The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Wednesday, February 28, 2024
(in North America and South America)

Thursday, February 29, 2024
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $3\left(\frac{5}{3}-\frac{1}{3}\right)$ is

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

2. If $x=2$, the value of $4x^2-3x^2$ is

   (A) 1 **(B)** 3 **(C)** 4 **(D)** 2 **(E)** 5

3. How many solid $1\times1\times1$ cubes are required to make a solid $2\times2\times2$ cube?

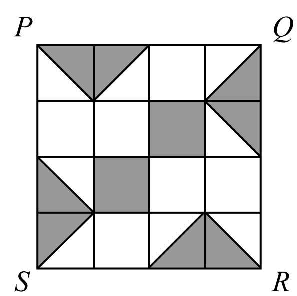
   (A) 4 **(B)** 7 **(C)** 6 **(D)** 5 **(E)** 8

4. Shuxin begins with 10 red candies, 7 yellow candies, and 3 blue candies. After eating some of the candies, there are equal numbers of red, yellow, and blue candies remaining. What is the smallest possible number of candies that Shuxin ate?

   (A) 11 **(B)** 7 **(C)** 17 **(D)** 20 **(E)** 14

5. Square $PQRS$ is divided into 16 smaller congruent squares, as shown. What fraction of $PQRS$ is shaded?

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{2}$
   (D) $\frac{1}{4}$ **(E)** $\frac{3}{8}$



6. How many integers are greater than $\sqrt{15}$ and less than $\sqrt{50}$?

   (A) 3 **(B)** 4 **(C)** 6 **(D)** 5 **(E)** 2

7. The line with equation $y=3x+6$ is reflected in the y-axis. What is the x-intercept of the new line?

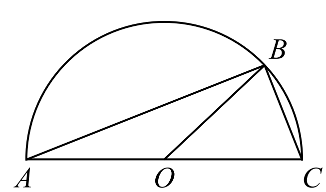
   (A) 2 **(B)** -2 **(C)** 6 **(D)** -6 **(E)** $\frac{1}{2}$

8. If $10^n=1000^{20}$, what is the value of n?

   (A) 1000 **(B)** 60 **(C)** 2000 **(D)** 300 **(E)** 102

9. In the diagram, a semi-circle has centre O and diameter AC. Also, B is a point on the circumference such that $\angle BAC=25^\circ$. The measure of $\angle BOC$ is

   (A) 60° **(B)** 55° **(C)** 45°
   (D) 50° **(E)** 65°



10. In a photograph, Aristotle, David, Flora, Munirah, and Pedro are seated in a random order in a row of 5 chairs. If David is seated in the middle of the row, what is the probability that Pedro is seated beside him?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{3}{5}$

Part B: Each correct answer is worth 6.

11. Figure 1 shows an arrangement of 3 lines with 1 intersection point, and Figure 2 shows an arrangement of 3 lines with 3 intersection points.

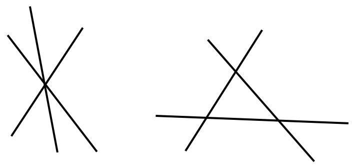


Figure 1 Figure 2

What is the maximum number of intersection points that can appear in an arrangement of 4 lines?

(A) 5 **(B)** 4 **(C)** 8 **(D)** 6 **(E)** 7

12. The numbers 5, 6, 10, 17, and 21 are rearranged so that the sum of the first three numbers is equal to the sum of the last three numbers. Which number is in the middle of this rearrangement?

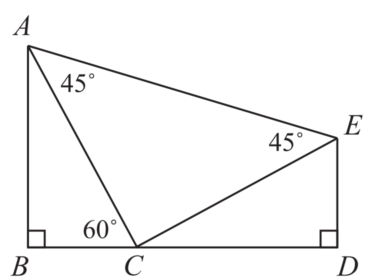
(A) 5 **(B)** 6 **(C)** 10 **(D)** 17 **(E)** 21

13. For some integers m and n, the expression $(x+m)(x+n)$ is equal to a quadratic expression in x with a constant term of -12. Which of the following cannot be a value of m?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. In diagram, point C is on side BD of quadrilateral $ABDE$. Also, AB and ED are perpendicular to BD, $\angle ACB = 60^\circ$, $\angle CAE = 45^\circ$, and $\angle AEC = 45^\circ$. If $AB = \sqrt{3}$, what is the perimeter of quadrilateral $ABDE$?

(A) $1+\sqrt{2}+2\sqrt{3}$
(B) $2+2\sqrt{2}+\sqrt{3}$
(C) $1+2\sqrt{2}+2\sqrt{3}$
(D) $2+\sqrt{2}+2\sqrt{3}$
(E) $2+2\sqrt{2}+2\sqrt{3}$



15. Anila's grandmother wakes up at the same time every day and follows this same routine:

    - She gets her coffee 1 hour after she wakes up. This takes 10 minutes.
    - She has a shower 2 hours after she wakes up. This takes 10 minutes.
    - She goes for a walk 3 hours after she wakes up. This takes 40 minutes.
    - She calls her granddaughter 4 hours after she wakes up. This takes 15 minutes.
    - She does some yoga 5 hours after she wakes up. This takes 30 minutes.

    If Anila's grandmother woke up 5 minutes ago, what will she be doing in 197 hours?

    (A) Getting her coffee **(B)** Having a shower **(C)** Doing some yoga
    (D) Going for a walk **(E)** Calling her granddaughter

16. Francesca put one of the integers $1, 2, 3, 4, 5, 6, 7, 8, 9$ in each of the nine squares of the grid shown. No integer is used twice. She calculated the product of the three integers in each row and wrote the products to the right of the corresponding rows. She calculated the product of the integers in each column and wrote the products below the corresponding columns. Finally, she erased the integers from the nine squares. Which integer was in the square marked N?

			56
			135
	N		48
21	108	160	

    (A) 3 **(B)** 4 **(C)** 6
    (D) 8 **(E)** 9

17. Each of the variables a, b, c, d, and e represents a positive integer with the properties that

$$b + d > a + d$$
$$c + e > b + e$$
$$b + d = c$$
$$a + c = b + e$$

    Which of the variables has the greatest value?

    (A) a **(B)** b **(C)** c **(D)** d **(E)** e

18. Suppose that x and y are real numbers that satisfy the two equations

$$3x + 2y = 6$$
$$9x^2 + 4y^2 = 468$$

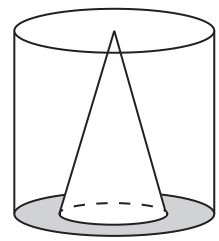
    What is the value of xy?

    (A) -36 **(B)** -12 **(C)** -32 **(D)** -120 **(E)** -32

19. Three standard six-sided dice are rolled. The sum of the three numbers rolled, S, is determined. The probability that $S > 5$ is closest to

    (A) 0.97 **(B)** 0.95 **(C)** 0.94 **(D)** 0.96 **(E)** 0.98

20. A cylinder contains some water. A solid cone with the same height and half the radius of the cylinder is submerged into the water until the circular face of the cone lies flat on the circular base of the cylinder, as shown. Once this is done, the depth of the water is half of the height of the cylinder. If the cone is then removed, the depth of the water will be what fraction of the height of the cylinder?



(The volume of a cylinder with radius r and height h is $\pi r^2 h$ and the volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.)

(A) $\frac{3}{16}$ **(B)** $\frac{41}{96}$ **(C)** $\frac{5}{16}$
(D) $\frac{3}{8}$ **(E)** $\frac{7}{16}$

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. A 3×3 table starts with every entry equal to 0 and is modified using the following steps:

(i) adding 1 to all three numbers in any row;

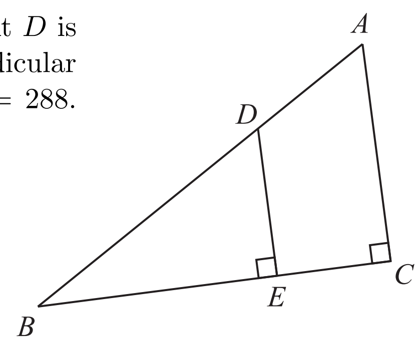
(ii) adding 2 to all three numbers in any column.

After step (i) has been used a total of a times and step (ii) has been used a total of b times, the table appears as shown. What is the value of $a + b$?

7	1	5
9	3	7
8	2	6

22. Four distinct integers a, b, c, and d are chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the greatest possible value of $ac + bd - ad - bc$?

23. In the diagram, $\triangle ABC$ is right-angled at C. Point D is on AB and point E is on BC so that DE is perpendicular to BC, $BE = AC$, $BD = 120$, and $DE + BC = 288$. What is the length of DE?



24. The integer N is the smallest positive integer that is a multiple of 2024, has more than 100 positive divisors (including 1 and N), and has fewer than 110 positive divisors (including 1 and N). What is the sum of the digits of N?

25. A sequence of 11 positive real numbers, a_1, a_2, a_3, $\ldots$, a_{11}, satisfies $a_1 = 4$ and $a_{11} = 1024$ and $a_n + a_{n-1} = \frac{5}{2}\sqrt{a_n \cdot a_{n-1}}$ for every integer n with $2 \le n \le 11$. For example when $n = 7$, $a_7 + a_6 = \frac{5}{2}\sqrt{a_7 \cdot a_6}$. There are S such sequences. What are the rightmost two digits of S?

2024 Fermat Contest (English)

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- Look at our free online courseware
- Use our free Problem Set Generator to create problem sets for curriculum support and enrichment
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results

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Fermat Contest

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Part A: Each correct answer is worth 5.

1. The value of $0.3 + 0.03$ is

   (A) 0.303 **(B)** 0.6 **(C)** 3.3 **(D)** 0.33 **(E)** 0.06

2. If $3 + x = 5$ and $-3 + y = 5$, then $x + y$ equals

   (A) 4 **(B)** 16 **(C)** 6 **(D)** 12 **(E)** 10

3. When $x = 2$, the expression $2x^2 + 3x^2$ equals

   (A) 14 **(B)** 10 **(C)** 12 **(D)** 22 **(E)** 20

4. The number of minutes in a week is closest to

   (A) 100 **(B)** 1000 **(C)** 10 000 **(D)** 100 000 **(E)** 1 000 000

5. Ava's machine takes four-digit positive integers as input. When the four-digit integer $ABCD$ is input, the machine outputs the integer $A \times B + C \times D$. For example, when the input is 1234, the output is $1 \times 2 + 3 \times 4 = 2 + 12 = 14$. When the input is 2023, the output is

   (A) 0 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

6. Vivek is painting three doors. The doors are numbered 1, 2 and 3. Each door is to be painted one colour: either black or gold. One possibility is that door 1 is painted black, door 2 is painted gold, and door 3 is painted gold. In total, how many different ways can the three doors be painted?

   (A) 8 **(B)** 7 **(C)** 5 **(D)** 4 **(E)** 3

7. Snacks are purchased for 17 soccer players. Juice boxes come in packs of 3 and cost \$2.00 per pack. Apples come in bags of 5 and cost \$4.00 per bag. Danny buys packs of juice boxes and bags of apples so that every player gets a juice box and an apple. What is the minimum amount of money that Danny spends?

   (A) \$26.00 **(B)** \$28.00 **(C)** \$24.00 **(D)** \$30.00 **(E)** \$36.00

8. A bicycle trip is 30 km long. Ari rides at an average speed of 20 km/h. Bri rides at an average speed of 15 km/h. If Ari and Bri begin at the same time, how many minutes after Ari finishes the trip will Bri finish?

   (A) 50 min **(B)** 40 min **(C)** 30 min **(D)** 20 min **(E)** 10 min

9. Three tanks contain water. The number of litres in each is shown in the table below:

Tank A	Tank B	Tank C
3600 L	1600 L	3800 L

Water is moved from each of Tank A and Tank C into Tank B so that each tank contains the same volume of water. How many litres of water are moved from Tank A to Tank B?

(A) 500 L **(B)** 600 L **(C)** 700 L **(D)** 800 L **(E)** 900 L

10. Points A, B, C, and D lie along a line, in that order. If $AB : AC = 1 : 5$, and $BC : CD = 2 : 1$, then the ratio $AB : CD$ is equal to

(A) $1 : 1$ **(B)** $1 : 2$ **(C)** $1 : 3$ **(D)** $2 : 5$ **(E)** $3 : 5$

Part B: Each correct answer is worth 6.

11. At the start of this month, Mathilde and Salah each had 100 coins. For Mathilde, this was 25% more coins than she had at the start of last month. For Salah, this was 20% fewer coins than he had at the start of last month. The total number of coins that they had at the start of last month was

(A) 180 **(B)** 185 **(C)** 190 **(D)** 200 **(E)** 205

12. A rectangle has length 8 cm and width π cm. A semi-circle has the same area as the rectangle. What is its radius?

(A) $4\sqrt{2}$ cm **(B)** 4 cm **(C)** 16 cm **(D)** 8 cm **(E)** 2 cm

13. If $a(x + 2) + b(x + 2) = 60$ and $a + b = 12$, then x is equal to

(A) 3 **(B)** 5 **(C)** 1 **(D)** 7 **(E)** 48

14. A line with a slope of 2 and a line with a slope of -4 each have a y-intercept of 6. The distance between the x-intercepts of these lines is

(A) 2 **(B)** 6 **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{2}$ **(E)** $\frac{9}{2}$

15. A sequence has 101 terms, each of which is a positive integer. If a term, n, is even, the next term is equal to $\frac{1}{2}n + 1$. If a term, n, is odd, the next term is equal to $\frac{1}{2}(n + 1)$. For example, if the first term is 7, then the second term is 4 and the third term is 3. If the first term is 16, the 101st term is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. Twenty-five cards are randomly arranged in a grid, as shown. Five of these cards have a 0 on one side and a 1 on the other side. The remaining twenty cards either have a 0 on both sides or a 1 on both sides. Loron chooses one row or one column and flips over each of the five cards in that row or column, leaving the rest of the cards untouched. After this operation, Loron determines the ratio of 0s to 1s facing upwards. No matter which row or column Loron chooses, it is *not possible* for this ratio to be

0	0	1	1	0
1	0	0	1	0
1	1	0	1	0
0	0	1	0	0
1	0	1	0	1

(A) 12 : 13 **(B)** 2 : 3 **(C)** 9 : 16
(D) 3 : 2 **(E)** 16 : 9

17. The positive divisors of 6 are 1, 2, 3, 6. The sum of the positive divisors of 1184 is

(A) 2394 **(B)** 2396 **(C)** 2398 **(D)** 2400 **(E)** 2402

18. A robotic grasshopper jumps 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After every fourth jump, the grasshopper restarts the sequence of jumps: 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After a total of n jumps, the position of the grasshopper is 162 cm to the west and 158 cm to the south of its original position. The sum of the squares of the digits of n is

(A) 22 **(B)** 29 **(C)** 17 **(D)** 14 **(E)** 13

19. If x and y are integers with $2x^2 + 8y = 26$, a possible value of $x - y$ is

(A) -8 **(B)** 26 **(C)** -16 **(D)** 22 **(E)** 30

20. If n is a positive integer, the symbol $n!$ (which is read "n factorial") represents the product of the integers from 1 to n, inclusive. For example, $5! = (1)(2)(3)(4)(5)$ or $5! = 120$, which ends with exactly 1 zero. For how many integers m, with $1 \leq m \leq 30$, is it possible to find a value of n so that $n!$ ends with exactly m zeros?

(A) 30 **(B)** 27 **(C)** 28 **(D)** 24 **(E)** 25

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Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in the circles and squares below with one number in each shape.

$$\boxed{x}\ \bigcirc\ \square\ \bigcirc\ \square\ \bigcirc\ \square\ \bigcirc\ \boxed{y}$$

Each integer must be used exactly once and the integer in each circle must be equal to the sum of the integers in the two neighbouring squares. If the integer x is placed in the leftmost square and the integer y is placed in the rightmost square, what is the largest possible value of $x + y$?

22. If x and y are positive real numbers with $\dfrac{1}{x+y} = \dfrac{1}{x} - \dfrac{1}{y}$, what is the value of $\left(\dfrac{x}{y} + \dfrac{y}{x}\right)^2$?

23. For each positive integer n, define $s(n)$ to equal the sum of the digits of n. For example, $s(2023) = 2 + 0 + 2 + 3$. The number of integers n with $100 \leq n \leq 999$ and $7 \leq s(n) \leq 11$ is S. What is the integer formed by the rightmost two digits of S?

24. Quadrilateral $ABCD$ has $\angle BCD = \angle DAB = 90^\circ$. The perimeter of $ABCD$ is 224 and its area is 2205. One side of $ABCD$ has length 7. The remaining three sides have integer lengths. The sum of the squares of the side lengths of $ABCD$ is S. What is the integer formed by the rightmost two digits of S?

25. A cube has edge length 4 m. One end of a rope of length 5 m is anchored to the centre of the top face of the cube. The area of the surface of the cube that can be reached by the other end of the rope is A m^2. What is the integer formed by the rightmost two digits of the integer closest to $100A$?

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Part A: Each correct answer is worth 5.

1. The value of $6 + (3 \times 6) - 12$ is

   (A) 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

2. The average (mean) of two numbers is 7. One of the numbers is 5. The other number is

   (A) 6 **(B)** 4 **(C)** 3 **(D)** 8 **(E)** 9

3. Gauravi walks every day. One Monday, she walks 500 m. On each day that follows, she increases her distance by 500 m from the previous day. On what day of the week will she walk exactly 4500 m?

   (A) Thursday **(B)** Friday **(C)** Tuesday **(D)** Monday **(E)** Wednesday

4. What is the largest number of squares with side length 2 that can be arranged, without overlapping, inside a square with side length 8?

   (A) 8 **(B)** 32 **(C)** 16 **(D)** 64 **(E)** 4

5. One integer is selected at random from the following list of 15 integers:

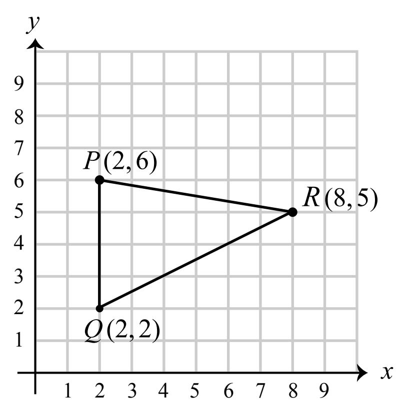
   $$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5$$

   The probability that the selected integer is equal to n is $\frac{1}{3}$. What is the value of n?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. In the diagram, points $P(2,6)$, $Q(2,2)$ and $R(8,5)$ form a triangle. The area of $\triangle PQR$ is

   (A) 24 **(B)** 14 **(C)** 21
   (D) 12 **(E)** 16



7. The expression $(1 + 2 + 3)(1 + \frac{1}{2} + \frac{1}{3})$ is equal to

   (A) 3 **(B)** 11 **(C)** 6 **(D)** $\frac{11}{6}$ **(E)** 12

8. If $10x + y = 75$ and $10y + x = 57$ for some positive integers x and y, the value of $x + y$ is

(A) 12 **(B)** 5 **(C)** 7 **(D)** 77 **(E)** 132

9. It takes Pearl 7 days to dig 4 holes. It takes Miguel 3 days to dig 2 holes. If they work together and each continues digging at these same rates, how many holes in total will they dig in 21 days?

(A) 35 **(B)** 22 **(C)** 12 **(D)** 26 **(E)** 28

10. If $2^{11} \times 6^5 = 4^x \times 3^y$ for some positive integers x and y, then the value of $x + y$ is

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

Part B: Each correct answer is worth 6.

11. Dhruv is older than Bev. Bev is older than Elcim. Elcim is younger than Andy. Andy is younger than Bev. Bev is younger than Cao. Who is the third oldest?

(A) Andy **(B)** Bev **(C)** Cao **(D)** Dhruv **(E)** Elcim

12. Suppose that d is an odd integer and e is an even integer. How many of the following expressions are equal to an odd integer?

$$d + d \qquad (e + e) \times d \qquad d \times d \qquad d \times (e + d)$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. Seven identical rectangles are used to create two larger rectangles, as shown in Figure A and Figure B.

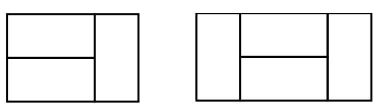


Figure A Figure B

The ratio of the perimeter of Figure A to the perimeter of Figure B is

(A) 2 : 3 **(B)** 3 : 4 **(C)** 3 : 5 **(D)** 4 : 5 **(E)** 5 : 6

14. Zebadiah has 3 red shirts, 3 blue shirts, and 3 green shirts in a drawer. Without looking, he randomly pulls shirts from his drawer one at a time. He would like a set of shirts that includes either 3 of the same colour or 3 of different colours. What is the minimum number of shirts that Zebadiah has to pull out to *guarantee* that he has such a set?

(A) 4 **(B)** 3 **(C)** 6 **(D)** 5 **(E)** 7

15. A positive integer a is input into a machine. If a is odd, the output is $a+3$. If a is even, the output is $a+5$. This process can be repeated using each successive output as the next input. For example, if the input is $a=1$ and the machine is used three times, the final output is 12. If the input is $a=15$ and the machine is used 51 times, the final output is

(A) 213 **(B)** 218 **(C)** 212 **(D)** 220 **(E)** 215

16. The remainder when 111 is divided by 10 is 1. The remainder when 111 is divided by the positive integer n is 6. The number of possible values of n is

(A) 5 **(B)** 8 **(C)** 7 **(D)** 6 **(E)** 4

17. An aluminum can in the shape of a cylinder is closed at both ends. Its surface area is 300 cm^2. If the radius of the can were doubled, its surface area would be 900 cm^2. If instead the height of the can were doubled, what would its surface area be?

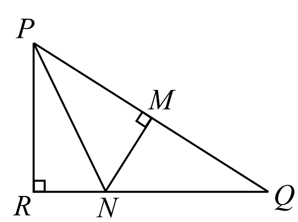
(The surface area of a cylinder with radius r and height h is equal to $2\pi r^2 + 2\pi rh$.)

(A) 450 cm^2 **(B)** 600 cm^2 **(C)** 750 cm^2 **(D)** 375 cm^2 **(E)** 300 cm^2

18. Aria and Bianca walk at different, but constant speeds. They each begin at 8:00 a.m. from the opposite ends of a road and walk directly toward the other's starting point. They pass each other at 8:42 a.m. Aria arrives at Bianca's starting point at 9:10 a.m. Bianca arrives at Aria's starting point at

(A) 9:30 a.m. **(B)** 9:35 a.m. **(C)** 9:40 a.m. **(D)** 9:45 a.m. **(E)** 9:50 a.m.

19. In the diagram, $\triangle PQR$ is right-angled at R, $PR=12$, and $QR=16$. Also, M is the midpoint of PQ and N is the point on QR so that MN is perpendicular to PQ. The area of $\triangle PNR$ is



(A) 21 **(B)** 17.5 **(C)** 36

(D) 16 **(E)** 21.5

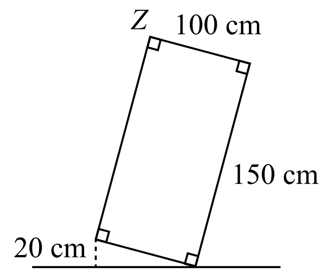
20. A sequence of numbers $t_1, t_2, t_3, \ldots$ has its terms defined by $t_n = \dfrac{1}{n} - \dfrac{1}{n+2}$ for every integer $n \geq 1$. For example, $t_4 = \dfrac{1}{4} - \dfrac{1}{6}$. What is the largest positive integer k for which the sum of the first k terms (that is, $t_1 + t_2 + \cdots + t_{k-1} + t_k$) is less than 1.499?

(A) 2000 **(B)** 1999 **(C)** 2002 **(D)** 2001 **(E)** 1998

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. Gustave has 15 steel bars of masses 1 kg, 2 kg, 3 kg, $\ldots$, 14 kg, 15 kg. He also has 3 bags labelled A, B, C. He places two steel bars in each bag so that the total mass in each bag is equal to M kg. How many different values of M are possible?

22. A rectangle with dimensions 100 cm by 150 cm is tilted so that one corner is 20 cm above a horizontal line, as shown. To the nearest centimetre, the height of vertex Z above the horizontal line is $(100 + x)$ cm. What is the value of x?



23. For how many positive integers k do the lines with equations $9x + 4y = 600$ and $kx - 4y = 24$ intersect at a point whose coordinates are positive integers?

24. There are functions $f(x)$ with the following properties:
    - $f(x) = ax^2 + bx + c$ for some integers a, b and c with $a > 0$, and
    - $f(p) = f(q) = 17$ and $f(p+q) = 47$ for some prime numbers p and q with $p < q$.

    For each such function, the value of $f(pq)$ is calculated. The sum of all possible values of $f(pq)$ is S. What are the rightmost two digits of S?

25. In the 3×3 grid shown, the central square contains the integer 5. The remaining eight squares contain a, b, c, d, e, f, g, h, which are each to be replaced with an integer from 1 to 9, inclusive. Integers can be repeated. There are N ways to complete the grid so that the sums of the integers along each row, along each column, and along the two main diagonals are all divisible by 5. What are the rightmost two digits of N?

a	b	c
d	5	e
f	g	h

2022 Fermat Contest (English)

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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Fermat Contest

(Grade 11)

Tuesday, February 23, 2021
(in North America and South America)

Wednesday, February 24, 2021
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
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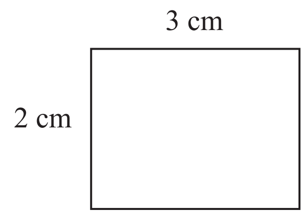
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. A rectangle has width 2 cm and length 3 cm.
The area of the rectangle is



(A) 2 cm^2 **(B)** 9 cm^2 **(C)** 5 cm^2
(D) 36 cm^2 **(E)** 6 cm^2

2. The expression $2 + 3 \times 5 + 2$ equals

(A) 19 **(B)** 27 **(C)** 35 **(D)** 17 **(E)** 32

3. The number equal to 25% of 60 is

(A) 10 **(B)** 15 **(C)** 20 **(D)** 12 **(E)** 18

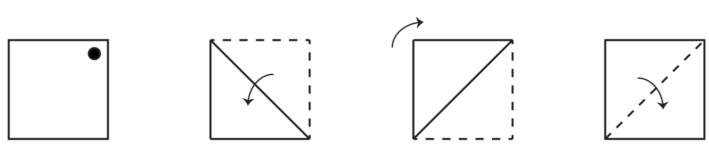
4. When $x = 2021$, the value of $\dfrac{4x}{x + 2x}$ is

(A) $\dfrac{3}{4}$ **(B)** $\dfrac{4}{3}$ **(C)** 2021 **(D)** 2 **(E)** 6

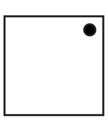
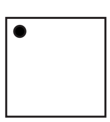
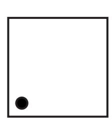
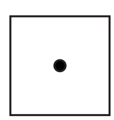
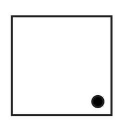
5. Which of the following integers *cannot* be written as a product of two integers, each greater than 1?

(A) 6 **(B)** 27 **(C)** 53 **(D)** 39 **(E)** 77

6. A square piece of paper has a dot in its top right corner and is lying on a table. The square is folded along its diagonal, then rotated 90° clockwise about its centre, and then finally unfolded, as shown.



The resulting figure is

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

7. For which of the following values of x is x greater than x^2 ?

(A) $x = -2$ **(B)** $x = -\frac{1}{2}$ **(C)** $x = 0$ **(D)** $x = \frac{1}{2}$ **(E)** $x = 2$

8. The digits in a two-digit positive integer are reversed. The new two-digit integer minus the original integer equals 54. What is the positive difference between the two digits of the original integer?

(A) 5 **(B)** 7 **(C)** 6 **(D)** 8 **(E)** 9

9. The line with equation $y = 2x - 6$ is translated upwards by 4 units. (That is, every point on the line is translated upwards by 4 units, forming a new line.) The x-intercept of the resulting line is

(A) 3 **(B)** $\frac{3}{2}$ **(C)** 4 **(D)** 1 **(E)** 2

10. If $3^x = 5$, the value of 3^{x+2} is

(A) 10 **(B)** 25 **(C)** 2187 **(D)** 14 **(E)** 45

Part B: Each correct answer is worth 6.

11. In the sum shown, P, Q and R represent three different single digits. The value of $P + Q + R$ is

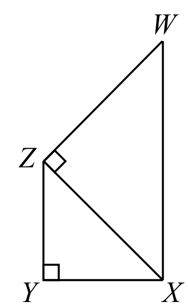
$$\begin{array}{cccc} & P & 7 & R \\ + & 3 & 9 & R \\ \hline & R & Q & 0 \end{array}$$

(A) 13 **(B)** 12 **(C)** 14
(D) 3 **(E)** 4

12. How many of the 20 perfect squares $1^2, 2^2, 3^2, \ldots, 19^2, 20^2$ are divisible by 9?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

13. In the diagram, each of $\triangle WXZ$ and $\triangle XYZ$ is an isosceles right-angled triangle. The length of WX is $6\sqrt{2}$. The perimeter of quadrilateral $WXYZ$ is closest to



(A) 18 **(B)** 20 **(C)** 23
(D) 25 **(E)** 29

14. Natascha cycles 3 times as fast as she runs. She spends 4 hours cycling and 1 hour running. The ratio of the distance that she cycles to the distance that she runs is

(A) $12 : 1$ **(B)** $7 : 1$ **(C)** $4 : 3$ **(D)** $16 : 9$ **(E)** $1 : 1$

15. Let a and b be positive integers for which $45a + b = 2021$. The minimum possible value of $a + b$ is

(A) 44 **(B)** 82 **(C)** 85 **(D)** 86 **(E)** 130

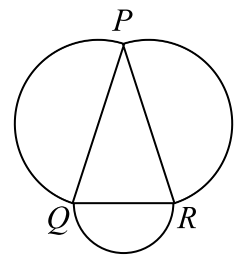
16. If n is a positive integer, the notation $n!$ (read "n factorial") is used to represent the product of the integers from 1 to n. That is, $n! = n(n-1)(n-2)\cdots(3)(2)(1)$. For example, $4! = 4(3)(2)(1) = 24$ and $1! = 1$. If a and b are positive integers with $b > a$, the ones (units) digit of $b! - a!$ cannot be

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

17. The set S consists of 9 distinct positive integers. The average of the two smallest integers in S is 5. The average of the two largest integers in S is 22. What is the greatest possible average of all of the integers of S?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

18. In the diagram, $\triangle PQR$ is an isosceles triangle with $PQ = PR$. Semi-circles with diameters PQ, QR and PR are drawn. The sum of the areas of these three semi-circles is equal to 5 times the area of the semi-circle with diameter QR. The value of $\cos(\angle PQR)$ is



(A) $\frac{1}{3}$ **(B)** $\frac{1}{\sqrt{8}}$ **(C)** $\frac{1}{\sqrt{12}}$

(D) $\frac{1}{\sqrt{15}}$ **(E)** $\frac{1}{\sqrt{10}}$

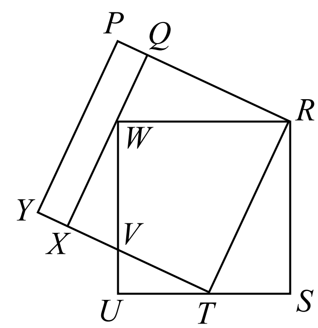
19. The real numbers x, y and z satisfy the three equations

$$\begin{aligned} x + y &= 7 \\ xz &= -180 \\ (x+y+z)^2 &= 4 \end{aligned}$$

If S is the sum of the two possible values of y, then $-S$ equals

(A) 56 **(B)** 14 **(C)** 36 **(D)** 34 **(E)** 42

20. In the diagram, $PRTY$ and $WRSU$ are squares. Point Q is on PR and point X is on TY so that $PQXY$ is a rectangle. Also, point T is on SU, point W is on QX, and point V is the point of intersection of UW and TY, as shown. If the area of rectangle $PQXY$ is 30, the length of ST is closest to



(A) 5 **(B)** 5.25 **(C)** 5.5

(D) 5.75 **(E)** 6

Part C: Each correct answer is worth 8.

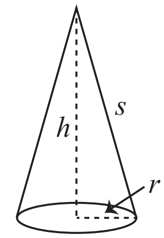
21. A function, f, has $f(2) = 5$ and $f(3) = 7$. In addition, f has the property that

$$f(m) + f(n) = f(mn)$$

for all positive integers m and n. (For example, $f(9) = f(3) + f(3) = 14$.) The value of $f(12)$ is

(A) 17 **(B)** 35 **(C)** 28 **(D)** 12 **(E)** 25

22. An unpainted cone has radius 3 cm and slant height 5 cm. The cone is placed in a container of paint. With the cone's circular base resting flat on the bottom of the container, the depth of the paint in the container is 2 cm. When the cone is removed, its circular base and the lower portion of its lateral surface are covered in paint. The fraction of the total surface area of the cone that is covered in paint can be written as $\frac{p}{q}$ where p and q are positive integers with no common divisor larger than 1. What is the value of $p + q$?



(The *lateral surface* of a cone is its external surface not including the circular base. A cone with radius r, height h, and slant height s has lateral surface area equal to πrs.)

(A) 59 **(B)** 61 **(C)** 63 **(D)** 65 **(E)** 67

23. In Figure 1, three unshaded dots are arranged to form an equilateral triangle, as shown. Figure 2 is formed by arranging three copies of Figure 1 to form the outline of a larger equilateral triangle and then filling the resulting empty space with 1 shaded dot. For each integer $n > 2$, Figure n is formed by first arranging three copies of Figure $n - 1$ to form the outline of a larger equilateral triangle and then filling the resulting empty space in the centre with an inverted triangle of shaded dots.

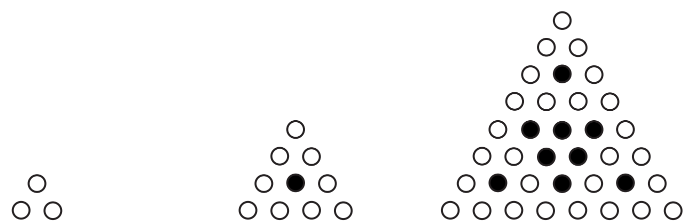


Figure 1 Figure 2 Figure 3

The smallest value of n for which Figure n includes at least 100 000 shaded dots is

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

24. A pair of real numbers (a, b) with $a^2 + b^2 \leq \frac{1}{4}$ is chosen at random. If p is the probability that the curves with equations $y = ax^2 + 2bx - a$ and $y = x^2$ intersect, then $100p$ is closest to

(A) 65 **(B)** 69 **(C)** 53 **(D)** 57 **(E)** 61

25. Let N be the number of triples (x, y, z) of positive integers such that $x < y < z$ and $xyz = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 17^2 \cdot 19^2$. When N is divided by 100, the remainder is

(A) 28 **(B)** 88 **(C)** 8 **(D)** 68 **(E)** 48

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Fermat Contest

(Grade 11)

Tuesday, February 25, 2020
(in North America and South America)

Wednesday, February 26, 2020
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

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Scoring:	There is *no penalty* for an incorrect answer.
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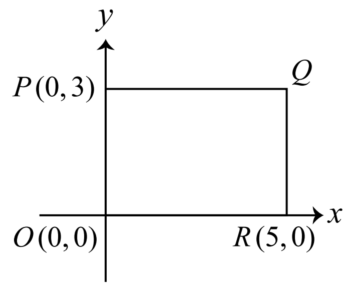
Part A: Each correct answer is worth 5.

1. The points $O(0,0)$, $P(0,3)$, Q, and $R(5,0)$ form a rectangle, as shown. The coordinates of Q are

   (A) $(5,5)$ **(B)** $(5,3)$ **(C)** $(3,3)$
   (D) $(2.5,1.5)$ **(E)** $(0,5)$



2. The value of $3\times 2020 + 2\times 2020 - 4\times 2020$ is

   (A) 6060 **(B)** 4040 **(C)** 8080 **(D)** 0 **(E)** 2020

3. For every real number x, the expression $(x+1)^2 - x^2$ is equal to

   (A) $2x+1$ **(B)** $2x-1$ **(C)** $(2x+1)^2$ **(D)** -1 **(E)** $x+1$

4. Ewan writes out a sequence where he counts by 11s starting at 3. The resulting sequence is $3, 14, 25, 36, \ldots$. A number that will appear in Ewan's sequence is

   (A) 113 **(B)** 111 **(C)** 112 **(D)** 110 **(E)** 114

5. The value of $\sqrt{\dfrac{\sqrt{81}+\sqrt{81}}{2}}$ is

   (A) 3 **(B)** 18 **(C)** 27 **(D)** 81 **(E)** 162

6. Anna thinks of an integer.

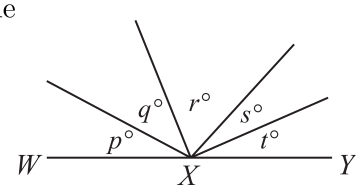
   - It is *not* a multiple of three.
   - It is *not* a perfect square.
   - The sum of its digits is a prime number.

   The integer that Anna is thinking of could be

   (A) 12 **(B)** 14 **(C)** 16 **(D)** 21 **(E)** 26

7. In the diagram, WXY is a straight angle. What is the average (mean) of p, q, r, s, and t?

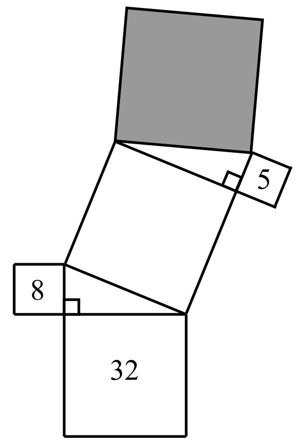
   (A) 30 **(B)** 36 **(C)** 60
   (D) 72 **(E)** 45



8. If $2^n = 8^{20}$, what is the value of n?

   (A) 10 **(B)** 60 **(C)** 40 **(D)** 16 **(E)** 17

9. The figure consists of five squares and two right-angled triangles. The areas of three of the squares are 5, 8 and 32, as shown. What is the area of the shaded square?



(A) 35 **(B)** 45 **(C)** 29
(D) 19 **(E)** 75

10. Positive integers s and t have the property that $s(s-t) = 29$. What is the value of $s+t$?

(A) 1 **(B)** 28 **(C)** 57 **(D)** 30 **(E)** 29

Part B: Each correct answer is worth 6.

11. In the 5×5 grid shown, 15 cells contain X's and 10 cells are empty. Any X may be moved to any empty cell. What is the smallest number of X's that must be moved so that each row and each column contains exactly three X's?

X	X	X	X	
X	X	X		X
X	X			
X	X		X	
		X	X	

(A) 1 **(B)** 2 **(C)** 3
(D) 4 **(E)** 5

12. Harriet ran a 1000 m course in 380 seconds. She ran the first 720 m of the course at a constant speed of 3 m/s. She ran the remaining part of the course at a constant speed of v m/s. What is the value of v?

(A) 2 **(B)** 1.5 **(C)** 3 **(D)** 1 **(E)** 4.5

13. In the list $2, x, y, 5$, the sum of any two adjacent numbers is constant. The value of $x - y$ is

(A) 1 **(B)** -3 **(C)** 3 **(D)** -1 **(E)** 0

14. In Rad's garden there are exactly 30 red roses, exactly 19 yellow roses, and no other roses. How many of the yellow roses does Rad need to remove so that $\frac{2}{7}$ of the roses in the garden are yellow?

(A) 5 **(B)** 6 **(C)** 4 **(D)** 8 **(E)** 7

15. Suppose that $N = 3x + 4y + 5z$, where x equals 1 or -1, and y equals 1 or -1, and z equals 1 or -1. How many of the following statements are true?

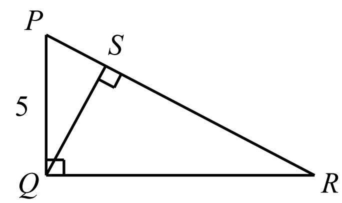
- N can equal 0.
- N is always odd.
- N cannot equal 4.
- N is always even.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

16. Suppose that x and y are real numbers with $-4 \leq x \leq -2$ and $2 \leq y \leq 4$. The greatest possible value of $\dfrac{x+y}{x}$ is

(A) 1 **(B)** -1 **(C)** $-\frac{1}{2}$ **(D)** 0 **(E)** $\frac{1}{2}$

17. In the diagram, $\triangle PQR$ is right-angled at Q and point S is on PR so that QS is perpendicular to PR. If the area of $\triangle PQR$ is 30 and $PQ = 5$, the length of QS is



(A) $\frac{60}{13}$ **(B)** 5 **(C)** $\frac{30}{13}$
(D) 4 **(E)** 3

18. Four teams play in a tournament in which each team plays exactly one game against each of the other three teams. At the end of each game, either the two teams tie or one team wins and the other team loses. A team is awarded 3 points for a win, 0 points for a loss, and 1 point for a tie. If S is the sum of the points of the four teams after the tournament is complete, which of the following values can S *not* equal?

(A) 13 **(B)** 17 **(C)** 11 **(D)** 16 **(E)** 15

19. When $(3 + 2x + x^2)(1 + mx + m^2x^2)$ is expanded and fully simplified, the coefficient of x^2 is equal to 1. What is the sum of all possible values of m?

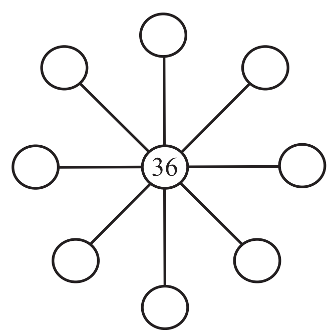
(A) $-\frac{4}{3}$ **(B)** $-\frac{2}{3}$ **(C)** 0 **(D)** $\frac{2}{3}$ **(E)** $\frac{4}{3}$

20. A cube has six faces. Each face has some dots on it. The numbers of dots on the six faces are 2, 3, 4, 5, 6, and 7. Harry removes one of the dots at random, with each dot equally likely to be removed. When the cube is rolled, each face is equally likely to be the top face. What is the probability that the top face has an odd number of dots on it?

(A) $\frac{4}{7}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{13}{27}$ **(D)** $\frac{11}{21}$ **(E)** $\frac{3}{7}$

Part C: Each correct answer is worth 8.

21. In the diagram, the central circle contains the number 36. Positive integers are to be written in the eight empty circles, one number in each circle, so that the product of the three integers along any straight line is 2592. If the nine integers in the circles must be all different, what is the largest possible sum of these nine integers?



(A) 160 **(B)** 176 **(C)** 178
(D) 195 **(E)** 216

22. Suppose that x and y are real numbers that satisfy the two equations:

$$x^2 + 3xy + y^2 = 909$$
$$3x^2 + xy + 3y^2 = 1287$$

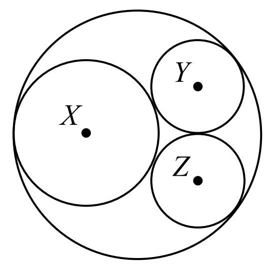
What is a possible value for $x + y$?

(A) 27 **(B)** 39 **(C)** 29 **(D)** 92 **(E)** 41

23. There are real numbers a and b for which the function f has the properties that $f(x) = ax + b$ for all real numbers x, and $f(bx + a) = x$ for all real numbers x. What is the value of $a + b$?

(A) 2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** -2

24. In the diagram, the circle with centre X is tangent to the largest circle and passes through the centre of the largest circle. The circles with centres Y and Z are each tangent to the other three circles, as shown. The circle with centre X has radius 1. The circles with centres Y and Z each have radius r. The value of r is closest to



(A) 0.93 **(B)** 0.91 **(C)** 0.95

(D) 0.87 **(E)** 0.89

25. Three real numbers x, y, z are chosen randomly, and independently of each other, between 0 and 1, inclusive. What is the probability that each of $x - y$ and $x - z$ is greater than $-\frac{1}{2}$ and less than $\frac{1}{2}$?

(A) $\frac{3}{4}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

2020
Fermat
Contest
(English)

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in MATHEMATICS and COMPUTING
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For students...

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The CENTRE for EDUCATION
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Fermat Contest

(Grade 11)

Tuesday, February 26, 2019
(in North America and South America)

Wednesday, February 27, 2019
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the remainder when 14 is divided by 5?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

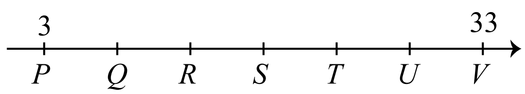
2. Which of the following is equal to $20(x+y)-19(y+x)$ for all values of x and y?

   (A) $39x+39y$ **(B)** $x+y$ **(C)** $39x+y$ **(D)** $x+39y$ **(E)** $19x-18y$

3. The value of $8-\dfrac{6}{4-2}$ is

   (A) 5 **(B)** 1 **(C)** $\dfrac{7}{2}$ **(D)** $\dfrac{17}{2}$ **(E)** 7

4. In the diagram, point P is on the number line at 3 and V is at 33. The number line between 3 and 33 is divided into six equal parts by the points Q, R, S, T, U.



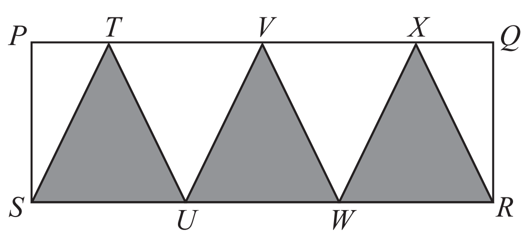
   What is the sum of the lengths of PS and TV?

   (A) 25 **(B)** 23 **(C)** 24 **(D)** 21 **(E)** 27

5. Mike rides his bicycle at a constant speed of 30 km/h. How many kilometres does Mike travel in 20 minutes?

   (A) 5 **(B)** 6 **(C)** 1.5 **(D)** 15 **(E)** 10

6. In the diagram, $PQRS$ is a rectangle. Also, $\triangle STU$, $\triangle UVW$ and $\triangle WXR$ are congruent.



   What fraction of the area of rectangle $PQRS$ is shaded?

   (A) $\frac{3}{7}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

7. The town of Cans is north of the town of Ernie. The town of Dundee is south of Cans but north of Ernie. The town of Arva is south of the town of Blythe and is north of both Dundee and Cans. The town that is the most north is

   (A) Arva **(B)** Blythe **(C)** Cans **(D)** Dundee **(E)** Ernie

8. The product $8 \times 48 \times 81$ is divisible by 6^k. The largest possible integer value of k is

(A) 7 **(B)** 6 **(C)** 5 **(D)** 4 **(E)** 3

9. The average of $\frac{1}{8}$ and $\frac{1}{6}$ is

(A) $\frac{1}{9}$ **(B)** $\frac{7}{24}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{48}$ **(E)** $\frac{7}{48}$

10. The digits 2, 3, 5, 7, and 8 can be used, each exactly once, to form many five-digit integers. Of these integers, N is the one that is as close as possible to 30 000. What is the tens digit of N?

(A) 2 **(B)** 5 **(C)** 3 **(D)** 8 **(E)** 7

Part B: Each correct answer is worth 6.

11. Line ℓ is perpendicular to the line with equation $y = x - 3$. Line ℓ has the same x-intercept as the line with equation $y = x - 3$. The y-intercept of line ℓ is

(A) -3 **(B)** $\frac{1}{3}$ **(C)** 3 **(D)** -1 **(E)** 0

12. The first part of the Genius Quiz has 30 questions and the second part has 50 questions. Alberto answered exactly 70% of the 30 questions in the first part correctly. He answered exactly 40% of the 50 questions in the second part correctly. The percentage of all of the questions on the quiz that Alberto answered correctly is closest to

(A) 59 **(B)** 57 **(C)** 48 **(D)** 51 **(E)** 41

13. Tanis looked at her watch and noticed that, at that moment, it was $8x$ minutes after 7:00 a.m. and $7x$ minutes before 8:00 a.m. for some value of x. What time was it at that moment?

(A) 7:08 a.m. **(B)** 7:40 a.m. **(C)** 7:32 a.m. **(D)** 7:36 a.m. **(E)** 7:31 a.m.

14. The letters A, B, C, D, and E are to be placed in the grid so that each of these letters appears exactly once in each row and exactly once in each column. Which letter will go in the square marked with $*$?

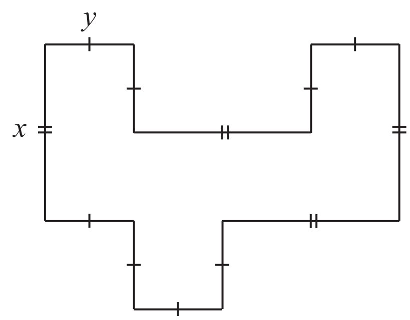
A				E
		C	A	
E		B	C	
	$*$			
B			D	

(A) A **(B)** B **(C)** C
(D) D **(E)** E

15. There are six identical red balls and three identical green balls in a pail. Four of these balls are selected at random and then these four balls are arranged in a line in some order. How many different-looking arrangements are possible?

(A) 15 **(B)** 16 **(C)** 10 **(D)** 11 **(E)** 12

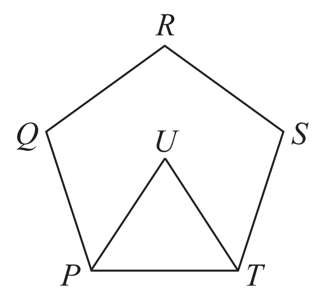
16. In the diagram, each line segment has length x or y. Also, each pair of adjacent sides is perpendicular.



If the area of the figure is 252 and $x = 2y$, the perimeter of the figure is

(A) 96 **(B)** 192 **(C)** 288 **(D)** 72 **(E)** 168

17. The five sides of a regular pentagon are all equal in length. Also, all interior angles of a regular pentagon have the same measure. In the diagram, $PQRST$ is a regular pentagon and $\triangle PUT$ is equilateral. The measure of obtuse $\angle QUS$ is



(A) 172° **(B)** 168° **(C)** 170°
(D) 176° **(E)** 174°

18. How many 7-digit positive integers are made up of the digits 0 and 1 only, and are divisible by 6?

(A) 16 **(B)** 11 **(C)** 21 **(D)** 10 **(E)** 33

19. The function f has the properties that $f(1) = 6$ and $f(2x + 1) = 3f(x)$ for every integer x. What is the value of $f(63)$?

(A) 4374 **(B)** 1162 **(C)** 54 **(D)** 1458 **(E)** 486

20. The vertices of an equilateral triangle lie on a circle with radius 2. The area of the triangle is

(A) $3\sqrt{3}$ **(B)** $4\sqrt{3}$ **(C)** $6\sqrt{3}$ **(D)** $5\sqrt{3}$ **(E)** $2\sqrt{3}$

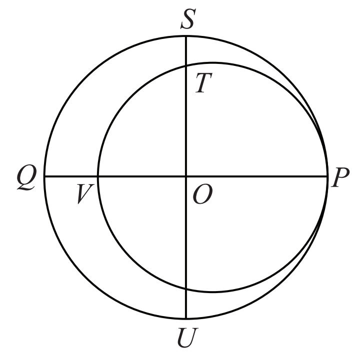
Part C: Each correct answer is worth 8.

21. In the multiplication shown, each of P, Q, R, S, and T is a digit. The value of $P + Q + R + S + T$ is

$$\begin{array}{rcccccc} & & P & Q & R & S & T\ 4 \\ \times & & & & & & 4 \\ \hline & 4 & P & Q & R & S & T \end{array}$$

(A) 14 **(B)** 20 **(C)** 16
(D) 17 **(E)** 13

22. In the diagram, two circles touch at P. Also, QP and SU are perpendicular diameters of the larger circle that intersect at O. Point V is on QP and VP is a diameter of the smaller circle. The smaller circle intersects SU at T, as shown. If $QV = 9$ and $ST = 5$, what is the sum of the lengths of the diameters of the two circles?



(A) 50 **(B)** 91 **(C)** 41 **(D)** 82 **(E)** 100

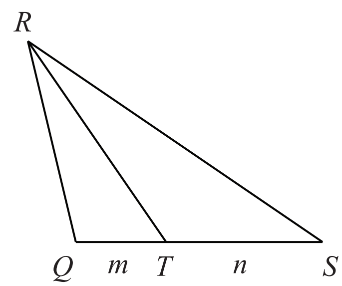
23. How many positive integers n with $n \leq 100$ can be expressed as the sum of four or more consecutive positive integers?

(A) 64 **(B)** 63 **(C)** 66 **(D)** 65 **(E)** 69

24. Consider the quadratic equation $x^2 - (r+7)x + r + 87 = 0$ where r is a real number. This equation has two distinct real solutions x which are both negative exactly when $p < r < q$, for some real numbers p and q. The value of $p^2 + q^2$ is

(A) 7618 **(B)** 698 **(C)** 1738 **(D)** 7508 **(E)** 8098

25. In $\triangle QRS$, point T is on QS with $\angle QRT = \angle SRT$. Suppose that $QT = m$ and $TS = n$ for some integers m and n with $n > m$ and for which $n + m$ is a multiple of $n - m$. Suppose also that the perimeter of $\triangle QRS$ is p and that the number of possible integer values for p is $m^2 + 2m - 1$. The value of $n - m$ is



(A) 4 **(B)** 1 **(C)** 3 **(D)** 2 **(E)** 5

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The CENTRE for EDUCATION
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Fermat Contest

(Grade 11)

Tuesday, February 27, 2018
(in North America and South America)

Wednesday, February 28, 2018
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2016 - 2017 + 2018 - 2019 + 2020$ is

   (A) 2012 **(B)** 2014 **(C)** 2016 **(D)** 2018 **(E)** 2020

2. On Monday, the minimum temperature in Fermatville was $-11°$C and the maximum temperature was $14°$C. What was the range of temperatures on Monday in Fermatville?

   (A) 3°C **(B)** 25°C **(C)** 14°C **(D)** 11°C **(E)** 23°C

3. If $x = -2$ and $y = -1$, the value of $(3x + 2y) - (3x - 2y)$ is

   (A) -4 **(B)** 12 **(C)** 0 **(D)** 4 **(E)** 8

4. How many integers are greater than $\frac{5}{7}$ and less than $\frac{28}{3}$?

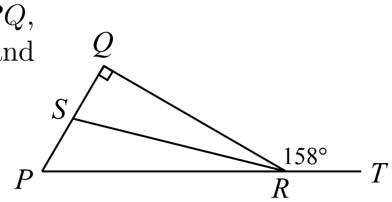
   (A) 1 **(B)** 9 **(C)** 5 **(D)** 7 **(E)** 3

5. The symbols $\heartsuit$ and ∇ represent different positive integers less than 20. If $\heartsuit \times \heartsuit \times \heartsuit = \nabla$, what is the value of $\nabla \times \nabla$?

   (A) 12 **(B)** 16 **(C)** 36 **(D)** 64 **(E)** 81

6. In the diagram, points R and S lie on PT and PQ, respectively. If $\angle PQR = 90°$, $\angle QRT = 158°$, and $\angle PRS = \angle QRS$, what is the measure of $\angle QSR$?

   (A) 34° **(B)** 22° **(C)** 68° **(D)** 11° **(E)** 79°



7. Bev is driving from Waterloo, ON to Marathon, ON. She has driven 312 km. She has 858 km still to drive. How much farther must she drive in order to be halfway from Waterloo to Marathon?

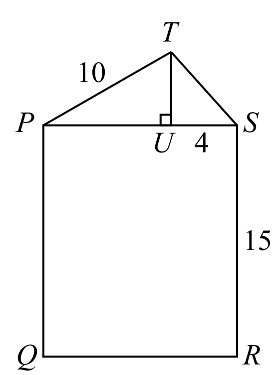
   (A) 585 km **(B)** 273 km **(C)** 312 km **(D)** 429 km **(E)** 196.5 km

8. For what value of k is the line through the points $(3, 2k + 1)$ and $(8, 4k - 5)$ parallel to the x-axis?

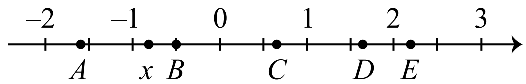
   (A) 3 **(B)** -4 **(C)** 2 **(D)** 0 **(E)** -1

9. In the diagram, $PQRS$ is a rectangle with $SR = 15$. Point T is above PS and point U is on PS so that TU is perpendicular to PS. If $PT = 10$ and $US = 4$ and the area of $PQRS$ is 180, what is the area of $\triangle PTS$?

   (A) 60 **(B)** 36 **(C)** 48 **(D)** 24 **(E)** 12



10. In the diagram, the number line between -2 and 3 is divided into 10 equal parts. The integers -1, 0, 1, 2 are marked on the line as are the numbers A, x, B, C, D, E. Which number best approximates the value of x^2?



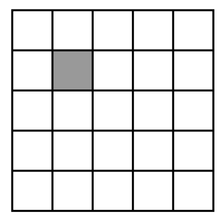
(A) A **(B)** B **(C)** C **(D)** D **(E)** E

Part B: Each correct answer is worth 6.

11. A bag contains 8 red balls, a number of white balls, and no other balls. If $\frac{5}{6}$ of the balls in the bag are white, then the number of white balls in the bag is

(A) 48 **(B)** 20 **(C)** 40 **(D)** 32 **(E)** 30

12. In the given 5×5 grid, many squares can be formed using the grid lines. How many of these squares contain the shaded 1×1 square?



(A) 15 **(B)** 16 **(C)** 11
(D) 12 **(E)** 14

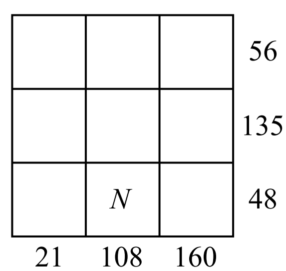
13. A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(A) 458 **(B)** 587 **(C)** 376 **(D)** 315 **(E)** 518

14. The line with equation $y = x$ is translated 3 units to the right and 2 units down. What is the y-intercept of the resulting line?

(A) -1 **(B)** -2 **(C)** -5 **(D)** 3 **(E)** 4

15. Francesca put the integers $1, 2, 3, 4, 5, 6, 7, 8, 9$ in the nine squares in the grid. She put one integer in each square and used no integer twice. She calculated the product of the three integers in each row and wrote the products to the right of the corresponding rows. She calculated the product of the integers in each column and wrote the products below the corresponding columns. Finally, she erased the integers from the nine squares. Which integer was in the square marked N?

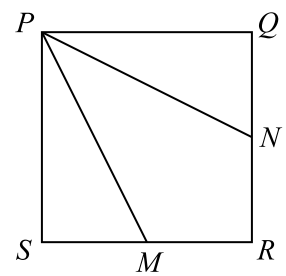


(A) 3 **(B)** 8 **(C)** 9
(D) 6 **(E)** 4

16. Points P and Q are two distinct points in the xy-plane. In how many different places in the xy-plane can a third point, R, be placed so that $PQ = QR = PR$?

(A) 6 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

17. In the diagram, square $PQRS$ has side length 2. Points M and N are the midpoints of SR and RQ, respectively. The value of $\cos(\angle MPN)$ is



(A) $\frac{4}{5}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{\sqrt{5}}{3}$
(D) $\frac{1}{3}$ **(E)** $\frac{\sqrt{3}}{2}$

18. Suppose that m and n are positive integers with $\sqrt{7+\sqrt{48}} = m + \sqrt{n}$. The value of $m^2 + n^2$ is

(A) 37 **(B)** 25 **(C)** 58 **(D)** 29 **(E)** 13

19. Radford and Peter ran a race, during which they both ran at a constant speed. Radford began the race 30 m ahead of Peter. After 3 minutes, Peter was 18 m ahead of Radford. Peter won the race exactly 7 minutes after it began. How far from the finish line was Radford when Peter won?

(A) 16 m **(B)** 64 m **(C)** 48 m **(D)** 82 m **(E)** 84 m

20. For how many positive integers x is $(x-2)(x-4)(x-6)\cdots(x-2016)(x-2018) \leq 0$? (The product on the left side of the inequality consists of 1009 factors of the form $x - 2k$ for integers k with $1 \leq k \leq 1009$.)

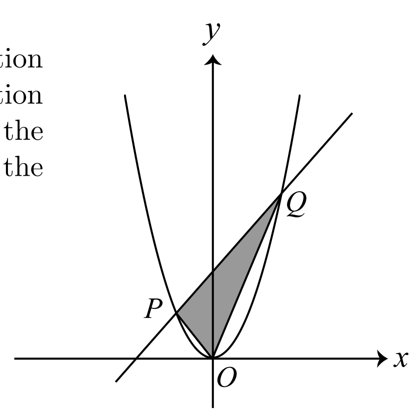
(A) 1009 **(B)** 1010 **(C)** 1514 **(D)** 1515 **(E)** 1513

Part C: Each correct answer is worth 8.

21. A sequence has terms $a_1, a_2, a_3, \ldots$. The first term is $a_1 = x$ and the third term is $a_3 = y$. The terms of the sequence have the property that every term after the first term is equal to 1 less than the sum of the terms immediately before and after it. That is, when $n \geq 1$, $a_{n+1} = a_n + a_{n+2} - 1$. The sum of the first 2018 terms in the sequence is

(A) $-x - 2y + 2023$ **(B)** $3x - 2y + 2017$ **(C)** y
(D) $x + y - 1$ **(E)** $2x + y + 2015$

22. Suppose that $k > 0$ and that the line with equation $y = 3kx + 4k^2$ intersects the parabola with equation $y = x^2$ at points P and Q, as shown. If O is the origin and the area of $\triangle OPQ$ is 80, then the slope of the line is



(A) 4 **(B)** 3 **(C)** $\frac{15}{4}$
(D) 6 **(E)** $\frac{21}{4}$

23. Suppose that a, b and c are integers with $(x-a)(x-6)+3 = (x+b)(x+c)$ for all real numbers x. The sum of all possible values of b is

(A) -12 **(B)** -24 **(C)** -14 **(D)** -8 **(E)** -16

24. Wayne has 3 green buckets, 3 red buckets, 3 blue buckets, and 3 yellow buckets. He randomly distributes 4 hockey pucks among the green buckets, with each puck equally likely to be put in each bucket. Similarly, he distributes 3 pucks among the red buckets, 2 pucks among the blue buckets, and 1 puck among the yellow buckets. Once he is finished, what is the probability that a green bucket contains more pucks than each of the other 11 buckets?

(A) $\frac{97}{243}$ **(B)** $\frac{89}{243}$ **(C)** $\frac{93}{243}$ **(D)** $\frac{95}{243}$ **(E)** $\frac{91}{243}$

25. For each positive digit D and positive integer k, we use the symbol $D_{(k)}$ to represent the positive integer having exactly k digits, each of which is equal to D. For example, $2_{(1)} = 2$ and $3_{(4)} = 3333$. There are N quadruples (P, Q, R, k) with P, Q and R positive digits, k a positive integer with $k \leq 2018$, and $P_{(2k)} - Q_{(k)} = \left(R_{(k)}\right)^2$. The sum of the digits of N is

(A) 10 **(B)** 9 **(C)** 11 **(D)** 12 **(E)** 13

2018 Fermat Contest (English)

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The CENTRE for EDUCATION
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Fermat Contest

(Grade 11)

Tuesday, February 28, 2017
(in North America and South America)

Wednesday, March 1, 2017
(outside of North America and South America)

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

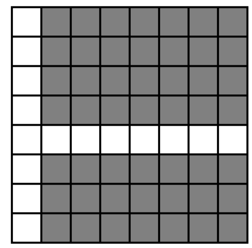
Part A: Each correct answer is worth 5.

1. The value of $6 \times 2017 - 2017 \times 4$ is

   (A) 2 **(B)** 20170 **(C)** 0 **(D)** 4034 **(E)** 24

2. In the diagram, how many 1×1 squares are shaded in the 8×8 grid?

   (A) 53 **(B)** 51 **(C)** 47
   (D) 45 **(E)** 49

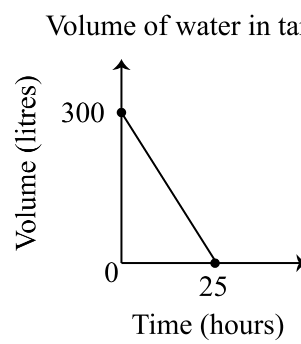


3. Three different numbers from the list 2, 3, 4, 6 have a sum of 11. What is the product of these numbers?

   (A) 24 **(B)** 72 **(C)** 36 **(D)** 48 **(E)** 32

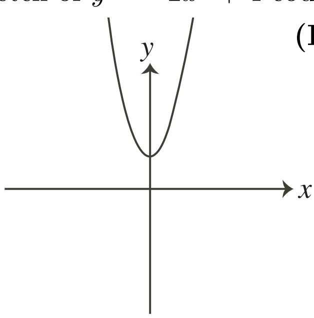
4. The graph shows the volume of water in a 300 L tank as it is being drained at a constant rate. At what rate is the water leaving the tank, in litres per hour?

   (A) 12 **(B)** 20 **(C)** 2.5
   (D) 5 **(E)** 15

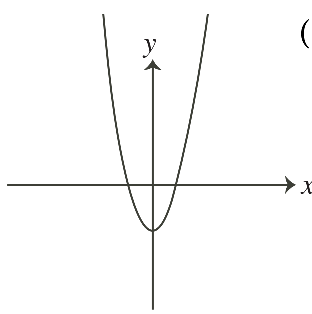


5. A sketch of $y = -2x^2 + 4$ could be

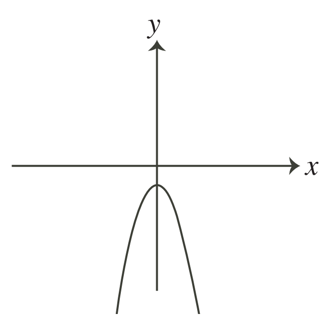
(A)



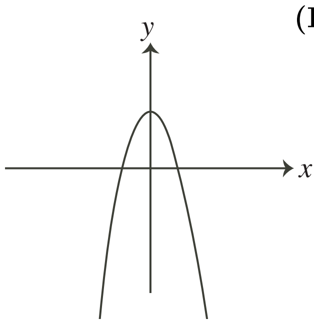
(B)



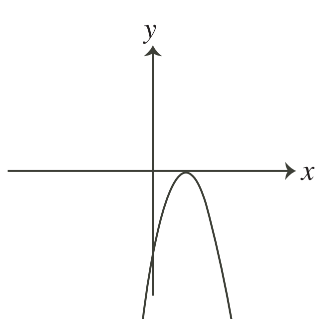
(C)



(D)



(E)



6. Emilia writes down the numbers 5, x and 9. Valentin calculates the mean (average) of each pair of these numbers and obtains 7, 10 and 12. The value of x is

(A) 5 **(B)** 15 **(C)** 3 **(D)** 25 **(E)** 1

7. If $x = 1$ is a solution of the equation $x^2 + ax + 1 = 0$, then the value of a is

(A) 3 **(B)** -1 **(C)** 1 **(D)** 2 **(E)** -2

8. If $\frac{1}{2n} + \frac{1}{4n} = \frac{3}{12}$, then n equals

(A) 6 **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** 2 **(E)** 3

9. Kamile turned her computer off at 5 p.m. Friday, at which point it had been on for exactly 100 hours. At what time had Kamile turned her computer on?

(A) 1 p.m. Tuesday **(B)** 9 p.m. Monday **(C)** 2 p.m. Tuesday
(D) 1 p.m. Monday **(E)** 9 p.m. Wednesday

10. The sum of four different positive integers is 100. The largest of these four integers is n. The smallest possible value of n is

(A) 26 **(B)** 50 **(C)** 28 **(D)** 27 **(E)** 94

Part B: Each correct answer is worth 6.

11. Last Thursday, each of the students in M. Fermat's class brought one piece of fruit to school. Each brought an apple, a banana, or an orange. In total, 20% of the students brought an apple and 35% brought a banana. If 9 students brought oranges, how many students were in the class?

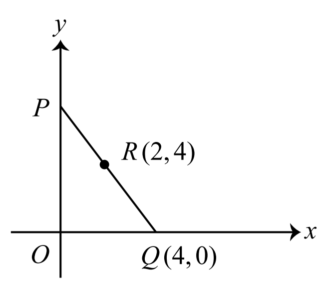
(A) 18 **(B)** 64 **(C)** 24 **(D)** 20 **(E)** 40

12. Digits are placed in the two boxes of $2\square\square$, with one digit in each box, to create a three-digit positive integer. In how many ways can this be done so that the three-digit positive integer is larger than 217?

(A) 81 **(B)** 82 **(C)** 83 **(D)** 92 **(E)** 93

13. In the diagram, P lies on the y-axis, Q has coordinates $(4, 0)$, and PQ passes through the point $R(2, 4)$. What is the area of $\triangle OPQ$?

(A) 8 **(B)** 12 **(C)** 32
(D) 24 **(E)** 16



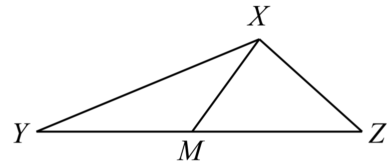
14. The expression

$$\left(1+\tfrac{1}{2}\right)\left(1+\tfrac{1}{3}\right)\left(1+\tfrac{1}{4}\right)\left(1+\tfrac{1}{5}\right)\left(1+\tfrac{1}{6}\right)\left(1+\tfrac{1}{7}\right)\left(1+\tfrac{1}{8}\right)\left(1+\tfrac{1}{9}\right)$$

is equal to

(A) 5 **(B)** $\frac{10}{9}$ **(C)** 9 **(D)** $9\frac{1}{8}$ **(E)** $\frac{1}{2}$

15. In the diagram, M is the midpoint of YZ, $\angle XMZ = 30°$, and $\angle XYZ = 15°$. The measure of $\angle XZY$ is



(A) 75° **(B)** 65° **(C)** 60°

(D) 80° **(E)** 85°

16. If $x + 2y = 30$, the value of $\frac{x}{5} + \frac{2y}{3} + \frac{2y}{5} + \frac{x}{3}$ is

(A) 8 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 30

17. Aaron has 144 identical cubes, each with edge length 1 cm. He uses all of the cubes to construct a solid rectangular prism, which he places on a flat table. If the perimeter of the base of the prism is 20 cm, what is the sum of all possible heights of the prism?

(A) 31 cm **(B)** 25 cm **(C)** 15 cm **(D)** 22 cm **(E)** 16 cm

18. For any positive real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. For example, $\lfloor 4.2 \rfloor = 4$ and $\lfloor 3 \rfloor = 3$. If $\lfloor x \rfloor \cdot x = 36$ and $\lfloor y \rfloor \cdot y = 71$ where $x, y > 0$, then $x + y$ equals

(A) $\frac{107}{8}$ **(B)** $\frac{119}{8}$ **(C)** $\frac{125}{9}$ **(D)** $\frac{107}{6}$ **(E)** $\frac{101}{7}$

19. A point is *equidistant from the coordinate axes* if the vertical distance from the point to the x-axis is equal to the horizontal distance from the point to the y-axis. The point of intersection of the vertical line $x = a$ with the line with equation $3x + 8y = 24$ is equidistant from the coordinate axes. What is the sum of all possible values of a?

(A) 0 **(B)** $-\frac{144}{55}$ **(C)** $-\frac{11}{5}$ **(D)** $\frac{24}{11}$ **(E)** 8

20. If m and n are positive integers with $n > 1$ such that $m^n = 2^{25} \times 3^{40}$, then $m + n$ is

(A) 209 962 **(B)** 1954 **(C)** 209 957 **(D)** 6598 **(E)** 1 049 760

Part C: Each correct answer is worth 8.

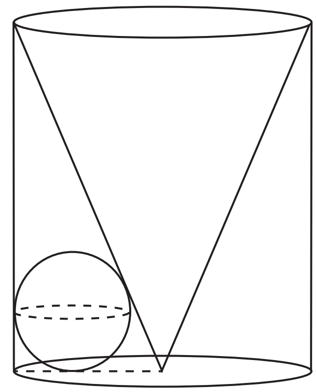
21. In the sum shown, each letter represents a different digit with $T \neq 0$ and $W \neq 0$. How many different values of U are possible?

$$\begin{array}{cccccc} & & W & X & Y & Z \\ + & & W & X & Y & Z \\ \hline & T & W & U & Y & V \end{array}$$

(A) 1 **(B)** 2 **(C)** 3

(D) 4 **(E)** 5

22. A cylinder has radius 12 and height 30. The top circular face of the cylinder is the base of a cone and the centre of the bottom circular base of the cylinder is the vertex of the cone. A sphere is placed inside so that it touches the cone, the base of the cylinder and the side of the cylinder as shown. Which of the following is closest to the radius of the sphere?



(A) 4.84 **(B)** 4.74 **(C)** 4.64 **(D)** 4.54 **(E)** 4.44

23. Sylvia chose positive integers a, b and c.

Peter determined the value of $a + \frac{b}{c}$ and got an answer of 101.

Paul determined the value of $\frac{a}{c} + b$ and got an answer of 68.

Mary determined the value of $\dfrac{a+b}{c}$ and got an answer of k.

The value of k is

(A) 13 **(B)** 168 **(C)** 152 **(D)** 12 **(E)** 169

24. Eight teams compete in a tournament. Each pair of teams plays exactly one game against each other. There are no ties. If the two possible outcomes of each game are equally likely, what is the probability that every team loses at least one game and wins at least one game?

(A) $\frac{1799}{2048}$ **(B)** $\frac{1831}{2048}$ **(C)** $\frac{1793}{2048}$ **(D)** $\frac{903}{1024}$ **(E)** $\frac{889}{1024}$

25. Let $r = \sqrt{\frac{\sqrt{53}}{2} + \frac{3}{2}}$. There is a unique triple of positive integers (a, b, c) such that

$$r^{100} = 2r^{98} + 14r^{96} + 11r^{94} - r^{50} + ar^{46} + br^{44} + cr^{40}$$

What is the value of $a^2 + b^2 + c^2$?

(A) 11 421 **(B)** 20 229 **(C)** 16 291 **(D)** 15 339 **(E)** 17 115

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The CENTRE for EDUCATION
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Fermat Contest

(Grade 11)

Wednesday, February 24, 2016
(in North America and South America)

Thursday, February 25, 2016
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

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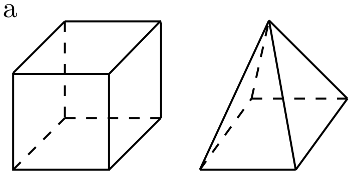
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. If $x = 3$, $y = 2x$ and $z = 3y$, the value of z is

(A) 8 **(B)** 9 **(C)** 6 **(D)** 18 **(E)** 15

2. A cube has 12 edges, as shown. How many edges does a square-based pyramid have?

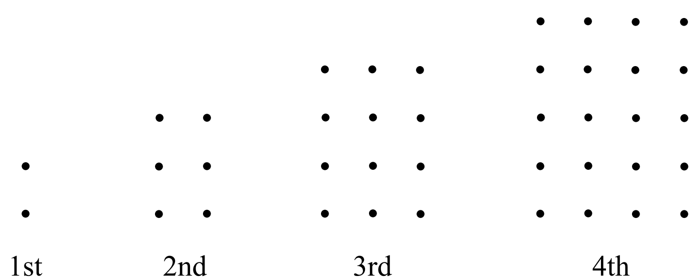


(A) 6 **(B)** 12 **(C)** 8
(D) 4 **(E)** 10

3. The expression $\dfrac{20 + 16 \times 20}{20 \times 16}$ equals

(A) 20 **(B)** 276 **(C)** 21 **(D)** $\dfrac{9}{4}$ **(E)** $\dfrac{17}{16}$

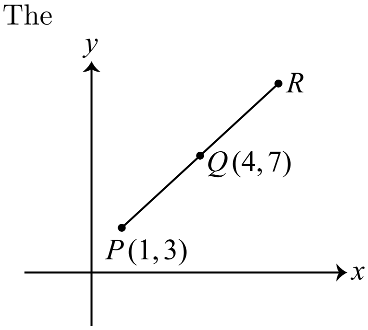
4. An *oblong number* is the number of dots in a rectangular grid with one more row than column. The first four oblong numbers are 2, 6, 12, and 20, and are represented below:



What is the 7th oblong number?

(A) 42 **(B)** 49 **(C)** 56 **(D)** 64 **(E)** 72

5. In the diagram, point Q is the midpoint of PR. The coordinates of R are



(A) $(2, 5)$ **(B)** $(7, 11)$ **(C)** $(6, 9)$
(D) $(8, 10)$ **(E)** $(9, 15)$

6. Carrie sends five text messages to her brother each Saturday and five text messages to her brother each Sunday. Carrie sends two text messages to her brother on each of the other days of the week. Over the course of four full weeks, how many text messages does Carrie send to her brother?

(A) 15 **(B)** 28 **(C)** 60 **(D)** 80 **(E)** 100

7. The value of $(-2)^3 - (-3)^2$ is

(A) -17 **(B)** 1 **(C)** -12 **(D)** 0 **(E)** -1

8. If $\sqrt{25-\sqrt{n}}=3$, the value of n is

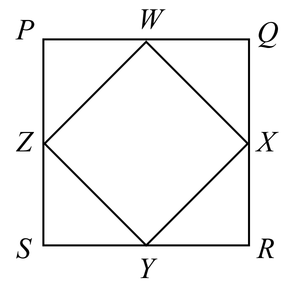
(A) 4 **(B)** 16 **(C)** 64 **(D)** 484 **(E)** 256

9. If $x\%$ of 60 is 12, then 15% of x is

(A) $\frac{3}{4}$ **(B)** $\frac{1}{3}$ **(C)** 4 **(D)** 3 **(E)** 9

10. In the diagram, square $PQRS$ has side length 2. Points W, X, Y, and Z are the midpoints of the sides of $PQRS$. What is the ratio of the area of square $WXYZ$ to the area of square $PQRS$?

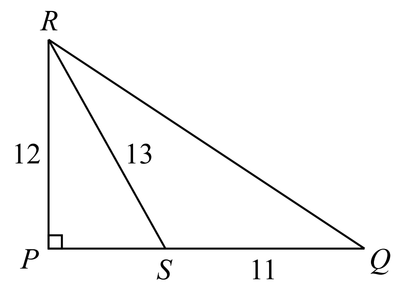
(A) $1:2$ **(B)** $2:1$ **(C)** $1:3$
(D) $1:4$ **(E)** $\sqrt{2}:2$



Part B: Each correct answer is worth 6.

11. In the diagram, $\triangle PQR$ is right-angled at P and $PR=12$. If point S is on PQ so that $SQ=11$ and $SR=13$, the perimeter of $\triangle QRS$ is

(A) 47 **(B)** 44 **(C)** 30
(D) 41 **(E)** 61



12. How many of the positive divisors of 128 are perfect squares larger than 1?

(A) 2 **(B)** 5 **(C)** 1 **(D)** 3 **(E)** 4

13. The numbers $4x, 2x-3, 4x-3$ are three consecutive terms in an arithmetic sequence. What is the value of x?

(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, $3, 5, 7, 9$ are the first four terms of an arithmetic sequence.)

(A) $\frac{3}{4}$ **(B)** $-\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $-\frac{3}{2}$ **(E)** $-\frac{3}{4}$

14. Suppose that a and b are integers with $4<a<b<22$. If the average (mean) of the numbers $4, a, b, 22$ is 13, then the number of possible pairs (a, b) is

(A) 10 **(B)** 8 **(C)** 7 **(D)** 9 **(E)** 6

15. Hicham runs 16 km in 1.5 hours. He runs the first 10 km at an average speed of 12 km/h. What is his average speed for the last 6 km?

(A) 8 km/h **(B)** 9 km/h **(C)** 10 km/h **(D)** 6 km/h **(E)** 12 km/h

16. If $x = 18$ is one of the solutions of the equation $x^2 + 12x + c = 0$, the other solution of this equation is

(A) $x = 216$ **(B)** $x = -6$ **(C)** $x = -30$ **(D)** $x = 30$ **(E)** $x = -540$

17. A total of n points are equally spaced around a circle and are labelled with the integers 1 to n, in order. Two points are called *diametrically opposite* if the line segment joining them is a diameter of the circle. If the points labelled 7 and 35 are diametrically opposite, then n equals

(A) 54 **(B)** 55 **(C)** 56 **(D)** 57 **(E)** 58

18. Suppose that x and y satisfy $\dfrac{x-y}{x+y} = 9$ and $\dfrac{xy}{x+y} = -60$.

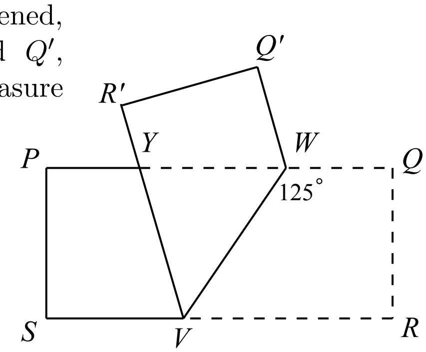
The value of $(x+y) + (x-y) + xy$ is

(A) 210 **(B)** -150 **(C)** 14160 **(D)** -14310 **(E)** -50

19. There are n students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the n students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the n students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the n students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to $n^2 - n$ is

(A) 11 **(B)** 12 **(C)** 20 **(D)** 13 **(E)** 10

20. In the diagram, $PQRS$ represents a rectangular piece of paper. The paper is folded along a line VW so that $\angle VWQ = 125°$. When the folded paper is flattened, points R and Q have moved to points R' and Q', respectively, and $R'V$ crosses PW at Y. The measure of $\angle PYV$ is



(A) 110° **(B)** 100° **(C)** 95°
(D) 105° **(E)** 115°

Part C: Each correct answer is worth 8.

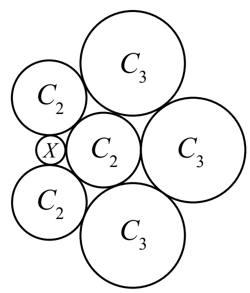
21. Box 1 contains one gold marble and one black marble. Box 2 contains one gold marble and two black marbles. Box 3 contains one gold marble and three black marbles. Whenever a marble is chosen randomly from one of the boxes, each marble in that box is equally likely to be chosen. A marble is randomly chosen from Box 1 and placed in Box 2. Then a marble is randomly chosen from Box 2 and placed in Box 3. Finally, a marble is randomly chosen from Box 3. What is the probability that the marble chosen from Box 3 is gold?

    (A) $\frac{11}{40}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{13}{40}$ **(D)** $\frac{7}{20}$ **(E)** $\frac{3}{8}$

22. If x and y are real numbers, the minimum possible value of the expression $(x+3)^2 + 2(y-2)^2 + 4(x-7)^2 + (y+4)^2$ is

    (A) 172 **(B)** 65 **(C)** 136 **(D)** 152 **(E)** 104

23. Seven coins of three different sizes are placed flat on a table, arranged as shown in the diagram. Each coin, except the centre one, touches three other coins. The centre coin touches all of the other coins. If the coins labelled C_3 have a radius of 3 cm, and those labelled C_2 have radius 2 cm, then the radius of the coin labelled X is closest to



    (A) 0.615 cm **(B)** 0.620 cm **(C)** 0.610 cm
    (D) 0.605 cm **(E)** 0.625 cm

24. For any real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. For example, $\lfloor 4.2 \rfloor = 4$ and $\lfloor 0.9 \rfloor = 0$.
    If S is the sum of all integers k with $1 \le k \le 999\,999$ and for which k is divisible by $\lfloor \sqrt{k} \rfloor$, then S equals

    (A) 999 500 000 **(B)** 999 000 000 **(C)** 999 999 000
    (D) 998 999 500 **(E)** 998 500 500

25. The set $A = \{1, 2, 3, \ldots, 2044, 2045\}$ contains 2045 elements. A subset S of A is called *triple-free* if no element of S equals three times another element of S. For example, $\{1, 2, 4, 5, 10, 2043\}$ is triple-free, but $\{1, 2, 4, 5, 10, 681, 2043\}$ is not triple-free. The triple-free subsets of A that contain the largest number of elements contain exactly 1535 elements. There are n triple-free subsets of A that contain exactly 1535 elements. The integer n can be written in the form $p^a q^b$, where p and q are distinct prime numbers and a and b are positive integers. If $N = p^2 + q^2 + a^2 + b^2$, then the last three digits of N are

    (A) 202 **(B)** 102 **(C)** 302 **(D)** 402 **(E)** 502

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Fermat Contest

(Grade 11)

Tuesday, February 24, 2015
(in North America and South America)

Wednesday, February 25, 2015
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

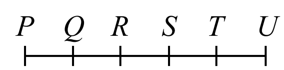
1. The average (mean) of the five numbers 8, 9, 10, 11, 12 is

   (A) 12.5 **(B)** 8 **(C)** 9.6 **(D)** 9 **(E)** 10

2. The value of $\dfrac{2 \times 3 + 4}{2 + 3}$ is

   (A) 2 **(B)** 5 **(C)** 8 **(D)** 4 **(E)** 11

3. Six points P, Q, R, S, T, U are equally spaced along a straight path. Emily walks from P to U and then back to P. At which point has she completed 70% of her walk?

   (A) T **(B)** Q **(C)** R
   (D) S **(E)** U

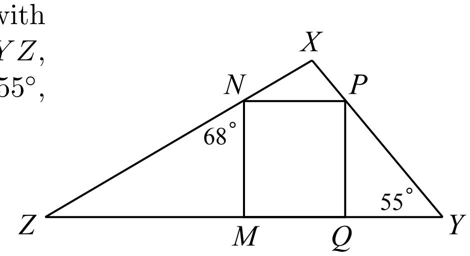
4. If $x = -3$, then $(x - 3)^2$ equals

   (A) 12 **(B)** 36 **(C)** -12 **(D)** 0 **(E)** -36

5. The points $P(3, -2)$, $Q(3, 1)$, $R(7, 1)$, and S form a rectangle. What are the coordinates of S?

   (A) $(-1, -2)$ **(B)** $(7, -2)$ **(C)** $(7, 4)$ **(D)** $(3, 7)$ **(E)** $(1, -2)$

6. In the diagram, $MNPQ$ is a rectangle with points M, N, P, and Q on the sides of $\triangle XYZ$, as shown. If $\angle ZNM = 68^\circ$ and $\angle XYZ = 55^\circ$, what is the measure of $\angle YXZ$?

   (A) 77° **(B)** 113° **(C)** 93°
   (D) 97° **(E)** 103°

7. Violet has one-half of the money she needs to buy her mother a necklace. After her sister gives her \$30, she has three-quarters of the amount she needs. Violet's father agrees to give her the rest. The amount that Violet's father will give her is

   (A) \$7.50 **(B)** \$15 **(C)** \$22.50 **(D)** \$30 **(E)** \$120

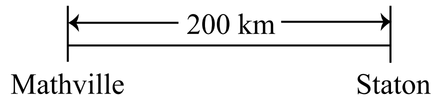
8. If x and y are positive integers with $3^x 5^y = 225$, then $x + y$ equals

   (A) 7 **(B)** 4 **(C)** 5 **(D)** 3 **(E)** 8

9. At Barker High School, a total of 36 students are on either the baseball team, the hockey team, or both. If there are 25 students on the baseball team and 19 students on the hockey team, how many students play both sports?

   (A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

10. Anca and Bruce left Mathville at the same time. They drove along a straight highway towards Staton. Bruce drove at 50 km/h. Anca drove at 60 km/h, but stopped along the way to rest. They both arrived at Staton at the same time. For how long did Anca stop to rest?



(A) 40 minutes **(B)** 10 minutes **(C)** 67 minutes
(D) 33 minutes **(E)** 27 minutes

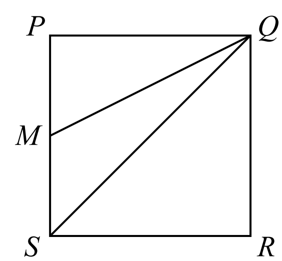
Part B: Each correct answer is worth 6.

11. Three-digit positive integers such as 789 and 998 use no digits other than 7, 8 and 9. In total, how many three-digit positive integers use no digits other than 7, 8 and 9?
(A) 36 **(B)** 6 **(C)** 9 **(D)** 18 **(E)** 27

12. If $\cos 60° = \cos 45° \cos \theta$ with $0° \leq \theta \leq 90°$, then θ equals
(A) $0°$ **(B)** $15°$ **(C)** $30°$ **(D)** $45°$ **(E)** $60°$

13. At the end of the year 2000, Steve had \$100 and Wayne had \$10 000. At the end of each following year, Steve had twice as much money as he did at the end of the previous year and Wayne had half as much money as he did at the end of the previous year. At the end of which year did Steve have more money than Wayne for the first time?
(A) 2002 **(B)** 2003 **(C)** 2004 **(D)** 2005 **(E)** 2006

14. In the diagram, $PQRS$ is a square and M is the midpoint of PS. The ratio of the area of $\triangle QMS$ to the area of square $PQRS$ is

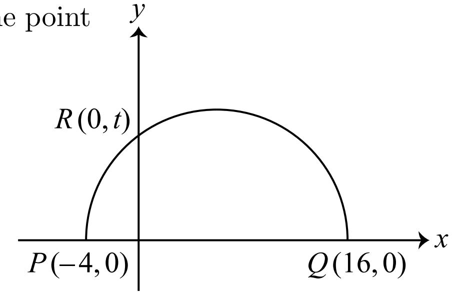


(A) $1:6$ **(B)** $1:4$ **(C)** $1:3$
(D) $1:8$ **(E)** $1:2$

15. A music test included 50 multiple choice questions. Zoltan's score was calculated by
- adding 4 points for each correct answer,
- subtracting 1 point for each incorrect answer, and
- adding 0 points for each unanswered question.

Zoltan answered 45 of the 50 questions and his score was 135 points. The number of questions that Zoltan answered *incorrectly* is
(A) 9 **(B)** 15 **(C)** 41 **(D)** 40 **(E)** 5

16. In the diagram, the line segment with endpoints $P(-4,0)$ and $Q(16,0)$ is the diameter of a semi-circle. If the point $R(0,t)$ is on the circle with $t > 0$, then t is



(A) 6 **(B)** 10 **(C)** 8
(D) 9 **(E)** 7

17. If a and b are two distinct numbers with $\dfrac{a+b}{a-b} = 3$, then $\dfrac{a}{b}$ equals

(A) -1 **(B)** 3 **(C)** 1 **(D)** 2 **(E)** 5

18. There are two values of k for which the equation $x^2 + 2kx + 7k - 10 = 0$ has two equal real roots (that is, has exactly one solution for x). The sum of these values of k is

(A) 0 **(B)** -3 **(C)** 3 **(D)** -7 **(E)** 7

19. The y-intercepts of three parallel lines are 2, 3 and 4. The sum of the x-intercepts of the three lines is 36. What is the slope of these parallel lines?

(A) $-\frac{1}{3}$ **(B)** $-\frac{2}{9}$ **(C)** $-\frac{1}{6}$ **(D)** -4 **(E)** $-\frac{1}{4}$

20. For how many integers a with $1 \le a \le 10$ is $a^{2014} + a^{2015}$ divisible by 5?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Part C: Each correct answer is worth 8.

21. Amina and Bert alternate turns tossing a fair coin. Amina goes first and each player takes three turns. The first player to toss a tail wins. If neither Amina nor Bert tosses a tail, then neither wins. What is the probability that Amina wins?

(A) $\frac{21}{32}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{11}{16}$ **(E)** $\frac{9}{16}$

22. Three distinct integers a, b and c satisfy the following three conditions:

- $abc = 17\,955$,
- a, b and c form an arithmetic sequence in that order, and
- $(3a+b)$, $(3b+c)$, and $(3c+a)$ form a geometric sequence in that order.

What is the value of $a+b+c$?

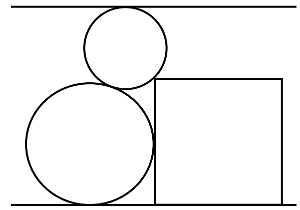
(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, $3, 5, 7$ is an arithmetic sequence with three terms.
A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

(A) -63 **(B)** -42 **(C)** $-68\,229$ **(D)** -48 **(E)** 81

23. How many pairs (x, y) of non-negative integers with $0 \le x \le y$ satisfy the equation $5x^2 - 4xy + 2x + y^2 = 624$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

24. In the diagram, two circles and a square lie between a pair of parallel lines that are a distance of 400 apart. The square has a side length of 279 and one of its sides lies along the lower line. The circles are tangent to each other, and each circle is tangent to one of the lines. Each circle also touches the square at only one point – the lower circle touches a side of the square and the upper circle touches a vertex of the square. If the upper circle has a radius of 65, then the radius of the lower circle is closest to



(A) 151 **(B)** 152 **(C)** 153 **(D)** 154 **(E)** 155

25. There are F fractions $\frac{m}{n}$ with the properties:

- m and n are positive integers with $m < n$,
- $\frac{m}{n}$ is in lowest terms,
- n is not divisible by the square of any integer larger than 1, and
- the shortest sequence of consecutive digits that repeats consecutively and indefinitely in the decimal equivalent of $\frac{m}{n}$ has length 6.

  (Note: The length of the shortest sequence of consecutive digits that repeats consecutively and indefinitely in $0.12\overline{745} = 0.12745745745745\ldots$ is 3 and the length of the shortest sequence of consecutive digits that repeats consecutively and indefinitely in $0.\overline{5}$ is 1.)

We define $G = F + p$, where the integer F has p digits. What is the sum of the squares of the digits of G?

(A) 170 **(B)** 168 **(C)** 217 **(D)** 195 **(E)** 181

2015 Fermat Contest (English)

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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Fermat Contest

(Grade 11)

Thursday, February 20, 2014
(in North America and South America)

Friday, February 21, 2014
(outside of North America and South America)

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $\dfrac{15-3^2}{3}$?

   (A) 2 **(B)** 4 **(C)** 12 **(D)** 48 **(E)** 3

2. The integer 2014 is between

   (A) 10^0 and 10^1 **(B)** 10^1 and 10^2 **(C)** 10^2 and 10^3
   (D) 10^3 and 10^4 **(E)** 10^4 and 10^5

3. If $x = 2$, then $(x + 2 - x)(2 - x - 2)$ equals

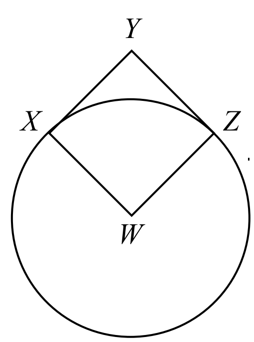
   (A) -12 **(B)** 4 **(C)** 0 **(D)** 12 **(E)** -4

4. Two positive integers x and y have $xy = 24$ and $x - y = 5$. The value of $x + y$ is

   (A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

5. In the diagram, square $WXYZ$ has area 9 and W is at the centre of a circle. If X and Z are on the circle, the area of the circle is

   (A) 3π **(B)** 6π **(C)** 9π
   (D) 18π **(E)** 81π

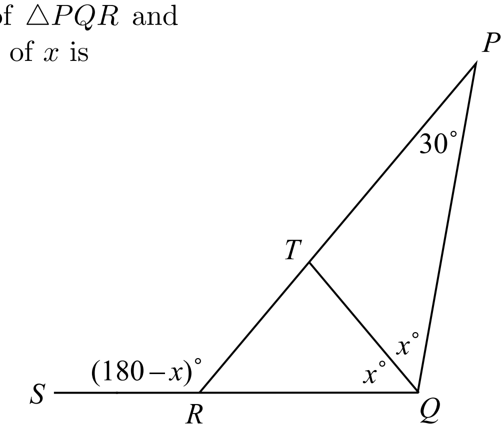


6. If 50% of N is 16, then 75% of N is

   (A) 12 **(B)** 6 **(C)** 20 **(D)** 24 **(E)** 40

7. In the diagram, point T is on side PR of $\triangle PQR$ and QRS is a straight line segment. The value of x is

   (A) 55 **(B)** 70 **(C)** 75
   (D) 60 **(E)** 50



8. In a group of five friends:

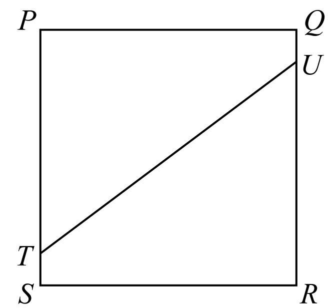
- Amy is taller than Carla.
- Dan is shorter than Eric but taller than Bob.
- Eric is shorter than Carla.

Who is the shortest?

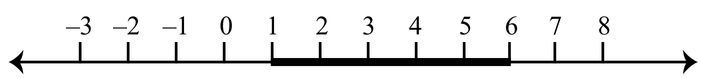
(A) Amy **(B)** Bob **(C)** Carla **(D)** Dan **(E)** Eric

9. In the diagram, $PQRS$ is a square with side length 8. Points T and U are on PS and QR respectively with $QU = TS = 1$. The length of TU is closest to

(A) 8.5 **(B)** 9.9 **(C)** 10 **(D)** 10.6 **(E)** 11.3



10. A line segment of length 5 lies along the number line initially between 1 and 6.



The line segment is rotated 180° about the point at 2. The resulting line segment lies between -2 and 3. This segment is then rotated 180° about the point at 1. The resulting line segment lies between

(A) -2 and 3 **(B)** -1 and 4 **(C)** 0 and 5 **(D)** -3 and 2 **(E)** -4 and 1

Part B: Each correct answer is worth 6.

11. If $a = \frac{2}{3}b$ and $b \neq 0$, then $\dfrac{9a+8b}{6a}$ is equal to

(A) $\dfrac{7}{2}$ **(B)** 9 **(C)** $\dfrac{3}{2}$ **(D)** $\dfrac{11}{2}$ **(E)** $\dfrac{17}{6}$

12. If $10^x \cdot 10^5 = 100^4$, what is the value of x?

(A) 1 **(B)** 35 **(C)** 11 **(D)** $\frac{4}{5}$ **(E)** 3

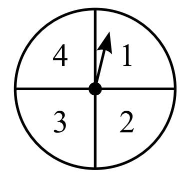
13. How many positive integers n between 10 and 1000 have the property that the sum of the digits of n is 3?

(A) 10 **(B)** 8 **(C)** 6 **(D)** 9 **(E)** 7

14. Last summer, Pat worked at a summer camp.
For each day that he worked, he earned \$100 and he was not charged for food.
For each day that he did not work, he was not paid and he was charged \$20 for food.
After 70 days, the money that he earned minus his food costs equalled \$5440.
On how many of these 70 days did Pat work?

(A) 60 **(B)** 68 **(C)** 50 **(D)** 57 **(E)** 34

15. On each spin of the spinner shown, the arrow is equally likely to stop on any one of the four numbers. Deanna spins the arrow on the spinner twice. She multiplies together the two numbers on which the arrow stops. Which product is most likely to occur?



(A) 2 **(B)** 4 **(C)** 6
(D) 8 **(E)** 12

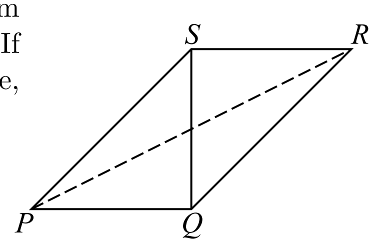
16. At the start of a 5 hour trip, the odometer in Jill's car indicates that her car had already been driven 13 831 km. The integer 13 831 is a palindrome, because it is the same when read forwards or backwards. At the end of the 5 hour trip, the odometer reading was another palindrome. If Jill never drove faster than 80 km/h, her greatest possible average speed was closest to

(A) 62 km/h **(B)** 20 km/h **(C)** 42 km/h **(D)** 16 km/h **(E)** 77 km/h

17. Sergio recently opened a store. One day, he determined that the average number of items sold per employee to date was 75. The next day, one employee sold 6 items, one employee sold 5 items, and one employee sold 4 items. The remaining employees each sold 3 items. This made the new average number of items sold per employee to date equal to 78.3. How many employees are there at the store?

(A) 50 **(B)** 5 **(C)** 20 **(D)** 40 **(E)** 30

18. A square is cut along a diagonal and reassembled to form the parallelogram $PQRS$ as shown in the diagram. If $PR = 90$ mm, what is the area of the original square, in mm^2?

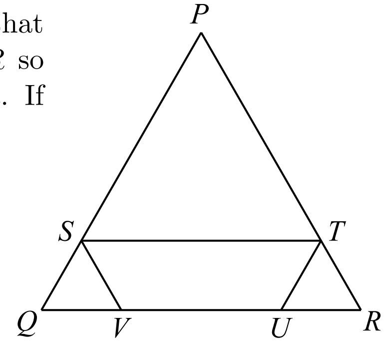


(A) 324 **(B)** 1620 **(C)** 1800
(D) 2025 **(E)** 2700

19. Max and Minnie each add up sets of three-digit positive integers. Each of them adds three different three-digit integers whose nine digits are all different. Max creates the largest possible sum. Minnie creates the smallest possible sum. The difference between Max's sum and Minnie's sum is

(A) 594 **(B)** 1782 **(C)** 1845 **(D)** 1521 **(E)** 2592

20. In the diagram, $\triangle PQR$ has $PQ = QR = RP = 30$. Points S and T are on PQ and PR, respectively, so that ST is parallel to QR. Points V and U are on QR so that TU is parallel to PQ and SV is parallel to PR. If $VS + ST + TU = 35$, the length of VU is



(A) 21 **(B)** 15 **(C)** 18
(D) 20 **(E)** 25

Part C: Each correct answer is worth 8.

21. A bin contains 10 kg of peanuts. 2 kg of peanuts are removed and 2 kg of raisins are added and thoroughly mixed in. Then 2 kg of this mixture are removed and 2 kg of raisins are added and thoroughly mixed in again. What is the ratio of the mass of peanuts to the mass of raisins in the final mixture?

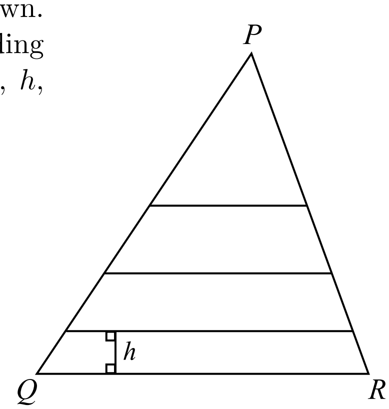
    (A) $3:2$ **(B)** $4:1$ **(C)** $5:1$ **(D)** $7:3$ **(E)** $16:9$

22. Jillian drives along a straight road that goes directly from her house (J) to her Grandfather's house (G). Some of this road is on flat ground and some is downhill or uphill. Her car travels downhill at 99 km/h, on flat ground at 77 km/h, and uphill at 63 km/h. It takes Jillian 3 hours and 40 minutes to drive from J to G. It takes her 4 hours and 20 minutes to drive from G to J. The distance between J and G, in km, is

    (A) $318\frac{2}{3}$ **(B)** 324 **(C)** 308 **(D)** $292\frac{3}{5}$ **(E)** $292\frac{1}{9}$

23. $\triangle PQR$ has $PQ = 150$ and $PR = QR = 125$, as shown. Three line segments are drawn parallel to QR, dividing $\triangle PQR$ into four sections of equal area. The height, h, of the bottom section is closest to

    (A) 16.7 **(B)** 16.9 **(C)** 16.5
    (D) 16.3 **(E)** 16.1



24. Mohammed has eight boxes numbered 1 to 8 and eight balls numbered 1 to 8. In how many ways can he put the balls in the boxes so that there is one ball in each box, ball 1 is not in box 1, ball 2 is not in box 2, and ball 3 is not in box 3?

    (A) 27 240 **(B)** 29 160 **(C)** 27 360 **(D)** 27 600 **(E)** 25 200

25. Points $P(r,s)$ and $Q(t,u)$ are on the parabola with equation $y = x^2 - \frac{1}{5}mx + \frac{1}{5}n$ so that $PQ = 13$ and the slope of PQ is $\frac{12}{5}$. For how many pairs (m,n) of positive integers with $n \leq 1000$ is $r + s + t + u = 27$?

    (A) 28 **(B)** 26 **(C)** 27 **(D)** 29 **(E)** 25

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Thank you for writing the 2014 Fermat Contest!
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Fermat Contest

(Grade 11)

Thursday, February 21, 2013
(in North America and South America)

Friday, February 22, 2013
(outside of North America and South America)

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

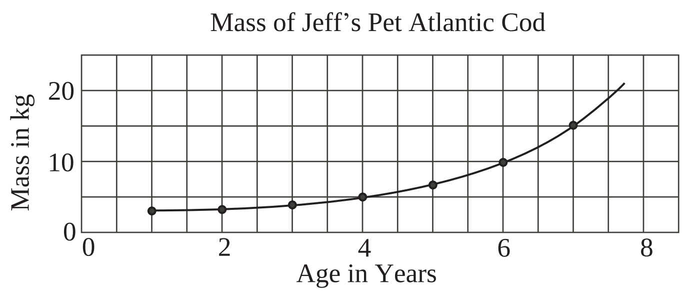
Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $\dfrac{10^2+6^2}{2}$?

   (A) 16 **(B)** 86 **(C)** 34 **(D)** 68 **(E)** 128

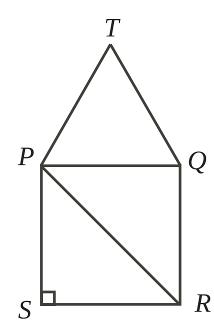
2. The graph shows the mass, in kilograms, of Jeff's pet Atlantic cod, given its age in years. What is the age of the cod when its mass is 15 kg?



   (A) 3 **(B)** 7 **(C)** 4 **(D)** 6 **(E)** 5

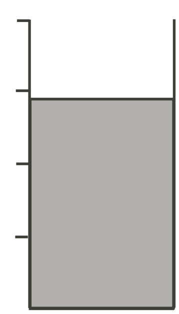
3. In the diagram, $PQRS$ is a square and PTQ is an equilateral triangle. What is the measure of $\angle TPR$?

   (A) 90° **(B)** 105° **(C)** 120°
   (D) 150° **(E)** 75°



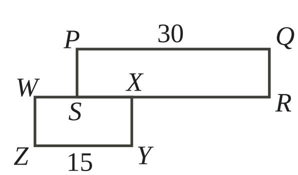
4. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?

   (A) 24 L **(B)** 28 L **(C)** 30 L
   (D) 36 L **(E)** 40 L



5. In the diagram, rectangle $PQRS$ has $PQ = 30$ and rectangle $WXYZ$ has $ZY = 15$. If S is on WX and X is on SR so that $SX = 10$, then WR equals

   (A) 20 **(B)** 25 **(C)** 55
   (D) 45 **(E)** 35



6. If $x = 11$, $y = 8$, and $2x + 3z = 5y$, what is the value of z?

(A) 6 **(B)** $\frac{62}{3}$ **(C)** 13 **(D)** 15 **(E)** $\frac{46}{5}$

7. If $(x + a)(x + 8) = x^2 + bx + 24$ for all values of x, then $a + b$ equals

(A) 32 **(B)** 144 **(C)** 40 **(D)** 14 **(E)** 16

8. Which number from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ must be removed so that the mean (average) of the numbers remaining in the set is 6.1?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

9. The regular price for a bicycle is \$320. The bicycle is on sale for 20% off. The regular price for a helmet is \$80. The helmet is on sale for 10% off. If Sandra bought both items on sale, what is her percentage savings on the total purchase?

(A) 18% **(B)** 12% **(C)** 15% **(D)** 19% **(E)** 22.5%

10. $PQRS$ is a square. The midpoint of PQ is M and the midpoint of RS is N. If the perimeter of rectangle $PMNS$ is 36, the area of square $PQRS$ is

(A) 81 **(B)** 72 **(C)** 324 **(D)** 144 **(E)** 36

Part B: Each correct answer is worth 6.

11. On Monday, Ramya read $\frac{1}{5}$ of a 300 page novel. On Tuesday, she read $\frac{4}{15}$ of the remaining pages. How many pages did she read in total on Monday and Tuesday?

(A) 124 **(B)** 60 **(C)** 252 **(D)** 80 **(E)** 64

12. An integer m is chosen at random from the list $-9, -7, -5, -3, -1, 1, 3, 5, 7, 9$. The probability that $m^4 > 100$ is

(A) $\frac{1}{5}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{3}{5}$

13. If $512^x = 64^{240}$, then x equals

(A) 80 **(B)** 30 **(C)** 360 **(D)** 160 **(E)** 237

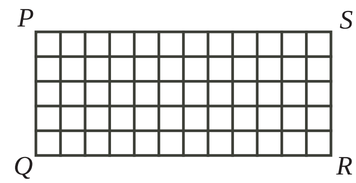
14. In a school fundraising campaign, 25% of the money donated came from parents. The rest of the money was donated by teachers and students. The ratio of the amount of money donated by teachers to the amount donated by students was $2 : 3$. The ratio of the amount of money donated by parents to the amount donated by students was

(A) $20 : 9$ **(B)** $5 : 6$ **(C)** $5 : 9$ **(D)** $1 : 2$ **(E)** $5 : 12$

15. The cookies in a cookie jar contain a total of 100 raisins. All but one of the cookies are the same size and contain the same number of raisins. One cookie is larger and contains one more raisin than each of the others. The number of cookies in the jar is between 5 and 10, inclusive. How many raisins are in the larger cookie?

(A) 10 **(B)** 11 **(C)** 20 **(D)** 17 **(E)** 12

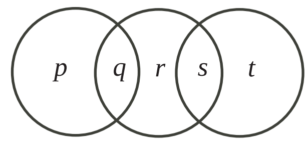
16. Rectangle $PQRS$ is divided into 60 identical squares, as shown. The length of the diagonal of each of these squares is 2. The length of QS is closest to



(A) 18 **(B)** 13 **(C)** 26

(D) 24 **(E)** 17

17. In the diagram, p, q, r, s, and t represent five consecutive integers, not necessarily in order. The two integers in the leftmost circle add to 63. The two integers in the rightmost circle add to 57. What is the value of r?



(A) 24 **(B)** 28 **(C)** 20

(D) 42 **(E)** 30

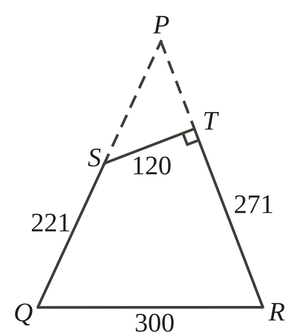
18. If m, n and p are positive integers with $m + \cfrac{1}{n + \cfrac{1}{p}} = \dfrac{17}{3}$, the value of n is

(A) 3 **(B)** 4 **(C)** 1 **(D)** 17 **(E)** 13

19. There are two ways of choosing six different numbers from the list $1, 2, 3, 4, 5, 6, 7, 8, 9$ so that the product of the six numbers is a perfect square. Suppose that these two perfect squares are m^2 and n^2, with m and n positive integers and $m \neq n$. What is the value of $m + n$?

(A) 108 **(B)** 11 **(C)** 61 **(D)** 56 **(E)** 144

20. Isosceles triangle PQR has $PQ = PR$ and $QR = 300$. Point S is on PQ and T is on PR so that ST is perpendicular to PR, $ST = 120$, $TR = 271$, and $QS = 221$. The area of quadrilateral $STRQ$ is

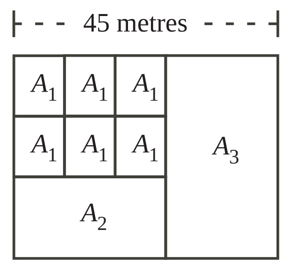


(A) 21 275 **(B)** 40 605 **(C)** 46 860

(D) 54 000 **(E)** 54 603

Part C: Each correct answer is worth 8.

21. A farmer has a rectangular field with width 45 metres. He divides the field into smaller rectangular animal enclosures in three different sizes, as shown.



Each enclosure labelled A_1 has the same dimensions. Also, the area of the enclosure labelled A_2 is 4 times the area of A_1, and the area of the enclosure labelled A_3 is 5 times the area of A_1. The lines in the diagram represent fences. The total length of all the fences is 360 metres. The area of A_1, in square metres, is closest to

(A) 143.4 **(B)** 150.0 **(C)** 175.2 **(D)** 162.7 **(E)** 405.0

22. Megan and Shana race against each other with the winner of each race receiving x gold coins and the loser receiving y gold coins. (There are no ties and x and y are integers with $x > y > 0$.) After several races, Megan has 42 coins and Shana has 35 coins. Shana has won exactly 2 races. The value of x is

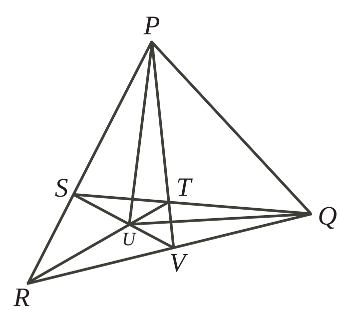
(A) 3 **(B)** 7 **(C)** 5 **(D)** 6 **(E)** 4

23. One bag contains 2 red marbles and 2 blue marbles. A second bag contains 2 red marbles, 2 blue marbles, and g green marbles, with $g > 0$. For each bag, Maria calculates the probability of randomly drawing two marbles of the same colour in two draws from that bag, without replacement. (Drawing two marbles *without replacement* means drawing two marbles, one after the other, without putting the first marble back into the bag.) If these two probabilities are equal, then the value of g is

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

24. In the diagram, $\triangle PQR$ has S on PR and V on RQ. Segments QS and PV intersect at T. Segments RT and SV intersect at U. If the area of $\triangle RST$ is 55, the area of $\triangle RTV$ is 66, and the area of $\triangle RSV$ is 77, then the area of $\triangle PQU$ is

(A) 869 **(B)** 836 **(C)** 840
(D) 864 **(E)** 847



25. For how many odd integers k between 0 and 100 does the equation

$$2^{4m^2} + 2^{m^2-n^2+4} = 2^{k+4} + 2^{3m^2+n^2+k}$$

have exactly two pairs of positive integers (m, n) that are solutions?

(A) 17 **(B)** 20 **(C)** 19 **(D)** 18 **(E)** 21

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Fermat Contest

(Grade 11)

Thursday, February 23, 2012
(in North America and South America)

Friday, February 24, 2012
(outside of North America and South America)

Time: 60 minutes
Calculators are permitted
Instructions

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3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. Which of the following is *not equal* to a whole number?

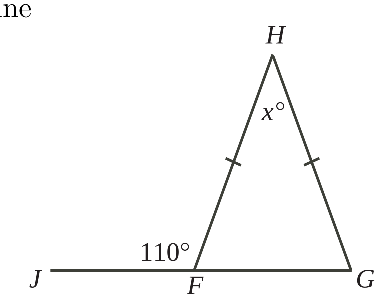
   (A) $\frac{60}{12}$ **(B)** $\frac{60}{8}$ **(C)** $\frac{60}{5}$ **(D)** $\frac{60}{4}$ **(E)** $\frac{60}{3}$

2. If $3 - 5 + 7 = 6 - x$, then x equals

   (A) -3 **(B)** -1 **(C)** 1 **(D)** 11 **(E)** 15

3. In the diagram, $HF = HG$ and JFG is a straight line segment. The value of x is

   (A) 45 **(B)** 35 **(C)** 55
   (D) 60 **(E)** 40

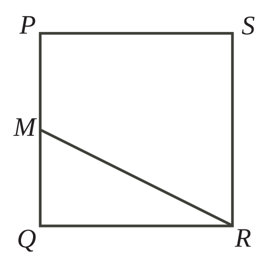


4. The value of $\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)$ is

   (A) $\frac{5}{3}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{13}{12}$ **(D)** $\frac{31}{12}$ **(E)** $\frac{16}{7}$

5. In the diagram, $PQRS$ is a square and M is the midpoint of PQ. The area of triangle MQR is 100. The area of the square $PQRS$ is

   (A) 200 **(B)** 500 **(C)** 300
   (D) 400 **(E)** 800

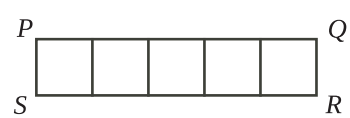


6. John ate a total of 120 peanuts over four consecutive nights. Each night he ate 6 more peanuts than the night before. How many peanuts did he eat on the fourth night?

   (A) 42 **(B)** 39 **(C)** 30 **(D)** 36 **(E)** 33

7. Five identical squares form rectangle $PQRS$, as shown. The perimeter of rectangle $PQRS$ is 48. What is the area of $PQRS$?

   (A) 45 **(B)** 9 **(C)** 80
   (D) 16 **(E)** 96



8. If $x = 2$ and $v = 3x$, then the value of $(2v - 5) - (2x - 5)$ is

   (A) 2 **(B)** 8 **(C)** -2 **(D)** -7 **(E)** 6

9. Mary and Sally were once the same height. Since then, Sally grew 20% taller and Mary's height increased by half as many centimetres as Sally's height increased. Sally is now 180 cm tall. How tall, in cm, is Mary now?

(A) 144 **(B)** 165 **(C)** 162 **(D)** 150 **(E)** 170

10. If $(2^a)(2^b) = 64$, then the mean (average) of a and b is

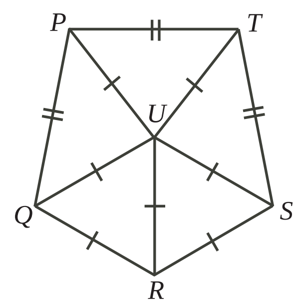
(A) 12 **(B)** $\frac{5}{2}$ **(C)** $\frac{3}{2}$ **(D)** 3 **(E)** 8

Part B: Each correct answer is worth 6.

11. There is one odd integer N between 400 and 600 that is divisible by both 5 and 11. The sum of the digits of N is

(A) 11 **(B)** 8 **(C)** 10 **(D)** 16 **(E)** 18

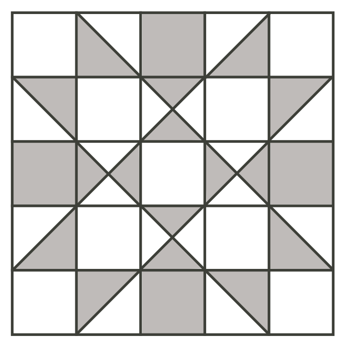
12. In the diagram, $\triangle QUR$ and $\triangle SUR$ are equilateral triangles. Also, $\triangle QUP$, $\triangle PUT$ and $\triangle TUS$ are isosceles triangles with $PU = QU = SU = TU$ and $QP = PT = TS$. The measure of $\angle UST$, in degrees, is



(A) 50 **(B)** 54 **(C)** 60

(D) 70 **(E)** 80

13. The diagram shows a square quilt that is made up of identical squares and two sizes of right-angled isosceles triangles. What percentage of the quilt is shaded?



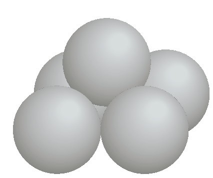
(A) 36% **(B)** 40% **(C)** 44%

(D) 48% **(E)** 50%

14. The product of the roots of the equation $(x-4)(x-2)+(x-2)(x-6)=0$ is

(A) 20 **(B)** 48 **(C)** 10 **(D)** 96 **(E)** 2

15. Oranges are placed in a pyramid-like stack with each layer completely filled. The base is a rectangle that is 5 oranges wide and 7 oranges long. Each orange, above the first layer, rests in a pocket formed by four oranges in the layer below, as shown. The last layer is a single row of oranges. The total number of oranges in the stack is



(A) 53 **(B)** 80 **(C)** 82

(D) 85 **(E)** 105

16. There are 30 people in a room, 60% of whom are men. If no men enter or leave the room, how many women must enter the room so that 40% of the total number of people in the room are men?

(A) 10 **(B)** 6 **(C)** 20 **(D)** 12 **(E)** 15

17. The expression $\dfrac{3^{2011}+3^{2011}}{3^{2010}+3^{2012}}$ is equal to

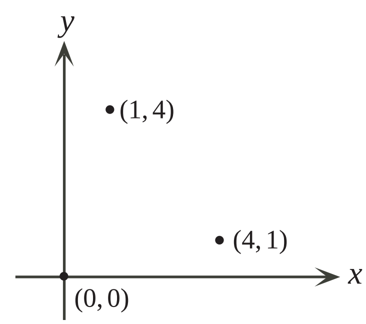
(A) $\frac{3}{5}$ **(B)** 1 **(C)** $\frac{9}{10}$ **(D)** $\frac{10}{3}$ **(E)** $\frac{2}{3}$

18. If N is the smallest positive integer whose digits have a product of 1728, then the sum of the digits of N is

(A) 28 **(B)** 26 **(C)** 18 **(D)** 27 **(E)** 21

19. The coordinates of three of the vertices of a parallelogram are $(0,0)$, $(1,4)$ and $(4,1)$. What is the area of this parallelogram?

(A) 15 **(B)** 19 **(C)** 16
(D) 17 **(E)** 12



20. Katie and Sarah run at different but constant speeds. They ran two races on a track that measured 100 m from start to finish. In the first race, when Katie crossed the finish line, Sarah was 5 m behind. In the second race, Katie started 5 m behind the original start line and they ran at the same speeds as in the first race. What was the outcome of the second race?

(A) Katie and Sarah crossed the finish line at the same time.
(B) When Katie crossed the finish line, Sarah was 0.25 m behind.
(C) When Katie crossed the finish line, Sarah was 0.26 m behind.
(D) When Sarah crossed the finish line, Katie was 0.25 m behind.
(E) When Sarah crossed the finish line, Katie was 0.26 m behind.

Part C: Each correct answer is worth 8.

21. If $x^2 = 8x + y$ and $y^2 = x + 8y$ with $x \neq y$, then the value of $x^2 + y^2$ is

(A) 9 **(B)** 49 **(C)** 63 **(D)** 21 **(E)** 56

22. In the country of Nohills, each pair of cities is connected by a straight (and flat) road. The chart to the right shows the distances along the straight roads between some pairs of cities. The distance along the straight road between city P and city R is closest to

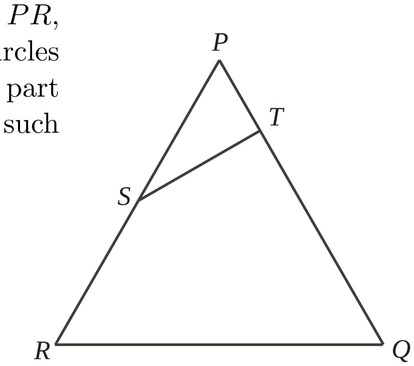
	P	Q	R	S
P	0	25		24
Q	25	0	25	7
R		25	0	18
S	24	7	18	0

(A) 30 **(B)** 25 **(C)** 27
(D) 24.5 **(E)** 24

23. A bowl contained 320 grams of pure white sugar. Mixture Y was formed by taking x grams of the white sugar out of the bowl, adding x grams of brown sugar to the bowl, and then mixing uniformly. In Mixture Y, the ratio of the mass of the white sugar to the mass of the brown sugar, expressed in lowest terms, was $w : b$. Mixture Z is formed by taking x grams of Mixture Y out of the bowl, adding x grams of brown sugar to the bowl, and then mixing uniformly. In Mixture Z, the ratio of the mass of the white sugar to the mass of the brown sugar is $49 : 15$. The value of $x + w + b$ is

(A) 48 **(B)** 49 **(C)** 139 **(D)** 76 **(E)** 104

24. In equilateral triangle PQR, S is the midpoint of PR, T is on PQ so that $PT = 1$ and $TQ = 3$. Many circles can be drawn inside quadrilateral $QRST$ so that no part extends outside of $QRST$. The radius of the largest such circle is closest to



(A) 1.00 **(B)** 1.10 **(C)** 1.15
(D) 1.05 **(E)** 1.37

25. There are many positive integers N with the following properties:

- the digits of N include at least one of each of the digits 3, 4, 5, and 6,
- the digits of N include no digits other than 3, 4, 5, and 6, and
- the sum of the digits of N is 900 and the sum of the digits of $2N$ is 900.

When the largest and smallest possible values of N are multiplied together, the number of digits in the resulting product is

(A) 408 **(B)** 400 **(C)** 432 **(D)** 416 **(E)** 424

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Fermat Contest

(Grade 11)

Thursday, February 24, 2011

UNIVERSITY OF WATERLOO

WATERLOO MATHEMATICS

STRONGER COMMUNITIES TOGETHER™

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. $\dfrac{2+3\times 6}{23+6}$ is equal to

   (A) 1 **(B)** $\frac{11}{29}$ **(C)** $\frac{36}{29}$ **(D)** $\frac{20}{29}$ **(E)** $\frac{5}{23}$

2. If $y = 77$, then $\dfrac{7y+77}{77}$ is equal to

   (A) 8 **(B)** 12 **(C)** 78 **(D)** 84 **(E)** 540

3. The area of the rectangle shown is 192. What is the perimeter of the rectangle?

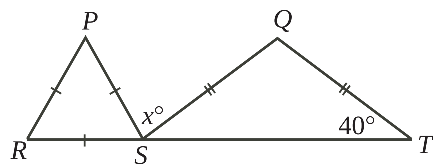
   (A) 64 **(B)** 384 **(C)** 192
   (D) 1728 **(E)** 32

4. If $\sqrt{n+9} = 25$, then n equals

   (A) 256 **(B)** -4 **(C)** 484 **(D)** 616 **(E)** 16

5. In the diagram, S lies on RT, $\angle QTS = 40^\circ$, $QS = QT$, and $\triangle PRS$ is equilateral. The value of x is

   (A) 50 **(B)** 60 **(C)** 80
   (D) 90 **(E)** 100



6. When three consecutive integers are added, the total is 27. When the same three integers are multiplied, the result is

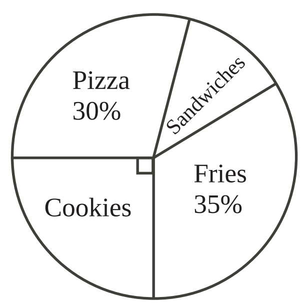
   (A) 504 **(B)** 81 **(C)** 720 **(D)** 729 **(E)** 990

7. The number halfway between $\frac{1}{12}$ and $\frac{1}{10}$ is

   (A) $\frac{1}{11}$ **(B)** $\frac{1}{120}$ **(C)** $\frac{11}{60}$ **(D)** $\frac{11}{120}$ **(E)** $\frac{1}{22}$

8. The circle graph shown illustrates the results of a survey taken by the Fermat H.S. Student Council to determine the favourite cafeteria food. How many of the 200 students surveyed said that their favourite food was sandwiches?

   (A) 10 **(B)** 20 **(C)** 35
   (D) 50 **(E)** 70

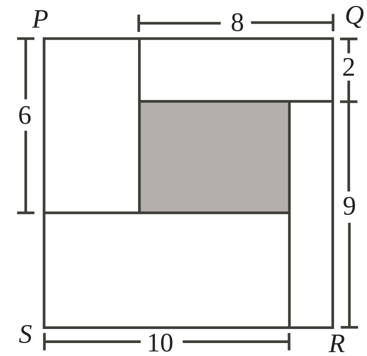


9. The set $S = \{1, 2, 3, \ldots, 49, 50\}$ contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many numbers remain in the set S?

(A) 8 **(B)** 9 **(C)** 16 **(D)** 17 **(E)** 18

10. In the diagram, $PQRS$ is a square. Square $PQRS$ is divided into five rectangles, as shown. The area of the shaded rectangle is

(A) 49 **(B)** 28 **(C)** 22
(D) 57 **(E)** 16



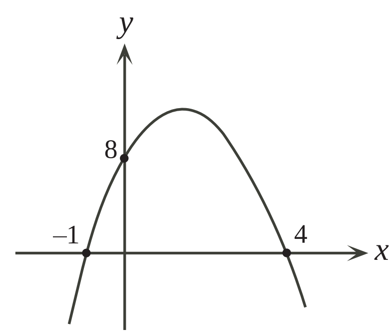
Part B: Each correct answer is worth 6.

11. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that Wally must buy to *guarantee* that he receives 3 gumballs of the same colour?

(A) 6 **(B)** 9 **(C)** 4 **(D)** 7 **(E)** 8

12. In the diagram, the parabola has x-intercepts -1 and 4, and y-intercept 8. If the parabola passes through the point $(3, w)$, what is the value of w?

(A) 4 **(B)** 5 **(C)** 6
(D) 7 **(E)** 8



13. Xavier, Yolanda, and Zixuan have a total of \$50. The ratio of the amount Xavier has to the total amount Yolanda and Zixuan have is 3 : 2. Yolanda has \$4 more than Zixuan. How much does Zixuan have?

(A) \$16 **(B)** \$8 **(C)** \$14 **(D)** \$13 **(E)** \$30

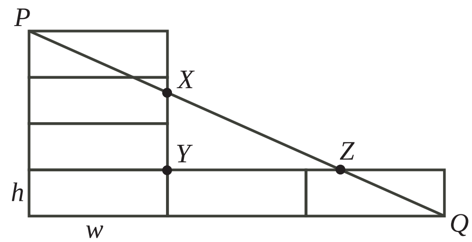
14. Which of the following *must* be an even integer?

(A) The average of two even integers
(B) The average of two prime numbers
(C) The average of two perfect squares
(D) The average of two multiples of 4
(E) The average of three consecutive integers

15. If m and n are consecutive positive integers and $n^2 - m^2 > 20$, then the minimum possible value of $n^2 + m^2$ is

(A) 29 **(B)** 181 **(C)** 265 **(D)** 23 **(E)** 221

16. Six identical rectangles with height h and width w are arranged as shown. Line segment PQ intersects the vertical side of one rectangle at X and the horizontal side of another rectangle at Z. If right-angled $\triangle XYZ$ has $YZ = 2XY$, then $\dfrac{h}{w}$ equals



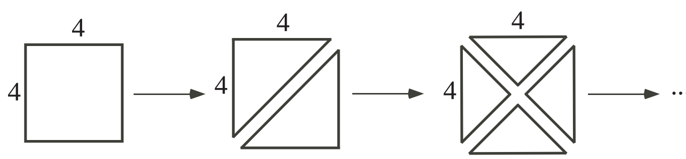
(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{8}$

(D) $\frac{1}{3}$ **(E)** $\frac{3}{4}$

17. If $3^{2x} = 64$, then 3^{-x} is equal to

(A) -32 **(B)** -8 **(C)** $\frac{1}{4096}$ **(D)** $\frac{1}{32}$ **(E)** $\frac{1}{8}$

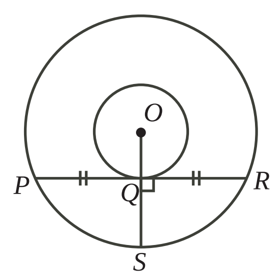
18. A 4×4 square piece of paper is cut into two identical pieces along its diagonal. The resulting triangular pieces of paper are each cut into two identical pieces.



Each of the four resulting pieces is cut into two identical pieces. Each of the eight new resulting pieces is finally cut into two identical pieces. The length of the longest edge of one of these final sixteen pieces of paper is

(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{\sqrt{2}}$ **(E)** $2\sqrt{2}$

19. In the diagram, the two circles are centred at O. Point S is on the larger circle. Point Q is the point of intersection of OS and the smaller circle. Line segment PR is a chord of the larger circle and touches (that is, is tangent to) the smaller circle at Q. Note that OS is the perpendicular bisector of PR. If $PR = 12$ and $QS = 4$, then the radius of the larger circle is



(A) 6.0 **(B)** 5.0 **(C)** 6.5

(D) 7.2 **(E)** 20.0

20. Three real numbers a, b and c have a sum of 114 and a product of 46 656. If $b = ar$ and $c = ar^2$ for some real number r, then the value of $a + c$ is

(A) 78 **(B)** 76 **(C)** 24 **(D)** 54 **(E)** 36

Part C: Each correct answer is worth 8.

21. The positive integers are arranged in increasing order in a triangle, as shown. Each row contains one more number than the previous row. The sum of the numbers in the row that contains the number 400 is

1
2 3
4 5 6
7 8 9 10
11 12 $\cdots$

(A) 10 990 **(B)** 12 209 **(C)** 9855
(D) 10 976 **(E)** 11 368

22. The number of pairs of positive integers (p, q), with $p + q \leq 100$, that satisfy the equation $\dfrac{p + q^{-1}}{p^{-1} + q} = 17$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

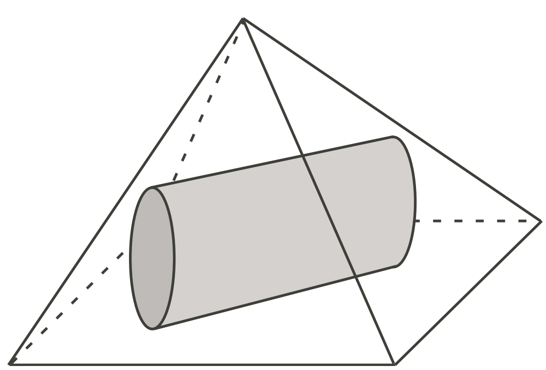
23. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

(A) $t < 3.9$ **(B)** $3.9 \leq t < 4.1$ **(C)** $4.1 \leq t < 4.3$
(D) $4.3 \leq t < 4.5$ **(E)** $4.5 \leq t$

24. Four numbers w, x, y, z satisfy $w < x < y < z$. Each of the six possible pairs of distinct numbers has a different sum. The four smallest sums are $1, 2, 3$, and 4. What is the sum of all possible values of z?

(A) 4 **(B)** $\frac{13}{2}$ **(C)** $\frac{17}{2}$ **(D)** $\frac{15}{2}$ **(E)** 7

25. A pyramid has a square base with side length 20. A right circular cylinder has a diameter of 10 and a length of 10. The cylinder is lying on its side, completely inside the pyramid. The central axis of the cylinder lies parallel to and directly above a diagonal of the pyramid's base. The midpoint of the central axis lies directly above the centre of the square base of the pyramid.



The smallest possible height of the pyramid is closest to

(A) 15.3 **(B)** 22.1 **(C)** 21.9 **(D)** 21.7 **(E)** 15.5

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Canadian
Mathematics
Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Thursday, February 25, 2010

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Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ is

   (A) 2 **(B)** $\frac{5}{13}$ **(C)** $\frac{5}{6}$ **(D)** 1 **(E)** $\frac{13}{6}$

2. The quantity "2% of 1" is equal to

   (A) $\frac{2}{100}$ **(B)** $\frac{2}{10}$ **(C)** 2 **(D)** 20 **(E)** 200

3. In the diagram, points P, Q, R, and S are arranged in order on a line segment. If $PQ = 1$, $QR = 2PQ$ and $RS = 3QR$, then the length of PS is

   (A) 7 **(B)** 6 **(C)** 9
   (D) 8 **(E)** 10

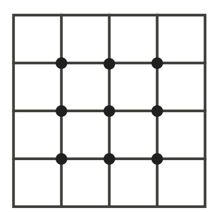
4. If $u = -6$ and $x = \frac{1}{3}(3 - 4u)$, then x equals

   (A) -23 **(B)** -7 **(C)** 9 **(D)** 2 **(E)** 25

5. If $2^x = 16$, then 2^{x+3} equals

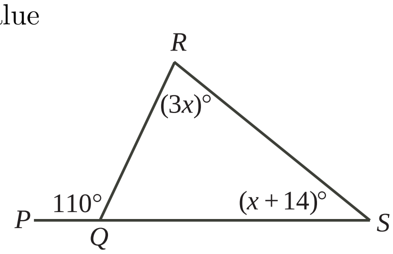
   (A) 19 **(B)** 48 **(C)** 22 **(D)** 128 **(E)** 2048

6. The nine interior intersection points on a 4 by 4 grid of squares are shown. How many interior intersection points are there on a 12 by 12 grid of squares?



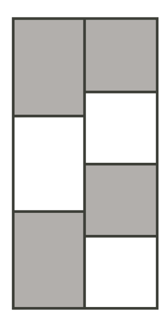
   (A) 100 **(B)** 121 **(C)** 132
   (D) 144 **(E)** 169

7. In the diagram, PQS is a straight line. What is the value of x?



   (A) 19 **(B)** 62 **(C)** 21.5
   (D) 24 **(E)** 32

8. A rectangle is divided into two vertical strips of equal width. The strip on the left is divided into three equal parts and the strip on the right is divided into four equal parts. Parts of the rectangle are then shaded as shown. What fraction of the original rectangle is shaded?



   (A) $\frac{3}{5}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{7}$
   (D) $\frac{7}{6}$ **(E)** $\frac{7}{12}$

9. The value of $k\nabla m$ is defined to be $k(k-m)$. For example, $7\nabla 2 = 7(7-2) = 35$. What is the value of $(5\nabla 1) + (4\nabla 1)$?

(A) 9 **(B)** 84 **(C)** 20 **(D)** 32 **(E)** 72

10. If $2x^2 = 9x - 4$ and $x \neq 4$, then the value of $2x$ is

(A) 4 **(B)** 1 **(C)** -1 **(D)** 0 **(E)** 2

Part B: Each correct answer is worth 6.

11. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?

(A) \$40 **(B)** \$100 **(C)** \$160 **(D)** \$1000 **(E)** \$1600

12. When k candies were distributed among seven people so that each person received the same number of candies and each person received as many candies as possible, there were 3 candies left over. If instead, $3k$ candies were distributed among seven people in this way, then the number of candies left over would have been

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

13. Fifty numbers have an average of 76. Forty of these numbers have an average of 80. The average of the other ten numbers is

(A) 60 **(B)** 4 **(C)** 72 **(D)** 40 **(E)** 78

14. Four friends went fishing one day and caught a total of 11 fish.
Each person caught at least one fish.
All of the following statements *could* be true.
Which one of the statements *must* be true?

(A) At least one person caught exactly one fish.
(B) At least one person caught exactly three fish.
(C) At least one person caught more than three fish.
(D) At least one person caught fewer than three fish.
(E) At least two people each caught more than one fish.

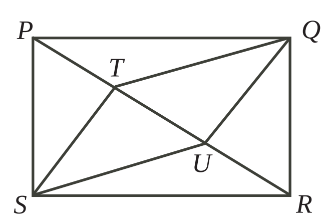
15. The number of positive integers p for which $-1 < \sqrt{p} - \sqrt{100} < 1$ is

(A) 19 **(B)** 21 **(C)** 38 **(D)** 39 **(E)** 41

16. Positive integers a and b satisfy $ab = 2010$. If $a > b$, the smallest possible value of $a - b$ is

(A) 37 **(B)** 119 **(C)** 191 **(D)** 1 **(E)** 397

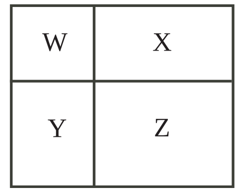
17. In the diagram, $PQRS$ is a rectangle with $PQ = 5$ and $QR = 3$. PR is divided into three segments of equal length by points T and U. The area of quadrilateral $STQU$ is



(A) $\frac{17}{3}$ **(B)** 5 **(C)** $\frac{5}{2}$

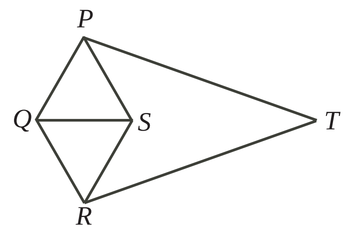
(D) $\frac{\sqrt{34}}{3}$ **(E)** $\sqrt{34}$

18. A rectangle is divided into four smaller rectangles, labelled W, X, Y, and Z, as shown. The perimeters of rectangles W, X and Y are 2, 3 and 5, respectively. What is the perimeter of rectangle Z?



(A) 6 **(B)** 7 **(C)** 4
(D) 8 **(E)** 7.5

19. In the diagram, $PQ = QR = RS = SP = SQ = 6$ and $PT = RT = 14$. The length of ST is



(A) $4\sqrt{10} - 3$ **(B)** 11 **(C)** $7\sqrt{3} - 3$
(D) 10 **(E)** $\sqrt{232 - 84\sqrt{3}}$

20. A square has side length 5. In how many different locations can point X be placed so that the distances from X to the four sides of the square are 1, 2, 3, and 4?

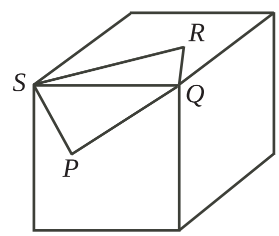
(A) 0 **(B)** 12 **(C)** 4 **(D)** 8 **(E)** 16

Part C: Each correct answer is worth 8.

21. If $\dfrac{x-y}{z-y} = -10$, then the value of $\dfrac{x-z}{y-z}$ is

(A) 11 **(B)** -10 **(C)** 9 **(D)** -9 **(E)** 10

22. A rectangular piece of paper, $PQRS$, has $PQ = 20$ and $QR = 15$. The piece of paper is glued flat on the surface of a large cube so that Q and S are at vertices of the cube. (Note that $\triangle QPS$ and $\triangle QRS$ lie flat on the front and top faces of the cube, respectively.) The shortest distance from P to R, as measured through the cube, is closest to

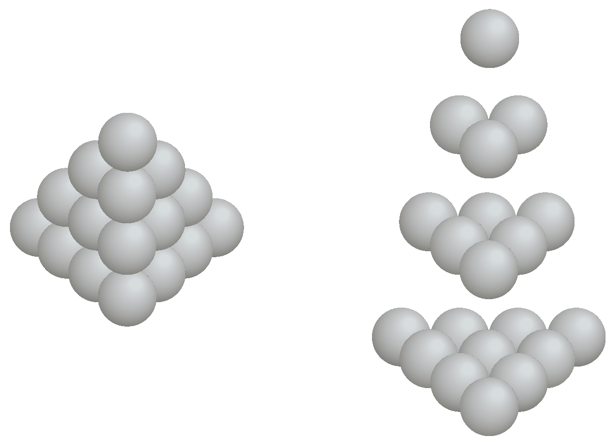


(A) 17.0 **(B)** 25.0 **(C)** 31.0
(D) 17.7 **(E)** 18.4

23. Let t_n equal the integer closest to $\sqrt{n}$.
For example, $t_1 = t_2 = 1$ since $\sqrt{1} = 1$ and $\sqrt{2} \approx 1.41$ and $t_3 = 2$ since $\sqrt{3} \approx 1.73$.
The sum $\dfrac{1}{t_1} + \dfrac{1}{t_2} + \dfrac{1}{t_3} + \dfrac{1}{t_4} + \cdots + \dfrac{1}{t_{2008}} + \dfrac{1}{t_{2009}} + \dfrac{1}{t_{2010}}$ equals

(A) $88\frac{1}{6}$ **(B)** $88\frac{1}{2}$ **(C)** $88\frac{2}{3}$ **(D)** $88\frac{1}{3}$ **(E)** 90

24. Spheres can be stacked to form a tetrahedron by using triangular layers of spheres. Each sphere touches the three spheres below it. The diagrams show a tetrahedron with four layers and the layers of such a tetrahedron. An *internal sphere* in the tetrahedron is a sphere that touches exactly three spheres in the layer above. For example, there is one internal sphere in the fourth layer, but no internal spheres in the first three layers.



A tetrahedron of spheres is formed with thirteen layers and each sphere has a number written on it. The top sphere has a 1 written on it and each of the other spheres has written on it the number equal to the sum of the numbers on the spheres in the layer above with which it is in contact. For the whole thirteen layer tetrahedron, the sum of the numbers on all of the internal spheres is

(A) 772 588 **(B)** 772 566 **(C)** 772 156 **(D)** 772 538 **(E)** 772 626

25. Alex chose positive integers a, b, c, d, e, f and completely multiplied out the polynomial product

$$(1-x)^a(1+x)^b(1-x+x^2)^c(1+x^2)^d(1+x+x^2)^e(1+x+x^2+x^3+x^4)^f$$

After she simplified her result, she discarded any term involving x to any power larger than 6 and was astonished to see that what was left was $1-2x$. If $a > d+e+f$ and $b > c+d$ and $e > c$, what value of a did she choose?

(A) 17 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 23

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Thank you for writing the 2010 Fermat Contest!
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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest *(Grade 11)*

Wednesday, February 18, 2009

C.M.C. Sponsors

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Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $3 + 3^3$ is

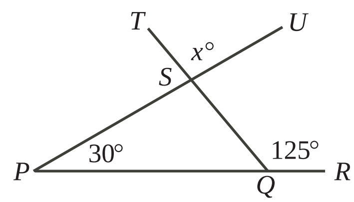
   (A) 12 **(B)** 18 **(C)** 216 **(D)** 30 **(E)** 36

2. If $3 \times 2 + 8 = \nabla + 5$, then ∇ equals

   (A) 14 **(B)** 25 **(C)** 19 **(D)** 17 **(E)** 9

3. In the diagram, PQR, QST and PSU are straight lines. The value of x is

   (A) 75 **(B)** 85 **(C)** 95
   (D) 125 **(E)** 155



4. If $w = 4$, $x = 9$, and $z = 25$ then $\sqrt{\frac{w}{x}} + \sqrt{\frac{x}{z}}$ equals

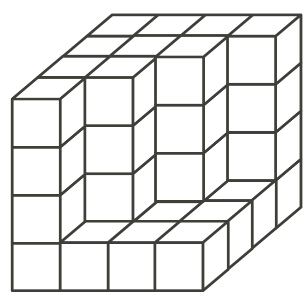
   (A) $\frac{5}{8}$ **(B)** $\frac{19}{15}$ **(C)** $\frac{77}{225}$ **(D)** $\frac{181}{225}$ **(E)** $\frac{2}{5}$

5. $1 - 4(3-1)^{-1}$ is equal to

   (A) -1 **(B)** $-\frac{3}{2}$ **(C)** 9 **(D)** 6 **(E)** $\frac{11}{3}$

6. Sixty-four identical cubes are stacked in a $4 \times 4 \times 4$ arrangement and then some of the cubes are removed from the front as shown. No cube hidden from sight has been removed. How many cubes remain in the arrangement?

   (A) 46 **(B)** 40 **(C)** 52
   (D) 55 **(E)** 49



7. If $n > 0$ and $\sqrt{n^2 + n^2 + n^2 + n^2} = 64$, then n equals

   (A) $\sqrt{8}$ **(B)** 16 **(C)** 4 **(D)** 32 **(E)** $\sqrt{2}$

8. Gavin has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from his collection that he can play in 3 hours?

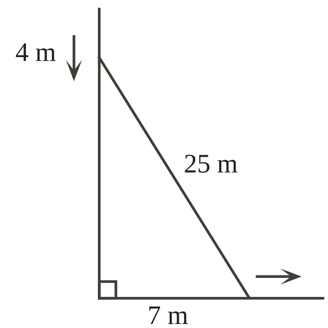
   (A) 100 **(B)** 36 **(C)** 56 **(D)** 60 **(E)** 45

9. In the diagram, any ♠ may be moved to any unoccupied space. What is the smallest number of ♠'s that must be moved so that each row and each column contains three ♠'s?

   (A) 1 **(B)** 2 **(C)** 3
   (D) 4 **(E)** 5

♠	♠	♠	♠	
♠	♠	♠		♠
♠	♠			
♠	♠	♠	♠	
		♠		

10. Judi leans a 25 m ladder against a vertical wall with the bottom of the ladder 7 m from the wall. (Please note that Judi is very strong – don't try this at home!) As she pulls the bottom of the ladder away from the wall, the top of the ladder slides 4 m down the wall. How far did she pull the bottom of the ladder from its original position?



(A) 4 m **(B)** 11 m **(C)** 2 m
(D) 13 m **(E)** 8 m

Part B: Each correct answer is worth 6.

11. Suppose m and n are positive integers with $m < n$. The value of $\frac{m+3}{n+3}$ will be

(A) equal to 1
(B) equal to 3
(C) less than the value of $\frac{m}{n}$
(D) greater than the value of $\frac{m}{n}$
(E) equal to the value of $\frac{m}{n}$

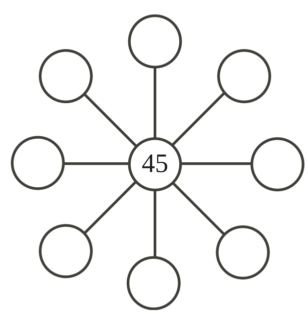
12. How many four-digit integers between 5000 and 6000 are there for which the thousands digit equals the sum of the other three digits? (The thousands digit of 5124 is 5.)

(A) 5 **(B)** 15 **(C)** 21 **(D)** 30 **(E)** 12

13. The number of integers x for which the value of $\frac{-6}{x+1}$ is an integer is

(A) 8 **(B)** 9 **(C)** 2 **(D)** 6 **(E)** 7

14. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?

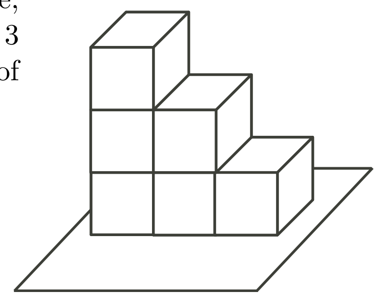


(A) 139 **(B)** 211 **(C)** 156
(D) 159 **(E)** 160

15. On Monday, 10% of the students at Dunkley S.S. were absent and 90% were present. On Tuesday, 10% of those who were absent on Monday were present and the rest of those absent on Monday were still absent. Also, 10% of those who were present on Monday were absent and the rest of those present on Monday were still present. What percentage of the students at Dunkley S.S. were present on Tuesday?

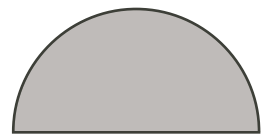
(A) 81% **(B)** 82% **(C)** 90% **(D)** 91% **(E)** 99%

16. Six dice are stacked on the floor as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?



(A) 69 **(B)** 88 **(C)** 89
(D) 91 **(E)** 96

17. In the diagram, the perimeter of the semicircular region is 20. (The perimeter includes both the semicircular arc and the diameter.) The area of the region is closest to



(A) 36.6 **(B)** 23.8 **(C)** 49.3
(D) 51.6 **(E)** 26.7

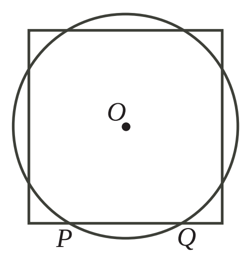
18. On Monday, Hank drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and took the same route. This time he drove at an average speed of 75 km/h and arrived 1 minute early. How long is his route to work?

(A) 30 km **(B)** 35 km **(C)** 45 km **(D)** 50 km **(E)** 60 km

19. If $2^x = 15$ and $15^y = 32$, the value of xy is

(A) 5 **(B)** 8 **(C)** 16 **(D)** 6 **(E)** 4

20. In the diagram, the circle and the square have the same centre O and equal areas. The circle has radius 1 and intersects one side of the square at P and Q. What is the length of PQ?



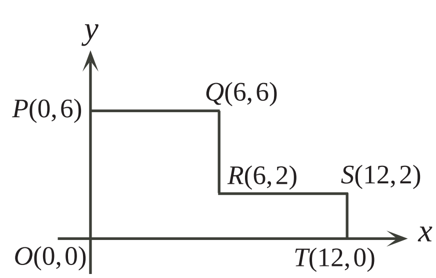
(A) $\sqrt{4-\pi}$ **(B)** 1 **(C)** $\sqrt{2}$
(D) $2-\sqrt{\pi}$ **(E)** $4-\sqrt{\pi}$

Part C: Each correct answer is worth 8.

21. At Matilda's birthday party, the ratio of people who ate ice cream to people who ate cake was 3 : 2. People who ate both ice cream and cake were included in both categories. If 120 people were at the party, what is the maximum number of people who could have eaten both ice cream and cake?

(A) 24 **(B)** 30 **(C)** 48 **(D)** 80 **(E)** 72

22. In the diagram, two straight lines are to be drawn through $O(0,0)$ so that the lines divide the figure $OPQRST$ into 3 pieces of equal area. The sum of the slopes of the lines will be



(A) $\frac{35}{24}$ **(B)** $\frac{7}{6}$ **(C)** $\frac{5}{4}$
(D) $\frac{4}{3}$ **(E)** $\frac{11}{8}$

23. Suppose that a, b, c, and d are positive integers that satisfy the equations

$$\begin{aligned} ab + cd &= 38 \\ ac + bd &= 34 \\ ad + bc &= 43 \end{aligned}$$

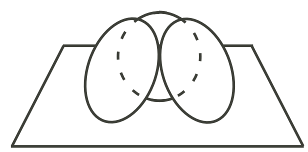
What is the value of $a + b + c + d$?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

24. Starting with the input (m, n), Machine A gives the output (n, m).
Starting with the input (m, n), Machine B gives the output $(m + 3n, n)$.
Starting with the input (m, n), Machine C gives the output $(m - 2n, n)$.
Natalie starts with the pair $(0, 1)$ and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with $(0, 1)$, she could use machines B, B, A, C, B in that order to obtain the output $(7, 6)$.) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?

(A) (2009,1016) **(B)** (2009,1004) **(C)** (2009,1002)
(D) (2009,1008) **(E)** (2009,1032)

25. In the diagram, three circles of radius 10 are tangent to each other and to a plane in three-dimensional space. Each of the circles is inclined at 45° to the plane. There are three points where the circles touch each other. These three points lie on a circle parallel to the plane. The radius of this circle is closest to



(A) 6.9 **(B)** 7.1 **(C)** 7.3
(D) 7.5 **(E)** 7.7

Canadian Mathematics Competition

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Tuesday, February 19, 2008

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Time: 60 minutes

Calculators are permitted

Instructions

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{1^2+2^2+3^2+4^2}{1\times 2\times 3}$ is

   (A) 110 **(B)** 22 **(C)** $\dfrac{50}{3}$ **(D)** 5 **(E)** 14

2. The value of $6\left(\frac{3}{2}+\frac{2}{3}\right)$ is

   (A) 13 **(B)** 6 **(C)** $\frac{13}{6}$ **(D)** $\frac{29}{3}$ **(E)** 5

3. If $1+2+3+4+5+x=21+22+23+24+25$, then the value of x is

   (A) 11 **(B)** 210 **(C)** 100 **(D)** 20 **(E)** 26

4. An empty truck weighs 9600 kg. When the truck is loaded with 40 identical crates, the total weight is 38 000 kg. The weight of each crate is

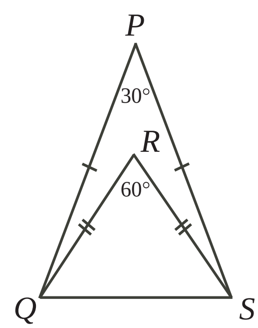
   (A) 460 kg **(B)** 950 kg **(C)** 1190 kg **(D)** 240 kg **(E)** 710 kg

5. If $\dfrac{18}{\sqrt{x}}=2$, then the value of x is

   (A) 81 **(B)** 36 **(C)** 18 **(D)** 9 **(E)** 3

6. In the diagram, what is the measure of $\angle PQR$?

   (A) 45° **(B)** 30° **(C)** 60°
   (D) 75° **(E)** 15°



7. If p is an odd integer and q is an even integer, which one of the following is an odd integer?

   (A) $2p+3q$ **(B)** $3p+2q$ **(C)** $4p+q$ **(D)** $2(p+3q)$ **(E)** pq

8. Two 3-digit integers, abc and def, have the following property:

$$\begin{array}{rcccc} & & a & b & c \\ + & & d & e & f \\ \hline & 1 & 0 & 0 & 0 \end{array}$$

   None of a, b, c, d, e, or f is 0. What is $a+b+c+d+e+f$?

   (A) 10 **(B)** 19 **(C)** 21 **(D)** 28 **(E)** 30

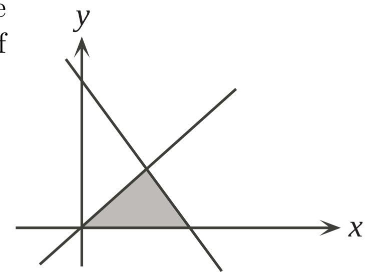
9. Beshmi invested $\frac{1}{5}$ of her savings in Company X, 42% in Company Y, and the remainder in Company Z. If Beshmi invested \$10 500 in Company Y, how much did she invest in Company Z?

(A) \$25 000 **(B)** \$15 500 **(C)** \$14 000 **(D)** \$9500 **(E)** \$5000

10. In the diagram, the shaded region is bounded by the x-axis and the lines $y = x$, and $y = -2x + 3$. The area of the shaded region is

(A) $\frac{3}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{4}$

(D) 1 **(E)** $\frac{\sqrt{10}}{4}$



Part B: Each correct answer is worth 6.

11. If $\dfrac{1}{x} = 2$ and $\dfrac{1}{x} + \dfrac{3}{y} = 3$, then the value of $x + y$ is

(A) 3 **(B)** $\frac{5}{6}$ **(C)** $\frac{7}{3}$ **(D)** $\frac{7}{2}$ **(E)** $\frac{4}{3}$

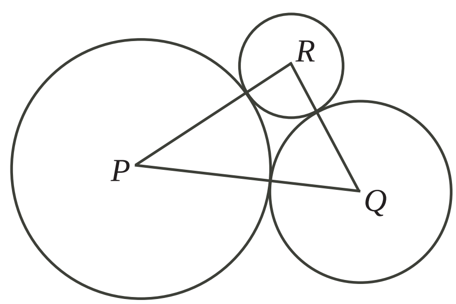
12. On seven tests, each out of 100 marks, Siobhan received marks of 69, 53, 69, 71, 78, x, and y. If her average mark on the seven tests is 66, then the minimum possible value of x is

(A) 22 **(B)** 68 **(C)** 61 **(D)** 53 **(E)** 0

13. In the diagram, the circles with centres P, Q and R have radii 3, 2 and 1 respectively. Each circle touches the other two as shown. The area of $\triangle PQR$ is

(A) 12 **(B)** 6 **(C)** 7.5

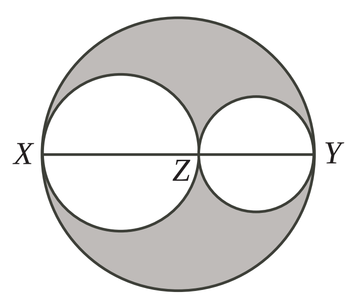
(D) 10 **(E)** 4



14. In the diagram, Z lies on XY and the three circles have diameters XZ, ZY and XY. If $XZ = 12$ and $ZY = 8$, then the ratio of the area of the shaded region to the area of the unshaded region is

(A) 12 : 25 **(B)** 12 : 13 **(C)** 1 : 1

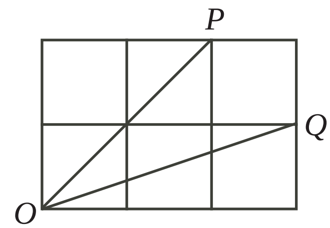
(D) 1 : 2 **(E)** 2 : 3



15. In a relay race, Ainslee runs the first lap in 72 seconds. Bridget runs the next lap at $\frac{9}{10}$ of Ainslee's speed. Cecilia runs the next lap at $\frac{4}{3}$ of Bridget's speed. Dana runs the last lap at $\frac{6}{5}$ of Cecilia's speed. What is their total time, to the nearest second?

(A) 4 minutes, 48 seconds
(B) 4 minutes, 22 seconds
(C) 5 minutes, 27 seconds
(D) 4 minutes, 37 seconds
(E) 3 minutes, 46 seconds

16. In the diagram, the six small squares all have side length 2. Lines are drawn from O to P and O to Q. The measure of $\angle POQ$ in degrees, accurate to one decimal place, is



(A) 15.0 **(B)** 25.5 **(C)** 26.6
(D) 22.5 **(E)** 30.0

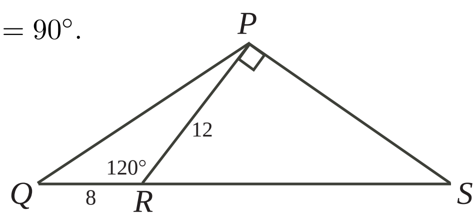
17. The difference between the squares of two consecutive integers is 199. The sum of the squares of these two consecutive integers is

(A) 19 801 **(B)** 39 601 **(C)** 19 602 **(D)** 20 201 **(E)** 19 405

18. An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term.
If the first four terms of an arithmetic sequence are a, $2a$, b, and $a - 6 - b$ for some numbers a and b, then the value of the 100th term is

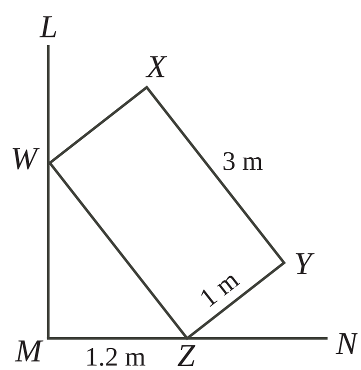
(A) -100 **(B)** -300 **(C)** 150 **(D)** -150 **(E)** 100

19. In the diagram, R is on QS and $QR = 8$.
Also, $PR = 12$, $\angle PRQ = 120°$, and $\angle RPS = 90°$.
What is the area of $\triangle QPS$?



(A) $72\sqrt{3}$ **(B)** 72 **(C)** 36
(D) $60\sqrt{3}$ **(E)** $96\sqrt{3}$

20. In the diagram, LM is perpendicular to MN. Rectangle $WXYZ$ has W on LM and Z on MN. Also, $YZ = 1$ m, $XY = 3$ m and $MZ = 1.2$ m. What is the distance from X to line MN, to the nearest hundredth of a metre?



(A) 2.75 m **(B)** 3.67 m **(C)** 3.15 m
(D) 3.26 m **(E)** 3.63 m

Part C: Each correct answer is worth 8.

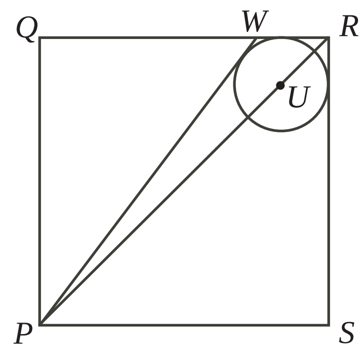
21. Suppose $N = 1 + 11 + 101 + 1001 + 10001 + \ldots + 1\overbrace{000\ldots0000}^{50 \text{ zeroes}}1$.
When N is calculated and written as a single integer, the sum of its digits is

(A) 58 **(B)** 99 **(C)** 55 **(D)** 50 **(E)** 103

22. For how many integers k do the parabolas with equations $y = -\frac{1}{8}x^2 + 4$ and $y = x^2 - k$ intersect on or above the x-axis?

(A) 9 **(B)** 32 **(C)** 33 **(D)** 36 **(E)** 37

23. Square $PQRS$ has side length 4 m. Point U is on PR with $PR = 4UR$. A circle centered at U touches two sides of the square. PW is a tangent to the circle, with W on QR. The length of PW, to the nearest thousandth of a metre, is

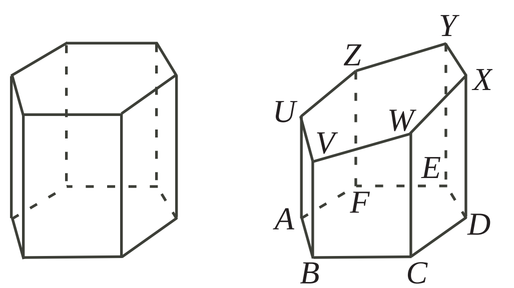


(A) 4.123 m **(B)** 4.472 m **(C)** 4.685 m
(D) 4.726 m **(E)** 4.767 m

24. The number of triples (a, b, c) of positive integers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{4}$ is

(A) 16 **(B)** 25 **(C)** 31 **(D)** 19 **(E)** 34

25. A right regular hexagonal prism is sliced as shown in the diagram. The bottom of the new solid is a regular hexagon $ABCDEF$. The six side faces are trapezoids perpendicular to $ABCDEF$. The top is a hexagon $UVWXYZ$ that is not necessarily a regular hexagon.



Of the six edges AU, BV, CW, DX, EY, and FZ, three have lengths 4, 7 and 10. The largest possible value for $AU + BV + CW + DX + EY + FZ$ is

(A) 42 **(B)** 51 **(C)** 69 **(D)** 78 **(E)** 91

2008 Fermat Contest (English)

Canadian Mathematics Competition

For students...

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest *(Grade 11)*

Tuesday, February 20, 2007

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Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{36-12}{12-4}$ is

   (A) 6 **(B)** 9 **(C)** 1 **(D)** 31 **(E)** 3

2. If $7x = 28$ and $x + w = 9$, what is the value of xw?

   (A) 9 **(B)** 20 **(C)** 18 **(D)** 52 **(E)** -252

3. Of the fractions $\dfrac{3}{4}$, $\dfrac{7}{8}$, $\dfrac{13}{16}$, and $\dfrac{1}{2}$, what is the difference between the largest and the smallest?

   (A) $\dfrac{3}{8}$ **(B)** $\dfrac{6}{7}$ **(C)** $\dfrac{5}{16}$ **(D)** $\dfrac{1}{16}$ **(E)** $\dfrac{1}{8}$

4. When $x = -5$, the value of $-2x^2 + \dfrac{5}{x}$ is

   (A) 99 **(B)** 101 **(C)** -51 **(D)** 19 **(E)** -49

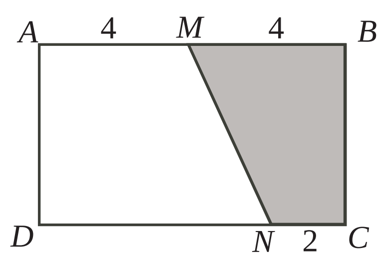
5. What is the value of $1^{-2} + 2^{-1}$?

   (A) $\frac{3}{2}$ **(B)** $\frac{1}{27}$ **(C)** 4 **(D)** -4 **(E)** 9

6. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is

   (A) 15 **(B)** 10 **(C)** 30

   (D) 12 **(E)** 16



7. The product of three positive integers is 42. The sum of two of these integers is 9. The third integer is

   (A) 1 **(B)** 7 **(C)** 6 **(D)** 3 **(E)** 2

8. Ivan trained for a cross-country meet.
   On Monday, he ran a certain distance.
   On Tuesday, he ran twice as far as he ran on Monday.
   On Wednesday, he ran half as far as he ran on Tuesday.
   On Thursday, he ran half as far as he ran on Wednesday.
   On Friday, he ran twice as far as he ran on Thursday.
   If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?

   (A) 55 km **(B)** 25 km **(C)** 27.5 km **(D)** 17.5 km **(E)** 50 km

9. If $\dfrac{1}{x+3} = 2$, then the value of $\dfrac{1}{x+5}$ is

   (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{4}$ **(E)** 4

10. A store normally sells each of its DVDs for \$20. At a sale, Phyllis buys two DVDs at the regular price and gets a third DVD for half price. This is the same rate of discount as getting

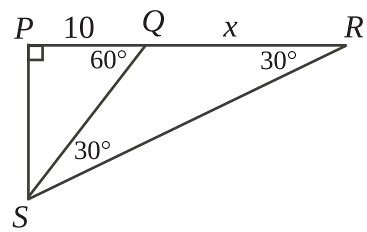
(A) 2 for the price of 1
(B) 3 for the price of 2
(C) 4 for the price of 3
(D) 5 for the price of 4
(E) 6 for the price of 5

Part B: Each correct answer is worth 6.

11. Five numbers in increasing order are 2, 5, x, 10, and y. The median of the numbers is 7 and the mean (average) is 8. The value of y is

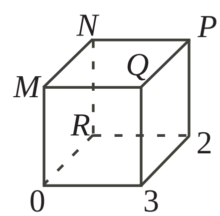
(A) 16 **(B)** 14 **(C)** 15 **(D)** 18 **(E)** 12

12. In the diagram, $PQ = 10$ and $QR = x$. The value of x is



(A) $10\sqrt{3}$ **(B)** 20 **(C)** $\frac{50}{3}$
(D) $\frac{20}{\sqrt{3}}$ **(E)** 10

13. In the diagram, each of the numbers 0, 1, 2, 3, 4, 5, 6, and 7 is to be used to label a vertex of the cube. The numbers 0, 2 and 3 are placed as shown. The sum of the numbers at the ends of each edge must be a prime number. (Note: 1 is not a prime number.) The value of $M + N + P + Q$ must be



(A) 16 **(B)** 17 **(C)** 18
(D) 19 **(E)** 22

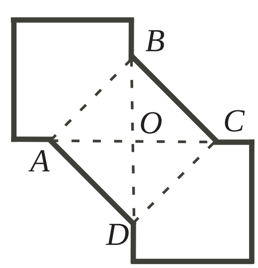
14. Two positive integers a and b have the property that if a is increased by 25%, the result will be *greater than* five times the value of b. What is the minimum possible value for $a + b$?

(A) 3 **(B)** 6 **(C)** 10 **(D)** 9 **(E)** 21

15. How many three-digit positive integers x are there with the property that x and $2x$ have only even digits? (One such number is $x = 420$, since $2x = 840$ and each of x and $2x$ has only even digits.)

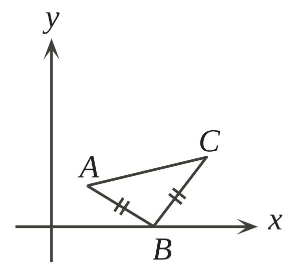
(A) 64 **(B)** 18 **(C)** 16 **(D)** 125 **(E)** 100

16. In the diagram, each of the three squares has a side length of 3. Two of the squares have a common vertex O, and O is the centre of the square labelled $ABCD$. The perimeter of the entire figure is closest to



(A) 21.5 **(B)** 22.0 **(C)** 22.5
(D) 24.0 **(E)** 30.0

17. In the diagram, $A(2,2)$ and $C(8,4)$ are two of the vertices of an isosceles right-angled triangle ABC. If the vertex B is located on the x-axis and $\angle ABC = 90°$, the x-coordinate of B is

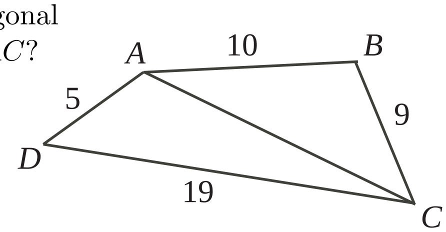


(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

18. Alphonso and Karen started out with the *same number* of apples. Karen gave twelve of her apples to Alphonso. Next, Karen gave half of her remaining apples to Alphonso. If Alphonso now has four times as many apples as Karen, how many apples does Karen now have?

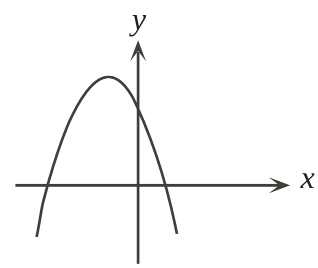
(A) 12 **(B)** 24 **(C)** 36 **(D)** 48 **(E)** 72

19. In the diagram, $ABCD$ is a quadrilateral with diagonal AC. Which of the following is a possible length for AC?



(A) 9 **(B)** 10 **(C)** 13 **(D)** 15 **(E)** 20

20. The graph of the function $y = ax^2 + bx + c$ is shown in the diagram. Which of the following must be positive?



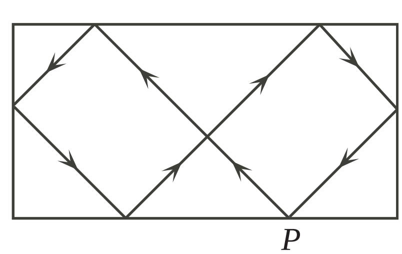
(A) a **(B)** bc **(C)** ab^2 **(D)** $b - c$ **(E)** $c - a$

Part C: Each correct answer is worth 8.

21. Five consecutive positive integers have the property that the sum of the second, third and fourth is a perfect square, while the sum of all five is a perfect cube. If m is the third of these five integers, then the minimum possible value of m satisfies

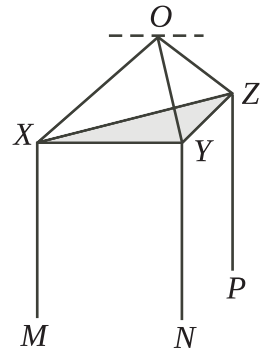
(A) $m \leq 200$
(B) $200 < m \leq 400$
(C) $400 < m \leq 600$
(D) $600 < m \leq 800$
(E) $m > 800$

22. A ball placed at point P on a rectangular billiard table is shot at an angle of 45° to the edge of the table. After successively bouncing off the edges of the table at 45° angles, it returns to point P, as shown. If the ball travels 7 m, the perimeter, in metres, of the table is closest to



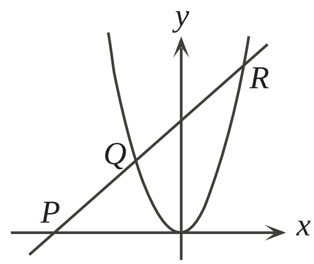
(A) 7.0 **(B)** 7.5 **(C)** 8.0 **(D)** 8.5 **(E)** 9.0

23. An ugly light fixture is hanging from point O on the ceiling. Wires OXM, OYN and OZP pass through the vertices of a very thin wooden equilateral triangle XYZ of side 60 cm. (A small bulb is attached to the end of each wire.) The plane of the wooden triangle is parallel to the ceiling. If each wire is 100 cm long and the lower end of each wire is 90 cm from the ceiling, what is the vertical distance between the wooden triangle and the ceiling?



(A) 40 cm **(B)** 45 cm **(C)** 50 cm
(D) 55 cm **(E)** 60 cm

24. A line with slope 1 passes through point P on the negative x-axis and intersects the parabola $y = x^2$ at points Q and R, as shown. If $PQ = QR$, then the y-intercept of PR is closest to



(A) 9.9 **(B)** 10.2 **(C)** 8.2
(D) 9.3 **(E)** 8.6

25. How many ordered pairs (b, g) of positive integers with $4 \leq b \leq g \leq 2007$ are there such that when b black balls and g gold balls are randomly arranged in a row, the probability that the balls on each end have the same colour is $\frac{1}{2}$?

(A) 60 **(B)** 62 **(C)** 58 **(D)** 61 **(E)** 59

Canadian Mathematics Competition

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Canadian
Mathematics
Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest *(Grade 11)*

Wednesday, February 22, 2006

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Canadian Institute of Actuaries

Time: 60 minutes

Calculators are permitted

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9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{1}{4 \times 5}$ is

   (A) 0.45 **(B)** 0.05 **(C)** 1.25 **(D)** 0.20 **(E)** 0.02

2. If $2x + 3x + 4x = 12 + 9 + 6$, then x equals

   (A) 6 **(B)** 3 **(C)** 1 **(D)** $\frac{1}{3}$ **(E)** $10\frac{1}{2}$

3. The value of $\dfrac{4^3}{10^2 - 6^2}$ is

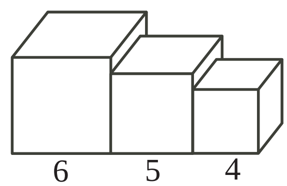
   (A) 1 **(B)** 0.5 **(C)** -35.36 **(D)** 1.5 **(E)** 4

4. The value of $\left(\sqrt{\sqrt{9} + \sqrt{1}}\right)^4$ is

   (A) $\sqrt{10}$ **(B)** 10 **(C)** 16 **(D)** 82 **(E)** 100

5. Three cubes have edges of lengths 4, 5 and 6.
   The average (mean) of their volumes is

   (A) 120 **(B)** 125 **(C)** 1125
   (D) 261 **(E)** 135



6. The regular price for a T-shirt is \$25 and the regular price for a pair of jeans is \$75. If the T-shirt is sold at a 30% discount and the jeans are sold at a 10% discount, then the total discount is

   (A) \$15 **(B)** \$20 **(C)** \$30 **(D)** \$36 **(E)** \$40

7. What is the smallest positive integer p for which $\sqrt{2^3 \times 5 \times p}$ is an integer?

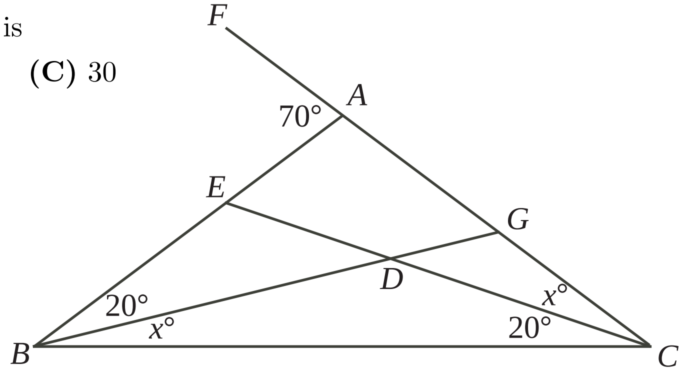
   (A) 2 **(B)** 5 **(C)** 10 **(D)** 1 **(E)** 20

8. If Corina had added the numbers P and Q correctly, the answer would have been 16. By mistake, she subtracted Q from P. Her answer was 4. What is the value of P?

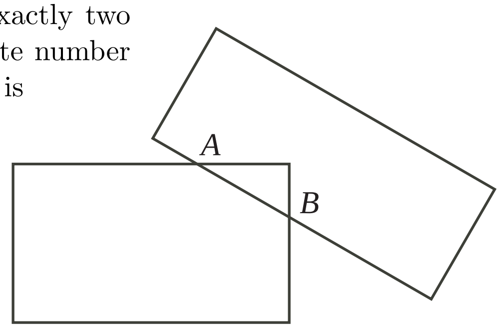
   (A) 4 **(B)** 5 **(C)** 8 **(D)** 10 **(E)** 16

9. In the diagram, the value of x is

   (A) 15 **(B)** 20 **(C)** 30
   (D) 35 **(E)** 50



10. In the diagram, two rectangles intersect at exactly two points, A and B. The maximum possible finite number of points of intersection of *any* two rectangles is



(A) 3 **(B)** 4 **(C)** 12 **(D)** 8 **(E)** 6

Part B: Each correct answer is worth 6.

11. If $\frac{a}{b} = 3$ and $\frac{b}{c} = 2$, then the value of $\frac{a-b}{c-b}$ is

(A) -4 **(B)** $-\frac{1}{3}$ **(C)** $\frac{2}{3}$ **(D)** 2 **(E)** 6

12. If $\left(2^4\right)\left(3^6\right) = 9\left(6^x\right)$, what is the value of x?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 216 **(E)** 8

13. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's *total* cost each year was the same. The cost of downloading the 360 songs in 2005 was

(A) \$144.00 **(B)** \$108.00 **(C)** \$80.00 **(D)** \$259.20 **(E)** \$72.00

14. If the system of equations

$$\begin{aligned} px + qy &= 8 \\ 3x - qy &= 38 \end{aligned}$$

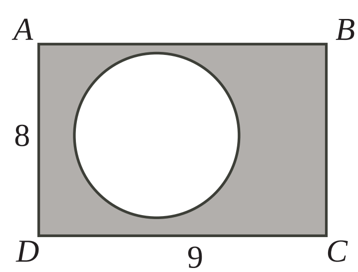
has the solution $(x, y) = (2, -4)$, then p is equal to

(A) -12 **(B)** 20 **(C)** 8 **(D)** 40 **(E)** 21.5

15. The points $(5, 3)$ and $(1, -1)$ are plotted on a sheet of graph paper. The sheet of graph paper is folded along a line so that the point $(5, 3)$ lands on top of the point $(1, -1)$. The equation of the line that represents the fold is

(A) $y = -x + 1$ **(B)** $y = -x + 2$ **(C)** $y = -x + 3$
(D) $y = -x + 4$ **(E)** $y = -x + 5$

16. In the diagram, $ABCD$ is a rectangle. If the area of the circle is equal to the area of the shaded region, the radius of the circle is



(A) $\sqrt{\frac{6}{\pi}}$ **(B)** $\frac{6}{\pi}$ **(C)** $\frac{6}{\sqrt{\pi}}$
(D) $\sqrt{\frac{18}{\pi}}$ **(E)** $\frac{18}{\pi}$

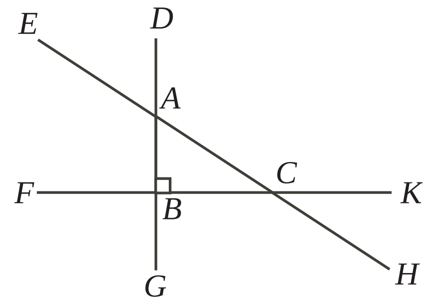
17. In seven term sequence, 5, p, q, 13, r, 40, x, each term after the third term is the sum of the preceding three terms. The value of x is

(A) 21 **(B)** 61 **(C)** 67 **(D)** 74 **(E)** 80

18. The front wheel of Georgina's bicycle has a diameter of 0.75 metres. She cycled for 6 minutes at a speed of 24 kilometres per hour. The number of complete rotations that the wheel made during this time is closest to

(A) 610 **(B)** 1020 **(C)** 1360 **(D)** 1700 **(E)** 5430

19. In the diagram, $\triangle ABC$ is right-angled. Side AB is extended in each direction to points D and G such that $DA = AB = BG$. Similarly, BC is extended to points F and K so that $FB = BC = CK$, and AC is extended to points E and H so that $EA = AC = CH$. The ratio of the area of the hexagon $DEFGHK$ to the area of $\triangle ABC$ is



(A) $4:1$ **(B)** $7:1$ **(C)** $9:1$

(D) $16:1$ **(E)** $13:1$

20. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes N marbles all at once. If he is to be sure that, no matter which choice of N marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of N?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

Part C: Each correct answer is worth 8.

21. For how many integers n, with $2 \leq n \leq 80$, is $\dfrac{(n-1)(n)(n+1)}{8}$ equal to an integer?

(A) 10 **(B)** 20 **(C)** 59 **(D)** 39 **(E)** 49

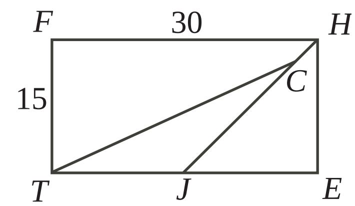
22. Quincy and Celine have to move 16 small boxes and 10 large boxes. The chart indicates the time that each person takes to move each type of box. They start moving the boxes at 9:00 a.m. The earliest time at which they can be finished moving all of the boxes is

	Celine	Quincy
small box	2 min.	3 min.
large box	6 min.	5 min.

(A) 9:41 a.m. **(B)** 9:42 a.m. **(C)** 9:43 a.m.

(D) 9:44 a.m. **(E)** 9:45 a.m.

23. Rectangle $TEHF$ has dimensions 15 m by 30 m, as shown. Tom the Cat begins at T, and Jerry the Mouse begins at J, the midpoint of TE. Jerry runs at 3 m/s in a straight line towards H. Tom starts at the same time as Jerry, and, running at 5 m/s in a straight line, arrives at point C at the same time as Jerry. The time, in seconds, that it takes Tom to catch Jerry is closest to

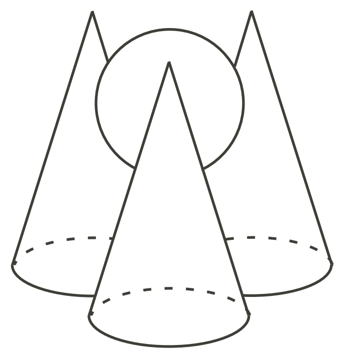


(A) 5.4 **(B)** 5.6 **(C)** 5.8
(D) 6.0 **(E)** 6.2

24. If a and b are positive integers such that $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 - 2b}$, then the smallest possible value of $a + b$ is

(A) 8 **(B)** 6 **(C)** 96 **(D)** 10 **(E)** 50

25. Three identical cones each have a radius of 50 and a height of 120. The cones are placed so that their circular bases are touching each other. A sphere is placed so that it rests in the space created by the three cones, as shown. If the top of the sphere is level with the tops of the cones, then the radius of the sphere is closest to



(A) 38.9 **(B)** 38.7 **(C)** 38.1
(D) 38.5 **(E)** 38.3

2006 Fermat Contest (English)

Canadian Mathematics Competition

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3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{150 + (150 \div 10)}{15 - 5}$ is

   (A) 6 **(B)** 3 **(C)** 146 **(D)** 151.5 **(E)** 16.5

2. $\frac{1}{2} - \frac{1}{3} + \frac{3}{9}$ equals

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{1}{9}$ **(E)** 0

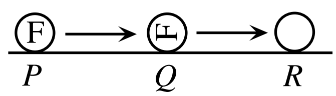
3. If $a = \frac{1}{2}$ and $b = \frac{2}{3}$, then $\dfrac{6a + 18b}{12a + 6b}$ equals

   (A) 9 **(B)** 7 **(C)** 10 **(D)** 6 **(E)** $\frac{3}{2}$

4. If $\sqrt{4 + 9 + x^2} = 7$, then a possible value for x is

   (A) 6 **(B)** 2 **(C)** 4 **(D)** 36 **(E)** 0

5. A Fermat coin rolls from P to Q to R, as shown. If the distance from P to Q is equal to the distance from Q to R, what is the orientation of the coin when it reaches R?



   (A)  **(B)**  **(C)** 

   (D)  **(E)** 

6. The sum of the first 2005 terms of the sequence 1, 2, 3, 4, 1, 2, 3, 4, . . . is

   (A) 5011 **(B)** 5110 **(C)** 5020 **(D)** 5010 **(E)** 501

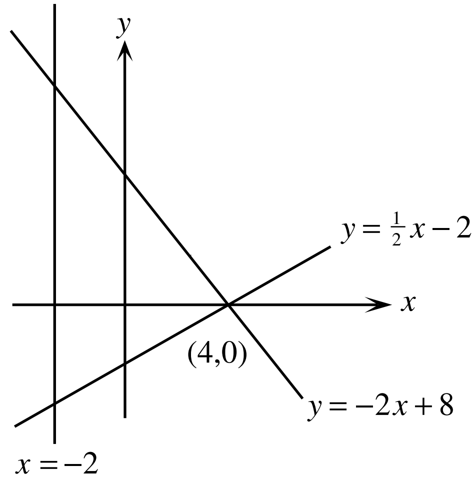
7. In triangle ABC, $\angle A$ is 21° more than $\angle B$, and $\angle C$ is 36° more than $\angle B$. The size of $\angle B$ is

   (A) 20° **(B)** 41° **(C)** 62° **(D)** 46° **(E)** 56°

8. Seven children, each with the same birthday, were born in seven consecutive years. The sum of the ages of the youngest three children is 42. What is the sum of the ages of the oldest three?

   (A) 51 **(B)** 54 **(C)** 57 **(D)** 60 **(E)** 63

9. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. The area of the triangle formed by these two lines and the line $x = -2$ is



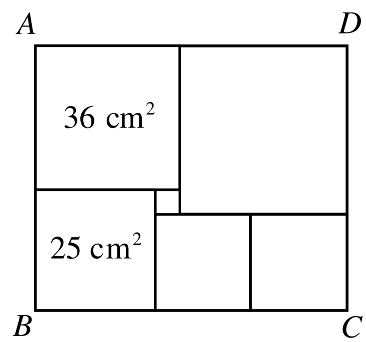
(A) 15 **(B)** 27 **(C)** 30
(D) 36 **(E)** 45

10. If 50% of P equals 20% of Q, then P, as a percent of Q, is

(A) 60% **(B)** 250% **(C)** 40% **(D)** 20% **(E)** 30%

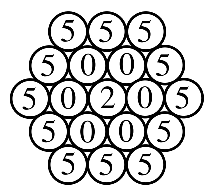
Part B: Each correct answer is worth 6.

11. Rectangle $ABCD$ is made up of six squares. The areas of two of the squares are shown. The perimeter of rectangle $ABCD$, in centimetres, is



(A) 50 **(B)** 44 **(C)** 46
(D) 52 **(E)** 48

12. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?



(A) 36 **(B)** 24 **(C)** 12
(D) 18 **(E)** 6

13. A circle is drawn so that no part of it lies outside a *regular* hexagon. If such a circle does not touch all six sides of the hexagon, what is the maximum number of sides that it could touch?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

14. The weight of a lioness is six times the weight of her female cub and four times the weight of her male cub. If the difference between the weights of the male and female cub is 14 kg, the weight of the lioness, in kg, is

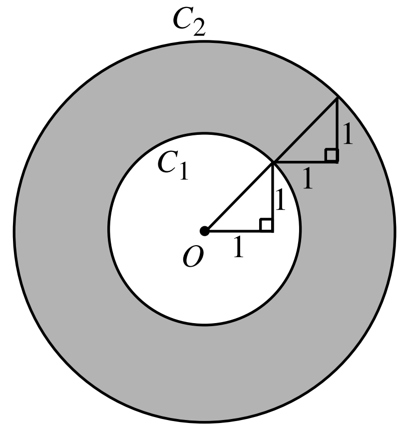
(A) 84 **(B)** 252 **(C)** 168 **(D)** 140 **(E)** 112

15. If $(x - 4)(5x + 2) = 0$, then the two possible values of $5x + 2$ are

(A) -4 and $\frac{2}{5}$ **(B)** 0 and -18 **(C)** 0 and 22 **(D)** 0 and 4 **(E)** 4 and 22

16. In the diagram, circles C_1 and C_2 each have center O. The area of the shaded region is

(A) 2π **(B)** 3π **(C)** 4π **(D)** 6π **(E)** 8π



17. A cylinder with radius 2 cm and height 8 cm is full of water. A second cylinder of radius 4 cm and height 8 cm is empty. If all of the water is poured from the first cylinder into the second cylinder, the depth of the water in the second cylinder will be

(A) 1 cm **(B)** 2 cm **(C)** 3 cm **(D)** 4 cm **(E)** 6 cm

18. A test has ten questions. Points are awarded as follows:
• Each correct answer is worth 3 points.
• Each unanswered question is worth 1 point.
• Each incorrect answer is worth 0 points.
A total score that is *not* possible is

(A) 11 **(B)** 13 **(C)** 17 **(D)** 23 **(E)** 29

19. Sam bicycles at 16 km/h and Chris bicycles at 24 km/h. At noon, Sam is 1 km north of Chris, and each begins to ride north. How many minutes will it take for Chris to catch Sam?

(A) $1\frac{1}{2}$ **(B)** $2\frac{1}{2}$ **(C)** $3\frac{3}{4}$ **(D)** $7\frac{1}{2}$ **(E)** 8

20. In triangle ABC, if $AB = AC = x + 1$ and $BC = 2x - 2$, where $x > 1$, then the area of the triangle is always equal to

(A) $(x-1)\sqrt{2x^2+2}$ **(B)** $2(x-1)$ **(C)** $\frac{1}{2}(x+1)^2$
(D) $(x+1)(x-1)$ **(E)** $2(x-1)\sqrt{x}$

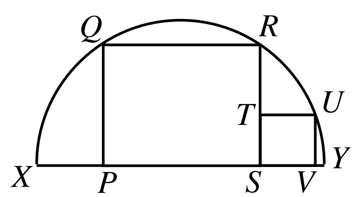
Part C: Each correct answer is worth 8.

21. Four *different* numbers a, b, c, and d are chosen from the list -1, -2, -3, -4, and -5. The largest possible value for the expression $a^b + c^d$ is

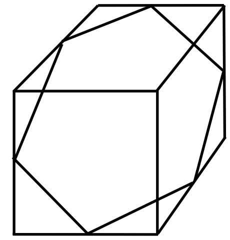
(A) $\frac{5}{4}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{31}{32}$ **(D)** $\frac{10}{9}$ **(E)** $\frac{26}{25}$

22. In the diagram, a semi-circle has diameter XY. Rectangle $PQRS$ is inscribed in the semi-circle with $PQ = 12$ and $QR = 28$. Square $STUV$ has T on RS, U on the semi-circle and V on XY. The area of $STUV$ is closest to

(A) 12 **(B)** 13 **(C)** 16
(D) 14 **(E)** 15



23. A solid cube of side length 4 cm is cut into two pieces by a plane that passed through the midpoints of six edges, as shown. To the nearest square centimetre, the surface area of each half cube created is



(A) 69 **(B)** 48 **(C)** 32 **(D)** 65 **(E)** 58

24. The arithmetic sequence a, $a+d$, $a+2d$, $a+3d$, ..., $a+(n-1)d$ has the following properties:

- When the first, third, and fifth, and so on terms are added, up to and including the last term, the sum is 320.
- When the first, fourth, seventh, and so on, terms are added, up to and including the last term, the sum is 224.

What is the sum of the whole sequence?

(A) 656 **(B)** 640 **(C)** 608 **(D)** 704 **(E)** 672

25. A *triline* is a line with the property that three times its slope is equal to the sum of its x-intercept and its y-intercept. For how many integers q with $1 \leq q \leq 10\,000$ is there at least one positive integer p so that there is exactly one triline through (p, q)?

(A) 60 **(B)** 57 **(C)** 58 **(D)** 61 **(E)** 59

N.B. This problem has been corrected from its original version with the addition of the underlined word "positive".

2005 Fermat Contest (English)

Canadian Mathematics Competition

For students...

Thank you for writing the 2005 Fermat Contest!
In 2004, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 18, 2004

C.M.C. Sponsors:

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*i*Anywhere Solutions

Time: 1 hour

Calculators are permitted.

Instructions

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{10}{10(11)-10^2}$ is

   (**A**) 100 (**B**) 2 (**C**) 10 (**D**) 1 (**E**) 11

2. $\sqrt{4^0+4^2+4^3}$ equals

   (**A**) 9 (**B**) 13 (**C**) 14 (**D**) 32 (**E**) 64

3. If $x=2-4+6$ and $y=1-3+5$, then $x-y$ equals

   (**A**) 0 (**B**) 1 (**C**) 5 (**D**) 3 (**E**) –1

4. A lemon loaf completely fills a pan measuring 20 cm by 18 cm by 5 cm. The loaf is cut into 25 pieces of equal volume. If the density of the loaf is $2\ \text{g}/\text{cm}^3$, how much does each of the 25 pieces weigh?

   (**A**) 72 g (**B**) 288 g (**C**) 36 g (**D**) 144 g (**E**) 720 g

5. If $\left(\dfrac{1}{2+3}\right)\left(\dfrac{1}{3+4}\right)=\dfrac{1}{x+5}$, the value of x is

   (**A**) 4 (**B**) 7 (**C**) 30 (**D**) 37 (**E**) 67

6. Three cans of juice fill $\frac{2}{3}$ of a one-litre jug. How many cans of juice are needed to completely fill 8 one-litre jugs?

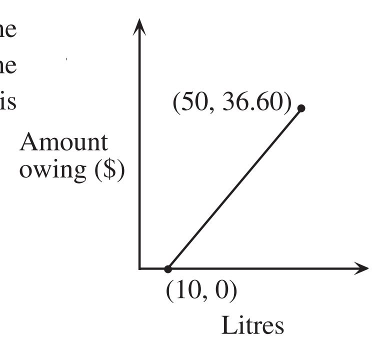
   (**A**) 36 (**B**) 12 (**C**) $\frac{16}{3}$ (**D**) 16 (**E**) 24

7. When $x=\frac{1}{5}$, the value of the expression $\dfrac{x^2-4}{x^2-2x}$ is

   (**A**) 0.4 (**B**) -0.52 (**C**) -5 (**D**) 10 (**E**) 11

8. Jane arrives at the Fermat Fuel Fill-up to fill up her gas tank. The graph shows the amount of gas that Jane had upon arrival, the amount that she purchased, and the cost of this purchase. What is the price per litre of the gas that she purchased?

   (**A**) 91.5¢ (**B**) 73.2¢ (**C**) 61.0¢
   (**D**) 53.2¢ (**E**) \$1.09



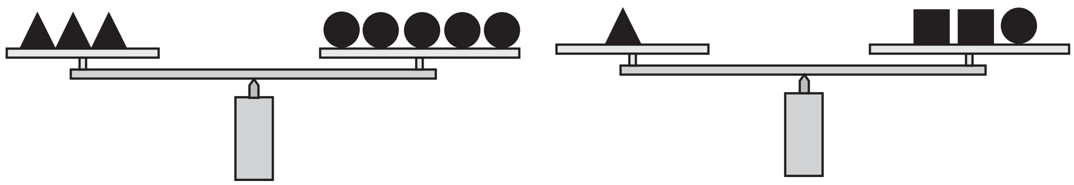
9. The table shows population information for two towns for the years 2003 and 2004.

Town	2003 Population	Percentage Change From 2003 to 2004
Cayleyville	10 000	4%
Pascalberg	25 000	–12%

What is the difference between the populations of the two towns in 2004?

(**A**) 12 400 (**B**) 11 600 (**C**) 17 600 (**D**) 13 800 (**E**) 17 400

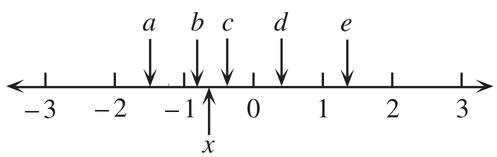
10. In the diagram, two equal-armed balances are shown. How many ■ would it take to balance one ●?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Part B: Each correct answer is worth 6.

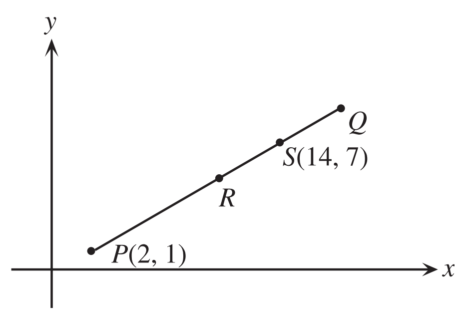
11. If x is located on the number line as shown, which letter best corresponds to the location of $-x^2$?



(**A**) a (**B**) b (**C**) c (**D**) d (**E**) e

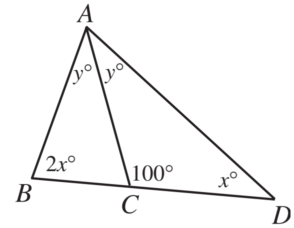
12. Point R is the midpoint of the line segment PQ and S is the midpoint of the line segment QR. If P has coordinates $(2,1)$ and S has coordinates $(14,7)$, then the coordinates of Q are

(**A**) $(8,4)$ (**B**) $(26,13)$ (**C**) $(10,5)$
(**D**) $(18,9)$ (**E**) $(16,11)$

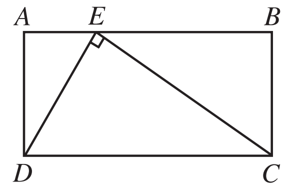


13. In the diagram, B, C and D lie on a straight line, with $\angle ACD = 100^\circ$, $\angle ADB = x^\circ$, $\angle ABD = 2x^\circ$, and $\angle DAC = \angle BAC = y^\circ$. The value of x is

(**A**) 10 (**B**) 45 (**C**) 30
(**D**) 50 (**E**) 20



14. In the diagram, $ABCD$ is a rectangle and point E lies on AB. Triangle DEC has $\angle DEC = 90°$, $DE = 3$ and $EC = 4$. The length of AD is

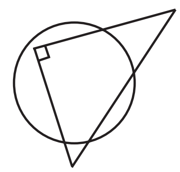


(**A**) 2.6 (**B**) 2.4 (**C**) 2.8
(**D**) 1.8 (**E**) 3.2

15. The graph of $x^2 - y^2 = 0$ is

(**A**) a straight line (**B**) a parabola (**C**) a circle
(**D**) a single point (**E**) two straight lines

16. A right-angled triangle has sides of length 6, 8 and 10. A circle is drawn so that the area inside the circle but outside this triangle equals the area inside the triangle but outside the circle. The radius of the circle is closest to



(**A**) 2.9 (**B**) 2.8 (**C**) 3.8
(**D**) 3.1 (**E**) 3.6

17. An increasing sequence is formed so that the difference between consecutive terms is a constant. If the first four terms of this sequence are x, y, $3x+y$, and $x+2y+2$, then the value of $y-x$ is

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

18. If $y = a(x-2)^2 + c$ and $y = (2x-5)(x-b)$ represent the same quadratic function, the value of b is

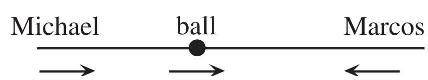
(**A**) 3 (**B**) $\frac{3}{2}$ (**C**) $\frac{4}{5}$ (**D**) $-\frac{5}{2}$ (**E**) $\frac{8}{5}$

19. A computer software retailer has 1200 copies of a new software package to sell. From past experience, she knows that:
- Half of them will sell right away at the original price she sets,
- Two-thirds of the remainder will sell later when their price is reduced by 40%, and
- The remaining copies will sell in a clearance sale at 75% off the original price.

In order to make a reasonable profit, the total sales revenue must be $72 000. To the nearest cent, what original price should she set?

(**A**) $60.01 (**B**) $75.01 (**C**) $79.13 (**D**) $80.90 (**E**) $240.01

20. A soccer ball rolls at 4 m/s towards Marcos in a direct line from Michael. The ball is 15 m ahead of Michael who is chasing it at 9 m/s. Marcos is 30 m away from the ball and is running towards it at 8 m/s. The distance between Michael and Marcos when the ball is touched for the first time by one of them is closest to



(**A**) 2.00 m (**B**) 2.25 m (**C**) 2.50 m
(**D**) 2.75 m (**E**) 3.00 m

continued ...

Part C: Each correct answer is worth 8.

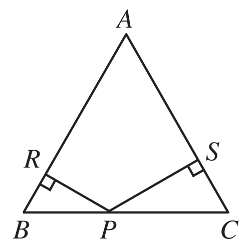
21. Bill and Jill are hired to paint a line on a road. If Bill works by himself, he could paint the line in B hours. If Jill works by herself, she could paint the line in J hours. Bill starts painting the line from one end, and Jill begins painting the line from the other end one hour later. They both work until the line is painted. Which is the following is an expression for the number of hours that Bill works?

(**A**) $\frac{B(J+1)}{B+J}$ (**B**) $J+1$ (**C**) $\frac{BJ}{B+J}+1$ (**D**) $\frac{B+J-1}{2}$ (**E**) $\frac{B(J-1)}{B+J}$

22. If k is the smallest positive integer such that $(2^k)(5^{300})$ has 303 digits when expanded, then the sum of the digits of the expanded number is

(**A**) 11 (**B**) 10 (**C**) 8 (**D**) 7 (**E**) 5

23. Triangle ABC is isosceles with $AB = AC$ and $BC = 65$ cm. P is a point on BC such that the perpendicular distances from P to AB and AC are 24 cm and 36 cm, respectively. The area of ΔABC, in cm^2, is

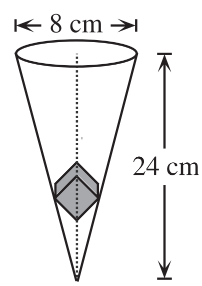


(**A**) 1254 (**B**) 1640 (**C**) 1950
(**D**) 2535 (**E**) 2942

24. The polynomial $f(x)$ satisfies the equation $f(x) - f(x-2) = (2x-1)^2$ for all x. If p and q are the coefficients of x^2 and x, respectively, in $f(x)$, then $p+q$ is equal to

(**A**) 0 (**B**) $\frac{5}{6}$ (**C**) $\frac{4}{3}$ (**D**) 1 (**E**) $\frac{2}{3}$

25. A steel cube has edges of length 3 cm, and a cone has a diameter of 8 cm and a height of 24 cm. The cube is placed in the cone so that one of its interior diagonals coincides with the axis of the cone. What is the distance, in cm, between the vertex of the cone and the closest vertex of the cube?



(**A**) $6\sqrt{6}-\sqrt{3}$ (**B**) $\frac{12\sqrt{6}-3\sqrt{3}}{4}$ (**C**) $6\sqrt{6}-2\sqrt{3}$
(**D**) $5\sqrt{3}$ (**E**) $6\sqrt{6}$

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PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at
http://www.cemc.uwaterloo.ca

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Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 19, 2003

C.M.C. Sponsors:

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Manulife Financial

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   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $3^3 - 3^2 + 3^1 - 3^0$ is

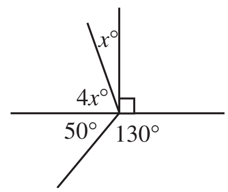
   (**A**) 18 (**B**) 6 (**C**) 9 (**D**) 40 (**E**) 20

2. If $a = 5$ and $a^2 + ab = 60$, the value of b is

   (**A**) 7 (**B**) 4 (**C**) 2 (**D**) 10 (**E**) 30

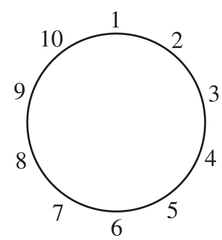
3. In the diagram, the value of x is

   (**A**) 22.5 (**B**) 25 (**C**) 20
   (**D**) 36 (**E**) 18



4. In the diagram, the numbers 1 to 10 are placed around a circle. Sandy crosses out 1, then 4, and then 7. Continuing in a clockwise direction, she crosses out every third number of those remaining, until only two numbers are left. The sum of these two numbers is

   (**A**) 13 (**B**) 10 (**C**) 8
   (**D**) 14 (**E**) 17



5. During hibernation, a bear loses 20% of its original mass. After hibernation, its mass is 220 kg. What was its mass, in kilograms, just before hibernation?

   (**A**) 176 (**B**) 264 (**C**) 240 (**D**) 275 (**E**) 1100

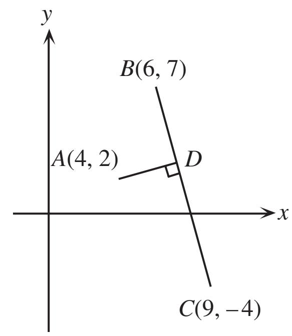
6. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that $\frac{5}{8}$ of the players are girls?

   (**A**) 6 (**B**) 3 (**C**) 5 (**D**) 8 (**E**) 7

7. A fish tank, in the shape of a rectangular prism, has a base measuring 20 cm by 40 cm with a height of 30 cm. The tank sits on a horizontal table and is half full of water. If 4000 cm^3 of water is added, what is the new depth of the water?

   (**A**) 5 cm (**B**) 15 cm (**C**) 20 cm (**D**) 25 cm (**E**) 10 cm

8. In the diagram, D is the point on BC so that AD is perpendicular to BC. The slope of AD is



(**A**) $\frac{3}{11}$ (**B**) 1 (**C**) $-\frac{15}{11}$
(**D**) $\frac{2}{7}$ (**E**) $\frac{2}{5}$

9. The average (mean) of $\frac{1}{5}$ and $\frac{1}{10}$ is $\frac{1}{x}$. The value of x is

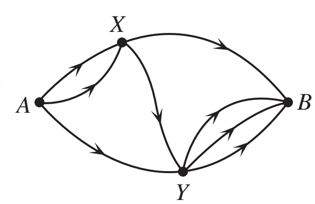
(**A**) $\frac{20}{3}$ (**B**) $\frac{3}{20}$ (**C**) 30 (**D**) $\frac{10}{3}$ (**E**) $\frac{2}{15}$

10. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

(**A**) 16 (**B**) 9 (**C**) 36 (**D**) 12 (**E**) 18

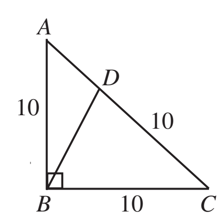
Part B: Each correct answer is worth 6.

11. In the diagram, it is only possible to travel along an edge in the direction indicated by the arrow. Hazel studied the figure, and determined all the possible routes from A to B. She selected one of these routes at random. What is the probability that she selected a route which passes through X?



(**A**) $\frac{8}{11}$ (**B**) $\frac{3}{11}$ (**C**) 1
(**D**) $\frac{9}{11}$ (**E**) $\frac{6}{11}$

12. In the diagram, $\angle ABC = 90^\circ$ and $AB = BC = CD = 10$. The length of AD is closest to

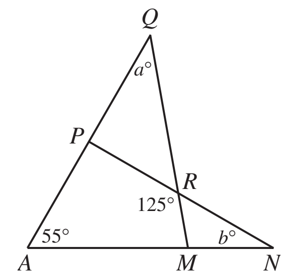


(**A**) 14 (**B**) 5 (**C**) 9
(**D**) 10 (**E**) 4

13. If $x + y = 1$ and $x - y = 3$, what is the value of $2^{x^2-y^2}$?

(**A**) 4 (**B**) 8 (**C**) 2 (**D**) 16 (**E**) 32

14. In the diagram, AMN, APQ, QRM, and PRN are all straight lines. The value of $a+b$ is



(**A**) 70 (**B**) 55 (**C**) 80
(**D**) 90 (**E**) 75

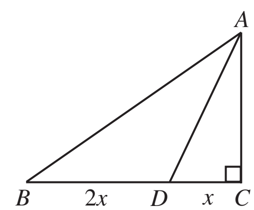
15. The side lengths of an equilateral triangle and a square are integers. If the triangle and the square have the same perimeter, which of the following is a possible side length of the triangle?

(**A**) 1 (**B**) 10 (**C**) 18 (**D**) 20 (**E**) 25

16. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is

(**A**) 18 (**B**) 19 (**C**) 23 (**D**) 25 (**E**) 22

17. In the diagram, ΔABC is right-angled at C. If $BD=2x$, $DC=x$, and $\angle ADC = 2(\angle ABC)$, then the length of AB is

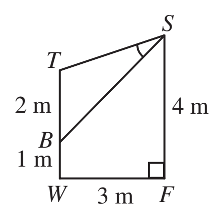


(**A**) $2\sqrt{2}x$ (**B**) $\sqrt{6}x$ (**C**) $2\sqrt{3}x$
(**D**) $3x$ (**E**) $4x$

18. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car's engine at a cost of \$400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs \$0.80 per litre, this distance, in kilometres, is between

(**A**) 10 000 and 14 000 (**B**) 14 000 and 18 000 (**C**) 18 000 and 22 000
(**D**) 22 000 and 26 000 (**E**) 26 000 and 30 000

19. In an art gallery, a 2 m high painting, BT, is mounted on a wall with its bottom edge 1 m above the floor. A spotlight is mounted at S, 3 m out from the wall and 4 m above the floor. The size of $\angle TSB$ is closest to



(**A**) 27° (**B**) 63° (**C**) 34°
(**D**) 45° (**E**) 18°

20. If a, b and c are positive, consecutive terms of a geometric sequence (that is, $\frac{c}{b}=\frac{b}{a}$), then the graph of $y=ax^2+bx+c$ is

(**A**) a curve that intersects the x-axis at two distinct points
(**B**) entirely below the x-axis
(**C**) entirely above the x-axis
(**D**) a straight line
(**E**) tangent to the x-axis

continued ...

Part C: Each correct answer is worth 8.

21. A sequence of numbers has 6 as its first term, and every term after the first is defined as follows: If a term, t, is even, the next term in the sequence is $\frac{1}{2}t$. If a term, s, is odd, the next term is $3s+1$. Thus, the first four terms in the sequence are 6, 3, 10, 5. The 100th term is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 6

22. Pentagon *ABCDE* is such that all five diagonals *AC*, *BD*, *CE*, *DA,* and *EB* lie entirely within the pentagon. If the area of each of the triangles *ABC*, *BCD*, *CDE*, *DEA*, and *EAB* is equal to 1, the area of the pentagon *ABCDE* is closest to

(**A**) 3.62 (**B**) 3.64 (**C**) 3.66 (**D**) 3.68 (**E**) 3.70

23. Three faces of a rectangular box meet at a corner of the box. The centres of these faces form the vertices of a triangle having side lengths of 4 cm, 5 cm and 6 cm. The volume of the box, in cm^3, is

(**A**) $45\sqrt{3}$ (**B**) $45\sqrt{6}$ (**C**) $90\sqrt{6}$ (**D**) 125 (**E**) $120\sqrt{2}$

24. When the expression $\left[(1+x)(1+2x^3)(1+4x^9)(1+8x^{27})(1+16x^{81})(1+32x^{243})(1+64x^{729})\right]^2$ is expanded and simplified, the coefficient of x^{2003} is

(**A**) 0 (**B**) 2^{28} (**C**) 2^{30} (**D**) 2^{29} (**E**) 2^{31}

25. The set $\{1,4,n\}$ has the property that when any two distinct elements are chosen and 2112 is added to their product, the result is a perfect square. If n is a positive integer, the number of possible values for n is

(**A**) 8 (**B**) 7 (**C**) 6 (**D**) 5 (**E**) 4

●●●●●

PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. **(Available in English only.)**

Volume 1
- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

Volume 2
- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

Volume 3
- over 235 problems and full solutions
- 12 topics
- for senior high school students

Volume 4
- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

Volume 5
- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

Volume 6
- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

Volume 7
NEW
- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10

Volume 8
NEW
- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12

Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is $20. **(Available in English only.)**

Orders should be addressed to: Canadian Mathematics Competition
Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2003.**

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 20, 2002

C.M.C. Sponsors:

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
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   There is *no penalty* for an incorrect answer.
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8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. If $x = 3$, the numerical value of $5 - 2x^2$ is

   (**A**) –1 (**B**) 27 (**C**) –13 (**D**) –31 (**E**) 3

2. $\dfrac{3^3 + 3}{2^2 + 2}$ is equal to

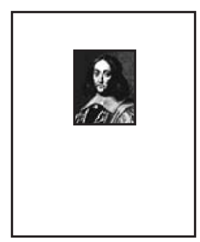
   (**A**) 3 (**B**) 6 (**C**) 2 (**D**) $\frac{3}{2}$ (**E**) 5

3. If it is now 9:04 a.m., in 56 hours the time will be

   (**A**) 9:04 a.m. (**B**) 5:04 p.m. (**C**) 5:04 a.m. (**D**) 1:04 p.m. (**E**) 1:04 a.m.

4. Which one of the following statements is **not** true?
   (**A**) 25 is a perfect square.
   (**B**) 31 is a prime number.
   (**C**) 3 is the smallest prime number.
   (**D**) 8 is a perfect cube.
   (**E**) 15 is the product of two prime numbers.

5. A rectangular picture of Pierre de Fermat, measuring 20 cm by 40 cm, is positioned as shown on a rectangular poster measuring 50 cm by 100 cm. What percentage of the area of the poster is covered by the picture?

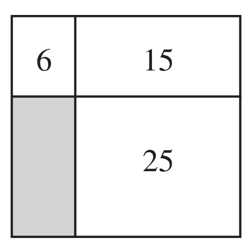


   (**A**) 24% (**B**) 16% (**C**) 20%
   (**D**) 25% (**E**) 40%

6. Gisa is taller than Henry but shorter than Justina. Ivan is taller than Katie but shorter than Gisa. The tallest of these five people is

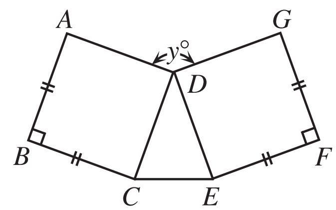
   (**A**) Gisa (**B**) Henry (**C**) Ivan (**D**) Justina (**E**) Katie

7. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is



   (**A**) 7 (**B**) 15 (**C**) 12
   (**D**) 16 (**E**) 10

8. In the diagram, $ABCD$ and $DEFG$ are squares with equal side lengths, and $\angle DCE = 70°$. The value of y is

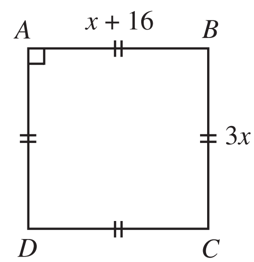


(**A**) 120 (**B**) 160 (**C**) 130
(**D**) 110 (**E**) 140

9. The numbers 1 through 20 are written on twenty golf balls, with one number on each ball. The golf balls are placed in a box, and one ball is drawn at random. If each ball is equally likely to be drawn, what is the probability that the number on the golf ball drawn is a multiple of 3?

(**A**) $\frac{3}{20}$ (**B**) $\frac{6}{20}$ (**C**) $\frac{10}{20}$ (**D**) $\frac{5}{20}$ (**E**) $\frac{1}{20}$

10. $ABCD$ is a square with $AB = x + 16$ and $BC = 3x$, as shown. The perimeter of $ABCD$ is



(**A**) 16 (**B**) 32 (**C**) 96
(**D**) 48 (**E**) 24

Part B: Each correct answer is worth 6.

11. A line passing through the points $(0,-2)$ and $(1,0)$ also passes through the point $(7,b)$. The numerical value of b is

(**A**) 12 (**B**) $\frac{9}{2}$ (**C**) 10 (**D**) 5 (**E**) 14

12. How many three-digit positive integers are perfect squares?

(**A**) 23 (**B**) 22 (**C**) 21 (**D**) 20 (**E**) 19

13. A "double-single" number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

(**A**) 81 (**B**) 18 (**C**) 72 (**D**) 64 (**E**) 90

14. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column m and row n. The value of $m+n$ is

	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Row 1	1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13	14
Row 3	15	16	17	18	19	20	21
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

(**A**) 290 (**B**) 291 (**C**) 292 (**D**) 293 (**E**) 294

15. In a sequence of positive numbers, each term after the first two terms is the sum of *all of the previous terms*. If the first term is a, the second term is 2, and the sixth term is 56, then the value of a is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

16. If $ac+ad+bc+bd=68$ and $c+d=4$, what is the value of $a+b+c+d$?

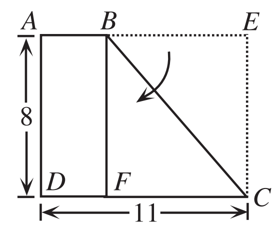
(**A**) 17 (**B**) 85 (**C**) 4 (**D**) 21 (**E**) 64

17. The average age of a group of 140 people is 24. If the average age of the males in the group is 21 and the average age of the females is 28, how many females are in the group?

(**A**) 90 (**B**) 80 (**C**) 70 (**D**) 60 (**E**) 50

18. A rectangular piece of paper $AECD$ has dimensions 8 cm by 11 cm. Corner E is folded onto point F, which lies on DC, as shown. The perimeter of trapezoid $ABCD$ is closest to

(**A**) 33.3 cm (**B**) 30.3 cm (**C**) 30.0 cm
(**D**) 41.3 cm (**E**) 35.6 cm

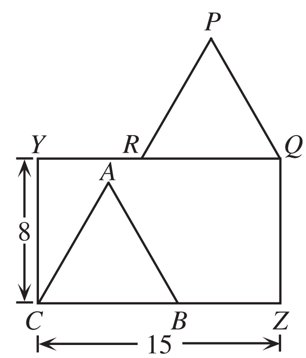


19. If $2^a 3^b = 8\left(6^{10}\right)$, where a and b are integers, then $b-a$ equals

(**A**) 0 (**B**) 23 (**C**) -13 (**D**) -7 (**E**) -3

20. In the diagram, $YQZC$ is a rectangle with $YC=8$ and $CZ=15$. Equilateral triangles ABC and PQR, each with side length 9, are positioned as shown with R and B on sides YQ and CZ, respectively. The length of AP is

(**A**) 10 (**B**) $\sqrt{117}$ (**C**) 9
(**D**) 8 (**E**) $\sqrt{72}$



Part C: Each correct answer is worth 8.

21. If $\sqrt{\frac{3}{1}\cdot\frac{5}{3}\cdot\frac{7}{5}\cdot\ \cdots\ \cdot\frac{2n+1}{2n-1}}=9$, then the value of n is

(**A**) 38 (**B**) 1 (**C**) 40 (**D**) 4 (**E**) 39

22. The function $f(x)$ has the property that $f(x+y)=f(x)+f(y)+2xy$, for all positive integers x and y. If $f(1)=4$, then the numerical value of $f(8)$ is

(**A**) 72 (**B**) 84 (**C**) 88 (**D**) 64 (**E**) 80

continued ...

23. The integers from 1 to 9 are listed on a blackboard. If an additional m eights and k nines are added to the list, the average of all of the numbers in the list is 7.3. The value of $k + m$ is

(**A**) 24 (**B**) 21 (**C**) 11 (**D**) 31 (**E**) 89

24. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to

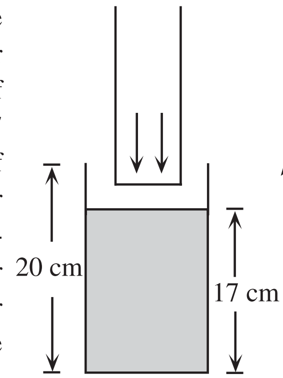


Figure 1

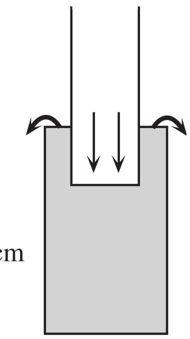


Figure 2

(**A**) 2.82 cm (**B**) 2.84 cm (**C**) 2.86 cm
(**D**) 2.88 cm (**E**) 2.90 cm

25. The lengths of all six edges of a tetrahedron are integers. The lengths of five of the edges are 14, 20, 40, 52, and 70. The number of possible lengths for the sixth edge is

(**A**) 9 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 21, 2001

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
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   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 20.

Part A: Each correct answer is worth 5.

1. If $x+2x+3x+4x=5$, then x equals

   (A) 10 **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{4}$ **(D)** 2 **(E)** $\frac{5}{9}$

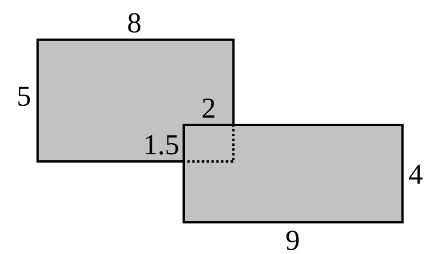
2. If $x=\frac{1}{4}$, which of the following has the largest value?

   (A) x **(B)** x^2 **(C)** $\frac{1}{2}x$ **(D)** $\dfrac{1}{x}$ **(E)** $\sqrt{x}$

3. In a school, 30 boys and 20 girls entered the Fermat competition. Certificates were awarded to 10% of the boys and 20% of the girls. Of the students who participated, the percentage that received certificates was

   (A) 14 **(B)** 15 **(C)** 16 **(D)** 30 **(E)** 50

4. Two rectangles overlap with their common region being a smaller rectangle, as shown. The total area of the shaded region is

   (A) 45 **(B)** 70 **(C)** 52
   (D) 79 **(E)** 73

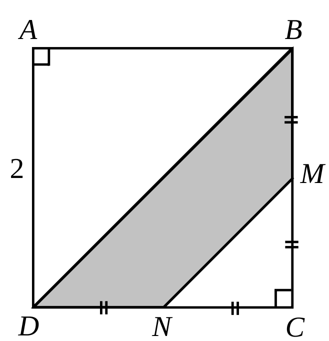
5. In ΔABC, $\angle A=3\angle B$ and $\angle B=2\angle C$. The measure of $\angle B$ is

   (A) $10°$ **(B)** $20°$ **(C)** $30°$ **(D)** $40°$ **(E)** $60°$

6. Pat gives half of his marbles to his best friend and then a third of those remaining to his sister. If his sister receives 9 marbles, then the number Pat keeps is

   (A) 27 **(B)** 54 **(C)** 18 **(D)** 36 **(E)** 9

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD. The area of the shaded region $BMND$ is

   (A) 1 **(B)** $2\sqrt{2}$ **(C)** $\frac{4}{3}$
   (D) $\frac{3}{2}$ **(E)** $4-\frac{3}{2}\sqrt{2}$

8. If $\sqrt{5+11-7}=\sqrt{5+11}-\sqrt{x}$, then the value of x is

   (A) 1 **(B)** 7 **(C)** -7 **(D)** 49 **(E)** 4

9. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

(**A**) 6 (**B**) 10 (**C**) 12 (**D**) 16 (**E**) 18

10. When a positive integer N is divided by 60, the remainder is 49. When N is divided by 15, the remainder is

(**A**) 0 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 8

Part B: Each correct answer is worth 6.

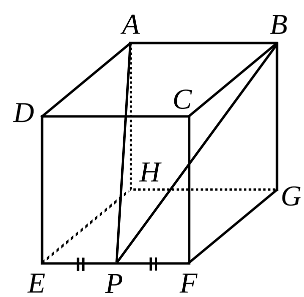
11. The fourth root of 2001200120012001 (that is, $\sqrt[4]{2001200120012001}$) is closest to

(**A**) 2 001 (**B**) 6 700 (**C**) 21 000 (**D**) 12 000 (**E**) 2 100

12. How many integer values of x satisfy $\frac{x-1}{3}<\frac{5}{7}<\frac{x+4}{5}$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

13. $ABCDEFGH$ is a cube having a side length of 2. P is the midpoint of EF, as shown. The area of ΔAPB is

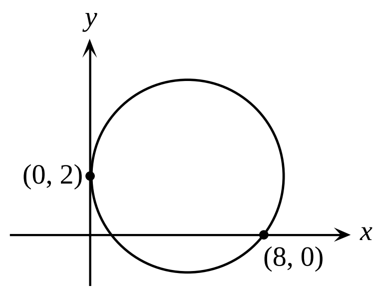


(A) $\sqrt{8}$ (**B**) 3 (**C**) $\sqrt{32}$
(**D**) $\sqrt{2}$ (**E**) 6

14. The last digit (that is, the units digit) of $(2002)^{2002}$ is

(**A**) 4 (**B**) 2 (**C**) 8 (**D**) 0 (**E**) 6

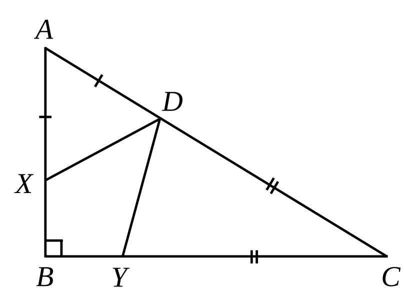
15. A circle is tangent to the y-axis at $(0, 2)$, and the larger of its x-intercepts is 8. The radius of the circle is



(**A**) $\frac{9}{2}$ (**B**) $\sqrt{17}$ (**C**) $\frac{17}{4}$

(**D**) $\frac{15}{4}$ (**E**) $\frac{\sqrt{17}}{2}$

16. In right triangle ABC, $AX = AD$ and $CY = CD$, as shown. The measure of $\angle XDY$ is

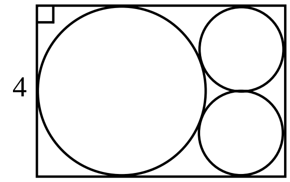


(**A**) 35° (**B**) 40° (**C**) 45°
(**D**) 50° (**E**) not determined by this information

17. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is

(**A**) 34 (**B**) 35 (**C**) 36 (**D**) 37 (**E**) 38

18. In the diagram, the two smaller circles have equal radii. Each of the three circles is tangent to the other two circles, and each is also tangent to sides of the rectangle. If the width of the rectangle is 4, then its length is



(**A**) $2+\sqrt{8}$ (**B**) $3+\sqrt{8}$ (**C**) $3+\sqrt{10}$
(**D**) $\sqrt{32}$ (**E**) $4+\sqrt{3}$

19. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?

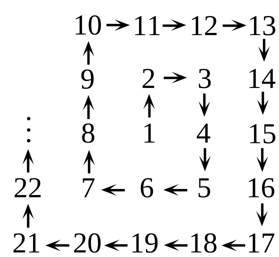
(**A**) $16\frac{2}{3}$ (**B**) 15 (**C**) $13\frac{1}{3}$ (**D**) 12 (**E**) $18\frac{3}{4}$

20. Point P is on the line $y=5x+3$. The coordinates of point Q are $(3,-2)$. If M is the midpoint of PQ, then M must lie on the line

(**A**) $y=\frac{5}{2}x-\frac{7}{2}$ (**B**) $y=5x+1$ (**C**) $y=-\frac{1}{5}x-\frac{7}{5}$ (**D**) $y=\frac{5}{2}x+\frac{1}{2}$ (**E**) $y=5x-7$

Part C: Each correct answer is worth 8.

21. A spiral of numbers is created, as shown, starting with 1. If the pattern of the spiral continues, in what configuration will the numbers 399, 400 and 401 appear?



(**A**) 399 → 400 → 401 (**B**) 401 ← 400 ← 399

(**C**) $\begin{array}{c} 401 \\ \uparrow \\ 400 \\ \uparrow \\ 399 \end{array}$ (**D**) $\begin{array}{c} 399 \\ \downarrow \\ 400 \\ \downarrow \\ 401 \end{array}$ (**E**) $\begin{array}{cc} 400 & \rightarrow 401 \\ \uparrow & \\ 399 & \end{array}$

22. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

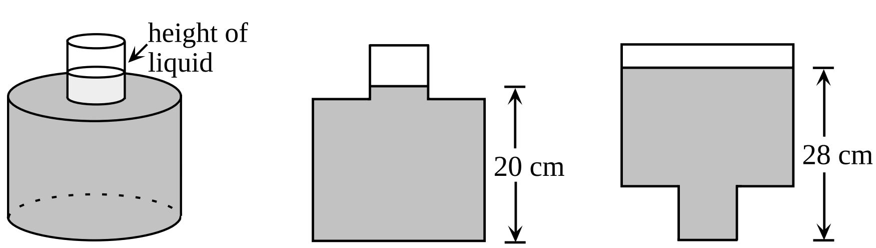


Figure A Figure B Figure C

(**A**) 29 (**B**) 30 (**C**) 31 (**D**) 32 (**E**) 48

continued ...

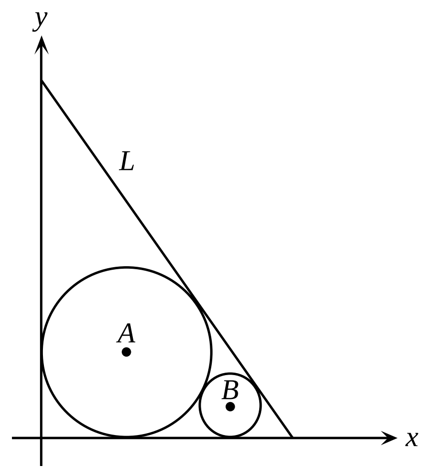
23. A sequence $t_1, t_2, ..., t_n, ...$ is defined as follows:

$$t_1 = 14$$
$$t_k = 24 - 5t_{k-1}, \text{ for each } k \geq 2.$$

For every positive integer n, t_n can be expressed as $t_n = p \cdot q^n + r$, where p, q and r are constants. The value of $p + q + r$ is

(**A**) -5 (**B**) -3 (**C**) 3 (**D**) 17 (**E**) 31

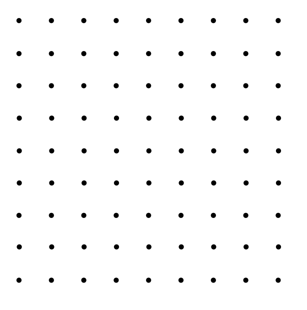
24. The circle with centre A has radius 3 and is tangent to both the positive x-axis and positive y-axis, as shown. Also, the circle with centre B has radius 1 and is tangent to both the positive x-axis and the circle with centre A. The line L is tangent to both circles. The y-intercept of line L is



(**A**) $3+6\sqrt{3}$ (**B**) $10+3\sqrt{2}$ (**C**) $8\sqrt{3}$

(**D**) $10+2\sqrt{3}$ (**E**) $9+3\sqrt{3}$

25. A square array of dots with 10 rows and 10 columns is given. Each dot is coloured either blue or red. Whenever two dots of the same colour are adjacent in the same row or column, they are joined by a line segment of the same colour as the dots. If they are adjacent but of different colours, they are then joined by a green line segment. In total, there are 52 red dots. There are 2 red dots at corners with an additional 16 red dots on the edges of the array. The remainder of the red dots are inside the array. There are 98 green line segments. The number of blue line segments is



(**A**) 36 (**B**) 37 (**C**) 38
(**D**) 39 (**E**) 40

Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 23, 2000

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer. Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each correct answer is worth 5.

1. The sum $29+12+23$ is equal to

   (**A**) 6^2 (**B**) 4^4 (**C**) 8^8 (**D**) 64^0 (**E**) 2^6

2. If the following sequence of five arrows repeats itself continuously, what arrow will be in the 48th position?

   $$\rightarrow, \nearrow, \nwarrow, \leftarrow, \downarrow$$

   (**A**) $\rightarrow$ (**B**) $\nearrow$ (**C**) $\nwarrow$ (**D**) $\leftarrow$ (**E**) $\downarrow$

3. A farmer has 7 cows, 8 sheep and 6 goats. How many more goats should be bought so that half of her animals will be goats?

   (**A**) 18 (**B**) 15 (**C**) 21 (**D**) 9 (**E**) 6

4. The square of 9 is divided by the cube root of 125. What is the remainder?

   (**A**) 6 (**B**) 3 (**C**) 16 (**D**) 2 (**E**) 1

5. The product of 2, 3, 5, and y is equal to its sum. What is the value of y?

   (**A**) $\frac{1}{3}$ (**B**) $\frac{10}{31}$ (**C**) $\frac{10}{29}$ (**D**) $\frac{3}{10}$ (**E**) $\frac{10}{3}$

6. A student uses a calculator to find an answer but instead of pressing the $\boxed{x^2}$ key presses the $\boxed{\sqrt{x}}$ key by mistake. The student's answer was 9. What should the answer have been?

   (**A**) 243 (**B**) 81 (**C**) 729 (**D**) 3 (**E**) 6561

7. The sum of the arithmetic series $(-300)+(-297)+(-294)+\ldots+306+309$ is

   (**A**) 309 (**B**) 927 (**C**) 615 (**D**) 918 (**E**) 18

8. In a school referendum, $\frac{3}{5}$ of a student body voted 'yes' and 28% voted 'no'. If there were no spoiled ballots, what percentage of the students did not vote?

   (**A**) 72% (**B**) 40% (**C**) 32% (**D**) 12% (**E**) 88%

9. The numbers 6, 14, x, 17, 9, y, 10 have a mean of 13. What is the value of $x+y$?

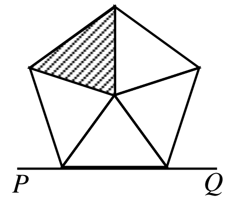
   (**A**) 20 (**B**) 21 (**C**) 23 (**D**) 25 (**E**) 35

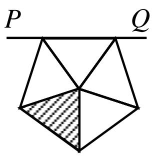
10. If $x(x(x+1)+2)+3=x^3+x^2+x-6$ then x is equal to

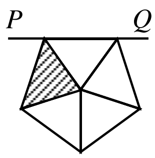
(**A**) 11 (**B**) -9 (**C**) -4 or 3 (**D**) -1 or 0 (**E**) -2

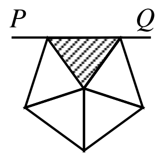
Part B: Each correct answer is worth 6.

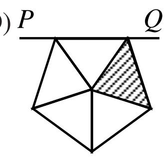
11. When the regular pentagon is reflected in the line PQ, and then rotated *clockwise* 144° about the centre of the pentagon, its position is

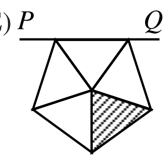


(**A**) 

(**B**) 

(**C**) 

(**D**) 

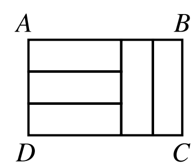
(**E**) 

12. If the expression $15^6 \times 28^5 \times 55^7$ was evaluated, it would end with a string of consecutive zeros. How many zeros are in this string?

(**A**) 10 (**B**) 18 (**C**) 26 (**D**) 13 (**E**) 5

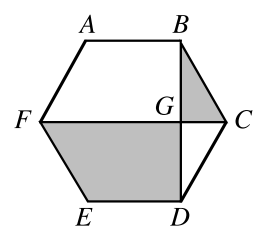
13. Rectangle $ABCD$ is divided into five congruent rectangles as shown. The ratio $AB:BC$ is

(**A**) 3:2 (**B**) 2:1 (**C**) 5:2
(**D**) 5:3 (**E**) 4:3



14. In the regular hexagon $ABCDEF$, two of the diagonals, FC and BD, intersect at G. The ratio of the area of quadrilateral $FEDG$ to the area of ΔBCG is

(**A**) $3\sqrt{3}:1$ (**B**) 4:1 (**C**) 6:1
(**D**) $2\sqrt{3}:1$ (**E**) 5:1



15. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?

(**A**) 160 (**B**) 304 (**C**) 28 (**D**) 56 (**E**) 64

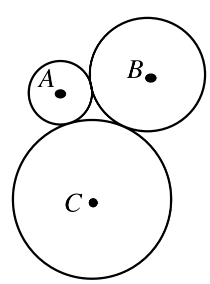
16. The digits 2, 2, 3, and 5 are randomly arranged to form a four digit number. What is the probability that the sum of the first and last digits is even?

(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{2}{3}$

17. Three circles have centres A, B and C with radii 2, 4 and 6 respectively. The circles are tangent to each other as shown. Triangle ABC has

(**A**) $\angle A$ obtuse (**B**) $\angle B = 90°$ (**C**) $\angle A = 90°$

(**D**) all angles acute (**E**) $\angle B = \angle C$



18. If $P = 3^{2000} + 3^{-2000}$ and $Q = 3^{2000} - 3^{-2000}$ then the value of $P^2 - Q^2$ is

(**A**) 3^{4000} (**B**) 2×3^{-4000} (**C**) 0 (**D**) 2×3^{4000} (**E**) 4

19. An ant walks inside a 18 cm by 150 cm rectangle. The ant's path follows straight lines which always make angles of $45°$ to the sides of the rectangle. The ant starts from a point X on one of the shorter sides. The first time the ant reaches the opposite side, it arrives at the mid-point. What is the distance, in centimetres, from X to the nearest corner of the rectangle?

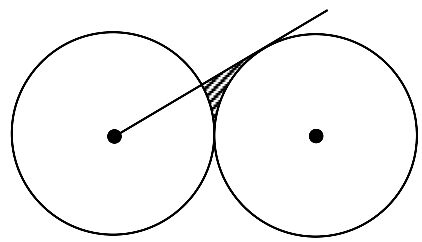
(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

20. Given $a + 2b + 3c + 4d + 5e = k$ and $5a = 4b = 3c = 2d = e$, find the smallest positive integer value for k so that a, b, c, d, and e are all positive integers.

(**A**) 87 (**B**) 522 (**C**) 10 (**D**) 120 (**E**) 60

Part C: Each question is worth 8 credits.

21. Two circles of radius 10 are tangent to each other. A tangent is drawn from the centre of one of the circles to the second circle. To the nearest integer, what is the area of the shaded region?

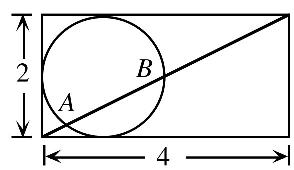


(**A**) 6 (**B**) 7 (**C**) 8
(**D**) 9 (**E**) 10

22. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either 'a' or 'b'. What is the value of $a + b$?

(**A**) 3 (**B**) 7 (**C**) 4 (**D**) 10 (**E**) 17

23. A circle is tangent to three sides of a rectangle having side lengths 2 and 4 as shown. A diagonal of the rectangle intersects the circle at points A and B. The length of AB is



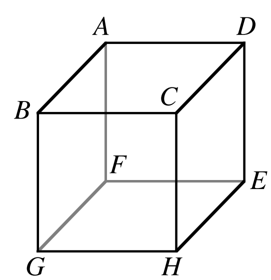
(**A**) $\sqrt{5}$ (**B**) $\frac{4\sqrt{5}}{5}$ (**C**) $\sqrt{5} - \frac{1}{5}$

(**D**) $\sqrt{5} - \frac{1}{6}$ (**E**) $\frac{5\sqrt{5}}{6}$

24. For the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$, the sum of the real y values that satisfy the equations is

(**A**) 20 (**B**) 2 (**C**) 5 (**D**) $\frac{55}{2}$ (**E**) $\frac{5}{2}$

25. The given cube is cut into four pieces by two planes. The first plane is parallel to face $ABCD$ and passes through the midpoint of edge BG. The second plane passes through the midpoints of edges AB, AD, HE, and GH. Determine the ratio of the volumes of the smallest and largest of the four pieces.



(**A**) 3:8 (**B**) 7:24 (**C**) 7:25
(**D**) 7:17 (**E**) 5:11

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 24, 1999

C.M.C. Sponsors:

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Manulife Financial

Equitable Life of Canada

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

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5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer. Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

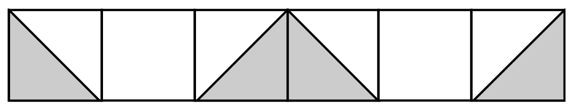
1. The value of $\left(\sqrt{25}-\sqrt{9}\right)^2$ is

   (**A**) 26 (**B**) 16 (**C**) 34 (**D**) 8 (**E**) 4

2. Today is Wednesday. What day of the week will it be 100 days from now?

   (**A**) Monday (**B**) Tuesday (**C**) Thursday (**D**) Friday (**E**) Saturday

3. Six squares are drawn and shaded as shown. What fraction of the total area is shaded?



   (**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{2}{5}$ (**E**) $\frac{2}{3}$

4. Turning a screwdriver $90°$ will drive a screw 3 mm deeper into a piece of wood. How many complete revolutions are needed to drive the screw 36 mm into the wood?

   (**A**) 3 (**B**) 4 (**C**) 6 (**D**) 9 (**E**) 12

5. A value of x such that $(5-3x)^5=-1$ is

   (**A**) $\frac{4}{3}$ (**B**) 0 (**C**) $\frac{10}{3}$ (**D**) $\frac{5}{3}$ (**E**) 2

6. The number which is 6 less than twice the square of 4 is

   (**A**) -26 (**B**) 10 (**C**) 26 (**D**) 38 (**E**) 58

7. The Partridge family pays each of their five children a weekly allowance. The average allowance for each of the three younger children is \$8. The two older children each receive an average allowance of \$13. The total amount of allowance money paid per week is

   (**A**) \$50 (**B**) \$52.50 (**C**) \$105 (**D**) \$21 (**E**) \$55

8. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

   (**A**) 71 (**B**) 72 (**C**) 255 (**D**) 316 (**E**) 436

9. In an election, Harold received 60% of the votes and Jacquie received all the rest. If Harold won by 24 votes, how many people voted?

   (**A**) 40 (**B**) 60 (**C**) 72 (**D**) 100 (**E**) 120

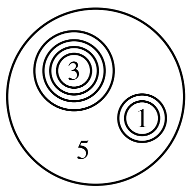
10. If x and y are each chosen from the set $\{1, 2, 3, 5, 10\}$, the largest possible value of $\frac{x}{y}+\frac{y}{x}$ is

(**A**) 2 (**B**) $12\frac{1}{2}$ (**C**) $10\frac{1}{10}$ (**D**) $2\frac{1}{2}$ (**E**) 20

Part B: Each question is worth 6 credits.

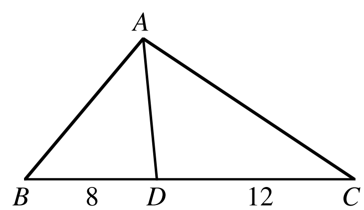
11. In *Circle Land*, the numbers 207 and 4520 are shown in the following way:

In *Circle Land*, what number does the following diagram represent?



(**A**) 30 105 (**B**) 30 150 (**C**) 3105 (**D**) 3015 (**E**) 315

12. The area of ΔABC is 60 square units. If $BD = 8$ units and $DC = 12$ units, the area (in square units) of ΔABD is

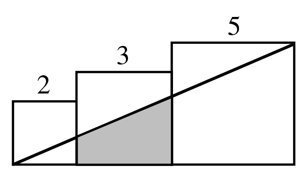


(**A**) 24 (**B**) 40 (**C**) 48
(**D**) 36 (**E**) 6

13. Crippin wrote four tests each with a maximum possible mark of 100. The average mark he obtained on these tests was 88. What is the lowest score he could have achieved on one of these tests?

(**A**) 88 (**B**) 22 (**C**) 52 (**D**) 0 (**E**) 50

14. Three squares have dimensions as indicated in the diagram. What is the area of the shaded quadrilateral?



(**A**) $\frac{21}{4}$ (**B**) $\frac{9}{2}$ (**C**) 5

(**D**) $\frac{15}{4}$ (**E**) $\frac{25}{4}$

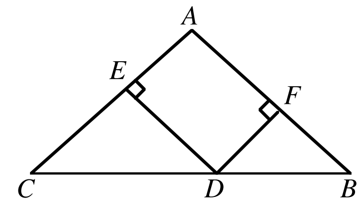
15. If $(a+b+c+d+e+f+g+h+i)^2$ is expanded and simplified, how many different terms are in the final answer?

(**A**) 36 (**B**) 9 (**C**) 45 (**D**) 81 (**E**) 72

16. If $px+2y=7$ and $3x+qy=5$ represent the same straight line, then p equals

(**A**) 5 (**B**) 7 (**C**) 21 (**D**) $\frac{21}{5}$ (**E**) $\frac{10}{7}$

17. In ΔABC, $AC=AB=25$ and $BC=40$. D is a point chosen on BC. From D, perpendiculars are drawn to meet AC at E and AB at F. $DE+DF$ equals



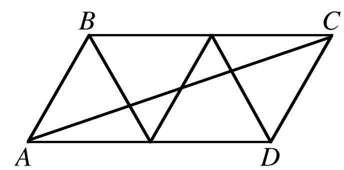
(**A**) 12 (**B**) 35 (**C**) 24

(**D**) 25 (**E**) $\frac{35}{2}\sqrt{2}$

18. The number of solutions (P,Q) of the equation $\frac{P}{Q}-\frac{Q}{P}=\frac{P+Q}{PQ}$, where P and Q are integers from 1 to 9 inclusive, is

(**A**) 1 (**B**) 8 (**C**) 16 (**D**) 72 (**E**) 81

19. Parallelogram $ABCD$ is made up of four equilateral triangles of side length 1. The length of diagonal AC is



(**A**) $\sqrt{5}$ (**B**) $\sqrt{7}$ (**C**) 3
(**D**) $\sqrt{3}$ (**E**) $\sqrt{10}$

20. If $a_1=\frac{1}{1-x}$, $a_2=\frac{1}{1-a_1}$, and $a_n=\frac{1}{1-a_{n-1}}$, for $n\geq 2$, $x\neq 1$ and $x\neq 0$, then a_{107} is

(**A**) $\frac{1}{1-x}$ (**B**) x (**C**) $-x$ (**D**) $\frac{x-1}{x}$ (**E**) $\frac{1}{x}$

Part C: Each question is worth 8 credits.

21. How many integers can be expressed as a sum of three distinct numbers if chosen from the set $\{4, 7, 10, 13, ..., 46\}$?

(**A**) 45 (**B**) 37 (**C**) 36 (**D**) 43 (**E**) 42

22. If $x^2+ax+48=(x+y)(x+z)$ and $x^2-8x+c=(x+m)(x+n)$, where y, z, m, and n are integers between -50 and 50, then the maximum value of ac is

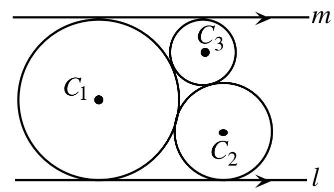
(**A**) 343 (**B**) 126 (**C**) 52 234 (**D**) 784 (**E**) 98 441

23. The sum of all values of x that satisfy the equation $\left(x^2-5x+5\right)^{x^2+4x-60}=1$ is

(**A**) -4 (**B**) 3 (**C**) 1 (**D**) 5 (**E**) 6

continued ...

24. Two circles C_1 and C_2 touch each other externally and the line l is a common tangent. The line m is parallel to l and touches the two circles C_1 and C_3. The three circles are mutually tangent. If the radius of C_2 is 9 and the radius of C_3 is 4, what is the radius of C_1?



(**A**) 10.4 (**B**) 11 (**C**) $8\sqrt{2}$
(**D**) 12 (**E**) $7\sqrt{3}$

25. Given that n is an integer, for how many values of n is $\dfrac{2n^2 - 10n - 4}{n^2 - 4n + 3}$ an integer?

(**A**) 9 (**B**) 7 (**C**) 6 (**D**) 4 (**E**) 5

Canadian Mathematics Competition

35th Anniversary 1963 – 1998

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 18, 1998

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

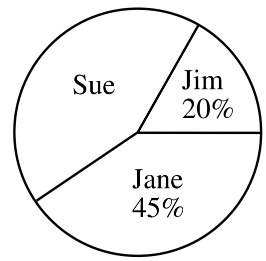
Part A: Each question is worth 5 credits.

1. The value of $\frac{1+2+3+4+5}{2+4+6+8+10}$ is

   (**A**) $\frac{1}{3}$ (**B**) 2.5 (**C**) $\frac{1}{2}$ (**D**) $\frac{11}{26}$ (**E**) $\frac{3}{8}$

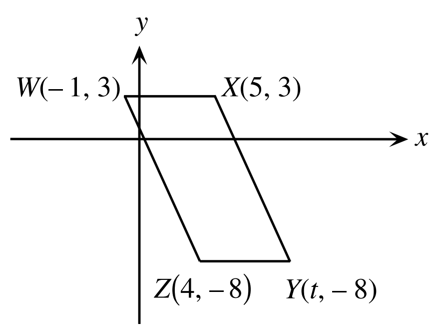
2. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?

   (**A**) 550 (**B**) 350 (**C**) 330
   (**D**) 450 (**E**) 935



3. If $WXYZ$ is a parallelogram, then t equals

   (**A**) 8 (**B**) 9 (**C**) 10
   (**D**) 11 (**E**) 12



4. The product of two positive integers p and q is 100. What is the largest possible value of $p+q$?

   (**A**) 52 (**B**) 101 (**C**) 20 (**D**) 29 (**E**) 25

5. If $\otimes$ is a new operation defined as $p \otimes q = p^2 - 2q$, what is the value of $7 \otimes 3$?

   (**A**) 43 (**B**) 8 (**C**) 141 (**D**) 36 (**E**) 26

6. The value of $\frac{1}{3}$ of 6^{30} is

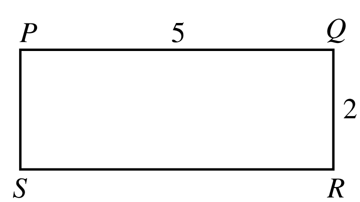
   (**A**) 6^{10} (**B**) 2^{30} (**C**) 2^{10} (**D**) 2×6^{29} (**E**) 2×6^{10}

7. The average (mean) of a list of 10 numbers is 0. If 72 and -12 are added to the list, the new average will be

   (**A**) 30 (**B**) 6 (**C**) 0 (**D**) 60 (**E**) 5

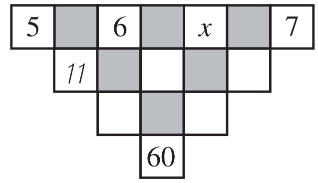
8. On a rectangular table 5 units long and 2 units wide, a ball is rolled from point P at an angle of $45°$ to PQ and bounces off SR. The ball continues to bounce off the sides at $45°$ until it reaches S. How many bounces of the ball are required?

   (**A**) 9 (**B**) 8 (**C**) 7
   (**D**) 5 (**E**) 4



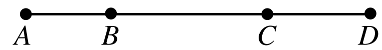
9. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of x must be

(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10



10. Four points are on a line segment, as shown. If $AB : BC = 1:2$ and $BC : CD = 8:5$, then $AB : BD$ equals

(**A**) $4:13$ (**B**) $1:13$ (**C**) $1:7$
(**D**) $3:13$ (**E**) $4:17$



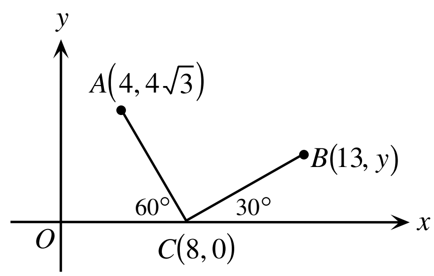
Part B: Each question is worth 6 credits.

11. The number of solutions (x, y) of the equation $3x + y = 100$, where x and y are positive integers, is

(**A**) 33 (**B**) 35 (**C**) 100 (**D**) 101 (**E**) 97

12. In the diagram, the value of y is

(**A**) $\frac{13}{2\sqrt{3}}$ (**B**) $\frac{5}{\sqrt{3}}$ (**C**) 2
(**D**) 12 (**E**) $\frac{\sqrt{3}}{5}$



13. Three-digit integers are formed using only the digits 1 and/or 2. The sum of all such integers formed is

(**A**) 1332 (**B**) 333 (**C**) 999 (**D**) 666 (**E**) 1665

14. Three straight lines, l_1, l_2 and l_3, have slopes $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. All three lines have the same y-intercept. If the sum of the x-intercepts of the three lines is 36, then the y-intercept is

(**A**) $\frac{-13}{12}$ (**B**) $\frac{-12}{13}$ (**C**) -4 (**D**) 4 (**E**) -9

15. If $-2 \le x \le 5$, $-3 \le y \le 7$, $4 \le z \le 8$, and $w = xy - z$, then the smallest value w may have is

(**A**) -14 (**B**) -18 (**C**) -19 (**D**) -22 (**E**) -23

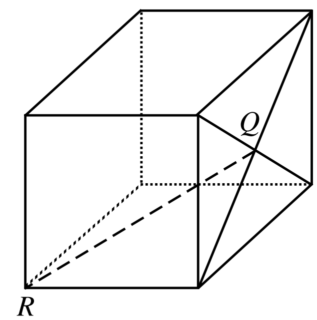
16. If $N = \left(7^{p+4}\right)\left(5^q\right)\left(2^3\right)$ is a perfect cube, where p and q are positive integers, the smallest possible value of $p+q$ is

(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 6 (**E**) 12

17. Using only digits 1, 2, 3, 4, and 5, a sequence is created as follows: one 1, two 2's, three 3's, four 4's, five 5's, six 1's, seven 2's, and so on.
The sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, ...
The 100th digit in the sequence is

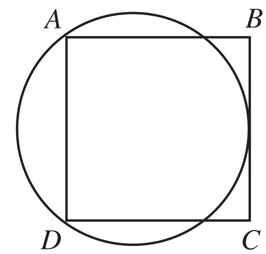
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

18. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is



(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$

19. Square $ABCD$ has sides of length 14. A circle is drawn through A and D so that it is tangent to BC, as shown. What is the radius of the circle?



(**A**) 8.5 (**B**) 8.75 (**C**) 9
(**D**) 9.25 (**E**) 9.5

20. A deck of 100 cards is numbered from 1 to 100. Each card has the same number printed on both sides. One side of each card is red and the other side is yellow. Barsby places all the cards, red side up, on a table. He first turns over every card that has a number divisible by 2. He then examines all the cards, and turns over every card that has a number divisible by 3. How many cards have the red side up when Barsby is finished?

(**A**) 83 (**B**) 17 (**C**) 66 (**D**) 50 (**E**) 49

Part C: Each question is worth 8 credits.

21. The numbers 123 456 789 and 999 999 999 are multiplied. How many of the digits in the final result are 9's?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 17

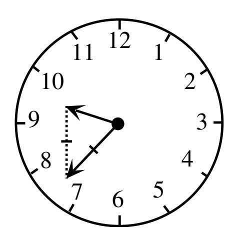
22. There are four unequal, positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

(**A**) 13 (**B**) 17 (**C**) 22 (**D**) 33 (**E**) 36

23. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug?

(**A**) 12 m^2 (**B**) 18 m^2 (**C**) 24 m^2 (**D**) 36 m^2 (**E**) 42 m^2

24. At some time between 9:30 and 10 o'clock the triangle determined by the minute hand and the hour hand is an isosceles triangle (see diagram). If the two equal angles in this triangle are each twice as large as the third angle, what is the time?



(**A**) 9:35 (**B**) 9:36 (**C**) 9:37
(**D**) 9:38 (**E**) 9:39

25. For each value of x, $f(x)$ is defined to be the minimum value of the three numbers $2x + 2$, $\frac{1}{2}x + 1$ and $-\frac{3}{4}x + 7$. What is the maximum value of $f(x)$?

(**A**) $\frac{2}{3}$ (**B**) 2 (**C**) $\frac{17}{5}$ (**D**) $\frac{62}{11}$ (**E**) 7

Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

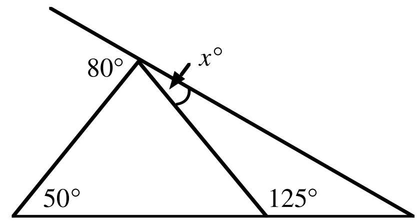
Fermat Contest (Grade 11)

Wednesday, February 19, 1997

Part A: Each question is worth 5 credits.

1. The value of $(1)^{10}+(-1)^8+(-1)^7+(1)^5$ is
(**A**) 0 (**B**) 4 (**C**) 1 (**D**) 16 (**E**) 2

2. The value of x is
(**A**) 15 (**B**) 20 (**C**) 25
(**D**) 30 (**E**) 35



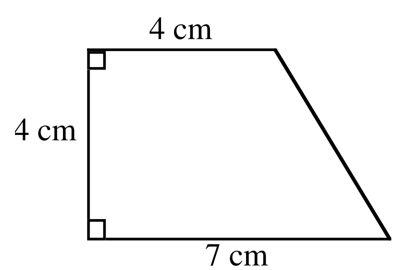
3. The greatest number of Mondays that can occur in 45 consecutive days is
(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

4. The product of a positive number, its square, and its reciprocal is $\frac{100}{81}$. What is the number?
(**A**) $\frac{81}{100}$ (**B**) $\frac{100}{81}$ (**C**) $\frac{9}{10}$ (**D**) $\frac{10}{9}$ (**E**) $\frac{10\,000}{6561}$

5. The sum of seven consecutive positive integers is 77. The largest of these integers is
(**A**) 8 (**B**) 11 (**C**) 14 (**D**) 15 (**E**) 17

6. If 2×10^3 is represented as $2E3$ on a certain calculator, how would the product of $2E3$ and $3E2$ be represented?
(**A**) $6E6$ (**B**) $6E5$ (**C**) $5E5$ (**D**) $2.3E3$ (**E**) $5E6$

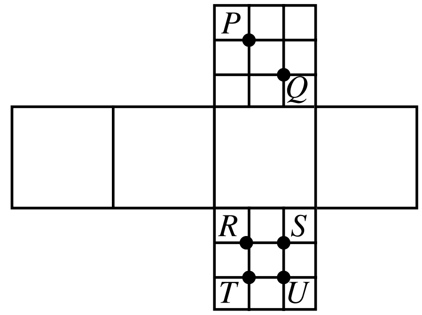
7. The perimeter of the figure shown is
(**A**) 19 cm (**B**) 22 cm (**C**) 21 cm
(**D**) 15 cm (**E**) 20 cm



8. Three of the vertices of a parallelogram are $(0,1)$, $(1,2)$, and $(2,1)$. The area of the parallelogram is
(**A**) 1 (**B**) 2 (**C**) $\sqrt{2}$ (**D**) $2\sqrt{2}$ (**E**) 4

9. If $10\le x\le 20$ and $40\le y\le 60$, the largest value of $\frac{x^2}{2y}$ is
(**A**) 5 (**B**) $\frac{5}{6}$ (**C**) $\frac{10}{3}$ (**D**) $\frac{5}{4}$ (**E**) 10

10. On a cube, two points are said to be diametrically opposite if the line containing the two points also contains the centre of the cube. The diagram below shows a pattern which could be folded into a cube. Which point would be diametrically opposite to point P?

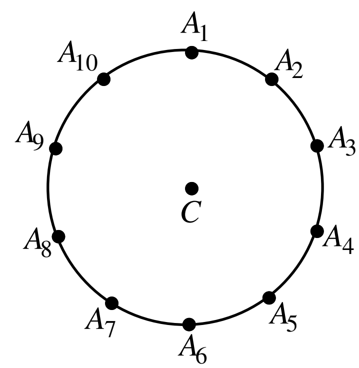


(**A**) Q (**B**) R (**C**) S
(**D**) T (**E**) U

Part B: Each question is worth 6 credits.

11. Five integers have an average of 69. The middle integer (the median) is 83. The most frequently occurring integer (the mode) is 85. The range of the five integers is 70. What is the second smallest of the five integers?
(**A**) 77 (**B**) 15 (**C**) 50 (**D**) 55 (**E**) 49

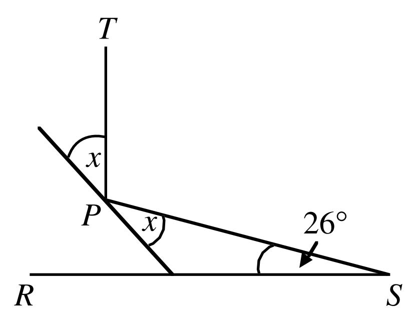
12. On a circle, ten points $A_1, A_2, A_3, \ldots, A_{10}$ are equally spaced. If C is the centre of the circle, what is the size, in degrees, of the angle A_1A_5C?



(**A**) 18 (**B**) 36 (**C**) 10
(**D**) 72 (**E**) 144

13. The digits 1, 2, 3, 4 can be arranged to form twenty-four different 4-digit numbers. If these twenty-four numbers are listed from smallest to largest, in what position is 3142?
(**A**) 13th (**B**) 14th (**C**) 15th (**D**) 16th (**E**) 17th

14. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. Then x is



(**A**) 26° (**B**) 13° (**C**) 64°
(**D**) 32° (**E**) 58°

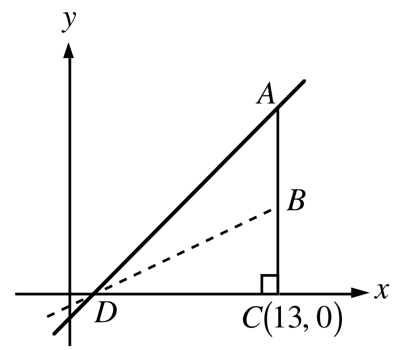
15. If $x^2yz^3 = 7^3$ and $xy^2 = 7^9$, then xyz equals
(**A**) 7^{10} (**B**) 7^9 (**C**) 7^8 (**D**) 7^6 (**E**) 7^4

16. The sum of the first 50 positive odd integers is 50^2. The sum of the first 50 positive even integers is
(**A**) 50^2 (**B**) 50^2+1 (**C**) 50^2+25 (**D**) 50^2+50 (**E**) 50^2+100

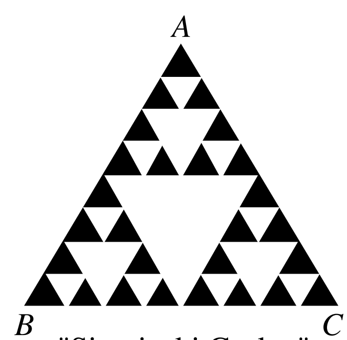
17. During 1996, the population of Sudbury decreased by 6% while the population of Victoria increased by 14%. At the end of the 1996, the populations of these cities were equal. What was the ratio of the population of Sudbury to the population of Victoria at the beginning of 1996?
(**A**) 47:57 (**B**) 57:47 (**C**) 53:43 (**D**) 53:57 (**E**) 43:47

18. Given $A=\{1, 2, 3, 5, 8, 13, 21, 34, 55\}$, how many of the numbers between 3 and 89 cannot be written as the sum of two elements of the set?
(**A**) 43 (**B**) 36 (**C**) 34 (**D**) 55 (**E**) 51

19. In the diagram, the equation of line AD is $y=\sqrt{3}(x-1)$. BD bisects $\angle ADC$. If the coordinates of B are (p, q), what is the value of q?
(**A**) 6 (**B**) 6.5 (**C**) $\frac{10}{\sqrt{3}}$
(**D**) $\frac{12}{\sqrt{3}}$ (**E**) $\frac{13}{\sqrt{3}}$



20. In the diagram, all triangles are equilateral. If $AB=16$, then the total area of all the black triangles is
(**A**) $37\sqrt{3}$ (**B**) $32\sqrt{3}$ (**C**) $27\sqrt{3}$
(**D**) $64\sqrt{3}$ (**E**) $\frac{64}{3}\sqrt{3}$



"Sierpinski Gasket"

Part C: Each question is worth 8 credits.

21. If $\dfrac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=11$, and a, b, and c are positive integers, then the number of ordered triples (a, b, c), such that $a+2b+c\le 40$, is
(**A**) 33 (**B**) 37 (**C**) 40 (**D**) 42 (**E**) 45

22. If $2x^2 - 2xy + y^2 = 289$, where x and y are integers and $x \geq 0$, the number of different ordered pairs (x, y) which satisfy this equation is

(**A**) 8 (**B**) 7 (**C**) 5 (**D**) 4 (**E**) 3

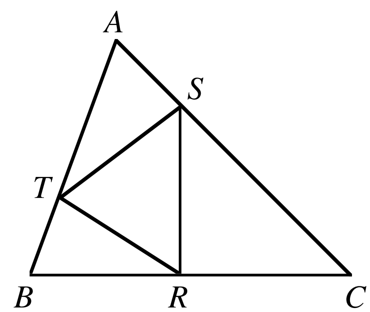
23. If $f(x) = px + q$ and $f(f(f(x))) = 8x + 21$, and if p and q are real numbers, then $p + q$ equals

(**A**) 2 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

24. The first term in a sequence of numbers is $t_1 = 5$. Succeeding terms are defined by the statement $t_n - t_{n-1} = 2n + 3$ for $n \geq 2$. The value of t_{50} is

(**A**) 2700 (**B**) 2702 (**C**) 2698 (**D**) 2704 (**E**) 2706

25. In triangle ABC, R is the mid-point of BC, $CS = 3SA$, and $\frac{AT}{TB} = \frac{p}{q}$. If w is the area of ΔCRS, x is the area of ΔRBT, z is the area of ΔATS, and $x^2 = wz$, then the value of $\frac{p}{q}$ is



(**A**) $\frac{\sqrt{21}-3}{2}$ (**B**) $\frac{\sqrt{21}+3}{2}$ (**C**) $\frac{\sqrt{21}-3}{6}$

(**D**) $\frac{\sqrt{105}+3}{6}$ (**E**) $\frac{\sqrt{105}-3}{6}$