R	R	I	I	K	N	P	Z	L	T
M	A	V	J	Q	A	Z	D	U	I	E
E	P	U	K	I	R	K	R	O	T	L
T	H	F	R	A	C	T	I	O	N	H
R	P	T	A	Z	I	N	K	J	G	J
Y	W	C	O	M	P	U	T	E	R	S

FRACTION	ANGLE	SYMMETRY	INPUT	SOFTWARE
TRIANGLE	GRAPH	PROGRAM	COMPUTER	ALGORITHM

Note that the word "ANGLE" appears twice in the grid: once as part of the word "TRIANGLE" and a second time on its own.

More Info:

Check the CEMC at Home webpage on Wednesday, June 17 for the solution to Can You Find the Terms?

CEMC at Home

Grade 4/5/6 - Tuesday, June 16, 2020
Can You Find the Terms? - Solution

C	P	R	O	G	R	A	M	Y	A	V
A	L	G	O	R	I	T	H	M	E	K
V	S	O	F	T	W	A	R	E	E	E
S	Z	E	U	M	I	D	N	L	U	O
Y	G	U	G	W	E	N	G	G	Q	U
M	R	R	I	I	K	N	P	Z	L	T
M	A	V	J	Q	A	Z	D	U	I	E
E	P	U	K	I	R	K	R	O	T	L
T	H	F	R	A	C	T	I	O	N	H
R	P	T	A	Z	I	N	K	J	G	J
Y	W	C	O	M	P	U	T	E	R	S

FRACTION	ANGLE	SYMMETRY	INPUT	SOFTWARE
TRIANGLE	GRAPH	PROGRAM	COMPUTER	ALGORITHM

CEMC at Home

Grade 4/5/6 - Wednesday, June 17, 2020
We've Got Your Number!

I am a nine-digit number. I contain each digit from 1 to 9 except for the digit 8, and I contain two appearances of the digit 5. Discover what number I am by using the following clues.

- I am less than 500 000 000.
- My ten millions digit and my ones (units) digit are the same.
- The sum of my hundred millions, ten millions, and millions digits is 18.
- My thousands digit is 1.
- My ten thousands digit is one more than my hundred thousands digit.
- My ones (units) digit is equal to the sum of my hundreds digit and my tens digit.
- My hundreds digit is 3.

More Info:

Check out the CEMC at Home webpage on Thursday, June 18 for a solution to We've Got Your Number!

This CEMC at Home resource is a past problem from Problem of the Week (POTW). POTW is a free, weekly resource that the CEMC provides for teachers, parents, and students during the school year. POTW is wrapped up for the current school year and will resume on September 17, 2020. To subscribe to POTW and to find more past problems and their solutions visit:
https://www.cemc.uwaterloo.ca/resources/potw.php

CEMC at Home

Grade 4/5/6 - Wednesday, June 17, 2020
We've Got Your Number - Solution

Problem:

I am a nine-digit number. I contain each digit from 1 to 9 except for the digit 8, and I contain two appearances of the digit 5. Discover what number I am by using the following clues.

- I am less than 500 000 000.
- My ten millions digit and my ones (units) digit are the same.
- The sum of my hundred millions, ten millions, and millions digits is 18.
- My thousands digit is 1.
- My ten thousands digit is one more than my hundred thousands digit.
- My ones (units) digit is equal to the sum of my hundreds digit and my tens digit.
- My hundreds digit is 3.

Solution:

The mystery number is **459 671 325**.

This can be reasoned from the clues in the following steps.

- Since the digit 5 occurs twice and the ten millions digit and the units digit are the same, then they are both 5. The number looks like $\underline{\ \ }\ \underline{5}\ \underline{\ \ }\quad \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\quad \underline{\ \ }\ \underline{\ \ }\ \underline{5}$
- Since the number is less than 500 000 000, the hundred millions digit must be 4 or less.
- Since the sum of the hundred millions, ten millions, and millions digits is 18, and the ten millions digit is 5, then the sum of the hundred millions digit and the millions digit must be 13. And since the hundred millions digit is 4 or less, the only combination that will work for the first 3 digits is 459. The number looks like $\underline{4}\ \underline{5}\ \underline{9}\quad \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\quad \underline{\ \ }\ \underline{\ \ }\ \underline{5}$
- The hundreds digit is 3, and the 5 in the ones (units) digit is the sum of the hundreds and tens digits; thus the tens digit is 2. The number now looks like $\underline{4}\ \underline{5}\ \underline{9}\quad \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\quad \underline{3}\ \underline{2}\ \underline{5}$
- The thousands digit is 1. The number now looks like $\underline{4}\ \underline{5}\ \underline{9}\quad \underline{\ \ }\ \underline{\ \ }\ \underline{1}\quad \underline{3}\ \underline{2}\ \underline{5}$
- All the digits have been used now except 6 and 7. Since the ten thousands digit is one more than the hundred thousands digit, they must be 7 and 6 respectively. The number we are looking for is $\underline{4}\ \underline{5}\ \underline{9}\quad \underline{6}\ \underline{7}\ \underline{1}\quad \underline{3}\ \underline{2}\ \underline{5}$

CEMC at Home

Grade 4/5/6 - Thursday, June 18, 2020
Games and Puzzles

The CEMC has created lots of resources that we hope you have found interesting over the last few months. We also know that there are a lot of online games and puzzles created by other organizations that make use of mathematics and logic. We've highlighted three examples below that you can explore for more mathematical fun!

Tower of Hanoi from Math Playground (https://www.mathplayground.com)

The Tower of Hanoi is a famous puzzle with wooden pegs and rings of different sizes. The goal of this puzzle is to move all of the rings to another peg using the fewest moves possible, but following certain rules. A tool is provided to help you solve the puzzle.

Deep Sea Duel from NCTM (https://www.nctm.org)

You are challenged to a game where you race to find numbers that add up to a particular value.

Deep Sea Math Mystery from Math Playground (https://www.mathplayground.com)

In this puzzle you need to use logic and number sense to figure out how many seashells belong to each sea creature.

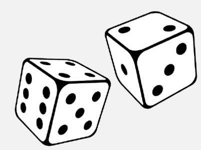
You can find other interesting mathematics related games and puzzles online. Share your favourites using any forum you are comfortable with.

CEMC at Home
Grade 4/5/6 - Friday, June 19, 2020
Relay Day - Part 1

As part of the CEMC's Canadian Team Mathematics Contest, students participate in Math Relays. Just like a relay in track, you "pass the baton" from teammate to teammate in order to finish the race, but in the case of a Math Relay, the "baton" you pass is actually a number!

Read the following set of problems carefully.

Problem 1: Two standard six-sided dice are rolled and the sum of the two top faces is calculated. What is the difference between the largest possible sum and the smallest possible sum?



Problem 2: *Replace N below with the number you receive.*
Marcia has N paper clips. Of these, 2 are pink, 1 is blue, 3 are yellow, and the rest are green. How many of Marcia's paper clips are green?

Problem 3: *Replace N below with the number you receive.*

- Atidya is 4 years older than Bharti.
- Atidya is 6 years younger than Dhruv.
- Dhruv is 9 years older than Chitra.

If Chitra is N years old, how old is Bharti?

Notice that you can answer Problem 1 without any additional information.

In order to answer Problem 2, you first need to know the mystery value of N. The value of N used in Problem 2 will be the *answer* to Problem 1. (For example, if the answer you got for Problem 1 was 5 then you would start Problem 2 by replacing N with 5 in the problem statement.)

Similarly, you need the answer to Problem 2 to answer Problem 3. The value of N in Problem 3 is the *answer* that you got in Problem 2.

Now try the relay! You can use this tool to check your answers.

Follow-up Activity: Can you come up with your own Math Relay?
What do you have to think about when making up the three problems in the relay?
Can you just find three math problems and put them together to form a relay?

In Part 1 of this resource, you were asked to complete a relay on your own. But, of course, relays are meant to be completed in teams! In a team relay, three different people are in charge of answering the problems. Player 1 answers Problem 1 and passes their answer to Player 2; Player 2 takes Player 1's answer and uses it to answer Problem 2; Player 2 passes their answer to Player 3; and so on.

In Part 2 of this resource, you will find instructions on how to run a relay game for your friends and family. We will provide a relay for you to use, but you can also come up with your own!

CEMC at Home

Grade 4 to 12 - Friday, June 19, 2020
Relay Day - Part 2

Relay for Family and Friends

In Part 1 of this resource, you learned how to do a Math Relay. Now, why not try one out with family and friends!

You can put together a relay team and

- play just for fun, not racing any other team, or
- compete against another team in your household (if you have at least 6 people in total), or
- compete with a team from another family or household by
  - timing your team and comparing times with other teams to declare a winner, or
  - competing live using a video chat.

Here are the instructions for how to play.

Relay Instructions:

1. Decide on a team of three players for the relay. The team will be competing together.
2. Find someone to help administer the relay; let's call them the "referee".
3. Each teammate will be assigned a number: 1, 2, or 3. Player 1 will be assigned Problem 1, Player 2 will be assigned Problem 2, and Player 3 will be assigned Problem 3.
4. The three teammates should not see any of the relay problems in advance and should not talk to each other during the relay.
5. Right before the relay starts, the referee should hand out the correct relay problem to each of the players, with the problem statement face down (not visible).
6. The referee will then start the relay. At this time *all three players* can start working on their problems.

   Think about what Player 2 and Player 3 can do before they receive the value of N (the answer from the previous question passed to them by their teammate).
7. When Player 1 thinks they have the correct answer to Problem 1, they record their answer on the answer sheet and pass the sheet to Player 2. When Player 2 thinks they have the correct answer to Problem 2, they record their answer to the answer sheet and pass the sheet to Player 3. When Player 3 thinks they have the correct answer to Problem 3, they record their answer on the answer sheet and pass the sheet to the referee.

8. If all three answers passed to the referee are correct, then the relay is complete! If at least one answer is incorrect, then the referee passes the sheet back to Player 3.

9. At any time during the relay, the players on the team can pass the answer sheet back and forth between them, as long as they write nothing but their current answers on it and do not discuss anything. (For example, if Player 2 is sure that Player 1's answer must be incorrect, then Player 2 can pass the answer sheet back to Player 1, silently. This is a cue for Player 1 to check their work and try again.)

See the next page for a relay for family and friends! This includes instructions for the referee. You can also come up with your own relays to play. You can find many more relays from past CTMC contests on the CEMC's Past Contests webpage.

Sample answer sheets are provided below for you to use for your relays if you wish.

Answer Sheets:

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Problem 1 Answer	
Problem 2 Answer	
Problem 3 Answer	

Relay For Three

Instructions for the Referee:

1. Multiple questions at different levels of difficulty are given for the different relay positions.
   - Assign one of the first three problems (marked "Problem 1") to Player 1.
   - Assign one of the next three problems (marked "Problem 2") to Player 2.
   - Assign one of the last three problems (marked "Problem 3") to Player 3.

   Choose a problem so that each player is comfortable with the level of their question. The level of difficulty of each question is represented using the following symbols:

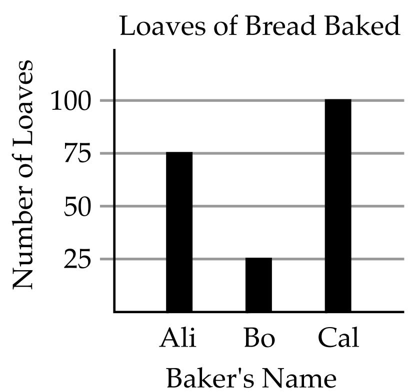
   - ● These questions should be accessible to most students in grade 4 or higher.
   - ■ These questions should be accessible to most students in grade 7 or higher.
   - ◆ These questions should be accessible to most students in grade 9 or higher.

2. Use this tool to find the answers for the relay problems in advance.

Relay Problems (to cut out):

Problem 1 ●

The graph shows the number of loaves of bread that three friends baked. How many loaves did Bo bake?

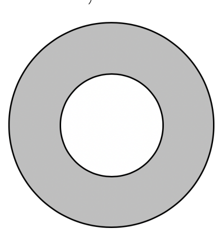


Problem 1 ■

An equilateral triangle has sides of length $x + 4$, $y + 11$, and 20. What is the value of $x + y$?

Problem 1 ◆

In the figure shown, two circles are drawn. If the radius of the larger circle is 10 and the area of the shaded region (in between the two circles) is 75π, then what is the *square* of the radius of the smaller circle?



Problem 2 ●

Replace N below with the number you receive.
Kwame writes the whole numbers in order from 1 to N (including 1 and N). How many times does Kwame write the digit '2'?

Problem 2 ■

Replace N below with the number you receive.
The total mass of three dogs is 43 kilograms. The largest dog has a mass of N kilograms, and the other two dogs have the same mass. What is the mass of each of the smaller dogs?

Problem 2 ◆

Replace N below with the number you receive.
The points $(6, 16)$, $(8, 22)$, and (x, N) lie on a straight line. Find the value of x.

Problem 3 ●

Replace N below with the number you receive.
You have some boxes of the same size and shape. If N oranges can fit in one box, how many oranges can fit in two boxes, in total?

Problem 3 ■

Replace N below with the number you receive.
One morning, a small farm sold 10 baskets of tomatoes, 2 baskets of peppers, and N baskets of zucchini. If the prices are as shown below, how much money, in dollars did the farm earn in total from these sales?

Basket of Tomatoes:	\$0.50
Basket of Peppers:	\$2.00
Basket of Zucchini:	\$1.00

Problem 3 ◆

Replace N with the number you receive.
Elise has N boxes, each containing x apples. She gives 12 apples to her sister. She then gives 20% of her remaining apples to her brother. After this, she has 120 apples left. What is the value of x?