The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Wednesday, February 28, 2024
(in North America and South America)

Thursday, February 29, 2024
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $2 \times 0 + 2 \times 4$ is equal to

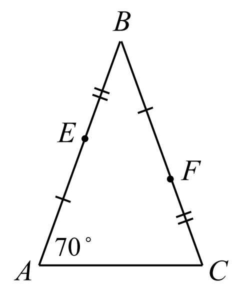
   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2. If $x = 3$, the value of $-(5x - 6x)$ is

   (A) -33 **(B)** 3 **(C)** -1 **(D)** 33 **(E)** 11

3. In $\triangle ABC$, points E and F are on AB and BC, respectively, such that $AE = BF$ and $BE = CF$. If $\angle BAC = 70^\circ$, the measure of $\angle ABC$ is

   (A) 40° **(B)** 50° **(C)** 60°
   (D) 70° **(E)** 30°

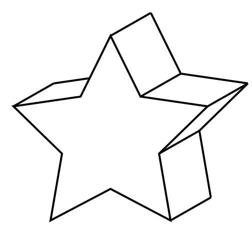


4. At Wednesday's basketball game, the Cayley Comets scored 90 points. At Friday's game, they scored 80% as many points as they scored on Wednesday. How many points did they score on Friday?

   (A) 60 **(B)** 72 **(C)** 75 **(D)** 78 **(E)** 82

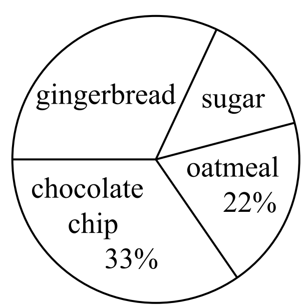
5. In the diagram, the two identical bases of the prism are shaped like a star. The area of each star-shaped base is 400 cm^2. The depth of the prism (that is, the distance between the star-shaped bases) is 8 cm. The volume of the prism is

   (A) 720 cm^3 **(B)** 1520 cm^3 **(C)** 3200 cm^3
   (D) 3600 cm^3 **(E)** 28 800 cm^3



6. Last year, Lloyd ate cookies in the percentages shown in the pie chart. The number of gingerbread cookies that he ate was two times the number of sugar cookies that he ate. What percentage of the cookies that he ate were gingerbread cookies?

   (A) 25% **(B)** 28% **(C)** 30%
   (D) 35% **(E)** 38%



7. If $\dfrac{1}{6} + \dfrac{1}{3} = \dfrac{1}{x}$, the value of x is

   (A) 9 **(B)** 6 **(C)** 18 **(D)** 2 **(E)** 3

8. Which of the following integers is equal to a perfect square?

   (A) 2^3 **(B)** 3^5 **(C)** 4^7 **(D)** 5^9 **(E)** 6^{11}

9. The sum of five consecutive *odd* integers is 125. The smallest of these integers is

(A) 17 **(B)** 19 **(C)** 21 **(D)** 25 **(E)** 29

10. Two standard six-sided dice are rolled. What is the probability that the product of the two numbers rolled is 12?

(A) $\frac{3}{36}$ **(B)** $\frac{4}{36}$ **(C)** $\frac{6}{36}$ **(D)** $\frac{8}{36}$ **(E)** $\frac{9}{36}$

Part B: Each correct answer is worth 6.

11. Arturo has an equal number of \$5 bills, of \$10 bills, and of \$20 bills. The total value of these bills is \$700. How many \$5 bills does Arturo have?

(A) 16 **(B)** 24 **(C)** 12 **(D)** 20 **(E)** 28

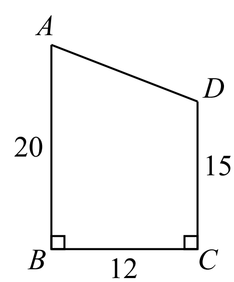
12. The Cayley Corner Store sells three types of toys: Exes, Wyes and Zeds. All Exes are identical, all Wyes are identical, and all Zeds are identical. The mass of 2 Exes equals the mass of 29 Wyes. The mass of 1 Zed equals the mass of 16 Exes. The mass of 1 Zed equals the mass of how many Wyes?

(A) 3.625 **(B)** 1.103 **(C)** 232 **(D)** 464 **(E)** 928

13. In the diagram, quadrilateral $ABCD$ has $AB = 20$, $BC = 12$, and $CD = 15$. Also, AB and CD are perpendicular to BC. The perimeter of quadrilateral $ABCD$ is

(A) 47 **(B)** 59 **(C)** 84

(D) 72 **(E)** 60



14. Ten numbers have an average (mean) of 87. Two of those numbers are 51 and 99. The average of the other eight numbers is

(A) 90 **(B)** 89 **(C)** 88 **(D)** 91 **(E)** 92

15. A rectangle has width x and length y, as shown in Figure 1. The rectangle is cut along the horizontal and vertical dotted lines in Figure 1 to produce four smaller rectangles as shown in Figure 2.

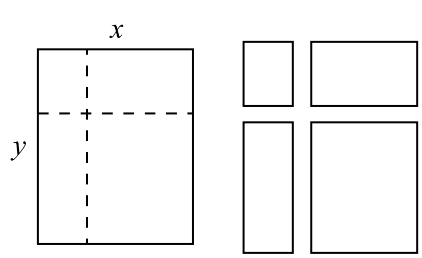


Figure 1 Figure 2

The sum of the perimeters of these four rectangles in Figure 2 is 24. The value of $x + y$ is

(A) 6 **(B)** 8 **(C)** 9.6 **(D)** 12 **(E)** 16

16. Suppose that $\sqrt{\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \cdots \times \frac{n-1}{n}} = \frac{1}{8}$. (The expression under the square root is the product of $n-1$ fractions.) The value of n is

(A) 81 **(B)** 64 **(C)** 16 **(D)** 256 **(E)** 100

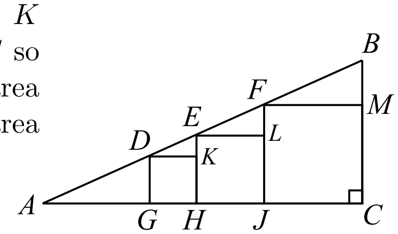
17. Each of the four digits of the integer 2024 is even. How many integers between 1000 and 9999, inclusive, have the property that all four of their digits are even?

(A) 500 **(B)** 625 **(C)** 96 **(D)** 54 **(E)** 256

18. The line with equation $y = 3x + 5$ is translated 2 units to the right. The equation of the resulting line is

(A) $y = 3x + 3$ **(B)** $y = 3x - 1$ **(C)** $y = 3x + 11$
(D) $y = 3x + 7$ **(E)** $y = 5x + 5$

19. In the diagram, $\triangle ABC$ is right-angled at C. Points D, E, F are on AB, points G, H, J are on AC, point K is on EH, point L is on FJ, and point M is on BC so that $DKHG$, $ELJH$ and $FMCJ$ are squares. The area of $DKHG$ is 16 and the area of $ELJH$ is 36. The area of square $FMCJ$ is



(A) 64 **(B)** 52 **(C)** 100
(D) 81 **(E)** 75

20. Jiwei and Hari entered a race. Hari finished the race in $\frac{4}{5}$ of the time it took Jiwei to finish. The next time that they raced the same distance, Jiwei increased his average speed from the first race by $x\%$, while Hari maintained the same average speed as in the first race. In this second race, Hari finished the race in the same amount of time that it took Jiwei to finish. The value of x is

(A) 20 **(B)** 25 **(C)** 35 **(D)** 40 **(E)** 50

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. A 3×3 table starts with every entry equal to 0 and is modified using the following steps:

(i) adding 1 to all three numbers in any row;

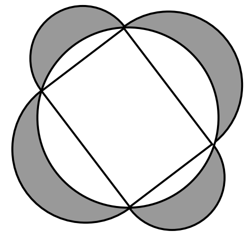
(ii) adding 2 to all three numbers in any column.

After step (i) has been used a total of a times and step (ii) has been used a total of b times, the table appears as shown. What is the value of $a + b$?

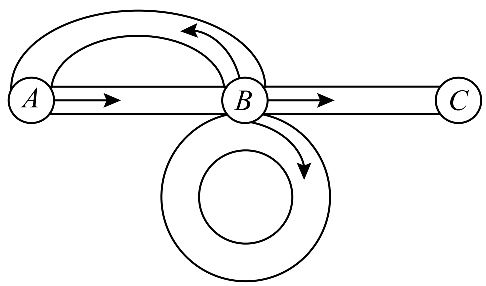
7	1	5
9	3	7
8	2	6

22. For how many integers m does the line with the equation $y = mx$ intersect the line segment with endpoints $(20, 24)$ and $(4, 202)$?

23. Four semi-circles are arranged so that their diameters form a 6 by 8 rectangle. A circle is drawn through the four vertices of the rectangle. In the diagram, the region inside the four semi-circles but outside the circle is shaded. The total area of the shaded region is A. What is the integer closest to A?

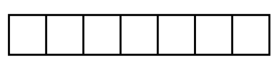


24. A park has four paths, as shown in the map below. It takes 2 minutes to walk along the path from A to B, 3 minutes to walk along the path from B to A, 3 minutes to walk along the path from B to C, and 3 minutes to walk around the path that begins and ends at B. Rasheeqa goes for a walk, starting at A, walking only in the directions indicated along the paths, never stopping to rest, and finishing at C.



If the walk takes a total of t minutes, how many possible values of t are there with $t \leq 100$?

25. Erin has an empty 1×7 grid consisting of 1×1 squares:

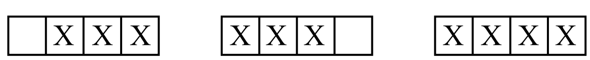


and follows the process below to construct a pattern:

(i) Place an X in any empty square.

(ii) If three or more consecutive squares each contain an X, stop and do not add any more X's; otherwise, go to step (i) and continue the process.

For example, in a smaller 1×4 grid, there are 3 different patterns that can be constructed:



(The last pattern may be obtained by placing X's, in order, in squares 1, 2, 4, and then 3.) By applying this process starting with the empy 1×7 grid, how many different possible patterns can Erin construct?

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For students...

Thank you for writing the 2024 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
- Use our free Problem Set Generator to create problem sets for curriculum support and enrichment
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results

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Part A: Each correct answer is worth 5.

1. What is the value of $\frac{1}{1} + \frac{2}{2} + \frac{3}{3}$?

   (A) 1 **(B)** 0 **(C)** 3 **(D)** 9 **(E)** 10

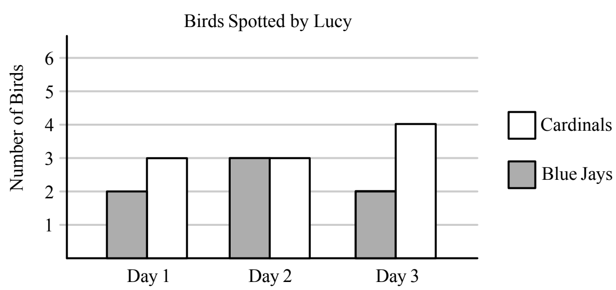
2. If $3n = 9 + 9 + 9$, then n equals

   (A) 3 **(B)** 6 **(C)** 18 **(D)** 9 **(E)** 27

3. One movie is 1 hour and 48 minutes long. A second movie is 25 minutes longer than the first. How long is the second movie?

   (A) 2 hours and 13 minutes
   (B) 1 hour and 48 minutes
   (C) 2 hours and 25 minutes
   (D) 2 hours and 3 minutes
   (E) 2 hours and 48 minutes

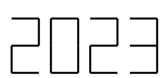
4. Lucy tracked the number of cardinals and the number of blue jays that she saw each day for three days. The graph below summarizes the birds that she saw.

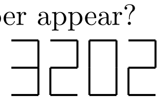


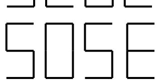
   How many more cardinals than blue jays did she see over the three days?

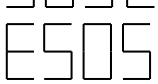
   (A) 7 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 1

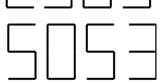
5. The number 2023 is written on one side of a classroom window using the style of numbers shown. If you were to stand on the other side of the window, how would the number appear?

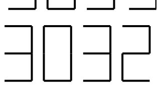


   (A) 

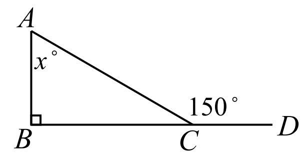
   (B) 

   (C) 

   (D) 

   (E) 

6. In the diagram, points B, C and D lie on a line. Also, $\angle ABC = 90°$ and $\angle ACD = 150°$. The value of x is



(A) 30 **(B)** 45 **(C)** 90
(D) 150 **(E)** 60

7. The surface area of a cube is 24. The volume of the cube is

(A) 4 **(B)** $3\sqrt{3}$ **(C)** 9 **(D)** 16 **(E)** 8

8. Charlie is making a necklace with yellow beads and green beads. She has already used 4 green beads and 0 yellow beads. How many yellow beads will she have to add so that $\frac{4}{5}$ of the total number of beads are yellow?

(A) 16 **(B)** 4 **(C)** 20 **(D)** 24 **(E)** 12

9. A positive number is increased by 60%. By what percentage should the result be decreased to return to the original value?

(A) 57.5% **(B)** 40% **(C)** 62.5% **(D)** 50% **(E)** 37.5%

10. Each of five doors is randomly either open or closed. What is the probability that exactly two of the five doors are open?

(A) $\frac{5}{16}$ **(B)** $\frac{13}{32}$ **(C)** $\frac{9}{25}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{7}{16}$

Part B: Each correct answer is worth 6.

11. Karim has 23 candies. He eats n candies and divides the remaining candies equally among his three children so that each child gets an integer number of candies. Which of the following is not a possible value of n?

(A) 2 **(B)** 5 **(C)** 9 **(D)** 11 **(E)** 14

12. A 6 m by 8 m rectangular field has a fence around it. There is a post at each of the four corners of the field. Starting at each corner, there is a post every 2 m along each side of the fence. How many posts are there?

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

13. The integer 2023 is equal to 7×17^2. Which of the following is the smallest positive perfect square that is a multiple of 2023?

(A) 2023^2 **(B)** 4×2023 **(C)** 7×2023 **(D)** 17×2023 **(E)** $7 \times 17 \times 2023$

14. Points A, B, C, and D are on a line in that order.
The distance from A to D is 24.
The distance from B to D is 3 times the distance from A to B.
Point C is halfway between B and D.
What is the distance from A to C?

(A) 12 **(B)** 8 **(C)** 16 **(D)** 9 **(E)** 15

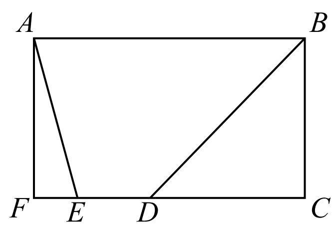
15. Suppose that $a = \dfrac{1}{n}$, where n is a positive integer with $n > 1$.

Which of the following statements is true?

(A) $a < \dfrac{1}{a} < a^2$ **(B)** $a^2 < a < \dfrac{1}{a}$ **(C)** $a < a^2 < \dfrac{1}{a}$

(D) $\dfrac{1}{a} < a < a^2$ **(E)** $\dfrac{1}{a} < a^2 < a$

16. In the diagram, $ABCF$ is a rectangle with $AB = 30$ cm and $AF = 14$ cm. Points E and D are on FC so that $FE = 5$ cm and the area of quadrilateral $ABDE$ is 266 cm^2. The length of DC is



(A) 17 cm **(B)** 16 cm **(C)** 19 cm

(D) 20 cm **(E)** 18 cm

17. Megan and Hana raced their remote control cars for 100 m.
The two cars started at the same time.
The average speed of Megan's car was $\frac{5}{4}$ m/s.
Hana's car finished 5 seconds before Megan's car.
What was the average speed of Hana's car?

(A) $\frac{4}{5}$ m/s **(B)** $\frac{5}{2}$ m/s **(C)** $\frac{4}{3}$ m/s **(D)** $\frac{6}{5}$ m/s **(E)** $\frac{3}{2}$ m/s

18. A factory makes chocolate bars. Five boxes, labelled V, W, X, Y, Z, are each packed with 20 bars. Each of the bars in three of the boxes has a mass of 100 g. Each of the bars in the other two boxes has a mass of 90 g. One bar is taken from box V, two bars are taken from box W, four bars are taken from box X, eight bars are taken from box Y, and sixteen bars are taken from box Z. The total mass of these bars taken from the boxes is 2920 g. The boxes containing the 90 g bars are labelled

(A) V and W **(B)** W and Z **(C)** X and Y **(D)** V and Z **(E)** W and Y

19. The average of a, b and c is 16. The average of c, d and e is 26. The average of a, b, c, d, and e is 20. The value of c is

(A) 10 **(B)** 20 **(C)** 21 **(D)** 26 **(E)** 30

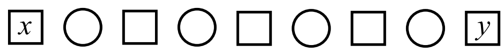
20. A robotic grasshopper jumps 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After every fourth jump, the grasshopper restarts the sequence of jumps: 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After a total of n jumps, the position of the grasshopper is 162 cm to the west and 158 cm to the south of its original position. The sum of the squares of the digits of n is

(A) 22 **(B)** 29 **(C)** 17 **(D)** 14 **(E)** 13

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

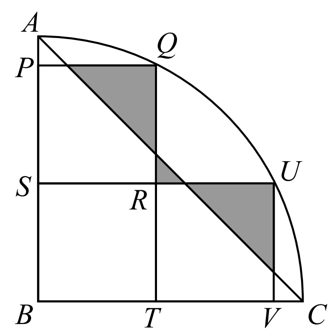
21. A line has equation $y = mx - 50$ for some positive integer m. The line passes through the point $(a, 0)$ for some positive integer a. What is the sum of all possible values of m?

22. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in the circles and squares below with one number in each shape.



Each integer must be used exactly once and the integer in each circle must be equal to the sum of the integers in the two neighbouring squares. If the integer x is placed in the leftmost square and the integer y is placed in the rightmost square, what is the largest possible value of $x + y$?

23. In the diagram, ABC is a quarter-circle centred at B. Each of square $PQRS$, square $SRTB$ and square $RUVT$ has side length 10. Points P and S are on AB, points T and V are on BC, and points Q and U are on the quarter-circle. Line segment AC is drawn. Three triangular regions are shaded, as shown. What is the integer closest to the total area of the shaded regions?



24. Carina is in a tournament in which no game can end in a tie. She continues to play games until she loses 2 games, at which point she is eliminated and plays no more games. The probability of Carina winning the first game is $\frac{1}{2}$. After she wins a game, the probability of Carina winning the next game is $\frac{3}{4}$. After she loses a game, the probability of Carina winning the next game is $\frac{1}{3}$. The probability that Carina wins 3 games before being eliminated from the tournament equals $\frac{a}{b}$, where the fraction $\frac{a}{b}$ is in lowest terms. What is the value of $a + b$?

25. A set consists of five different odd positive integers, each greater than 2. When these five integers are multiplied together, their product is a five-digit integer of the form $AB0AB$, where A and B are digits with $A \neq 0$ and $A \neq B$. (The hundreds digit of the product is zero.) For example, the integers in the set $\{3, 5, 7, 13, 33\}$ have a product of 45 045. In total, how many different sets of five different odd positive integers have these properties?

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The CENTRE for EDUCATION
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Cayley Contest

(Grade 10)

Wednesday, February 23, 2022
(in North America and South America)

Thursday, February 24, 2022
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $2 + (0 \times 2^2)$ is equal to

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 6

2. The integer 119 is a multiple of

   (A) 2 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 11

3. Which of the following fractions has the greatest value?

   (A) $\frac{3}{10}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{5}{23}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{1}{2}$

4. The pattern of shapes $\triangle, \square, \heartsuit, \diamondsuit, \bigcirc$ is repeated to form the sequence

   $$\triangle, \square, \heartsuit, \diamondsuit, \bigcirc, \triangle, \square, \heartsuit, \diamondsuit, \bigcirc, \ldots$$

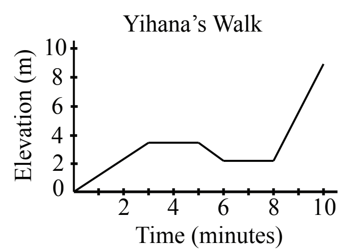
   The 22nd shape in the sequence is

   (A) $\square$ **(B)** $\bigcirc$ **(C)** $\diamondsuit$ **(D)** $\triangle$ **(E)** $\heartsuit$

5. The expression $(5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5) + (5 \times 5)$ is equal to

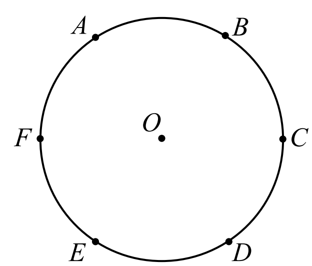
   (A) 5 **(B)** 10 000 **(C)** 625 **(D)** 19 525 **(E)** 125

6. Yihana walks for 10 minutes. A graph of her elevation in metres versus time in minutes is shown. The length of time for which she was walking uphill is

   (A) 5 minutes **(B)** 6 minutes **(C)** 4 minutes
   (D) 7 minutes **(E)** 8 minutes

7. Points A, B, C, D, E, and F are evenly spaced around the circle with centre O, as shown. The measure of $\angle AOC$ is

   (A) 90° **(B)** 150° **(C)** 144°
   (D) 120° **(E)** 108°

8. A rectangle has positive integer side lengths and an area of 24. The perimeter of the rectangle *cannot* be

   (A) 20 **(B)** 22 **(C)** 28 **(D)** 50 **(E)** 36

9. The operation $a\nabla b$ is defined by $a\nabla b = \dfrac{a+b}{a-b}$ for all integers a and b with $a \neq b$. For example, $2\nabla 3 = \dfrac{2+3}{2-3} = -5$. If $3\nabla b = -4$, what is the value of b?

   (A) 5 **(B)** -7 **(C)** 7 **(D)** -5 **(E)** 3

10. If x is 20% of y and x is 50% of z, then what percentage is z of y?

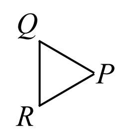
(A) 70% **(B)** 10% **(C)** 30% **(D)** 40% **(E)** 60%

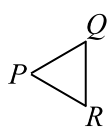
Part B: Each correct answer is worth 6.

11. A store sells jellybeans at a fixed price per gram. The price for 250 g of jellybeans is \$7.50. What mass of jellybeans sells for \$1.80?

(A) 6 g **(B)** 54 g **(C)** 60 g **(D)** 120 g **(E)** 190 g

12. An equilateral triangle is made of cardboard and lies on a table. Paola stands in front of the table and sees the triangle in the position shown. She flips the triangle over, keeping edge QR on the table throughout the flip. From this position, Paola then flips the triangle again, this time keeping edge PR on the table throughout the flip. What is the resulting position of the triangle that Paola sees?



(A)


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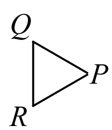
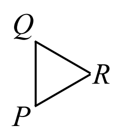
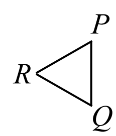
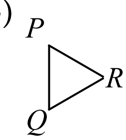


(C)


(D)


(E)


13. Two identical smaller cubes are stacked next to a larger cube. Each of the two smaller cubes has a volume of 8. The combined height of the smaller cubes equals the height of the larger cube. What is the volume of the larger cube?

(A) 16 **(B)** 32 **(C)** 125 **(D)** 48 **(E)** 64

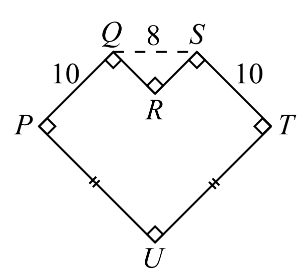
14. The integer 48 178 includes the block of digits 178. The three integers 51 870, 19 728 and 38 717 do not include the block of digits 178. How many integers between 10 000 and 100 000 include the block of digits 178?

(A) 280 **(B)** 300 **(C)** 270 **(D)** 310 **(E)** 260

15. The integers a, b and c satisfy the equations $a + 5 = b$ and $5 + b = c$ and $b + c = a$. The value of b is

(A) -30 **(B)** -20 **(C)** -10 **(D)** 0 **(E)** 5

16. In the diagram, hexagon $PQRSTU$ has interior right angles at P, Q, S, T, and U and an exterior right angle at R. Also, $PU = UT$, $PQ = ST = 10$, and $QS = 8$. The perimeter of $PQRSTU$ is closest to



(A) 48 **(B)** 56 **(C)** 63

(D) 71 **(E)** 72

17. Zebadiah has 3 red shirts, 3 blue shirts, and 3 green shirts in a drawer. Without looking, he randomly pulls shirts from his drawer one at a time. He would like a set of shirts that includes either 3 of the same colour or 3 of different colours. What is the minimum number of shirts that Zebadiah has to pull out to *guarantee* that he has such a set?

(A) 4 **(B)** 3 **(C)** 6 **(D)** 5 **(E)** 7

18. At the beginning of the first day, a box contains 1 black ball, 1 gold ball, and no other balls. At the end of each day, for each gold ball in the box, 2 black balls and 1 gold ball are added to the box; this means that at the end of the first day, there are 5 balls in the box. If no balls are removed from the box, how many balls are in the box at the end of the seventh day?

(A) 395 **(B)** 371 **(C)** 389 **(D)** 377 **(E)** 383

19. The area of the triangular region bounded by the x-axis, the y-axis and the line with equation $y = 2x - 6$ is one-quarter of the area of the triangular region bounded by the x-axis, the line with equation $y = 2x - 6$ and the line with equation $x = d$, where $d > 0$. What is the value of d?

(A) 9 **(B)** 6 **(C)** 8 **(D)** $3 + 3\sqrt{2}$ **(E)** 15

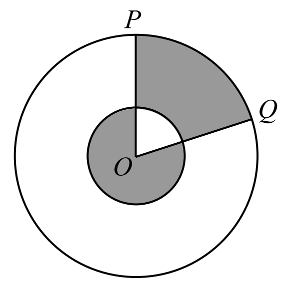
20. If m and n are positive integers that satisfy the equation $3m^3 = 5n^5$, the smallest possible value for $m + n$ is

(A) 900 **(B)** 450 **(C)** 720 **(D)** 810 **(E)** 630

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. There are exactly four ordered pairs of positive integers (x, y) that satisfy the equation $20x + 11y = 881$. Mehdi writes down the four values of y and adds the smallest and largest of these values. What is this sum?

22. In the diagram, two circles are centred at O. The smaller circle has a radius of 1 and the larger circle has a radius of 3. Points P and Q are placed on the larger circle so that the areas of the two shaded regions are equal. If $\angle POQ = x^\circ$, what is the value of x?



23. Andreas, Boyu, Callista, and Diane each randomly choose an integer from 1 to 9, inclusive. Each of their choices is independent of the other integers chosen and the same integer can be chosen by more than one person. The probability that the sum of their four integers is even is equal to $\dfrac{N}{6561}$ for some positive integer N. What is the sum of the squares of the digits of N?

24. A cube with edge length 8 is balanced on one of its vertices on a horizontal table such that the diagonal from this vertex through the interior of the cube to the farthest vertex is vertical. When the sun is directly above the top vertex, the shadow of the cube on the table is a regular hexagon. The area of this shadow can be written in the form $a\sqrt{b}$, where a and b are positive integers and b is not divisible by any perfect square larger than 1. What is the value of $a + b$?

25. There are T tokens arranged in a circle for some positive integer T. Moving clockwise around the circle, the tokens are labelled, in order, with the integers from 1 to T. Starting from the token labelled 1, Évariste:

(i) Removes the token at the current position.
(ii) Moves clockwise to the next remaining token.
(iii) Moves clockwise again to the next remaining token.
(iv) Repeats steps (i) to (iii) until only one token remains.

When $T = 337$, the number on the last remaining token is L. There are other integers T for which the number on the last remaining token is also L. What are the rightmost two digits of the smallest possible value of T?

2022 Cayley Contest (English)

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- Find your school's contest results

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in MATHEMATICS and COMPUTING
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Cayley Contest

(Grade 10)

Tuesday, February 23, 2021
(in North America and South America)

Wednesday, February 24, 2021
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

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4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

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Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $\dfrac{2+4}{1+2}$ is equal to

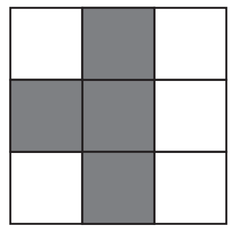
(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

2. The ones (units) digit of 542 is 2. When 542 is multiplied by 3, the ones (units) digit of the result is

(A) 9 **(B)** 3 **(C)** 5 **(D)** 4 **(E)** 6

3. Some of the 1×1 squares in a 3×3 grid are shaded, as shown. What is the perimeter of the shaded region?

(A) 10 **(B)** 14 **(C)** 8
(D) 18 **(E)** 20



4. If $3x + 4 = x + 2$, the value of x is

(A) 0 **(B)** -4 **(C)** -3 **(D)** -1 **(E)** -2

5. Which of the following is equal to 110% of 500?

(A) 610 **(B)** 510 **(C)** 650 **(D)** 505 **(E)** 550

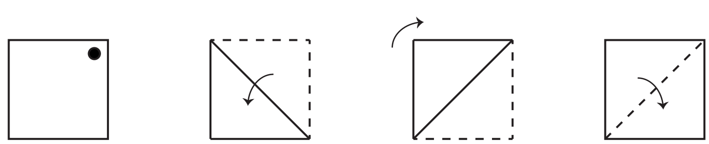
6. Eugene swam on Sunday, Monday and Tuesday. On Monday, he swam for 30 minutes. On Tuesday, he swam for 45 minutes. His average swim time over the three days was 34 minutes. For how many minutes did he swim on Sunday?

(A) 20 **(B)** 25 **(C)** 27 **(D)** 32 **(E)** 37.5

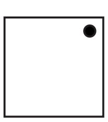
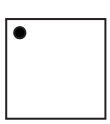
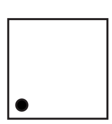
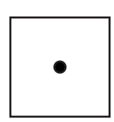
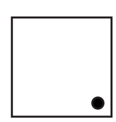
7. For which of the following values of x is $x^3 < x^2$?

(A) $x = \frac{5}{3}$ **(B)** $x = \frac{3}{4}$ **(C)** $x = 1$ **(D)** $x = \frac{3}{2}$ **(E)** $x = \frac{21}{20}$

8. A square piece of paper has a dot in its top right corner and is lying on a table. The square is folded along its diagonal, then rotated 90° clockwise about its centre, and then finally unfolded, as shown.

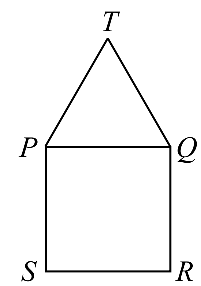


The resulting figure is

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

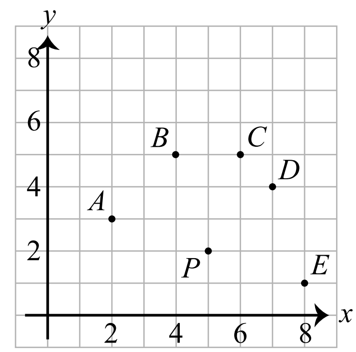
9. In 12 years, Janice will be 8 times as old as she was 2 years ago. How old is Janice now?
(A) 4 **(B)** 8 **(C)** 10 **(D)** 2 **(E)** 6

10. In the diagram, pentagon $TPSRQ$ is constructed from equilateral $\triangle PTQ$ and square $PQRS$. The measure of $\angle STR$ is equal to
(A) 10° **(B)** 15° **(C)** 20°
(D) 30° **(E)** 45°



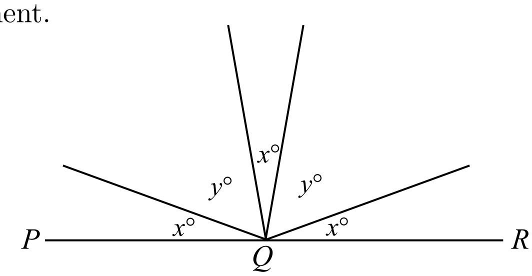
Part B: Each correct answer is worth 6.

11. In the diagram, which of the following points is at a different distance from P than the rest of the points?
(A) A **(B)** B **(C)** C
(D) D **(E)** E



12. If $x = 2$ and $y = x^2 - 5$ and $z = y^2 - 5$, then z equals
(A) -6 **(B)** -8 **(C)** 4 **(D)** 76 **(E)** -4

13. In the diagram, PQR is a straight line segment. If $x + y = 76$, what is the value of x?
(A) 28 **(B)** 30 **(C)** 35
(D) 36 **(E)** 38



14. The line with equation $y = 2x - 6$ is reflected in the y-axis. What is the x-intercept of the resulting line?
(A) -12 **(B)** 6 **(C)** -6 **(D)** -3 **(E)** 0

15. Amy bought and then sold $15n$ avocados, for some positive integer n. She made a profit of \$100. (Her *profit* is the difference between the total amount that she earned by selling the avocados and the total amount that she spent in buying the avocados.) She paid \$2 for every 3 avocados. She sold every 5 avocados for \$4. What is the value of n?
(A) 100 **(B)** 20 **(C)** 50 **(D)** 30 **(E)** 8

16. If $3^x = 5$, the value of 3^{x+2} is
(A) 10 **(B)** 25 **(C)** 2187 **(D)** 14 **(E)** 45

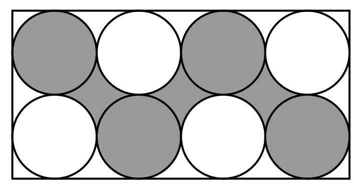
17. A group of friends are sharing a bag of candy.
On the first day, they eat $\frac{1}{2}$ of the candies in the bag.
On the second day, they eat $\frac{2}{3}$ of the remaining candies.
On the third day, they eat $\frac{3}{4}$ of the remaining candies.
On the fourth day, they eat $\frac{4}{5}$ of the remaining candies.
On the fifth day, they eat $\frac{5}{6}$ of the remaining candies.
At the end of the fifth day, there is 1 candy remaining in the bag.
How many candies were in the bag before the first day?

(A) 512 **(B)** 720 **(C)** 1024 **(D)** 1440 **(E)** 2048

18. Elina and Gustavo leave Cayley H.S. at 3:00 p.m. Elina runs north at a constant speed of 12 km/h. Gustavo walks east at a constant speed of 5 km/h. After 12 minutes, Elina and Gustavo change direction and travel directly towards each other, still at 12 km/h and 5 km/h, respectively. The time that they will meet again is closest to

(A) 3:24 p.m. **(B)** 3:35 p.m. **(C)** 3:25 p.m. **(D)** 3:29 p.m. **(E)** 3:21 p.m.

19. In the diagram, eight circles, each of radius 1, are drawn inside a rectangle. Four of the circles are tangent to two sides of the rectangle and to two other circles. Four of the circles are tangent to one side of the rectangle and to three other circles. A region has been shaded, as shown. It consists of three spaces (each space bounded by a different set of four circles), as well as four of the circles themselves. The area of this region is closest to



(A) 12 **(B)** 13 **(C)** 14
(D) 15 **(E)** 16

20. How many four-digit positive integers are divisible by both 12 and 20, but are not divisible by 16?

(A) 111 **(B)** 113 **(C)** 125 **(D)** 150 **(E)** 149

Part C: Each correct answer is worth 8.

21. The variables a, b, c, d, e, and f represent the numbers 4, 12, 15, 27, 31, and 39 in some order. Suppose that

$$a + b = c$$
$$b + c = d$$
$$c + e = f$$

The value of $a + c + f$ is

(A) 58 **(B)** 70 **(C)** 73 **(D)** 82 **(E)** 85

22. The cells of a 3×3 grid are to be filled with integers so that the average value of the entries along each row, each column, and each diagonal is the same. The integers 10, 64 and 70 are entered, as shown. When the remaining six squares are filled in to complete the grid, what integer replaces x?

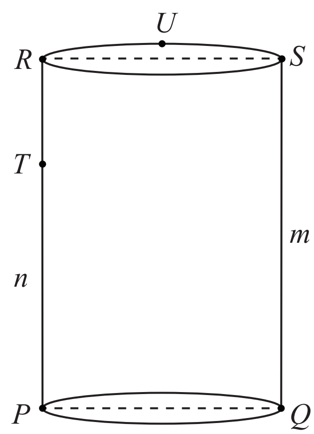
64	10	
70		
	x	

(A) 78 **(B)** 82 **(C)** 86 **(D)** 90 **(E)** 94

23. A special six-sided die has its faces numbered 1 through 6 and has the property that rolling each number x is x times as likely as rolling a 1. For example, the probability of rolling a 5 is 5 times the probability of rolling a 1, while the probability of rolling a 2 is 2 times the probability of rolling a 1. Robbie and Francine play a game where they each roll this die three times, and the total of their three rolls is their score. The winner is the player with the highest score; if the two players are tied, neither player wins. After two rolls each, Robbie has a score of 8 and Francine has a score of 10. The probability that Robbie will win can be written in lowest terms as $\dfrac{r}{400+s}$, where r and s are positive integers. What is value of $r+s$?

(A) 96 **(B)** 86 **(C)** 76 **(D)** 66 **(E)** 56

24. In the diagram, PQ is a diameter of the circular base of the cylinder. RS is a diameter of the top face of the cylinder and is directly above PQ, as shown. Point U is on the circumference of the top face, halfway between R and S. Point T is on the cylinder and is directly above P. Suppose that $QS = m$ and $PT = n$, where m and n are integers with $1 < n < m$. If $QU = 9\sqrt{33}$ and $UT = 40$, what is the remainder when the integer equal to QT^2 is divided by 100?



(A) 29 **(B)** 49 **(C)** 9 **(D)** 89 **(E)** 69

25. The points $J(2,7)$, $K(5,3)$ and $L(r,t)$ form a triangle whose area is less than or equal to 10. Let $\mathcal{R}$ be the region formed by all such points L with $0 \le r \le 10$ and $0 \le t \le 10$. When written as a fraction in lowest terms, the area of $\mathcal{R}$ is equal to $\dfrac{300+a}{40-b}$ for some positive integers a and b. The value of $a+b$ is

(A) 82 **(B)** 71 **(C)** 60 **(D)** 49 **(E)** 93

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Cayley Contest

(Grade 10)

Tuesday, February 25, 2020
(in North America and South America)

Wednesday, February 26, 2020
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{20-20}{20+20}$ is

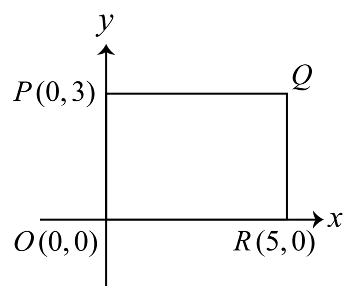
   (A) 0 **(B)** 1 **(C)** 10 **(D)** -2 **(E)** 2

2. When $x=3$ and $y=4$, the value of $xy-x$ is

   (A) 3 **(B)** 4 **(C)** 12 **(D)** 9 **(E)** 15

3. The points $O(0,0)$, $P(0,3)$, Q, and $R(5,0)$ form a rectangle, as shown. The coordinates of Q are

   (A) $(5,5)$ **(B)** $(5,3)$ **(C)** $(3,3)$ **(D)** $(2.5,1.5)$ **(E)** $(0,5)$

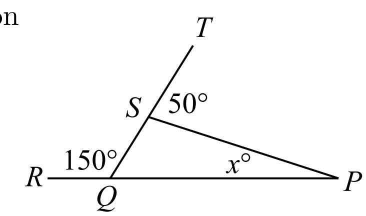


4. Which of the following numbers is less than $\frac{1}{20}$?

   (A) $\frac{1}{15}$ **(B)** $\frac{1}{25}$ **(C)** 0.5 **(D)** 0.055 **(E)** $\frac{1}{10}$

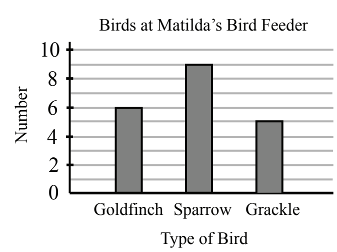
5. In the diagram, point Q lies on PR and point S lies on QT. What is the value of x?

   (A) 10 **(B)** 30 **(C)** 50 **(D)** 40 **(E)** 20



6. Matilda counted the birds that visited her bird feeder yesterday. She summarized the data in the bar graph shown. The percentage of birds that were goldfinches is

   (A) 15% **(B)** 20% **(C)** 30% **(D)** 45% **(E)** 60%



7. The average of the two positive integers m and n is 5. What is the largest possible value for n?

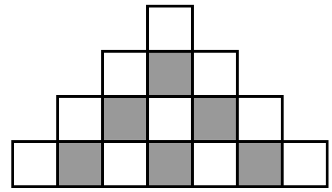
   (A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

8. Roman wins a contest with a prize of \$200. He gives 30% of the prize to Jackie. He then splits 15% *of what remains* equally between Dale and Natalia. How much money does Roman give Dale?

   (A) \$10.50 **(B)** \$15.00 **(C)** \$4.50 **(D)** \$25.50 **(E)** \$59.50

9. Shaded and unshaded squares are arranged in rows so that:

- the first row consists of one unshaded square,
- each row begins with an unshaded square,
- the squares in each row alternate between unshaded and shaded, and
- each row after the first has two more squares than the previous row.

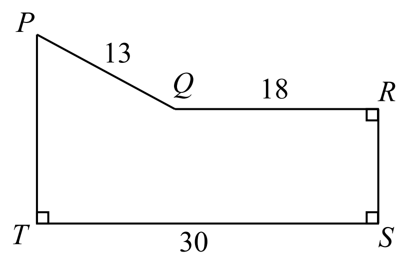


The first 4 rows are shown. The number of shaded squares in the 2020th row is

(A) 2022 **(B)** 2021 **(C)** 2020
(D) 2019 **(E)** 2018

10. In the diagram, pentagon $PQRST$ has $PQ = 13$, $QR = 18$, $ST = 30$, and a perimeter of 82. Also, $\angle QRS = \angle RST = \angle STP = 90°$. The area of the pentagon $PQRST$ is

(A) 306 **(B)** 297 **(C)** 288
(D) 279 **(E)** 270



Part B: Each correct answer is worth 6.

11. The sum of the first 9 positive integers is 45; in other words,

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

What is the sum of the first 9 positive multiples of 5? In other words, what is the value of $5 + 10 + 15 + \cdots + 40 + 45$?

(A) 225 **(B)** 250 **(C)** 180 **(D)** 150 **(E)** 450

12. The volume of a rectangular prism is 21. Its length, width and height are all different positive integers. The sum of its length, width and height is

(A) 11 **(B)** 13 **(C)** 15 **(D)** 9 **(E)** 17

13. If $2^n = 8^{20}$, what is the value of n?

(A) 10 **(B)** 60 **(C)** 40 **(D)** 16 **(E)** 17

14. Juliana chooses three different numbers from the set $\{-6, -4, -2, 0, 1, 3, 5, 7\}$ and multiplies them together to obtain the integer n. What is the greatest possible value of n?

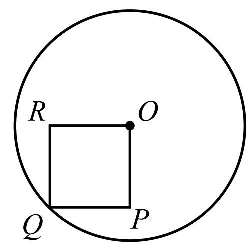
(A) 168 **(B)** 0 **(C)** 15 **(D)** 105 **(E)** 210

15. A bag contains only green, yellow and red marbles. The ratio of green marbles to yellow marbles to red marbles in the bag is $3 : 4 : 2$. If 63 of the marbles in the bag are *not* red, the number of red marbles in the bag is

(A) 14 **(B)** 18 **(C)** 27 **(D)** 36 **(E)** 81

16. In the diagram, the circle has centre O and square $OPQR$ has vertex Q on the circle. If the area of the circle is 72π, the area of the square is

(A) 38 **(B)** 48 **(C)** 25
(D) 12 **(E)** 36



17. Carley made treat bags. Each bag contained exactly 1 chocolate, 1 mint, and 1 caramel. The chocolates came in boxes of 50. The mints came in boxes of 40. The caramels came in boxes of 25. Carley made no incomplete treat bags and there were no unused chocolates, mints or caramels. What is the minimum total number of boxes that Carley could have bought?

(A) 19 **(B)** 17 **(C)** 44 **(D)** 25 **(E)** 9

18. Nate is driving to see his grandmother. If he drives at a constant speed of 40 km/h, he will arrive 1 hour late. If he drives at a constant speed of 60 km/h, he will arrive 1 hour early. At what constant speed should he drive to arrive just in time?

(A) 56 km/h **(B)** 80 km/h **(C)** 54 km/h **(D)** 48 km/h **(E)** 58 km/h

19. A multiple choice test has 10 questions on it. Each question answered correctly is worth 5 points, each unanswered question is worth 1 point, and each question answered incorrectly is worth 0 points. How many of the integers between 30 and 50, inclusive, are *not* possible total scores?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 5

20. For how many pairs (m, n) with m and n integers satisfying $1 \leq m \leq 100$ and $101 \leq n \leq 205$ is $3^m + 7^n$ divisible by 10?

(A) 2600 **(B)** 2626 **(C)** 2601 **(D)** 2650 **(E)** 2625

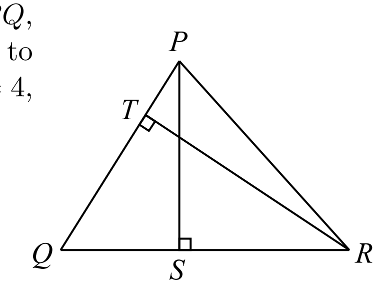
Part C: Each correct answer is worth 8.

21. How many points (x, y), with x and y both integers, are on the line with equation $y = 4x + 3$ and inside the region bounded by $x = 25$, $x = 75$, $y = 120$, and $y = 250$?

(A) 44 **(B)** 36 **(C)** 40 **(D)** 32 **(E)** 48

22. In the diagram, points S and T are on sides QR and PQ, respectively, of $\triangle PQR$ so that PS is perpendicular to QR and RT is perpendicular to PQ. If $PT = 1$, $TQ = 4$, and $QS = 3$, what is the length of SR?

(A) 3 **(B)** $\frac{11}{3}$ **(C)** $\frac{15}{4}$
(D) $\frac{7}{2}$ **(E)** 4



23. Ricardo wants to arrange three 1s, three 2s, two 3s, and one 4 to form nine-digit positive integers with the properties that

- when reading from left to right, there is at least one 1 before the first 2, at least one 2 before the first 3, and at least one 3 before the 4, and
- no digit 2 can be next to another 2.

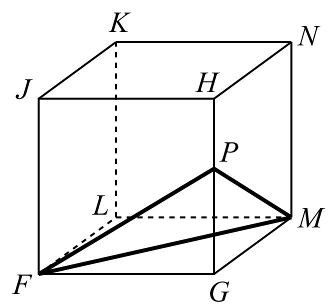
(For example, the integer 121 321 234 satisfies these properties.) In total, how many such nine-digit positive integers can Ricardo make?

(A) 278 **(B)** 260 **(C)** 254 **(D)** 272 **(E)** 266

24. A cube with vertices $FGHJKLMN$ has edge length 200. Point P is on HG, as shown. The shortest distance from G to a point inside $\triangle PFM$ is 100. Which of the following is closest to the length of HP?

(A) 53 **(B)** 55 **(C)** 57

(D) 59 **(E)** 61



25. How many positive integers $n \leq 20\,000$ have the properties that $2n$ has 64 positive divisors including 1 and $2n$, and $5n$ has 60 positive divisors including 1 and $5n$?

(A) 4 **(B)** 5 **(C)** 3 **(D)** 2 **(E)** 6

2020 Cayley Contest (English)

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The CENTRE for EDUCATION
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Cayley Contest

(Grade 10)

Tuesday, February 26, 2019
(in North America and South America)

Wednesday, February 27, 2019
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

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	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

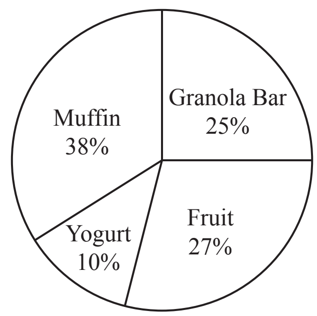
1. The expression $2 \times 0 + 1 - 9$ equals

   (A) -8 **(B)** -6 **(C)** -7 **(D)** -11 **(E)** 0

2. Kai will celebrate his 25th birthday in March 2020. In what year was Kai born?

   (A) 1975 **(B)** 1990 **(C)** 1995 **(D)** 2000 **(E)** 1955

3. Yesterday, each student at Cayley S.S. was given a snack. Each student received either a muffin, yogurt, fruit, or a granola bar. No student received more than one of these snacks. The percentages of the students who received each snack are shown in the circle graph. What percentage of students *did not* receive a muffin?



   (A) 27% **(B)** 38% **(C)** 52% **(D)** 62% **(E)** 78%

4. The expression $(2 \times \frac{1}{3}) \times (3 \times \frac{1}{2})$ equals

   (A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** 1 **(D)** 5 **(E)** 6

5. If $10d + 8 = 528$, then $2d$ is equal to

   (A) 104 **(B)** 76 **(C)** 96 **(D)** 41 **(E)** 520

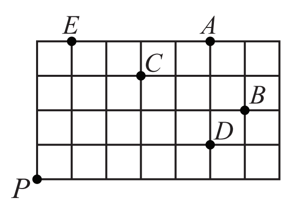
6. The line with equation $y = x + 4$ is translated down 6 units. The y-intercept of the resulting line is

   (A) 6 **(B)** 4 **(C)** 10 **(D)** -6 **(E)** -2

7. The three numbers 2, x, and 10 have an average of x. What is the value of x?

   (A) 5 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 6

8. Alain travels on the 4×7 grid shown from point P to one of the points A, B, C, D, or E. Alain can travel only right or up, and only along gridlines. To which point should Alain travel in order to travel the shortest distance?



   (A) A **(B)** B **(C)** C **(D)** D **(E)** E

9. If $(pq)(qr)(rp) = 16$, then a possible value for pqr is

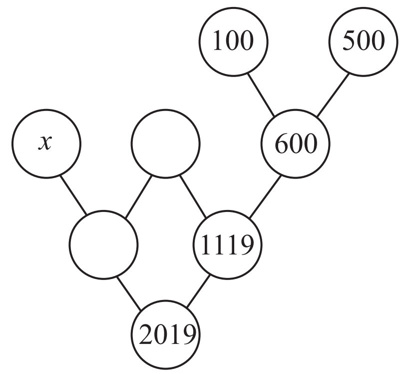
   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

10. Matilda and Ellie divide a white wall in their bedroom in half, each taking half of the wall. Matilda paints half of her section red. Ellie paints one third of her section red. The fraction of the entire wall that is painted red is

   (A) $\frac{5}{12}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{2}$

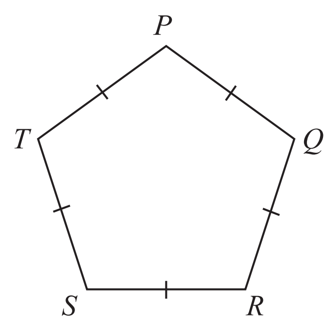
Part B: Each correct answer is worth 6.

11. In the diagram, numbers are to be placed in the circles so that each circle that is connected to two circles above it will contain the sum of the numbers contained in the two circles above it. What is the value of x?



(A) 481 **(B)** 381 **(C)** 281 **(D)** 581 **(E)** 681

12. In a regular pentagon, the measure of each interior angle is 108°. If $PQRST$ is a regular pentagon, then the measure of $\angle PRS$ is



(A) 72° **(B)** 54° **(C)** 60° **(D)** 45° **(E)** 80°

13. In the addition problem shown, m, n, p, and q represent positive digits. When the problem is completed correctly, the value of $m+n+p+q$ is

$$\begin{array}{r} n\,6\,3 \\ 7\,p\,2 \\ +\quad 5\,8\,q \\ \hline m\,0\,4\,2 \end{array}$$

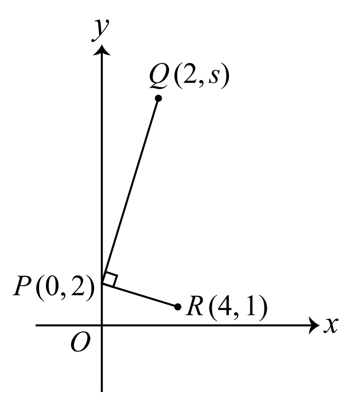
(A) 23 **(B)** 24 **(C)** 21 **(D)** 22 **(E)** 20

14. The letters A, B, C, D, and E are to be placed in the grid so that each of these letters appears exactly once in each row and exactly once in each column. Which letter will go in the square marked with $*$?

A				E
		C	A	
E		B	C	
	*			
B			D	

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

15. In the diagram, the line segments PQ and PR are perpendicular. The value of s is

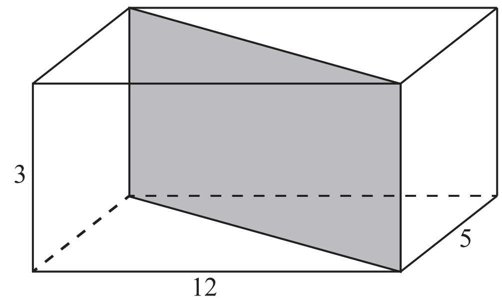


(A) 6 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 9.5

16. Kaukab is standing in a cafeteria line. In the line, the number of people that are ahead of her is equal to two times the number of people that are behind her. There are n people in the line. A possible value of n is

(A) 23 **(B)** 20 **(C)** 24 **(D)** 21 **(E)** 25

17. A solid wooden rectangular prism measures $3 \times 5 \times 12$. The prism is cut in half by a vertical cut through four vertices, as shown. This cut creates two congruent triangular-based prisms. When these prisms are pulled apart, what is the surface area of one of these triangular-based prisms?



(A) 135 **(B)** 111 **(C)** 114 **(D)** 150 **(E)** 90

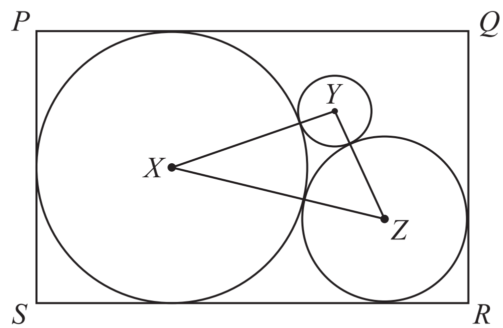
18. Carl and André are running a race. Carl runs at a constant speed of x m/s. André runs at a constant speed of y m/s. Carl starts running, and then André starts running 20 s later. After André has been running for 10 s, he catches up to Carl. The ratio $y : x$ is equivalent to

(A) $20 : 1$ **(B)** $2 : 1$ **(C)** $1 : 3$ **(D)** $3 : 1$ **(E)** $1 : 2$

19. If x and y are positive integers with $xy = 6$, the sum of all of the possible values of $\dfrac{2^{x+y}}{2^{x-y}}$ is

(A) 4180 **(B)** 4160 **(C)** 4164 **(D)** 4176 **(E)** 4128

20. In the diagram, each of the circles with centres X, Y and Z is tangent to the two other circles. Also, the circle with centre X touches three sides of rectangle $PQRS$ and the circle with centre Z touches two sides of rectangle $PQRS$, as shown.



If $XY = 30$, $YZ = 20$ and $XZ = 40$, the area of rectangle $PQRS$ is closest to

(A) 3900 **(B)** 4100 **(C)** 4050 **(D)** 4000 **(E)** 3950

Part C: Each correct answer is worth 8.

21. In the multiplication shown, each of P, Q, R, S, and T is a digit. The value of $P + Q + R + S + T$ is

$$\begin{array}{r} P\,Q\,R\,S\,T\,4 \\ \times \qquad\quad 4 \\ \hline 4\,P\,Q\,R\,S\,T \end{array}$$

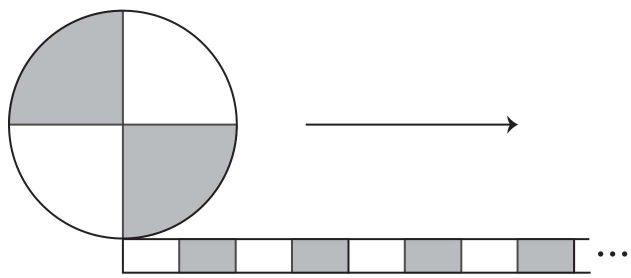
(A) 14 **(B)** 20 **(C)** 16 **(D)** 17 **(E)** 13

22. Seven friends are riding the bus to school:
- Cha and Bai are on 2 different buses.
- Bai, Abu and Don are on 3 different buses.
- Don, Gia and Fan are on 3 different buses.
- Abu, Eva and Bai are on 3 different buses.
- Gia and Eva are on 2 different buses.
- Fan, Cha and Gia are on 3 different buses.
- Cha and Eva are on 2 different buses.

What is the least possible number of buses on which the friends could be riding?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

23. A path of length 38 m consists of 19 unshaded stripes, each of length 1 m, alternating with 19 shaded stripes, each of length 1 m. A circular wheel of radius 2 m is divided into four quarters which are alternately shaded and unshaded. The wheel rolls at a constant speed along the path from the starting position shown.



The wheel makes exactly 3 complete revolutions. The percentage of time during which a shaded section of the wheel is touching a shaded part of the path is closest to

(A) 20% **(B)** 18% **(C)** 24% **(D)** 22% **(E)** 26%

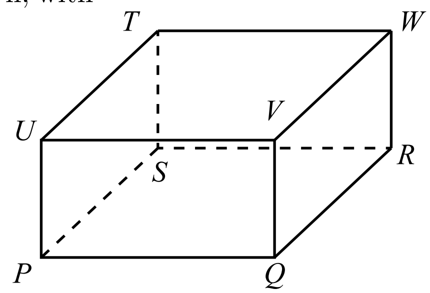
24. Roberta chooses an integer r from the set $\{2,3,4,5,6,7,8,9\}$, an integer s from the set $\{22,33,44,55,66,77,88,99\}$, and an integer t from the set $\{202,303,404,505,606,707,808,909\}$. How many possible values are there for the product rst?

(A) 85 **(B)** 81 **(C)** 90 **(D)** 84 **(E)** 80

25. For how many positive integers x does there exist a rectangular prism $PQRSTUVW$, labelled as shown, with $PR = 1867$, $PV = 2019$, and $PT = x$?

(A) 1980 **(B)** 1982 **(C)** 1984

(D) 1983 **(E)** 1981



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The CENTRE for EDUCATION
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Cayley Contest

(Grade 10)

Tuesday, February 27, 2018
(in North America and South America)

Wednesday, February 28, 2018
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

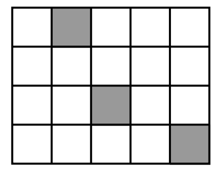
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. If $3 \times n = 6 \times 2$, then n equals

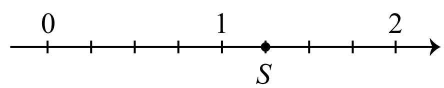
   (A) 6 **(B)** 2 **(C)** 9 **(D)** 5 **(E)** 4

2. In the diagram, 3 of the 1×1 squares that make up the 4×5 grid are shaded. How many additional 1×1 squares need to be shaded so that one-half of all of the 1×1 squares are shaded?



   (A) 5 **(B)** 9 **(C)** 7
   (D) 6 **(E)** 8

3. In the diagram, the number line between 0 and 2 is divided into 8 equal parts. The numbers 1 and S are marked on the line. What is the value of S?

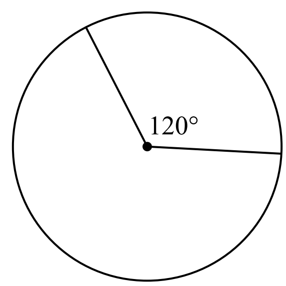


   (A) 1.1 **(B)** 0.75 **(C)** 1.2
   (D) 1.25 **(E)** 1.15

4. Which of the following is equal to 9^4?

   (A) 3^2 **(B)** 3^4 **(C)** 3^6 **(D)** 3^8 **(E)** 3^{10}

5. In the diagram, a sector of a circle has central angle 120°. The area of the whole circle is 9π. What is the area of this sector?



   (A) 2π **(B)** 3π **(C)** 4π
   (D) 6π **(E)** $\frac{9}{2}\pi$

6. If $x = 2018$, then the expression $x^2 + 2x - x(x+1)$ equals

   (A) -2018 **(B)** 2018 **(C)** 10090 **(D)** -10090 **(E)** 4039

7. At 8:00 a.m., there were 24 cars in a parking lot.
   At 9:00 a.m., there were 48 cars in the same parking lot.
   What is the percentage increase in number of cars in the parking lot between 8:00 a.m. and 9:00 a.m.?

   (A) 20% **(B)** 48% **(C)** 72% **(D)** 100% **(E)** 124%

8. For what value of k is the line through the points $(3, 2k+1)$ and $(8, 4k-5)$ parallel to the x-axis?

   (A) -1 **(B)** 3 **(C)** 2 **(D)** 0 **(E)** -4

9. The three numbers $5, a, b$ have an average (mean) of 33. What is the average of a and b?

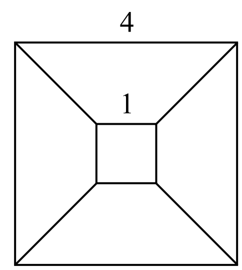
(A) 38 **(B)** 14 **(C)** 28 **(D)** 33 **(E)** 47

10. Glenda, Helga, Ioana, Julia, Karl, and Liu participated in the 2017 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Helga's and Julia's numbers were even. Karl's and Liu's numbers were prime numbers. Glenda's number was a perfect square. What was Ioana's number?

(A) 11 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 12

Part B: Each correct answer is worth 6.

11. A large square has side length 4. It is divided into four identical trapezoids and a small square, as shown. The small square has side length 1. What is the area of each trapezoid?



(A) $\frac{2}{3}$ **(B)** 3 **(C)** $\frac{9}{2}$
(D) $\frac{15}{4}$ **(E)** 15

12. In an unusual country, there are three kinds of coins: Exes, Wyes and Zeds. In this country, the value of 2 Exes equals the value of 29 Wyes, and the value of 1 Zed equals the value of 16 Exes. The value of 1 Zed equals the value of how many Wyes?

(A) 3.625 **(B)** 1.103 **(C)** 232 **(D)** 464 **(E)** 928

13. The number of integer values of x for which $\dfrac{3}{x+1}$ is an integer is

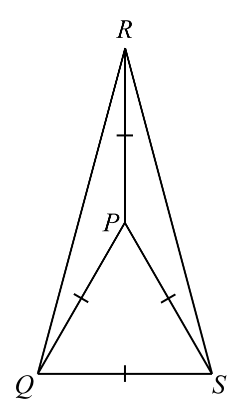
(A) 4 **(B)** 3 **(C)** 5 **(D)** 1 **(E)** 6

14. Including the endpoints, how many points on the line segment joining $(-9,-2)$ and $(6,8)$ have coordinates that are both integers?

(A) 2 **(B)** 7 **(C)** 16 **(D)** 11 **(E)** 6

15. In the diagram, $\triangle PQS$ is equilateral. Also, $\triangle PQR$ and $\triangle PSR$ are isosceles with $PQ = PR = PS$. If $\angle RPQ = \angle RPS$, the measure of $\angle QRS$ is

(A) 30° **(B)** 60° **(C)** 15°
(D) 20° **(E)** 45°



16. A ladder has 5 rungs. Elisabeth can climb up by 1 or 2 rungs at a time. In how many different ways can she climb up to the fifth rung of the ladder?

(A) 10 **(B)** 9 **(C)** 7 **(D)** 6 **(E)** 8

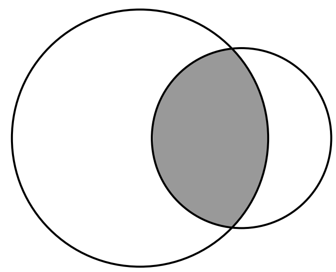
17. If $\dfrac{x-y}{x+y}=5$, then $\dfrac{2x+3y}{3x-2y}$ equals

(A) 1 **(B)** 0 **(C)** $\frac{2}{3}$ **(D)** $\frac{15}{2}$ **(E)** $\frac{12}{5}$

18. A quadrilateral is bounded by the lines with equations $x=0$, $x=4$, $y=x-2$, and $y=x+3$. The area of this quadrilateral is

(A) 16 **(B)** 24 **(C)** 4 **(D)** $20\sqrt{2}$ **(E)** 20

19. In the diagram, two circles overlap. The area of the overlapped region is $\frac{3}{5}$ of the area of the small circle and $\frac{6}{25}$ of the area of the large circle. The ratio of the area of the small circle to the area of the large circle is



(A) 18 : 125 **(B)** 1 : 3 **(C)** 5 : 12

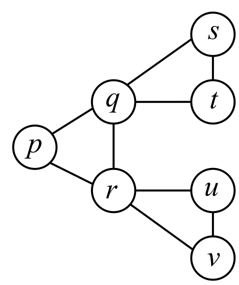
(D) 2 : 5 **(E)** 1 : 4

20. Abigail chooses an integer at random from the set $\{2,4,6,8,10\}$. Bill chooses an integer at random from the set $\{2,4,6,8,10\}$. Charlie chooses an integer at random from the set $\{2,4,6,8,10\}$. What is the probability that the product of their three integers is *not* a power of 2?

(A) $\frac{117}{125}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{98}{125}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{64}{125}$

Part C: Each correct answer is worth 8.

21. In the diagram, each of p,q,r,s,t,u,v is to be replaced with 1, 2 or 3 so that p, q and r are all different, q, s and t are all different, and r, u and v are all different. What is the maximum possible value of $s+t+u+v$?



(A) 8 **(B)** 9 **(C)** 11

(D) 7 **(E)** 10

22. If n is a positive integer, the symbol $n!$ (read "n factorial") represents the product of the integers from 1 to n. For example, $4!=(1)(2)(3)(4)$ or $4!=24$. If x and y are integers and $\dfrac{30!}{36^x25^y}$ is equal to an integer, what is the maximum possible value of $x+y$?

(A) 10 **(B)** 47 **(C)** 17 **(D)** 26 **(E)** 13

23. A container in the shape of a triangular prism stands on one of its triangular faces. Three spheres of radius 1 are placed inside the container, each touching the triangular bottom. Each sphere touches two of the rectangular faces of the container and each sphere touches the other two spheres. A fourth sphere of radius 1 is placed on top of the three spheres, touching each of the three spheres and the top of the prism. The volume of the prism is closest to

(A) 48.00 **(B)** 47.75 **(C)** 47.50 **(D)** 47.25 **(E)** 47.00

24. There are more than 1 000 000 ways in which n identical black socks and $2n$ identical gold socks can be arranged in a row so that there are at least 2 gold socks between any 2 black socks. The sum of the digits of the smallest possible value of n is

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

25. There are N sequences with 15 terms and the following properties:

- each term is an integer,
- at least one term is between -16 and 16, inclusive,
- the 15 terms have at most two different values,
- the sum of every six consecutive terms is positive, and
- the sum of every eleven consecutive terms is negative.

The value of N is

(A) 48 **(B)** 72 **(C)** 64 **(D)** 80 **(E)** 56

2018 Cayley Contest (English)

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The CENTRE for EDUCATION
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Cayley Contest

(Grade 10)

Tuesday, February 28, 2017
(in North America and South America)

Wednesday, March 1, 2017
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

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	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $6 \times 111 - 2 \times 111$ equals

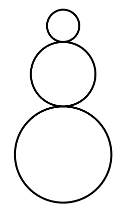
   (A) 222 **(B)** 333 **(C)** 444 **(D)** 555 **(E)** 666

2. The value of $\dfrac{5^2 - 9}{5 - 3}$ is

   (A) 4 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** 8 **(E)** -2

3. A snowman is built by stacking three spheres with their centres aligned vertically. The spheres have radii of 10 cm, 20 cm and 30 cm. How tall is the snowman?

   (A) 90 cm **(B)** 100 cm **(C)** 110 cm **(D)** 120 cm **(E)** 130 cm

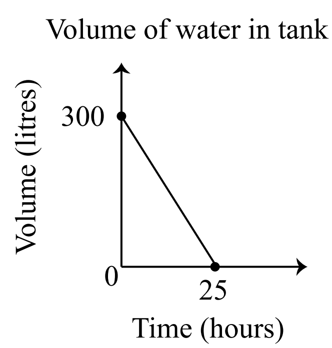


4. Which of the following fractions has the greatest value?

   (A) $\frac{44444}{55555}$ **(B)** $\frac{5555}{6666}$ **(C)** $\frac{666}{777}$ **(D)** $\frac{77}{88}$ **(E)** $\frac{8}{9}$

5. The graph shows the volume of water in a 300 L tank as it is being drained at a constant rate. At what rate is the water leaving the tank, in litres per hour?

   (A) 12 **(B)** 20 **(C)** 2.5 **(D)** 5 **(E)** 15



6. Penelope folds a piece of paper in half, creating two layers of paper. She folds the paper in half again, creating a total of four layers of paper. If she continues to fold the paper in half, which of the following is a possible number of layers that could be obtained?

   (A) 10 **(B)** 12 **(C)** 14 **(D)** 16 **(E)** 18

7. The operation $\Diamond$ is defined by $a \Diamond b = a^2b - ab^2$. The value of $2 \Diamond 7$ is

   (A) -140 **(B)** -70 **(C)** 0 **(D)** 70 **(E)** 140

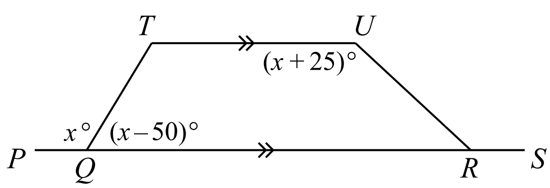
8. Each of three cards is labelled with three numbers. Which of the following groups of three cards has the properties that the first and second cards have exactly one number in common, the first and third cards have exactly one number in common, and the second and third cards have exactly one number in common?

(A) $\boxed{135}$ $\boxed{367}$ $\boxed{246}$
(B) $\boxed{147}$ $\boxed{234}$ $\boxed{245}$
(C) $\boxed{234}$ $\boxed{257}$ $\boxed{124}$
(D) $\boxed{147}$ $\boxed{234}$ $\boxed{257}$
(E) $\boxed{135}$ $\boxed{147}$ $\boxed{235}$

9. A restaurant bill, including 13% tax but not including a tip, is \$226. The server is paid a tip of 15% based on the bill before tax. How much is the tip that the server is paid?

(A) \$32.87 **(B)** \$29.49 **(C)** \$30.00 **(D)** \$28.00 **(E)** \$44.07

10. In the diagram, TU is parallel to PS and points Q and R lie on PS.
Also, $\angle PQT = x°$, $\angle RQT = (x - 50)°$, and $\angle TUR = (x + 25)°$.

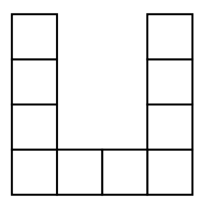


What is the measure of $\angle URS$?

(A) 115° **(B)** 140° **(C)** 135° **(D)** 130° **(E)** 120°

Part B: Each correct answer is worth 6.

11. The figure shown is made up of 10 identical squares. If the area of the figure is 160 cm^2, what is the perimeter of the figure?



(A) 72 cm **(B)** 80 cm **(C)** 88 cm
(D) 64 cm **(E)** 100 cm

12. The mean (average) of the three integers p, q and r is 9.
The mean of the two integers s and t is 14.
The mean of the five integers p, q, r, s, and t is

(A) 11 **(B)** 11.5 **(C)** 12 **(D)** 10 **(E)** 13

13. In the addition shown, each of X, Y and Z represents a digit. What is the value of $X + Y + Z$?

(A) 10 **(B)** 15 **(C)** 22
(D) 20 **(E)** 8

$$\begin{array}{cccc} & X & Y & Z \\ & X & Y & Z \\ + & & Y & Z \\ \hline 1 & 6 & 7 & 5 \end{array}$$

14. Igor is shorter than Jie. Faye is taller than Goa. Jie is taller than Faye. Han is shorter than Goa. Who is the tallest?

(A) Faye **(B)** Goa **(C)** Han **(D)** Igor **(E)** Jie

15. A bag contains red, blue and purple marbles, and does not contain any other marbles. The ratio of the number of red marbles to the number of blue marbles is $4:7$. The ratio of the number of blue marbles to the number of purple marbles is $2:3$. There are 32 red marbles in the bag. In total, how many marbles are there in the bag?

(A) 162 **(B)** 129 **(C)** 176 **(D)** 164 **(E)** 172

16. If $x + 2y = 30$, the value of $\frac{x}{5} + \frac{2y}{3} + \frac{2y}{5} + \frac{x}{3}$ is

(A) 8 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 30

17. The positive integers r, s and t have the property that $r \times s \times t = 1230$. What is the smallest possible value of $r + s + t$?

(A) 51 **(B)** 52 **(C)** 54 **(D)** 58 **(E)** 53

18. The number of integers n for which $\frac{1}{7} \leq \frac{6}{n} \leq \frac{1}{4}$ is

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 24

19. Two lines with slopes $\frac{1}{4}$ and $\frac{5}{4}$ intersect at $(1, 1)$. What is the area of the triangle formed by these two lines and the vertical line $x = 5$?

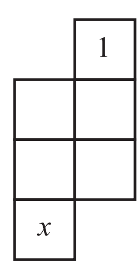
(A) 5 **(B)** 10 **(C)** 8 **(D)** 12 **(E)** 15

20. Car X and Car Y are travelling in the same direction in two different lanes on a long straight highway. Car X is travelling at a constant speed of 90 km/h and has a length of 5 m. Car Y is travelling at a constant speed of 91 km/h and has a length of 6 m. Car Y starts behind Car X and eventually passes Car X. The length of time between the instant when the front of Car Y is lined up with the back of Car X and the instant when the back of Car Y is lined up with the front of Car X is t seconds. The value of t is

(A) 39.6 **(B)** 18.0 **(C)** 21.6 **(D)** 46.8 **(E)** 32.4

Part C: Each correct answer is worth 8.

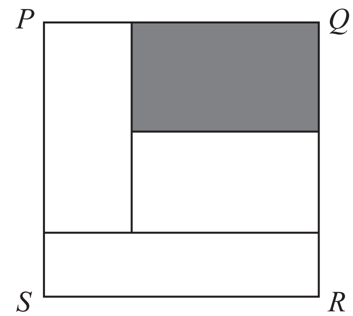
21. The integers 1 to 6 are to be inserted into the grid shown. No two integers that differ by 1 may be in squares that share an edge. If the 1 is inserted as shown, how many different integers can be placed in the box labelled x?



(A) 1 **(B)** 3 **(C)** 5

(D) 0 **(E)** 2

22. In the diagram, square $PQRS$ has side length 42 and is divided into four non-overlapping rectangles. If each of these four rectangles has the same perimeter, what is the area of the shaded rectangle?

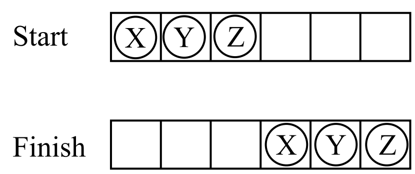


(A) 252 **(B)** 432 **(C)** 441
(D) 490 **(E)** 540

23. The triangle with side lengths 6, 8 and 10 is right-angled, while the triangle with side lengths 6, 8 and 9 is an acute triangle and the triangle with side lengths 6, 8 and 11 is an obtuse triangle. An obtuse triangle with positive area has side lengths 10, 17 and x. If x is an integer, what is the sum of all possible values of x?

(A) 161 **(B)** 198 **(C)** 63 **(D)** 323 **(E)** 224

24. Three coins are placed in the first three of six squares, as shown. A move consists of moving one coin one space to the right, assuming that this space is empty. (No coin can jump over another coin, so the order of the coins will never change.) How many different sequences of moves can be used to move the three coins from the first three squares to the last three squares?



(A) 44 **(B)** 40 **(C)** 42
(D) 48 **(E)** 50

25. A positive integer n with $n \geq 3$ is called a *Nella number* if there exists a positive integer x with $x < n$ and there exists a positive integer m such that

- m is not divisible by x or by $x + 1$, and
- m is divisible by every other positive integer between 1 and n inclusive.

For example, $n = 7$ is a Nella number. How many Nella numbers n are there with $50 \leq n \leq 2017$?

(A) 393 **(B)** 394 **(C)** 395 **(D)** 396 **(E)** 397

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For students...

Thank you for writing the 2017 Cayley Contest! Each year, more than 235 000 students from more than 75 countries register to write the CEMC's Contests.

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Cayley Contest

(Grade 10)

Wednesday, February 24, 2016
(in North America and South America)

Thursday, February 25, 2016
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
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6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

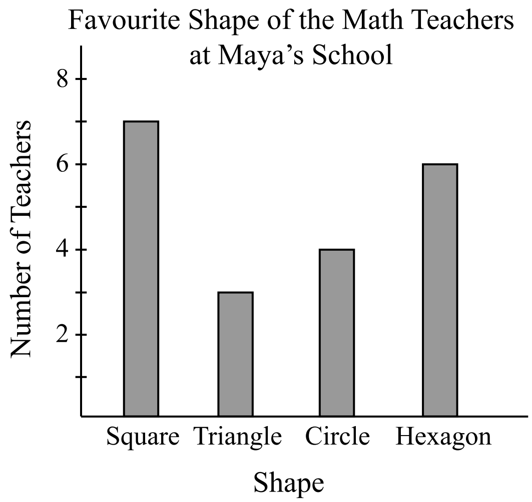
Part A: Each correct answer is worth 5.

1. The value of $(3+2)-(2+1)$ is

   (A) 8 **(B)** 3 **(C)** 1 **(D)** 5 **(E)** 2

2. Maya asked the 20 math teachers at her school to tell her their favourite shape. She represented their answers on the bar graph shown. The number of teachers who *did not* pick "Square" as their favourite shape was

   (A) 14
   (B) 12
   (C) 15
   (D) 16
   (E) 13



3. The expression $\sqrt{5^2-4^2}$ is equal to

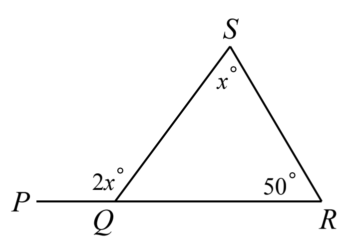
   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 9

4. If each of Bill's steps is $\frac{1}{2}$ metre long, how many steps does Bill take to walk 12 metres in a straight line?

   (A) 9 **(B)** 12 **(C)** 16 **(D)** 24 **(E)** 36

5. In the diagram, Q is on PR. The value of x is

   (A) 50 **(B)** 80 **(C)** 100
   (D) 16.7 **(E)** 130



6. If the line that passes through the points $(2,7)$ and $(a,3a)$ has a slope of 2, the value of a is

   (A) $\frac{5}{2}$ **(B)** 10 **(C)** 3 **(D)** $\frac{11}{5}$ **(E)** $\frac{12}{5}$

7. A soccer team played three games. Each game ended in a win, loss, or tie. (If a game finishes with both teams having scored the same number of goals, the game ends in a tie.) In total, the team scored more goals than were scored against them. Which of the following combinations of outcomes is not possible for this team?

(A) 2 wins, 0 losses, 1 tie
(B) 1 win, 2 losses, 0 ties
(C) 0 wins, 1 loss, 2 ties
(D) 1 win, 1 loss, 1 tie
(E) 1 win, 0 losses, 2 ties

8. The first five letters of the alphabet are assigned the values $A = 1$, $B = 2$, $C = 3$, $D = 4$, and $E = 5$. The value of a word equals the sum of the values of its letters. For example, the value of BAD is $2 + 1 + 4 = 7$. Which of the following words has the largest value?

(A) BAD **(B)** CAB **(C)** DAD **(D)** BEE **(E)** BED

9. Grace writes a sequence of 20 numbers. The first number is 43 and each number after the first is 4 less than the number before it, so her sequence starts $43, 39, 35, \ldots$. How many of the numbers that Grace writes are positive?

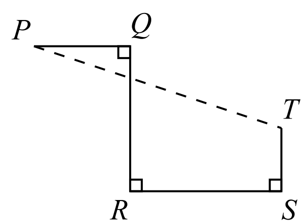
(A) 11 **(B)** 9 **(C)** 13 **(D)** 15 **(E)** 12

10. Five students play chess matches against each other. Each student plays three matches against each of the other students. How many matches are played in total?

(A) 15 **(B)** 8 **(C)** 30 **(D)** 60 **(E)** 16

Part B: Each correct answer is worth 6.

11. In the diagram, PQ is perpendicular to QR, QR is perpendicular to RS, and RS is perpendicular to ST. If $PQ = 4$, $QR = 8$, $RS = 8$, and $ST = 3$, then the distance from P to T is



(A) 16 **(B)** 12 **(C)** 17
(D) 15 **(E)** 13

12. Alejandro has a box that contains 30 balls, numbered from 1 to 30. He randomly selects a ball from the box where each ball is equally likely to be chosen. Which of the following is most likely?

(A) He selects a ball whose number is a multiple of 10.
(B) He selects a ball whose number is odd.
(C) He selects a ball whose number includes the digit 3.
(D) He selects a ball whose number is a multiple of 5.
(E) He selects a ball whose number includes the digit 2.

13. Which of the following fractions is both larger than $\frac{1}{6}$ and smaller than $\frac{1}{4}$?

(A) $\frac{5}{12}$ **(B)** $\frac{5}{36}$ **(C)** $\frac{5}{24}$ **(D)** $\frac{5}{60}$ **(E)** $\frac{5}{48}$

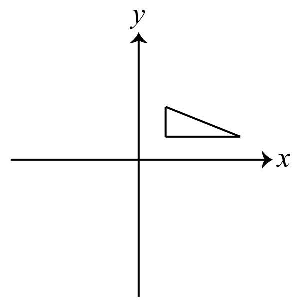
14. The number of zeros in the integer equal to $\left(10^{100}\right) \times \left(100^{10}\right)$ is

(A) 120 **(B)** 200 **(C)** 220 **(D)** 300 **(E)** 110

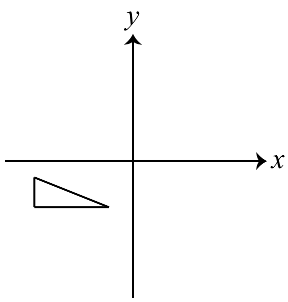
15. What is the tens digit of the smallest positive integer that is divisible by each of 20, 16 and 2016?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

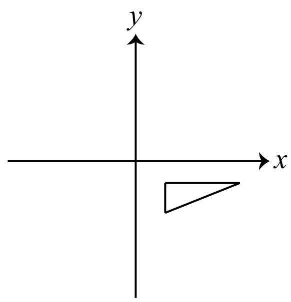
16. The triangle shown is reflected in the x-axis and the resulting triangle is reflected in the y-axis. Which of the following best represents the final position of the triangle?



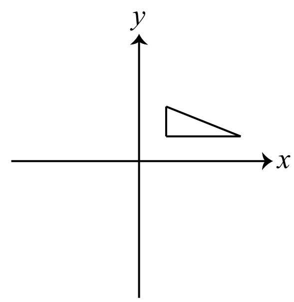
(A)



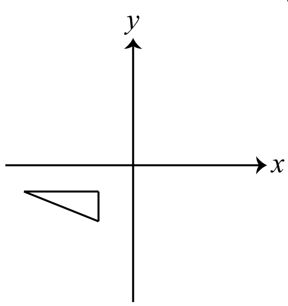
(B)



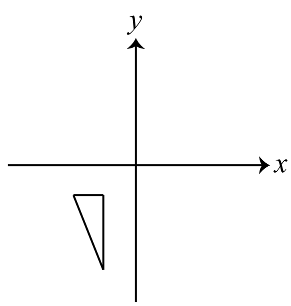
(C)



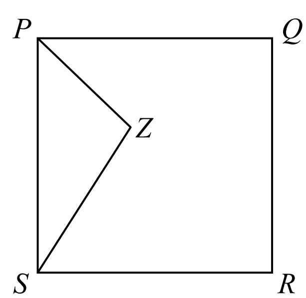
(D)



(E)



17. In the diagram, the perimeter of square $PQRS$ is 120 and the perimeter of $\triangle PZS$ is $2x$. Which of the following expressions in terms of x is equal to the perimeter of pentagon $PQRSZ$?



(A) $120 + 2x$ **(B)** $40 + 2x$ **(C)** $60 + 2x$

(D) $90 + 2x$ **(E)** $30 + 2x$

18. When three positive integers are added in pairs, the resulting sums are 998, 1050 and 1234. What is the difference between the largest and smallest of the three original positive integers?

(A) 262 **(B)** 248 **(C)** 224 **(D)** 250 **(E)** 236

19. A total of n points are equally spaced around a circle and are labelled with the integers 1 to n, in order. Two points are called *diametrically opposite* if the line segment joining them is a diameter of the circle. If the points labelled 7 and 35 are diametrically opposite, then n equals

(A) 54 **(B)** 55 **(C)** 56 **(D)** 57 **(E)** 58

20. There are n students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the n students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the n students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the n students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to $n^2 - n$ is

(A) 11 **(B)** 12 **(C)** 20 **(D)** 13 **(E)** 10

Part C: Each correct answer is worth 8.

21. In her last basketball game, Jackie scored 36 points. These points raised the average (mean) number of points that she scored per game from 20 to 21. To raise this average to 22 points, how many points must Jackie score in her next game?

(A) 38 **(B)** 22 **(C)** 23 **(D)** 36 **(E)** 37

22. Alain and Louise are driving on a circular track with radius 25 km. Alain leaves the starting line first, going clockwise around the track at a speed of 80 km/h. Fifteen minutes after Alain starts, Louise leaves the same starting line, going counter-clockwise around the track at a speed of 100 km/h. For how many hours will Louise have been driving when the two of them pass each other for the fourth time?

(A) $\frac{50\pi-6}{45}$ **(B)** $\frac{4\pi+1}{4}$ **(C)** $\frac{10\pi-1}{9}$ **(D)** $\frac{15\pi+6}{16}$ **(E)** $\frac{25\pi-1}{24}$

23. Suppose that $PQRSTUVW$ is a regular octagon. (A *regular octagon* is an octagon with eight equal side lengths and eight equal interior angles.) There are 70 ways in which four of its sides can be chosen at random. If four of its sides are chosen at random and each of these sides is extended infinitely in both directions, what is the probability that they will meet to form a quadrilateral that contains the octagon?

(A) $\frac{1}{2}$ **(B)** $\frac{19}{35}$ **(C)** $\frac{37}{70}$ **(D)** $\frac{17}{35}$ **(E)** $\frac{18}{35}$

24. What is the sum of all numbers q which can be written in the form $q = \dfrac{a}{b}$ where a and b are positive integers with $b \leq 10$ and for which there are exactly 19 integers n that satisfy $\sqrt{q} < n < q$?

(A) 871.5 **(B)** 743.5 **(C)** 777.5 **(D)** 808.5 **(E)** 1106.5

25. A new language uses only the letters A, B, C, D, and E. The letters A and E are called *vowels*, while the letters B, C and D are called *consonants*. A sequence of letters is called a *word* if it does not include the same letter twice in a row, and it does not include two vowels in a row. How many words are there in this language that are 10 letters long and that begin with a vowel?

(A) 199 680 **(B)** 199 968 **(C)** 199 584 **(D)** 199 872 **(E)** 199 776

2016 Cayley Contest (English)

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Cayley Contest

(Grade 10)

Tuesday, February 24, 2015
(in North America and South America)

Wednesday, February 25, 2015
(outside of North America and South America)

UNIVERSITY OF WATERLOO

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Part A: Each correct answer is worth 5.

1. The value of $2 \times 2015 - 2015$ is

   (A) 2015 **(B)** 4030 **(C)** 6045 **(D)** 0 **(E)** -2015

2. The expression $\sqrt{1} + \sqrt{9}$ is equal to

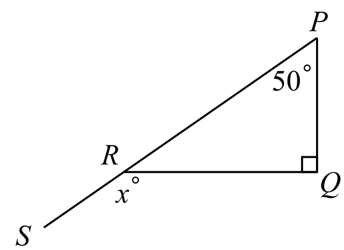
   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. The base of a rectangular box measures 2 cm by 5 cm. The volume of the box is 30 cm^3. What is the height of the box?

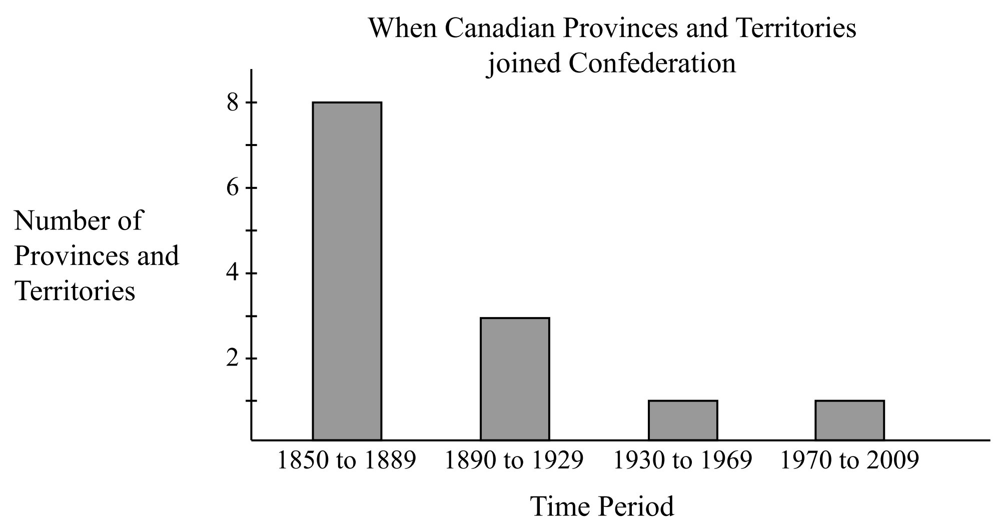
   (A) 1 cm **(B)** 2 cm **(C)** 3 cm **(D)** 4 cm **(E)** 5 cm

4. In the diagram, R lies on line segment PS. The value of x is

   (A) 120 **(B)** 130 **(C)** 135 **(D)** 140 **(E)** 150



5. The bar graph shows the number of provinces and territories that joined Canadian Confederation during each of four 40 year time periods.



   If one of the 13 provinces or territories is chosen at random, what is the probability that it joined Canadian Confederation between 1890 and 1969?

   (A) $\frac{12}{13}$ **(B)** $\frac{4}{13}$ **(C)** $\frac{5}{13}$ **(D)** $\frac{3}{13}$ **(E)** $\frac{2}{13}$

6. If $a^2 = 9$, then a^4 equals

   (A) 27 **(B)** 81 **(C)** 243 **(D)** 729 **(E)** 2187

7. The expression $3 + \frac{1}{10} + \frac{4}{100}$ is *not* equal to

(A) $3\frac{14}{100}$ **(B)** 3.14 **(C)** $3\frac{5}{110}$ **(D)** $3\frac{7}{50}$ **(E)** $\frac{157}{50}$

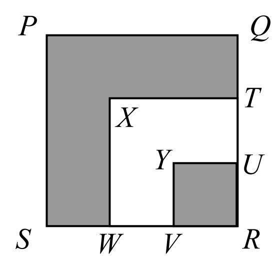
8. Violet has one-half of the money she needs to buy her mother a necklace. After her sister gives her \$30, she has three-quarters of the amount she needs. Violet's father agrees to give her the rest. The amount that Violet's father will give her is

(A) \$7.50 **(B)** \$15 **(C)** \$22.50 **(D)** \$30 **(E)** \$120

9. John goes for a jog every 3 days. He went for a jog on Monday, January 5. He went for his next jog on January 8. What was the date of the next Monday on which he went for a jog?

(A) January 12 **(B)** January 19 **(C)** January 26
(D) February 2 **(E)** February 9

10. In the diagram, square $PQRS$ is 3×3. Points T and U are on side QR with $QT = TU = UR = 1$. Points V and W are on side RS with $RV = VW = WS = 1$. Line segments TX and UY are perpendicular to QR and line segments VY and WX are perpendicular to RS. The ratio of the shaded area to the unshaded area is



(A) $2 : 1$ **(B)** $7 : 3$ **(C)** $7 : 4$
(D) $5 : 4$ **(E)** $3 : 1$

Part B: Each correct answer is worth 6.

11. The operation $\otimes$ is defined by $a \otimes b = \dfrac{a}{b} + \dfrac{b}{a}$. What is the value of $4 \otimes 8$?

(A) $\dfrac{1}{2}$ **(B)** 1 **(C)** $\dfrac{5}{4}$ **(D)** 2 **(E)** $\dfrac{5}{2}$

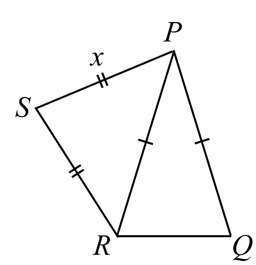
12. The points $(-1, q)$ and $(-3, r)$ are on a line parallel to $y = \frac{3}{2}x + 1$. What is the value of $r - q$?

(A) 3 **(B)** $\frac{4}{3}$ **(C)** $-\frac{3}{4}$ **(D)** $-\frac{4}{3}$ **(E)** -3

13. At Barker High School, a total of 36 students are on either the baseball team, the hockey team, or both. If there are 25 students on the baseball team and 19 students on the hockey team, how many students play both sports?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

14. In the diagram, $\triangle PQR$ is isosceles with $PQ = PR$ and $\triangle PRS$ is isosceles with $PS = SR = x$. Also, the perimeter of $\triangle PQR$ is 22, the perimeter of $\triangle PRS$ is 22, and the perimeter of $PQRS$ is 24. What is the value of x?



(A) 7.5 **(B)** 6.5 **(C)** 7
(D) 6 **(E)** 8

15. If n is a positive integer, the symbol $n!$ (read "n factorial") represents the product of the integers from 1 to n. For example, $4! = (1)(2)(3)(4)$ or $4! = 24$. The ones (units) digit of the sum $1! + 2! + 3! + 4! + 5! + 6! + 7! + 8! + 9! + 10!$ is

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

16. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the sum $a + b + c$ equals

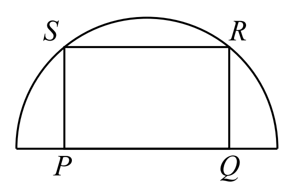
a	13	b
19	c	11
12	d	16

(A) 49 **(B)** 54 **(C)** 47
(D) 50 **(E)** 46

17. For the first 30 minutes of a trip, Deanna drove at a constant speed. For the next 30 minutes, she drove at a constant speed 20 km/h faster than her original speed. If the total distance that she travelled was 100 km, how fast did she drive for the first 30 minutes?

(A) 80 km/h **(B)** 90 km/h **(C)** 100 km/h **(D)** 110 km/h **(E)** 120 km/h

18. In the diagram, rectangle $PQRS$ has side PQ on the diameter of the semicircle with R and S on the semicircle. If the diameter of the semicircle is 20 and the length of PQ is 16, then the length of PS is



(A) 6 **(B)** 7 **(C)** 8
(D) 9 **(E)** 10

19. A bank teller has some stacks of bills. The total value of the bills in each stack is \$1000. Every stack contains at least one \$20 bill, at least one \$50 bill, and no other types of bills. If no two stacks have the same number of \$20 bills, what is the maximum possible number of stacks that the teller could have?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 4 **(E)** 8

20. For how many integers n is $72\left(\frac{3}{2}\right)^n$ equal to an integer?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Part C: Each correct answer is worth 8.

21. The average of a list of three consecutive odd integers is 7. When a fourth positive integer, m, different from the first three, is included in the list, the average of the list is an integer. What is the sum of the three smallest possible values of m?

(A) 6 **(B)** 9 **(C)** 21 **(D)** 29 **(E)** 33

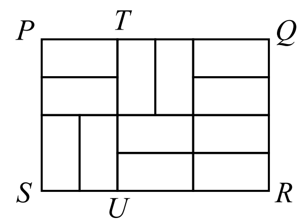
22. Six players compete in a chess tournament. Each player plays exactly two games against every other player. In each game, the winning player earns 1 point and the losing player earns 0 points; if the game results in a draw (tie), each player earns $\frac{1}{2}$ point. What is the minimum possible number of points that a player needs to earn in order to guarantee that he has more points than every other player?

(A) 8 **(B)** $8\frac{1}{2}$ **(C)** 9 **(D)** $9\frac{1}{2}$ **(E)** 10

23. Nylah has her living room lights on a timer. Each evening, the timer switches the lights on randomly at exactly 7:00 p.m., 7:30 p.m., 8:00 p.m., 8:30 p.m., or 9:00 p.m. Later in the evening, the timer switches the lights off at any random time between 11 p.m. and 1 a.m. For example, the lights could be switched on at exactly 7:30 p.m. and off at any one of the infinite number of possible times between 11 p.m. and 1 a.m. On a given night, Nylah's lights are on for t hours. What is the probability that $4 < t < 5$?

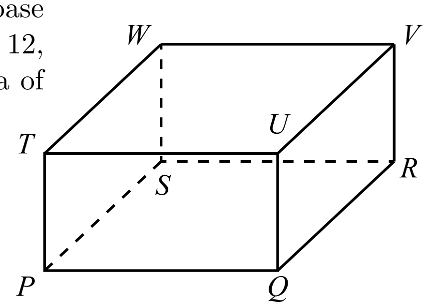
(A) $\frac{1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{3}{10}$ **(E)** $\frac{7}{20}$

24. In the diagram, a rectangular ceiling $PQRS$ measures 6 m by 4 m and is to be completely covered using 12 rectangular tiles, each measuring 1 m by 2 m. If there is a beam, TU, that is positioned so that $PT = SU = 2$ m and that cannot be crossed by any tile, then the number of possible arrangements of tiles is



(A) 180 **(B)** 190 **(C)** 185
(D) 170 **(E)** 175

25. Rectangular prism $PQRSWTUV$ has a square base $PQRS$. Point X is on the face $TUVW$ so that $PX = 12$, $QX = 10$ and $RX = 8$. The maximum possible area of rectangle $PQUT$ is closest to



(A) 67.84 **(B)** 67.82 **(C)** 67.90
(D) 67.86 **(E)** 67.88

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Thank you for writing the 2015 Cayley Contest! Each year, more than 200 000 students from more than 60 countries register to write the CEMC's Contests.

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Cayley Contest

(Grade 10)

Thursday, February 20, 2014
(in North America and South America)

Friday, February 21, 2014
(outside of North America and South America)

UNIVERSITY OF WATERLOO WATERLOO MATHEMATICS Deloitte.

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, http://www.cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2000 - 80 + 200 - 120$ is

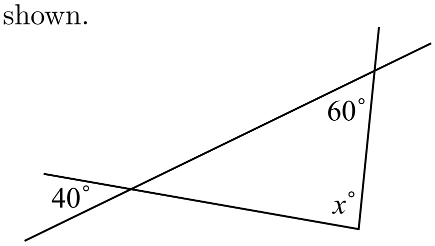
   (A) 2000 **(B)** 1600 **(C)** 2100 **(D)** 1860 **(E)** 1760

2. If $(2)(3)(4) = 6x$, what is the value of x?

   (A) 2 **(B)** 6 **(C)** 12 **(D)** 3 **(E)** 4

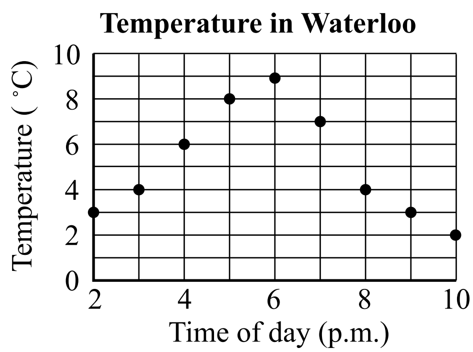
3. In the diagram, three line segments intersect as shown. The value of x is

   (A) 40 **(B)** 60 **(C)** 80 **(D)** 100 **(E)** 120



4. At 2 p.m., Sanjay measures the temperature to be 3°C. He measures the temperature every hour after this until 10 p.m. He plots the temperatures that he measures on the graph shown. At what time after 2 p.m. does he again measure a temperature of 3°C?

   (A) 9 p.m. **(B)** 5 p.m. **(C)** 8 p.m. **(D)** 10 p.m. **(E)** 7 p.m.



5. If $2n + 5 = 16$, the expression $2n - 3$ equals

   (A) 8 **(B)** 10 **(C)** 18 **(D)** 14 **(E)** 7

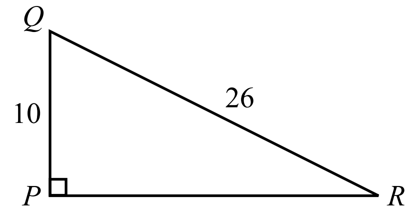
6. When the numbers 3, $\frac{5}{2}$ and $\sqrt{10}$ are listed in order from smallest to largest, the list is

   (A) $3, \frac{5}{2}, \sqrt{10}$ **(B)** $\frac{5}{2}, 3, \sqrt{10}$ **(C)** $\sqrt{10}, \frac{5}{2}, 3$ **(D)** $\frac{5}{2}, \sqrt{10}, 3$ **(E)** $3, \sqrt{10}, \frac{5}{2}$

7. Meg started with the number 100. She increased this number by 20% and then increased the resulting number by 50%. Her final result was

   (A) 120 **(B)** 240 **(C)** 187.5 **(D)** 200 **(E)** 180

8. In the diagram, $\triangle PQR$ has $\angle RPQ = 90^\circ$, $PQ = 10$, and $QR = 26$. The area of $\triangle PQR$ is



(A) 100 **(B)** 120 **(C)** 130
(D) 60 **(E)** 312

9. In a group of five friends:
   - Amy is taller than Carla.
   - Dan is shorter than Eric but taller than Bob.
   - Eric is shorter than Carla.

   Who is the shortest?

   (A) Amy **(B)** Bob **(C)** Carla **(D)** Dan **(E)** Eric

10. Consider the following flowchart:

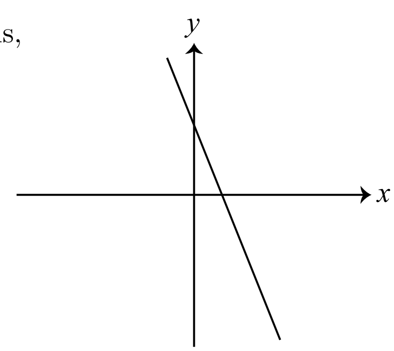
`INPUT` → Subtract 8 → ☐ → Divide by 2 → ☐ → Add 16 → `OUTPUT`

If the `OUTPUT` is 32, the `INPUT` must have been

(A) 16 **(B)** 28 **(C)** 36 **(D)** 40 **(E)** 32

Part B: Each correct answer is worth 6.

11. A line intersects the positive x-axis and positive y-axis, as shown. A possible equation of this line is



(A) $y = 2x + 7$ **(B)** $y = 4$ **(C)** $y = -3x - 5$
(D) $y = 5x - 2$ **(E)** $y = -2x + 3$

12. If $x = 2y$ and $y \neq 0$, then $(x - y)(2x + y)$ equals

(A) $5y^2$ **(B)** y^2 **(C)** $3y^2$ **(D)** $6y^2$ **(E)** $4y^2$

13. In a factory, Erika assembles 3 calculators in the same amount of time that Nick assembles 2 calculators. Also, Nick assembles 1 calculator in the same amount of time that Sam assembles 3 calculators. How many calculators in total can be assembled by Nick, Erika and Sam in the same amount of time as Erika assembles 9 calculators?

(A) 30 **(B)** 24 **(C)** 27 **(D)** 81 **(E)** 33

14. Storage space on a computer is measured in gigabytes (GB) and megabytes (MB), where 1 GB = 1024 MB. Julia has an empty 300 GB hard drive and puts 300 000 MB of data onto it. How much storage space on the hard drive remains empty?

(A) 72 MB **(B)** 720 MB **(C)** 7200 MB **(D)** 7.2 GB **(E)** 72 GB

15. In the 4×4 grid shown, each of the four symbols has a different value. The sum of the values of the symbols in each row is given to the right of that row. What is the value of $\blacklozenge$?

$\heartsuit$	$\triangle$	$\triangle$	$\heartsuit$	26
$\triangle$	$\triangle$	$\triangle$	$\triangle$	24
$\square$	$\blacklozenge$	$\heartsuit$	$\blacklozenge$	27
$\square$	$\heartsuit$	$\square$	$\triangle$	33

(A) 5 **(B)** 6 **(C)** 7
(D) 8 **(E)** 9

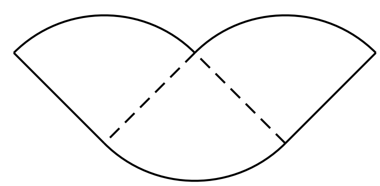
16. The table shows the results of a poll which asked each student how many hamburgers he or she ate at the end of year class party.

Number of hamburgers	0	1	2	3	4
Number of students	12	14	8	4	2

What is the average (mean) number of hamburgers eaten per student?

(A) 1.8 **(B)** 2 **(C)** 1.25 **(D)** 2.5 **(E)** 8

17. A circle with area 36π is cut into quarters and three of the pieces are arranged as shown. What is the perimeter of the resulting figure?



(A) $6\pi + 12$ **(B)** $9\pi + 12$ **(C)** $9\pi + 18$
(D) $27\pi + 12$ **(E)** $27\pi + 24$

18. At the post office, Sonita bought some 2 ¢ stamps and she bought ten times as many 1 ¢ stamps as 2 ¢ stamps. She also bought some 5 ¢ stamps. She did not buy any other stamps. The total value of the stamps that she bought was 100 ¢. How many stamps did Sonita buy in total?

(A) 66 **(B)** 30 **(C)** 44 **(D)** 63 **(E)** 62

19. Two different numbers are randomly selected from the set $\{-3, -1, 0, 2, 4\}$ and then multiplied together. What is the probability that the product of the two numbers chosen is 0?

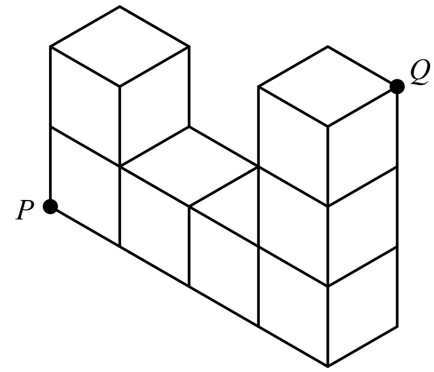
(A) $\frac{1}{10}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

20. If $wxyz$ is a four-digit positive integer with $w \neq 0$, the *layer sum* of this integer equals $wxyz + xyz + yz + z$. For example, the layer sum of 4089 is $4089 + 089 + 89 + 9 = 4276$. If the layer sum of $wxyz$ equals 2014, what is the value of $w + x + y + z$?

(A) 12 **(B)** 15 **(C)** 11 **(D)** 13 **(E)** 10

Part C: Each correct answer is worth 8.

21. In the diagram, the shape consists of seven identical cubes with edge length 1. Entire faces of the cubes are attached to one another, as shown. What is the distance between P and Q?



(A) $\sqrt{20}$ **(B)** $\sqrt{26}$ **(C)** $\sqrt{14}$
(D) $\sqrt{18}$ **(E)** $\sqrt{30}$

22. A five-digit positive integer is created using each of the odd digits 1, 3, 5, 7, 9 once so that

- the thousands digit is larger than the hundreds digit,
- the thousands digit is larger than the ten thousands digit,
- the tens digit is larger than the hundreds digit, and
- the tens digit is larger than the units digit.

How many such five-digit positive integers are there?

(A) 12 **(B)** 8 **(C)** 16 **(D)** 14 **(E)** 10

23. Three friends are in the park. Bob and Clarise are standing at the same spot and Abe is standing 10 m away. Bob chooses a random direction and walks in this direction until he is 10 m from Clarise. What is the probability that Bob is closer to Abe than Clarise is to Abe?

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{\pi}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{6}$

24. For each positive integer n, define $S(n)$ to be the smallest positive integer divisible by each of the positive integers $1, 2, 3, \ldots, n$. For example, $S(5) = 60$. How many positive integers n with $1 \leq n \leq 100$ have $S(n) = S(n+4)$?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

25. Point P is on the y-axis with y-coordinate greater than 0 and less than 100. A circle is drawn through P, $Q(4,4)$ and $O(0,0)$. How many possible positions for P are there so that the radius of this circle is an integer?

(A) 2 **(B)** 68 **(C)** 66 **(D)** 65 **(E)** 67

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Cayley Contest

(Grade 10)

Thursday, February 21, 2013
(in North America and South America)

Friday, February 22, 2013
(outside of North America and South America)

Time: 60 minutes
Calculators are permitted
Instructions

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{8+4}{8-4}$ is

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2. The expression $2^3 + 2^2 + 2^1$ is equal to

   (A) 6 **(B)** 10 **(C)** 14 **(D)** 18 **(E)** 22

3. If $x + \sqrt{81} = 25$, then x equals

   (A) 16 **(B)** 56 **(C)** 9 **(D)** 35 **(E)** 4

4. How many of the four integers 222, 2222, 22 222, and 222 222 are multiples of 3?

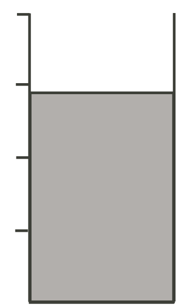
   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

5. A rectangular field has a length of 20 metres and a width of 5 metres. If its length is increased by 10 m, by how many square metres will its area be increased?

   (A) 10 **(B)** 20 **(C)** 50 **(D)** 75 **(E)** 100

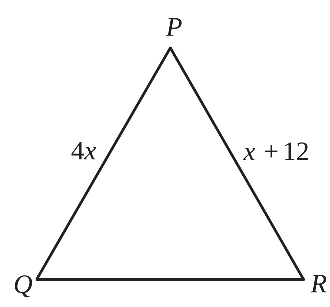
6. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?

   (A) 24 L **(B)** 28 L **(C)** 30 L
   (D) 36 L **(E)** 40 L



7. In the diagram, $\triangle PQR$ is an equilateral triangle. If $PQ = 4x$ and $PR = x + 12$, what is the value of x?

   (A) 48 **(B)** 16 **(C)** 4
   (D) 32 **(E)** 12

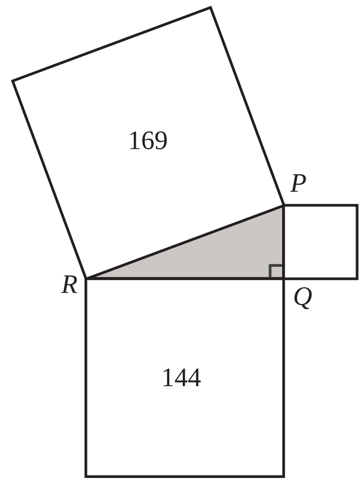


8. The symbol $\diamond$ is defined so that $a \diamond b = \dfrac{a+b}{a \times b}$. For example, $2 \diamond 5 = \dfrac{2+5}{2 \times 5} = \dfrac{7}{10}$.

   What is the value of $3 \diamond 6$?

   (A) 9 **(B)** $\dfrac{1}{18}$ **(C)** $\dfrac{1}{6}$ **(D)** 2 **(E)** $\dfrac{1}{2}$

9. In the diagram, $\triangle PQR$ has a right angle at Q. A square is drawn on each side of the triangle. The area of the square on side QR is 144. The area of the square on side PR is 169. What is the area of the square on side PQ?



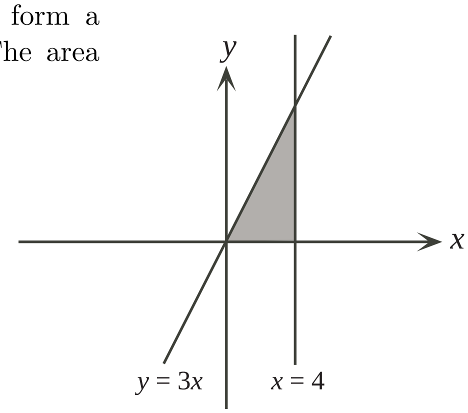
(A) 16 **(B)** 12 **(C)** 13
(D) 36 **(E)** 25

10. Barry has three sisters. The average age of the three sisters is 27. The average age of Barry and his three sisters is 28. What is Barry's age?

(A) 1 **(B)** 30 **(C)** 4 **(D)** 29 **(E)** 31

Part B: Each correct answer is worth 6.

11. The lines with equations $x = 4$ and $y = 3x$ form a triangle with the positive x-axis, as shown. The area of the triangle is



(A) 12 **(B)** 24 **(C)** 36
(D) 48 **(E)** 60

12. If $a(x + b) = 3x + 12$ for all values of x, then $a + b$ equals

(A) 12 **(B)** 15 **(C)** 8 **(D)** 7 **(E)** 13

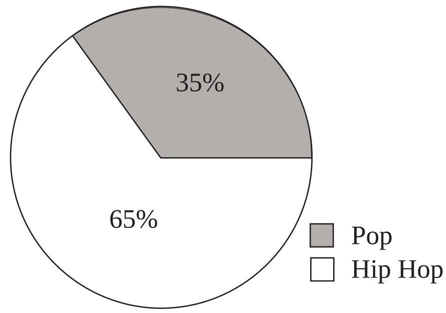
13. An integer x is chosen so that $3x + 1$ is an even integer. Which of the following must be an odd integer?

(A) $x + 3$ **(B)** $x - 3$ **(C)** $2x$ **(D)** $7x + 4$ **(E)** $5x + 3$

14. Integers greater than 1000 are created using the digits 2, 0, 1, 3 exactly once in each integer. What is the difference between the largest and the smallest integers that can be created in this way?

(A) 2187 **(B)** 2333 **(C)** 1980 **(D)** 3209 **(E)** 4233

15. The graph shows styles of music on a playlist. Country music songs are added to the playlist so that 40% of the songs are now Country. If the ratio of Hip Hop songs to Pop songs remains the same, what percentage of the total number of songs are now Hip Hop?

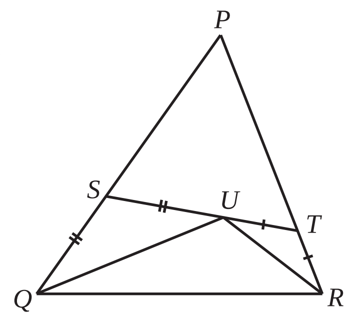


(A) 7 **(B)** 15 **(C)** 21
(D) 35 **(E)** 39

16. When $5^{35} - 6^{21}$ is evaluated, the units (ones) digit is

(A) 1 **(B)** 9 **(C)** 2 **(D)** 5 **(E)** 6

17. In the diagram, $PQ = 19$, $QR = 18$, and $PR = 17$. Point S is on PQ, point T is on PR, and point U is on ST so that $QS = SU$ and $UT = TR$. The perimeter of $\triangle PST$ is equal to

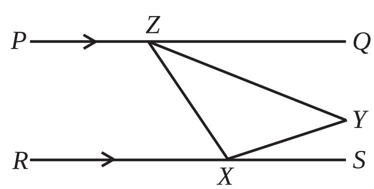


(A) 36 **(B)** 35 **(C)** 37
(D) 34 **(E)** 38

18. A two-digit positive integer x has the property that when 109 is divided by x, the remainder is 4. What is the sum of all such two-digit positive integers x?

(A) 36 **(B)** 56 **(C)** 50 **(D)** 71 **(E)** 35

19. In the diagram, PQ is parallel to RS. Also, Z is on PQ and X is on RS. If Y is located between PQ and RS so that $\angle YXS = 20°$ and $\angle ZYX = 50°$, what is the measure of $\angle QZY$?



(A) 30° **(B)** 20° **(C)** 40°
(D) 50° **(E)** 60°

20. Jack and Jill exercise along the same route. Jill jogs the first half of the route at 6 km/h, runs the remainder of the route at 12 km/h and takes a total time of x hours. Jack walks the first third of the route at 5 km/h, runs the remainder at 15 km/h and takes a total time of y hours. Which of the following is equal to $\frac{x}{y}$?

(A) $\frac{9}{8}$ **(B)** $\frac{7}{5}$ **(C)** $\frac{15}{16}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{10}{9}$

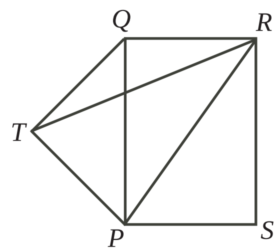
Part C: Each correct answer is worth 8.

21. In the addition shown, the letters X, Y, and Z each represent a different non-zero digit. The digit X is

$$\begin{array}{r} X\ X\ X \\ Y\ Y\ Y \\ +\ Z\ Z\ Z \\ \hline Z\ Y\ Y\ X \end{array}$$

(A) 1 **(B)** 2 **(C)** 7 **(D)** 8 **(E)** 9

22. In the diagram, $PQRS$ is a rectangle. Point T is outside the rectangle so that $\triangle PTQ$ is an isosceles right-angled triangle with hypotenuse PQ. If $PQ = 4$ and $QR = 3$, then the area of $\triangle PTR$ is

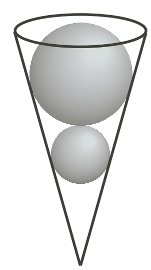


(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

23. One bag contains 2 red marbles and 2 blue marbles. A second bag contains 2 red marbles, 2 blue marbles, and g green marbles, with $g > 0$. For each bag, Maria calculates the probability of randomly drawing two marbles of the same colour in two draws from that bag, without replacement. (Drawing two marbles *without replacement* means drawing two marbles, one after the other, without putting the first marble back into the bag.) If these two probabilities are equal, then the value of g is

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

24. A cone is filled with water. Two solid spheres are placed in the cone as shown in the diagram and water spills out. (The spheres are touching each other, each sphere touches the cone all of the way around, and the top of the top sphere is level with the top of the cone.) The larger sphere has radius twice that of the smaller sphere. If the volume of the water remaining in the cone is 2016π, what is the radius of the smaller sphere? (The volume of a sphere with radius r is $\frac{4}{3}\pi r^3$. The volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.)



(A) $2\sqrt{2}$ **(B)** 6 **(C)** 8 **(D)** $6\sqrt{2}$ **(E)** $4\sqrt[3]{2}$

25. A positive integer has k *trailing zeros* if its last k digits are all zero and it has a non-zero digit immediately to the left of these k zeros. For example, the number 1 030 000 has 4 trailing zeros. Define $Z(m)$ to be the number of trailing zeros of the positive integer m. Lloyd is bored one day, so makes a list of the value of $n - Z(n!)$ for each integer n from 100 to 10 000, inclusive. How many integers appear in his list at least three times?
(Note: If n is a positive integer, the symbol $n!$ (read "n factorial") is used to represent the product of the integers from 1 to n. That is, $n! = n(n-1)(n-2)\cdots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$.)

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

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The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
www.cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Thursday, February 23, 2012
(in North America and South America)

Friday, February 24, 2012
(outside of North America and South America)

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
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   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

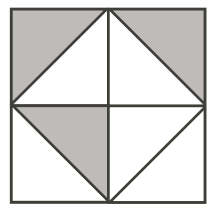
1. The value of $\dfrac{5-2}{2+1}$ is

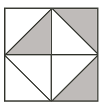
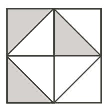
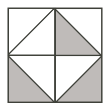
   (A) 3 **(B)** 1 **(C)** -1 **(D)** -5 **(E)** 5

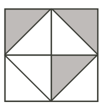
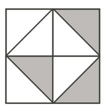
2. The average of 1, 3 and x is 3. What is the value of x?

   (A) 4 **(B)** 5 **(C)** 2 **(D)** 3 **(E)** 1

3. Which of the following is obtained by rotating the figure to the right clockwise by 90°?



   (A)  **(B)**  **(C)** 

   (D)  **(E)** 

4. The value of $(-1)^3 + (-1)^2 + (-1)$ is

   (A) 2 **(B)** 1 **(C)** -3 **(D)** -1 **(E)** -2

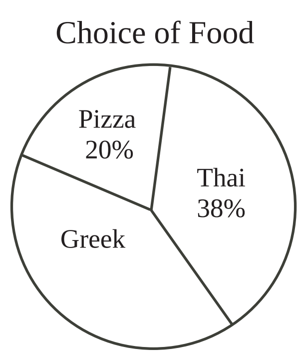
5. If $\sqrt{100-x} = 9$, then x equals

   (A) 9 **(B)** 91 **(C)** $\sqrt{19}$ **(D)** 97 **(E)** 19

6. A basket contains 12 apples, 15 bananas and no other fruit. If 3 more bananas are added to the basket, what fraction of the fruit in the basket will be bananas?

   (A) $\frac{2}{5}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{5}{9}$

7. The circle graph shows the results of asking 150 students to choose pizza, Thai food, or Greek food. How many students chose Greek food?



   (A) 78 **(B)** 32 **(C)** 48

   (D) 58 **(E)** 63

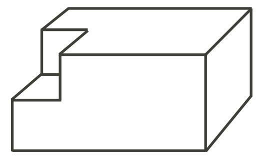
8. The product $\left(1-\dfrac{1}{3}\right)\left(1-\dfrac{1}{4}\right)\left(1-\dfrac{1}{5}\right)$ is equal to

   (A) $\frac{2}{5}$ **(B)** $\frac{1}{60}$ **(C)** 1 **(D)** 3 **(E)** $\frac{59}{60}$

9. A class of 30 students was asked what they did on their winter holiday.
20 students said that they went skating.
9 students said that they went skiing.
Exactly 5 students said that they went skating and went skiing.
How many students did not go skating and did not go skiing?

(A) 1 **(B)** 6 **(C)** 11 **(D)** 19 **(E)** 4

10. A solid rectangular prism has dimensions 4 by 2 by 2. A 1 by 1 by 1 cube is cut out of the corner creating the new solid shown. What is the surface area of the new solid?



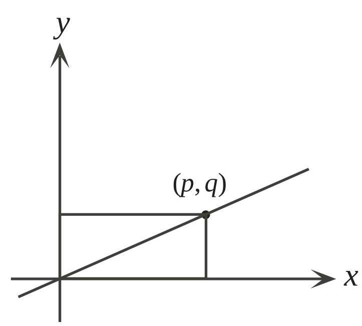
(A) 34 **(B)** 37 **(C)** 40
(D) 15 **(E)** 39

Part B: Each correct answer is worth 6.

11. Matilda has a summer job delivering newspapers.
She earns \$6.00 an hour plus \$0.25 per newspaper delivered.
Matilda delivers 30 newspapers per hour.
How much money will she earn during a 3 hour shift?

(A) \$40.50 **(B)** \$18.75 **(C)** \$13.50 **(D)** \$25.50 **(E)** \$28.50

12. The point (p, q) is on the line $y = \frac{2}{5}x$, as shown. Also, the area of the rectangle shown is 90. What is the value of p?

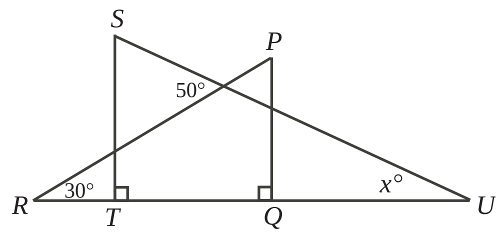


(A) 12 **(B)** 9 **(C)** 10
(D) 15 **(E)** 30

13. There is one odd integer N between 400 and 600 that is divisible by both 5 and 11. The sum of the digits of N is

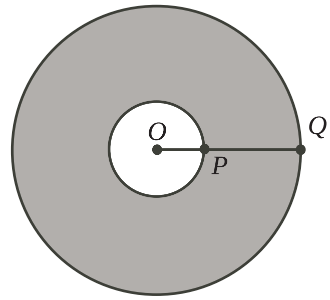
(A) 11 **(B)** 8 **(C)** 10 **(D)** 16 **(E)** 18

14. In the diagram, $\triangle PQR$ and $\triangle STU$ overlap so that $RTQU$ forms a straight line segment. What is the value of x?



(A) 10 **(B)** 20 **(C)** 30
(D) 40 **(E)** 50

15. In the diagram, each of the two circles has centre O. Also, $OP : PQ = 1 : 2$. If the radius of the larger circle is 9, what is the area of the shaded region?



(A) 12π **(B)** 36π **(C)** 54π **(D)** 72π **(E)** 81π

16. The equation $y = ax^2 + bx + c$ was used to create the table of values below:

x	0	1
y	8	9

What is the value of $a + b$?

(A) -2 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** -5

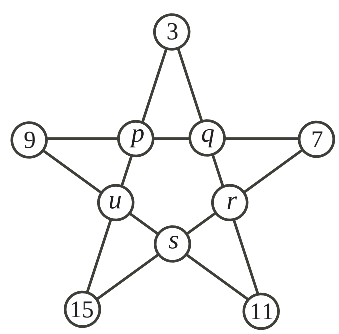
17. A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?

(A) $\frac{8}{15}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{6}{13}$ **(E)** $\frac{1}{4}$

18. Six consecutive integers are written on a blackboard. When one of them is erased the sum of the remaining five integers is 2012. What is the sum of the digits of the integer that was erased?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 9 **(E)** 11

19. In the star shown, the sum of the four integers along each straight line is to be the same. Five numbers have been entered. The five missing numbers are 19, 21, 23, 25, and 27. Which number is represented by q?



(A) 25 **(B)** 21 **(C)** 23 **(D)** 27 **(E)** 19

20. If N is the smallest positive integer whose digits have a product of 2700, then the sum of the digits of N is

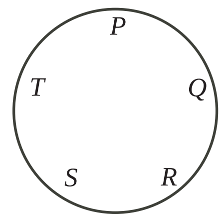
(A) 23 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27

Part C: Each correct answer is worth 8.

21. If x and y are positive integers with $x > y$ and $x + xy = 391$, what is the value of $x + y$?

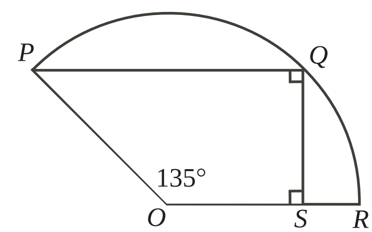
(A) 38 **(B)** 39 **(C)** 40 **(D)** 41 **(E)** 42

22. Five monkeys are seated around a table. Their seats are labelled P, Q, R, S, and T, in clockwise order, as shown. The five monkeys are randomly numbered Monkey 1, Monkey 2, Monkey 3, Monkey 4, and Monkey 5. Monkey 1 remains in its seat. The remaining four monkeys then sit themselves in the remaining seats so that they are seated in clockwise order as Monkey 1, Monkey 2, Monkey 3, Monkey 4, and Monkey 5. What is the probability that the Monkey originally in seat R moves to seat P?



(A) $\frac{1}{20}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{3}{20}$
(D) $\frac{1}{5}$ **(E)** $\frac{1}{4}$

23. In the diagram, points P, Q and R lie on a circle with centre O and radius 12, and point S lies on OR. If $\angle POR = 135°$, the area of trapezoid $OPQS$ is closest to



(A) 216 **(B)** 144 **(C)** 108
(D) 112.5 **(E)** 114.6

24. Six friends will exchange books in their book club. Each friend has one book to give to a friend, and will receive one book from a different friend. (No two friends trade books with each other.) In how many ways can the books be exchanged?

(A) 200 **(B)** 120 **(C)** 140 **(D)** 240 **(E)** 160

25. The digits of the positive integer n include no 9s, exactly four 8s, exactly three 7s, exactly two 6s, and some other digits. If the sum of the digits of n is 104 and the sum of the digits of $2n$ is 100, then the number of times that the digit 5 occurs in n is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

2012 Cayley Contest (English)

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Cayley Contest

(Grade 10)

Thursday, February 24, 2011

STRONGER COMMUNITIES TOGETHER™

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Part A: Each correct answer is worth 5.

1. The value of $(5+2)+(8+6)+(4+7)+(3+2)$ is

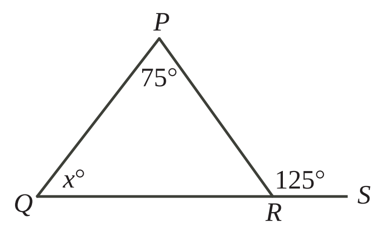
   (A) 35 **(B)** 37 **(C)** 40 **(D)** 45 **(E)** 47

2. If $(-1)(2)(x)(4)=24$, then x equals

   (A) 4 **(B)** -3 **(C)** -1 **(D)** 2 **(E)** -4

3. In the diagram, R lies on line segment QS. What is the value of x?

   (A) 50 **(B)** 55 **(C)** 75
   (D) 100 **(E)** 105



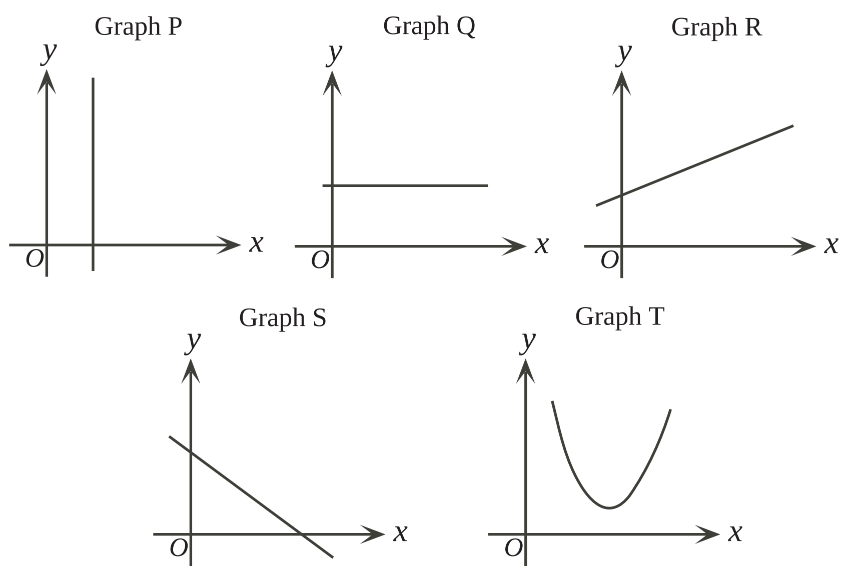
4. When a number is tripled, then decreased by 5, the result is 16. What is the original number?

   (A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

5. The expression $\sqrt{13+\sqrt{7+\sqrt{4}}}$ is equal to

   (A) 7 **(B)** 8 **(C)** 6 **(D)** 4 **(E)** 5

6. Which of the five graphs is linear with a slope of 0?



   (A) Graph P **(B)** Graph Q **(C)** Graph R **(D)** Graph S **(E)** Graph T

7. After a fair die with faces numbered 1 to 6 is rolled, the number on the top face is x. Which of the following is most likely?

(A) x is greater than 2 **(B)** x equals 4 or 5 **(C)** x is even
(D) x is less than 3 **(E)** x equals 3

8. If 2.4×10^8 is doubled, then the result is equal to

(A) 2.4×20^8 **(B)** 4.8×20^8 **(C)** 4.8×10^8 **(D)** 2.4×10^{16} **(E)** 4.8×10^{16}

9. A proposed new \$5 coin is called the “foonie”. The foonie’s two faces are identical and each has area 5 cm^2. The thickness of the foonie is 0.5 cm. How many foonies are in a stack that has a volume of 50 cm^3?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 40

10. The Athenas are playing a 44 game season. Each game results in a win or a loss, and cannot end in a tie. So far, they have 20 wins and 15 losses. In order to make the playoffs, they must win at least 60% of all of their games. What is the smallest number of their remaining games that they must win to make the playoffs?

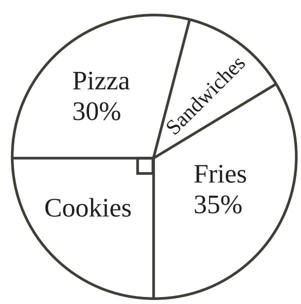
(A) 8 **(B)** 9 **(C)** 5 **(D)** 6 **(E)** 7

Part B: Each correct answer is worth 6.

11. The operation “∇” is defined by $(a,b)\nabla(c,d) = ac + bd$.
For example $(1,2)\nabla(3,4) = (1)(3) + (2)(4) = 11$.
The value of $(3,1)\nabla(4,2)$ is

(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 24

12. The circle graph shown illustrates the results of a survey taken by the Cayley H.S. Student Council to determine the favourite cafeteria food. How many of the 200 students surveyed said that their favourite food was sandwiches?



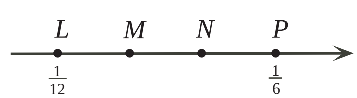
(A) 10 **(B)** 20 **(C)** 35
(D) 50 **(E)** 70

13. In the subtraction shown, K, L, M, and N are digits. What is the value of $K + L + M + N$?

$$\begin{array}{rcccc} & 5 & K & 3 & L \\ - & M & 4 & N & 1 \\ \hline & 4 & 4 & 5 & 1 \end{array}$$

(A) 20 **(B)** 19 **(C)** 16
(D) 13 **(E)** 9

14. On the number line, points M and N divide LP into three equal parts. What is the value at M?



(A) $\frac{1}{7}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{9}$
(D) $\frac{1}{10}$ **(E)** $\frac{1}{11}$

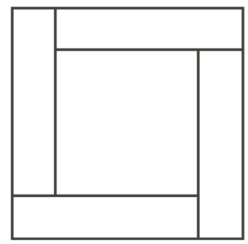
15. The points $Q(1,-1)$, $R(-1,0)$ and $S(0,1)$ are three vertices of a parallelogram. The coordinates of the fourth vertex of the parallelogram could be

(A) $(-2,2)$ **(B)** $(0,-1)$ **(C)** $(0,0)$ **(D)** $(\frac{3}{2},\frac{1}{2})$ **(E)** $(-1,1)$

16. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that Wally must buy to *guarantee* that he receives 3 gumballs of the same colour?

(A) 6 **(B)** 9 **(C)** 4 **(D)** 7 **(E)** 8

17. Four congruent rectangles and a square are assembled without overlapping to form a large square, as shown. Each of the rectangles has a perimeter of 40 cm. The total area of the large square is

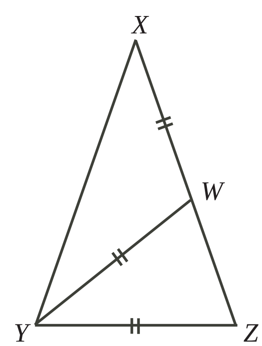


(A) 160 cm^2 **(B)** 200 cm^2 **(C)** 400 cm^2
(D) 800 cm^2 **(E)** 1600 cm^2

18. When 100 is divided by 12, the remainder is 4.
When 100 is divided by a positive integer x, the remainder is 10.
When 1000 is divided by x, the remainder is

(A) 10 **(B)** 100 **(C)** 0 **(D)** 1 **(E)** 90

19. In the diagram, $\triangle XYZ$ is isosceles with $XY = XZ$. Also, point W is on XZ so that $XW = WY = YZ$. The measure of $\angle XYW$ is



(A) 18° **(B)** 30° **(C)** 45°
(D) 36° **(E)** 60°

20. For how many positive integers n, with $n \leq 100$, is n^3+5n^2 the square of an integer?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Part C: Each correct answer is worth 8.

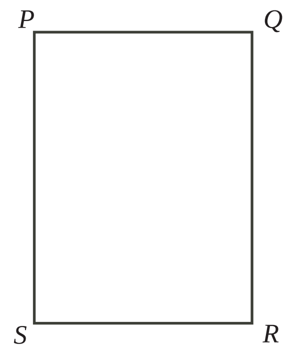
21. Suppose that x and y are positive numbers with

$$\begin{aligned} xy &= \tfrac{1}{9} \\ x(y+1) &= \tfrac{7}{9} \\ y(x+1) &= \tfrac{5}{18} \end{aligned}$$

What is the value of $(x+1)(y+1)$?

(A) $\frac{11}{6}$ **(B)** $\frac{8}{9}$ **(C)** $\frac{16}{9}$ **(D)** $\frac{10}{9}$ **(E)** $\frac{35}{18}$

22. The top section of an 8 cm by 6 cm rectangular sheet of paper is folded along a straight line so that when the top section lies flat on the bottom section, corner P lies on top of corner R. The length of the crease, in cm, is



(A) 6.25 **(B)** 7 **(C)** 7.5
(D) 7.4 **(E)** 10

23. A *Fano table* is a table with three columns where

- each entry is an integer taken from the list $1, 2, 3, \ldots, n$, and
- each row contains three different integers, and
- for each possible pair of distinct integers from the list $1, 2, 3, \ldots, n$, there is exactly one row that contains both of these integers.

1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

The number of rows in the table will depend on the value of n. For example, the table shown is a Fano table with $n = 7$. (Notice that 2 and 6 appear in the same row only once, as does every other possible pair of the numbers $1, 2, 3, 4, 5, 6, 7$.) For how many values of n with $3 \leq n \leq 12$ can a Fano table be created?

(A) 2 **(B)** 3 **(C)** 5
(D) 6 **(E)** 7

24. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

(A) $t < 3.9$ **(B)** $3.9 \leq t < 4.1$ **(C)** $4.1 \leq t < 4.3$
(D) $4.3 \leq t < 4.5$ **(E)** $t \geq 4.5$

25. Two numbers a and b with $0 \leq a \leq 1$ and $0 \leq b \leq 1$ are chosen at random. The number c is defined by $c = 2a + 2b$. The numbers a, b and c are each rounded to the nearest integer to give A, B and C, respectively. (For example, if $a = 0.432$ and $b = 0.5$, then $c = 1.864$, and so $A = 0$, $B = 1$, and $C = 2$.) What is the probability that $2A + 2B = C$?

(A) $\frac{15}{32}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{3}{4}$

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

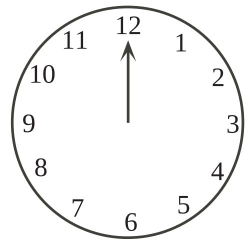
Part A: Each correct answer is worth 5.

1. The value of $6 + 4 \div 2$ is

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2. The minute hand on a clock points at the 12. The minute hand then rotates $120°$ clockwise. Which number will it be pointing at?

   (A) 6 **(B)** 2 **(C)** 4
   (D) 3 **(E)** 5



3. If $x + \sqrt{25} = \sqrt{36}$, then x equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 11

4. When simplified, $\dfrac{1}{2 + \frac{2}{3}}$ is equal to

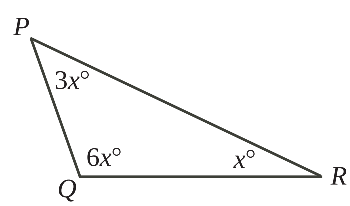
   (A) $\frac{1}{8}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{8}$

5. A rectangle has a length of $\frac{3}{5}$ and an area of $\frac{1}{3}$. What is the width of the rectangle?

   (A) $\frac{1}{5}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{14}{15}$ **(D)** $\frac{15}{14}$ **(E)** $\frac{9}{5}$

6. What is the measure of the largest angle in $\triangle PQR$?

   (A) $144°$ **(B)** $96°$ **(C)** $120°$
   (D) $60°$ **(E)** $108°$

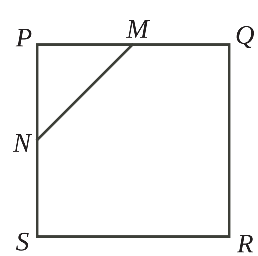


7. The mean (average) of 5 consecutive integers is 9. What is the smallest of these 5 integers?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. Square $PQRS$ has an area of 900. M is the midpoint of PQ and N is the midpoint of PS. What is the area of triangle PMN?

   (A) 100 **(B)** 112.5 **(C)** 150
   (D) 225 **(E)** 180



9. Which of the following lines, when drawn together with the x-axis and the y-axis, encloses an isosceles triangle?

(A) $y = 4x + 4$ **(B)** $y = \frac{1}{2}x + 4$ **(C)** $y = -x + 4$
(D) $y = 2x + 4$ **(E)** $y = -3x + 4$

10. There are 400 students at Pascal H.S., where the ratio of boys to girls is $3 : 2$. There are 600 students at Fermat C.I., where the ratio of boys to girls is $2 : 3$. When considering all the students from both schools, what is the ratio of boys to girls?

(A) $2 : 3$ **(B)** $12 : 13$ **(C)** $1 : 1$ **(D)** $6 : 5$ **(E)** $3 : 2$

Part B: Each correct answer is worth 6.

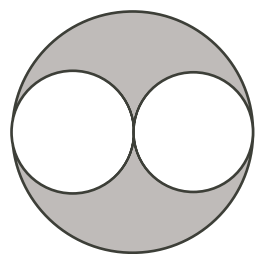
11. If x and y are positive integers with $x + y = 31$, then the largest possible value of xy is

(A) 240 **(B)** 238 **(C)** 255 **(D)** 248 **(E)** 242

12. The price of each item at the Gauss Gadget Store has been reduced by 20% from its original price. An MP3 player has a sale price of \$112. What would the same MP3 player sell for if it was on sale for 30% off of its original price?

(A) \$78.40 **(B)** \$100.80 **(C)** \$89.60 **(D)** \$168.00 **(E)** \$98.00

13. In the diagram, the smaller circles touch the larger circle and touch each other at the centre of the larger circle. The radius of the larger circle is 6. What is the area of the shaded region?



(A) 27π **(B)** 6π **(C)** 9π
(D) 18π **(E)** 36π

14. How many ordered pairs (a, b) of positive integers satisfy $a^2 + b^2 = 50$?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 7

15. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?

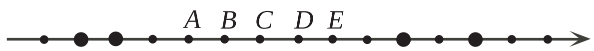
(A) \$40 **(B)** \$100 **(C)** \$160 **(D)** \$1000 **(E)** \$1600

16. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of x?

	5	
9		17
x		

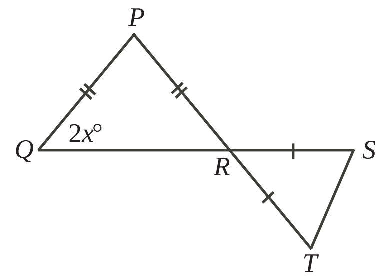
(A) 7 **(B)** 11 **(C)** 13
(D) 15 **(E)** 19

17. In the diagram, the number line is marked at consecutive integers, but the numbers themselves are not shown. The four larger dots represent two numbers that are multiples of 3 and two numbers that are multiples of 5. Which point represents a number which is a multiple of 15?



(A) A **(B)** B **(C)** C **(D)** D **(E)** E

18. In the diagram, R is the point of intersection of PT and QS, $PQ = PR$, and $RS = RT$. If $\angle PQR = 2x^\circ$, then the measure of $\angle RST$, in degrees, is



(A) $45 - x$ **(B)** $90 + \frac{1}{2}x$ **(C)** $90 - \frac{1}{2}x$
(D) $45 + 2x$ **(E)** $90 - x$

19. How many 3-digit positive integers have exactly one even digit?

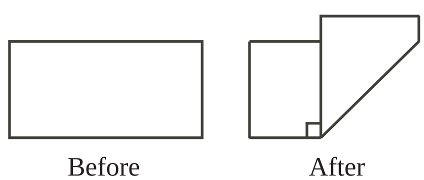
(A) 350 **(B)** 450 **(C)** 375 **(D)** 75 **(E)** 125

20. What is the largest positive integer n that satisfies $n^{200} < 3^{500}$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Part C: Each correct answer is worth 8.

21. A rectangular piece of paper measures 17 cm by 8 cm. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is the area of the new figure?



(A) 104 cm^2 **(B)** 81 cm^2 **(C)** 72 cm^2 **(D)** 168 cm^2 **(E)** 64 cm^2

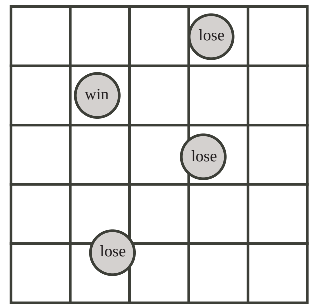
22. A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is

(A) 2155 **(B)** 2153 **(C)** 2151 **(D)** 2149 **(E)** 2147

23. Connie has a number of gold bars, all of different weights. She gives the 24 lightest bars, which weigh 45% of the total weight, to Brennan. She gives the 13 heaviest bars, which weigh 26% of the total weight, to Maya. She gives the rest of the bars to Blair. How many bars did Blair receive?

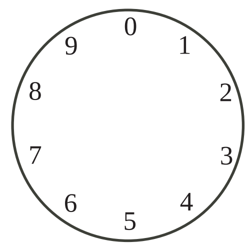
(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

24. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?



(A) $\frac{25}{441}$ **(B)** $\frac{1}{25}$ **(C)** $\frac{1}{49}$
(D) $\frac{5}{147}$ **(E)** $\frac{4\pi}{25}$

25. Steve places a counter at 0 on the diagram. On his first move, he moves the counter 1^1 step clockwise to 1. On his second move, he moves 2^2 steps clockwise to 5. On his third move, he moves 3^3 steps clockwise to 2. He continues in this manner, moving n^n steps clockwise on his nth move. At which position will the counter be after 1234 moves?



(A) 1 **(B)** 3 **(C)** 5
(D) 7 **(E)** 9

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Canadian
Mathematics
Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Wednesday, February 18, 2009

Time: 60 minutes

Calculators are permitted

Instructions

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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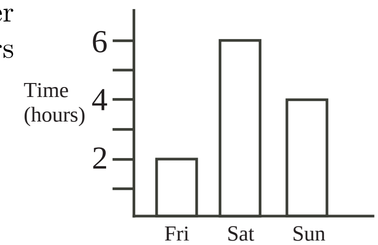
Part A: Each correct answer is worth 5.

1. The value of $\dfrac{10^2 - 10}{9}$ is

(A) 10 **(B)** 1 **(C)** 7 **(D)** 2009 **(E)** 11

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?



(A) 2 **(B)** 4 **(C)** 6
(D) 8 **(E)** 10

3. If $3(-2) = \nabla + 2$, then ∇ equals

(A) -2 **(B)** 0 **(C)** -8 **(D)** -6 **(E)** -4

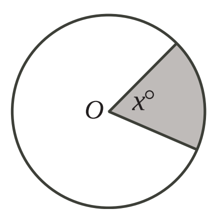
4. If $\sqrt{5+n} = 7$, the value of n is

(A) 4 **(B)** 9 **(C)** 24 **(D)** 44 **(E)** 74

5. $3^2 + 4^2 + 12^2$ is equal to

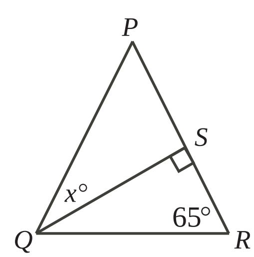
(A) 13^2 **(B)** 19^2 **(C)** 17^2 **(D)** 15^2 **(E)** 11^2

6. In the diagram, the centre of the circle is O. The area of the shaded region is 20% of the area of the circle. The value of x is



(A) 18 **(B)** 45 **(C)** 60
(D) 72 **(E)** 90

7. In the diagram, $PQ = PR$ and $\angle QRP = 65^\circ$. The value of x is



(A) 45 **(B)** 30 **(C)** 50
(D) 60 **(E)** 40

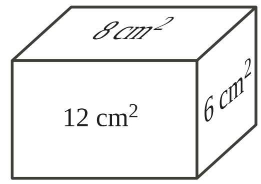
8. When three consecutive positive integers are multiplied together, the answer is always

(A) odd **(B)** a multiple of 6 **(C)** a multiple of 12
(D) a multiple of 4 **(E)** a multiple of 5

9. If Francis spends $\frac{1}{3}$ of his day sleeping, $\frac{1}{4}$ of his day studying and $\frac{1}{8}$ of his day eating, how many hours in the day does he have left?

(A) 4 **(B)** 6 **(C)** 5 **(D)** 7 **(E)** 9

10. The front of a rectangular prism has an area of 12 cm^2, the side has an area of 6 cm^2, and the top has area 8 cm^2. The volume of the prism in cm^3, is



(A) 24 **(B)** 26 **(C)** 48 **(D)** 72 **(E)** 52

Part B: Each correct answer is worth 6.

11. Gillian has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from her collection that she can play in 3 hours?

(A) 100 **(B)** 36 **(C)** 56 **(D)** 60 **(E)** 80

12. In the table shown, a sequence starts with 2 in the top left corner. Moving across each row, each box is filled with a number 3 greater than the number to its left. The leftmost number in each row is 3 greater than the greatest in the previous row. When all of the boxes are filled in, the value of x is

2	5	8	11	14	17
20	23	26	29	32	35
38					
					x

(A) 101 **(B)** 104 **(C)** 107 **(D)** 110 **(E)** 113

13. Filipa plays a game. She starts with a row of 15 squares and a coin on the centre square. Filipa then rolls a die. If she rolls an even number, she moves the coin that many squares to the right; if she rolls an odd number, she moves the coin that many squares to the left. If the results of six rolls were $1, 2, 3, 4, 5, 6$, where would her coin be located?

(A) On the square where it started
(B) 1 square to the right of where it started
(C) 2 squares to the right of where it started
(D) 2 squares to the left of where it started
(E) 3 squares to the right of where it started

14. A positive integer larger than 2 is called composite if it is not prime. What is the smallest prime number that is the sum of three *different* composite numbers?

(A) 11 **(B)** 13 **(C)** 17 **(D)** 19 **(E)** 23

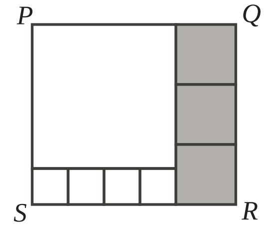
15. A list of 5 positive integers has all of the following properties:

- the only integer in the list that occurs more than once is 8,
- its median is 9, and
- its average (mean) is 10.

What is the largest possible integer that could appear in the list?
(Note: The median of a set of five positive integers is the middle integer when the set is arranged in increasing order.)

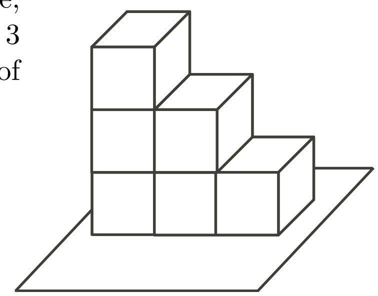
(A) 15 **(B)** 16 **(C)** 17 **(D)** 24 **(E)** 25

16. Rectangle $PQRS$ is divided into eight squares, as shown. The side length of each shaded square is 10. What is the length of the side of the largest square?



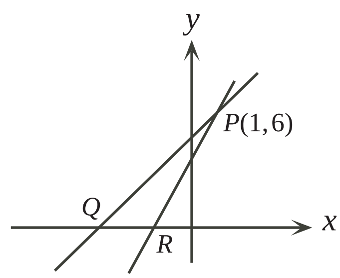
(A) 18 **(B)** 24 **(C)** 16
(D) 23 **(E)** 25

17. Six dice are stacked on the floor as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?



(A) 69 **(B)** 88 **(C)** 89
(D) 91 **(E)** 96

18. A line with slope equal to 1 and a line with slope equal to 2 intersect at the point $P(1,6)$, as shown. The area of $\triangle PQR$ is



(A) 6 **(B)** 9 **(C)** 12
(D) 15 **(E)** 18

19. How many integers n are there with the property that the product of the digits of n is 0, where $5000 \leq n \leq 6000$?

(A) 332 **(B)** 270 **(C)** 301 **(D)** 272 **(E)** 299

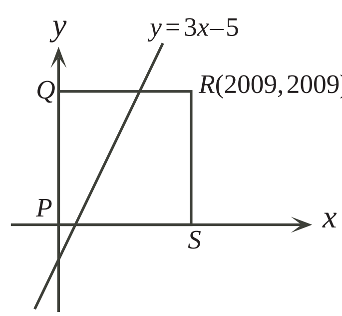
20. On Monday, Hank drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and took the same route. This time he drove at an average speed of 75 km/h and arrived 1 minute early. How long is his route to work?

(A) 30 km **(B)** 35 km **(C)** 45 km **(D)** 50 km **(E)** 60 km

---

Part C: Each correct answer is worth 8.

21. A *lattice point* is a point with integer coordinates. (For example, $(1,4)$ is a lattice point but $(\frac{3}{2},4)$ is not.) The line $y = 3x - 5$ passes through square $PQRS$ as shown in the diagram. If the coordinates of R are $(2009, 2009)$, then the number of lattice points on the line which are inside the square is



(A) 666 **(B)** 667 **(C)** 668
(D) 669 **(E)** 670

22. Suppose that a, b and c are three numbers with

$$\begin{aligned} a+b &= 3 \\ ac+b &= 18 \\ bc+a &= 6 \end{aligned}$$

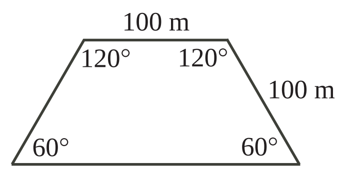
The value of c is

(A) 2 **(B)** 11 **(C)** 3 **(D)** 6 **(E)** 7

23. Angela and Barry share a piece of land. The ratio of the area of Angela's portion to the area of Barry's portion is $3:2$. They each grow corn and peas on their piece of land. The entire piece of land is covered by corn and peas in the ratio $7:3$. On Angela's portion of the land, the ratio of corn to peas is $4:1$. What is the ratio of corn to peas for Barry's portion?

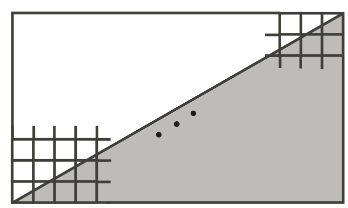
(A) $11:9$ **(B)** $2:3$ **(C)** $3:2$ **(D)** $3:7$ **(E)** $1:4$

24. The field shown has been planted uniformly with wheat. At harvest, the wheat at any point in the field is brought to the nearest point on the field's perimeter. The fraction of the crop that is brought to the longest side is



(A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$
(D) $\frac{2}{5}$ **(E)** $\frac{4}{9}$

25. Unit squares are arranged to form a rectangular grid that is m units wide and n units tall, where m and n are positive integers with $2n < m < 3n$. The region below one of the diagonals of the rectangle is shaded as shown. For certain pairs m and n, there is a unit square in the grid that is not completely shaded but whose shaded area is greater than 0.999. The smallest possible value of mn for which this is true satisfies



(A) $496 \leq mn \leq 500$
(B) $501 \leq mn \leq 505$
(C) $506 \leq mn \leq 510$
(D) $511 \leq mn \leq 515$
(E) $516 \leq mn \leq 520$

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Tuesday, February 19, 2008

C.M.C. Sponsors

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Time: 60 minutes

Calculators are permitted

Instructions

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Part A: Each correct answer is worth 5.

1. What is the value of $3^2 - 2^2 + 1^2$?

   (A) 8 **(B)** -2 **(C)** 10 **(D)** -5 **(E)** 6

2. $\dfrac{\sqrt{25-16}}{\sqrt{25}-\sqrt{16}}$ is equal to

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Which of the following numbers is closest to 1?

   (A) $\frac{3}{4}$ **(B)** 1.2 **(C)** 0.81 **(D)** $1\frac{1}{3}$ **(E)** $\frac{7}{10}$

4. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected at random. What is the probability that it is blue?

   (A) $\frac{5}{26}$ **(B)** $\frac{3}{13}$ **(C)** $\frac{7}{26}$ **(D)** $\frac{4}{13}$ **(E)** $\frac{6}{13}$

5. The 5-digit number $52\,28\square$ is a multiple of 6. Which digit is represented by $\square$?

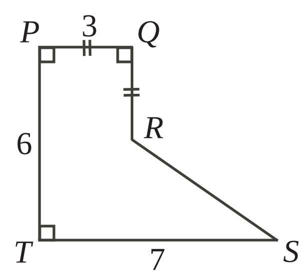
   (A) 0 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 8

6. If $\dfrac{40}{x} - 1 = 19$, then x is equal to

   (A) -1 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** -2

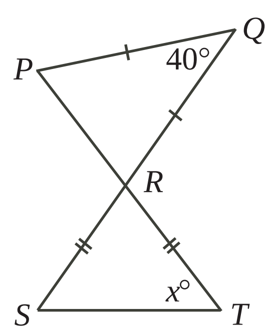
7. In the diagram, what is the perimeter of polygon $PQRST$?

   (A) 24 **(B)** 23 **(C)** 25
   (D) 26 **(E)** 27



8. In the diagram, PRT and QRS are straight lines. What is the value of x?

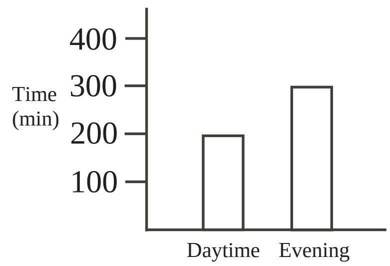
   (A) 45 **(B)** 50 **(C)** 55
   (D) 60 **(E)** 65



9. If $a = 7$ and $b = 13$, the number of even positive integers less than ab is

   (A) $\dfrac{ab-1}{2}$ **(B)** $\dfrac{ab}{2}$ **(C)** $ab - 1$ **(D)** $\dfrac{a+b}{4}$ **(E)** $(a-1)(b-1)$

10. Vivian's cell phone bill includes the graph showing her cell phone use for the month. She is charged



- $20 per month, plus
- 10¢ per minute for daytime calls, plus
- 5¢ per minute for evening calls after the first 200 evening minutes. (The first 200 evening minutes are free.)

What is her total cell phone bill for the month shown?

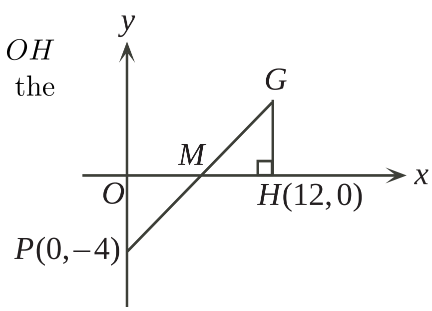
(A) $25 **(B)** $40 **(C)** $45 **(D)** $70 **(E)** $75

Part B: Each correct answer is worth 6.

11. Lex has $2.65. He has only dimes (worth $0.10 each) and quarters (worth $0.25 each). If Lex has more quarters than dimes, how many coins does he have in total?

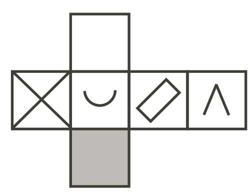
(A) 12 **(B)** 13 **(C)** 16 **(D)** 19 **(E)** 22

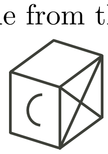
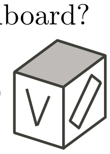
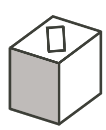
12. The line from G through the midpoint M of OH intersects the y-axis at $P(0, -4)$. What are the coordinates of G?

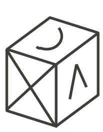
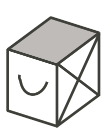


(A) $(12, 3)$ **(B)** $(12, 7)$ **(C)** $(12, 5)$ **(D)** $(12, 6)$ **(E)** $(12, 4)$

13. The diagram shows a piece of cardboard that can be folded to make a cube. The cardboard has designs on one side only. Which one of the following cubes can be made from this cardboard?



(A)  **(B)**  **(C)** 

(D)  **(E)** 

14. The first term of a sequence is 20.
If a term in the sequence is t and t is even, the next term is $\frac{1}{2}t$.
If a term in the sequence is t and t is odd, the next term is $3t + 1$.
Therefore, the first three terms in the sequence are 20, 10, 5.
What is the 10th term of the sequence?

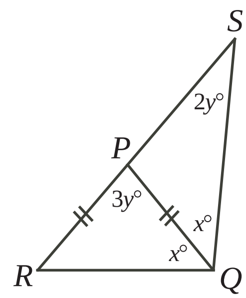
(A) 2 **(B)** 4 **(C)** 5 **(D)** 1 **(E)** 8

15. If x and y are two-digit positive integers with $xy = 555$, what is $x + y$?

(A) 52 **(B)** 116 **(C)** 66 **(D)** 555 **(E)** 45

16. In the diagram, P is on RS so that QP bisects $\angle SQR$. Also, $PQ = PR$, $\angle RSQ = 2y^\circ$, and $\angle RPQ = 3y^\circ$. The measure of $\angle RPQ$ is

(A) 90° **(B)** 108° **(C)** 120°
(D) 60° **(E)** 72°



17. If $3 \le p \le 10$ and $12 \le q \le 21$, then the difference between the largest and smallest possible values of $\dfrac{p}{q}$ is

(A) $\dfrac{29}{42}$ **(B)** $\dfrac{29}{5}$ **(C)** $\dfrac{19}{70}$ **(D)** $\dfrac{19}{12}$ **(E)** $\dfrac{19}{84}$

18. In the board game "Silly Bills", there are \$1, \$2 and \$3 bills.
There are 11 more \$2 bills than \$1 bills.
There are 18 fewer \$3 bills than \$1 bills.
If there is \$100 in total, then how many \$1 bills are there in the board game?

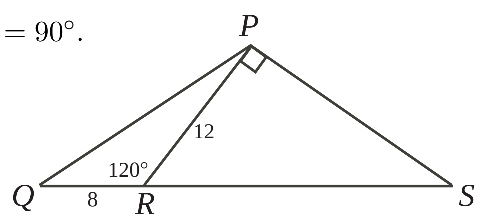
(A) 11 **(B)** 14 **(C)** 22 **(D)** 33 **(E)** 40

19. A box contains apple and pears.
An equal number of apples and pears are rotten.
$\frac{2}{3}$ of all of the apples are rotten.
$\frac{3}{4}$ of all of the the pears are rotten.
What fraction of the total number of pieces of fruit in the box is rotten?

(A) $\frac{17}{24}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{12}{17}$ **(E)** $\frac{5}{7}$

20. In the diagram, R is on QS and $QR = 8$.
Also, $PR = 12$, $\angle PRQ = 120^\circ$, and $\angle RPS = 90^\circ$.
What is the area of $\triangle QPS$?

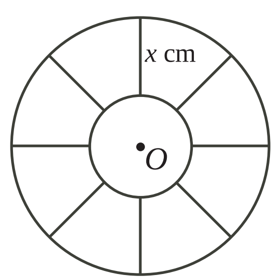
(A) $72\sqrt{3}$ **(B)** 72 **(C)** 36
(D) $60\sqrt{3}$ **(E)** $96\sqrt{3}$



Part C: Each correct answer is worth 8.

21. The circular window shown in the diagram has nine panes of equal area. The inner circular pane has radius 20 cm and the same centre, O, as the outer circle. The eight lines separating the outer panes are of equal length, x cm, and all, if extended, would pass through O. What is the value of x, to the nearest tenth?

(A) 40.0 **(B)** 36.6 **(C)** 30.0
(D) 20.0 **(E)** 43.2



22. Suppose $N = 1 + 11 + 101 + 1001 + 10001 + \ldots + 1\overbrace{000\ldots0000}^{50 \text{ zeroes}}1$.
When N is calculated and written as a single integer, the sum of its digits is

(A) 50 **(B)** 99 **(C)** 55 **(D)** 58 **(E)** 103

23. If x and y are integers with $(y-1)^{x+y} = 4^3$, then the number of possible values for x is

(A) 8 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

24. A cube has edges of length 1 cm and has a dot marked in the centre of the top face. The cube is sitting on a flat table. The cube is rolled, without lifting or slipping, in one direction so that at least two of its vertices are always touching the table. The cube is rolled until the dot is again on the top face. The length, in centimetres, of the path travelled by the dot is

(A) π **(B)** 2π **(C)** $\sqrt{2}\pi$ **(D)** $\sqrt{5}\pi$ **(E)** $\left(\dfrac{1+\sqrt{5}}{2}\right)\pi$

25. The average value of $(a-b)^2 + (b-c)^2 + (c-d)^2 + (d-e)^2 + (e-f)^2 + (f-g)^2$ over all possible arrangements (a, b, c, d, e, f, g) of the seven numbers 1, 2, 3, 11, 12, 13, 14 is

(A) 398 **(B)** 400 **(C)** 396 **(D)** 392 **(E)** 394

Canadian Mathematics Competition

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Thank you for writing the 2008 Cayley Contest!
In 2007, more than 86 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Tuesday, February 20, 2007

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer.
	Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $8 + 2(3^2)$ is

   (A) 26 **(B)** 90 **(C)** 41 **(D)** 44 **(E)** 60

2. The value of $\dfrac{7+21}{14+42}$ is

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

3. If $3x - 2x + x = 3 - 2 + 1$, then x equals

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. The table shows the pay Leona earned for two different shifts at the same fixed hourly rate. How much will she earn for a five hour shift at this rate?

Shift	Total Pay
3 hours	\$24.75
6 hours	\$49.50

   (A) \$43.75 **(B)** \$46.25 **(C)** \$38.75 **(D)** \$36.25 **(E)** \$41.25

5. $\frac{1}{4}$ of 100 is equal to

   (A) 20% of 200 **(B)** 10% of 250 **(C)** 15% of 100 **(D)** 25% of 50 **(E)** 5% of 300

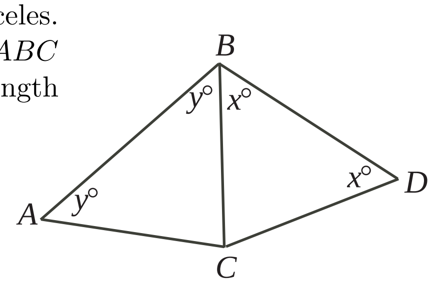
6. If $a = 2$ and $b = 5$, which of the following expressions has the greatest value?

   (A) $\dfrac{a}{b}$ **(B)** $\dfrac{b}{a}$ **(C)** $a - b$ **(D)** $b - a$ **(E)** $\frac{1}{2}a$

7. The mean (average) of 6, 9 and 18 is equal to the mean (average) of 12 and y. What is the value of y?

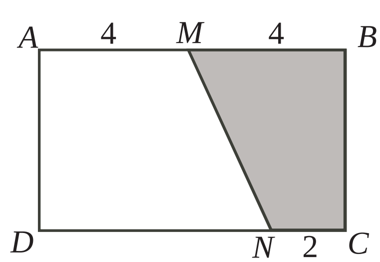
   (A) 22 **(B)** 21 **(C)** 10 **(D)** 11 **(E)** 5

8. In the diagram, triangles ABC and CBD are isosceles. The perimeter of $\triangle CBD$ is 19, the perimeter of $\triangle ABC$ is 20, and the length of BD is 7. What is the length of AB?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

9. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is

   (A) 15 **(B)** 10 **(C)** 30 **(D)** 12 **(E)** 16

10. The first term in a sequence is x. Each of the following terms is obtained by doubling the previous term and then adding 4. If the third term is 52, then x equals

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Part B: Each correct answer is worth 6.

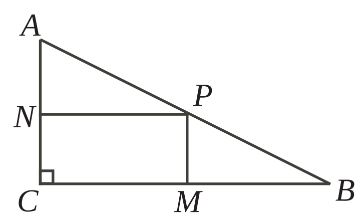
11. Ivan trained for a cross-country meet.
On Monday, he ran a certain distance.
On Tuesday, he ran twice as far as he ran on Monday.
On Wednesday, he ran half as far as he ran on Tuesday.
On Thursday, he ran half as far as he ran on Wednesday.
On Friday, he ran twice as far as he ran on Thursday.
If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?

(A) 55 km **(B)** 25 km **(C)** 27.5 km **(D)** 17.5 km **(E)** 50 km

12. The point $(0, 0)$ is reflected in the vertical line $x = 1$. When its image is then reflected in the line $y = 2$, the resulting point is

(A) $(0, 0)$ **(B)** $(2, 0)$ **(C)** $(4, 4)$ **(D)** $(2, 2)$ **(E)** $(2, 4)$

13. In the diagram, $\triangle ABC$ is right-angled at C. Also, points M, N and P are the midpoints of sides BC, AC and AB, respectively. If the area of $\triangle APN$ is 2 cm^2, then the area of $\triangle ABC$ is

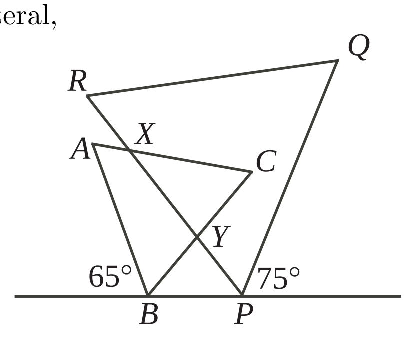


(A) 8 cm^2 **(B)** 16 cm^2 **(C)** 6 cm^2
(D) 4 cm^2 **(E)** 12 cm^2

14. If $\dfrac{3}{x-3} + \dfrac{5}{2x-6} = \dfrac{11}{2}$, then the value of $2x - 6$ is

(A) 2 **(B)** 12 **(C)** 6 **(D)** 8 **(E)** 10

15. In the diagram, if $\triangle ABC$ and $\triangle PQR$ are equilateral, then $\angle CXY$ equals



(A) 30° **(B)** 35° **(C)** 40°
(D) 45° **(E)** 50°

16. At Springfield University, there are 10 000 students, and there are as many male students as female students. Each student is enrolled either in the Arts program or Science program (but not in both); 60% of the students are in the Arts program. Also, 40% of the Science students are male. To the nearest percent, what percentage of the Arts students are female?

(A) 50% **(B)** 52% **(C)** 26% **(D)** 65% **(E)** 43%

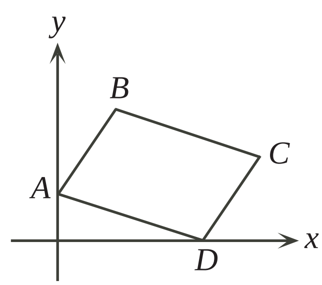
17. On an island there are two types of inhabitants: Heroes who always tell the truth and Villains who always lie. Four inhabitants are seated around a table. When each is asked "Are you a Hero or a Villain?", all four reply "Hero". When asked "Is the person on your right a Hero or a Villain?", all four reply "Villain". How many Heroes are present?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

18. There are a certain number of red balls, green balls and blue balls in a bag. Of the balls in the bag, $\frac{1}{3}$ are red and $\frac{2}{7}$ are blue. The number of green balls in the bag is 8 less than twice the number of blue balls. The number of green balls in the bag is

(A) 12 **(B)** 16 **(C)** 20 **(D)** 24 **(E)** 28

19. In the diagram, the four points have coordinates $A(0,1)$, $B(1,3)$, $C(5,2)$, and $D(4,0)$. What is the area of quadrilateral $ABCD$?



(A) 9 **(B)** 3 **(C)** 6
(D) $\sqrt{85}$ **(E)** $2\sqrt{5}+2\sqrt{17}$

20. What is the largest integer n for which $3(n^{2007}) < 3^{4015}$?

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

Part C: Each correct answer is worth 8.

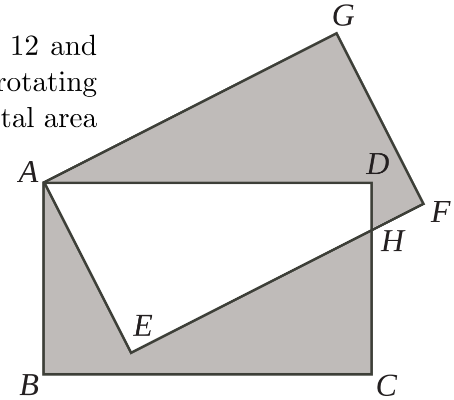
21. In a soccer league with 6 teams (P, Q, R, S, T, W), each team must eventually play each other team exactly once. So far, P has played one match, Q has played 2 matches, R has played 3 matches, S has played 4 matches, and T has played 5 matches. How many matches has W played so far?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. Five positive integers are listed in increasing order. The difference between any two consecutive numbers in the list is three. The fifth number is a multiple of the first number. How many different such lists of five integers are there?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

23. In the diagram, $ABCD$ is rectangle with $AB = 12$ and $BC = 18$. Rectangle $AEFG$ is formed by rotating $ABCD$ about A through an angle of 30°. The total area of the shaded regions is closest to

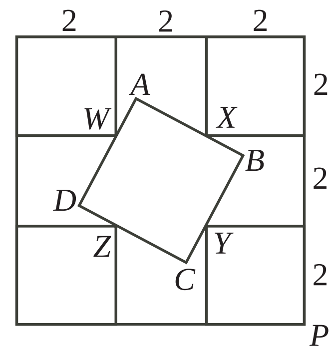


(A) 202.8 **(B)** 203.1 **(C)** 203.4
(D) 203.7 **(E)** 204.0

24. The number 8 is the sum and product of the numbers in the collection of four positive integers $\{1, 1, 2, 4\}$, since $1 + 1 + 2 + 4 = 8$ and $1 \times 1 \times 2 \times 4 = 8$. The number 2007 can be made up from a collection of n positive integers that multiply to 2007 and add to 2007. What is the smallest value of n with $n > 1$?

(A) 1171 **(B)** 1337 **(C)** 1551 **(D)** 1777 **(E)** 1781

25. In the diagram, four squares of side length 2 are placed in the corners of a square of side length 6. Each of the points W, X, Y, and Z is a vertex of one of the small squares. Square $ABCD$ can be constructed with sides passing through W, X, Y, and Z. The maximum possible distance from A to P is closest to



(A) 5.2 **(B)** 5.4 **(C)** 5.6

(D) 5.8 **(E)** 6.0

Canadian Mathematics Competition

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Wednesday, February 22, 2006

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Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2}\right)$ is

   (A) $\frac{3}{8}$ **(B)** 1 **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{3}{4}$

2. The value of $\left(\sqrt{100} - \sqrt{36}\right)^2$ is

   (A) 16 **(B)** 256 **(C)** 8 **(D)** 1024 **(E)** 4096

3. The value of $43 - 41 + 39 - 37 + 35 - 33 + 31 - 29$ is

   (A) 8 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

4. If $a = -3$ and $b = 2$, the value of $a(b - 3)$ is

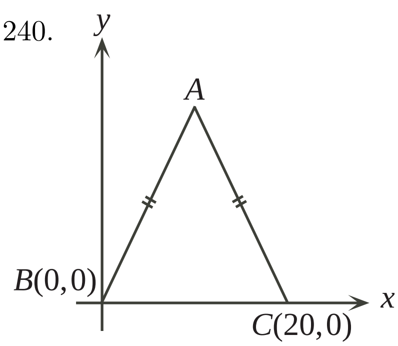
   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. In the four term sequence $0.001,\ 0.02,\ 0.4,\ x$, each term after the first is found by multiplying the previous term by the same number. What is the value of x?

   (A) 0.8 **(B)** 8 **(C)** 80 **(D)** 8.8 **(E)** 0.08

6. In the diagram, $\triangle ABC$ is isosceles and its area is 240. The y-coordinate of A is

   (A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 48

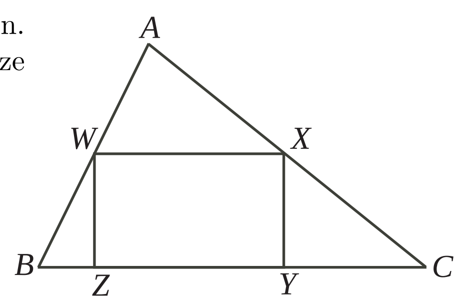


7. If $\dfrac{6}{x+1} = \dfrac{3}{2}$, then x equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. A rectangle is drawn inside $\triangle ABC$, as shown. If $\angle BWZ = 22^\circ$ and $\angle CXY = 65^\circ$, then the size of $\angle BAC$ is

   (A) 87° **(B)** 90° **(C)** 93° **(D)** 104° **(E)** 82°



9. The lengths of the three sides of a triangle are 7, $x + 4$ and $2x + 1$. The perimeter of the triangle is 36. What is the length of the longest side of the triangle?

   (A) 7 **(B)** 12 **(C)** 17 **(D)** 15 **(E)** 16

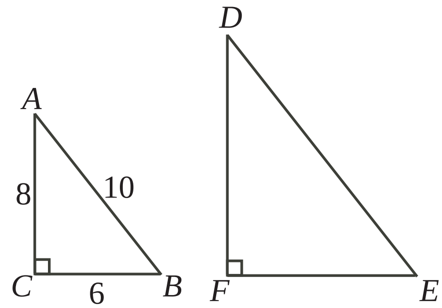
10. A class of 30 students recently wrote a test. If 20 students scored 80, 8 students scored 90, and 2 students scored 100, then the class average on this test was

(A) 90 **(B)** 84 **(C)** 82 **(D)** 86 **(E)** 88

Part B: Each correct answer is worth 6.

11. $\triangle ABC$ has side lengths 6, 8 and 10, as shown. Each of the side lengths of $\triangle ABC$ is increased by 50%, forming a new triangle, $\triangle DEF$. The area of $\triangle DEF$ is

(A) 24 **(B)** 48 **(C)** 108
(D) 12 **(E)** 54



12. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?

(A) 60 **(B)** 80 **(C)** 112 **(D)** 63 **(E)** 48

13. If $x + 1 = y - 8$ and $x = 2y$, then the value of $x + y$ is

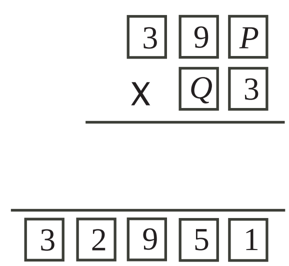
(A) -18 **(B)** 0 **(C)** -9 **(D)** -27 **(E)** -36

14. If $x = -3$, which of the following expressions has the smallest value?

(A) $x^2 - 3$ **(B)** $(x-3)^2$ **(C)** x^2 **(D)** $(x+3)^2$ **(E)** $x^2 + 3$

15. In the multiplication shown, P and Q each represent a single digit, and the product is 32 951. What is the value of $P + Q$?

(A) 14 **(B)** 12 **(C)** 15
(D) 13 **(E)** 11



16. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's *total* cost each year was the same. The cost of downloading the 360 songs in 2005 was

(A) \$144.00 **(B)** \$108.00 **(C)** \$80.00 **(D)** \$259.20 **(E)** \$72.00

17. If w is a positive integer and $w^3 = 9w$, then w^5 is equal to

(A) 59 049 **(B)** 243 **(C)** 1024 **(D)** 3125 **(E)** 32

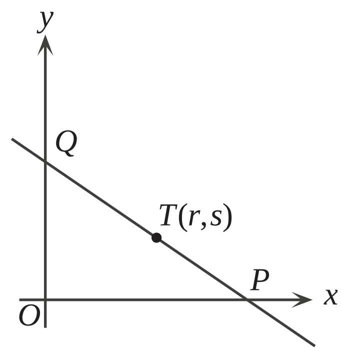
18. In a right-angled triangle, the sum of the squares of the three side lengths is 1800. The length of its hypotenuse is

(A) $\sqrt{1800}$ **(B)** $\frac{1}{2}\sqrt{1800}$ **(C)** 90 **(D)** 30 **(E)** 45

19. In a bin at the Cayley Convenience Store, there are 200 candies. Of these candies, 90% are black and the rest are gold. After Yehudi eats some of the black candies, 80% of the remaining candies in the bin are black. How many black candies did Yehudi eat?

(A) 2 **(B)** 20 **(C)** 40 **(D)** 100 **(E)** 160

20. The line $y = -\frac{3}{4}x + 9$ crosses the x-axis at P and the y-axis at Q. Point $T(r, s)$ is on line segment PQ. If the area of $\triangle POQ$ is three times the area of $\triangle TOP$, then the value of $r + s$ is



(A) 7 **(B)** 10 **(C)** 11
(D) 14 **(E)** 18

Part C: Each correct answer is worth 8.

21. If p, q and r are positive integers and $p + \dfrac{1}{q + \dfrac{1}{r}} = \dfrac{25}{19}$, then q equals

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. A positive integer is called *multiplicatively perfect* if it is equal to the product of its proper divisors. For example, 10 is multiplicatively perfect since its proper divisors are 1, 2 and 5, and it is true that $1 \times 2 \times 5 = 10$. How many multiplicatively perfect integers are there between 2 and 30?

(A) 9 **(B)** 5 **(C)** 8 **(D)** 6 **(E)** 4

23. Quincy and Celine have to move 16 small boxes and 10 large boxes. The chart indicates the time that each person takes to move each type of box. They start moving the boxes at 9:00 a.m. The earliest time at which they can be finished moving all of the boxes is

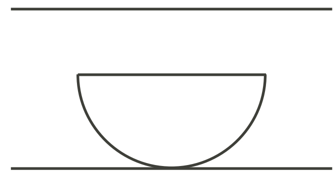
	Celine	Quincy
small box	2 min.	3 min.
large box	6 min.	5 min.

(A) 9:41 a.m. **(B)** 9:42 a.m. **(C)** 9:43 a.m.
(D) 9:44 a.m. **(E)** 9:45 a.m.

24. Anne and Brenda play a game which begins with a pile of n toothpicks. They alternate turns with Anne going first. On each player's turn, she must remove 1, 3 or 4 toothpicks from the pile. The player who removes the last toothpick wins the game. For which of the following values of n does Brenda have a winning strategy? (In a game, a player has a winning strategy if, regardless of what the other player does, there are moves that she can make which *guarantee* that she will win.)

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

25. A semi-circle of radius 8 cm, rocks back and forth along a line. The distance between the line on which the semi-circle sits and the line above is 12 cm. As it rocks without slipping, the semi-circle touches the line above at two points. (When the semi-circle hits the line above, it immediately rocks back in the other direction.) The distance between these two points, in millimetres, is closest to



(A) 55 **(B)** 53 **(C)** 51

(D) 49 **(E)** 47

Canadian Mathematics Competition

For students...

Thank you for writing the 2006 Cayley Contest!
In 2005, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for Galois Contest which will be written on April 20, 2006.
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- **Learn about workshops and resources we offer for teachers**
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Canadian Mathematics Competition

An activity of the Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 23, 2005

Time: 60 minutes

Calculators are permitted.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $a + 1 + a - 2 + a + 3 + a - 4$ is equal to

   (A) $10a$ **(B)** 0 **(C)** $4a - 2$ **(D)** $4a + 2$ **(E)** $-2a$

2. The value of $(\frac{4}{5})(\frac{5}{6})(\frac{6}{7})(\frac{7}{8})(\frac{8}{9})$ is

   (A) $\frac{4}{9}$ **(B)** 1 **(C)** $\frac{6}{7}$ **(D)** 36 **(E)** $\frac{36}{25}$

3. When 45 is divided by 7, the remainder is 3. What is the remainder when 70 is divided by 17?

   (A) 1 **(B)** 12 **(C)** 15 **(D)** 2 **(E)** 11

4. If $\dfrac{3}{x+10} = \dfrac{1}{2x}$, then x equals

   (A) $\frac{1}{2}$ **(B)** 10 **(C)** -4 **(D)** 2 **(E)** -8

5. A teacher writes five different possible values for $(5^2 - 4^2)^3$ on the board and asks her class to decide which is correct. The correct value is

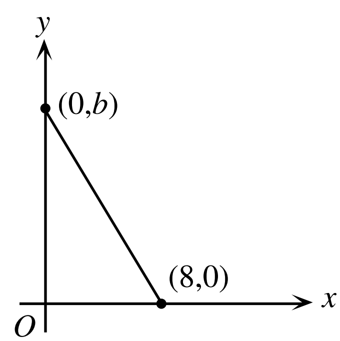
   (A) 1 **(B)** 8 **(C)** $11\,529$ **(D)** 216 **(E)** 729

6. Last week, a charity fundraiser had 8 volunteers who each worked 40 hours and who each raised \$18 per hour. This week, 12 volunteers, each working 32 hours, raised the same total amount of money. How much did each volunteer raise per hour this week?

   (A) \$9 **(B)** \$12 **(C)** \$15 **(D)** \$21 **(E)** \$24

7. In the diagram, the line segment has slope $-\frac{3}{2}$. The value of b is

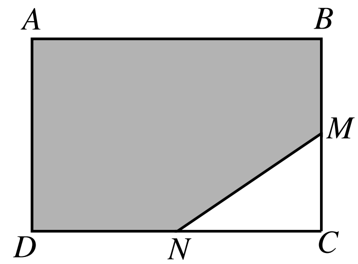
   (A) 10 **(B)** 12 **(C)** 6
   (D) 16 **(E)** 20



8. Jack went running last Saturday morning. He ran the first 12 km at 12 km/h and the second 12 km at 6 km/h. Jill ran the same route at a constant speed, and took the same length of time as Jack. Jill's speed in km/h was

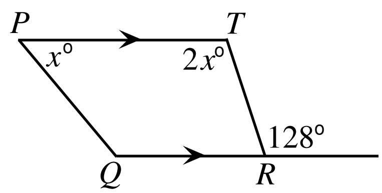
   (A) 8 **(B)** 9 **(C)** 6 **(D)** 12 **(E)** 24

9. $ABCD$ is a rectangle, with M the midpoint of BC and N the midpoint of CD. If $CM = 4$ and $NC = 5$, what percent of the area of the rectangle is shaded?



(A) 70 **(B)** 78 **(C)** 80 **(D)** 87.5 **(E)** 75

10. In the diagram, PT is parallel to QR. What is the size of $\angle PQR$?



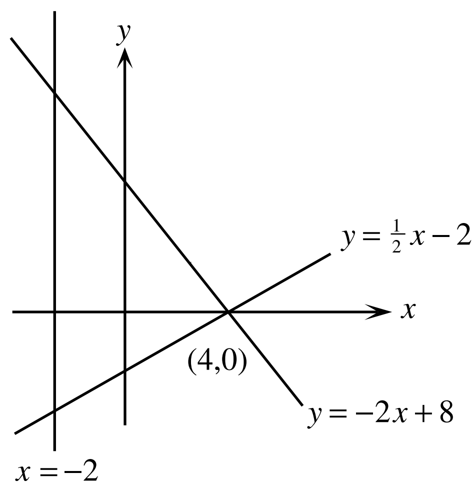
(A) 116° **(B)** 168° **(C)** 138° **(D)** 144° **(E)** 122°

Part B: Each correct answer is worth 6.

11. During a football game, Matt kicked the ball three times. His longest kick was 43 metres and the three kicks averaged 37 metres. If the other two kicks were the same length, the distance, in metres, that each travelled was

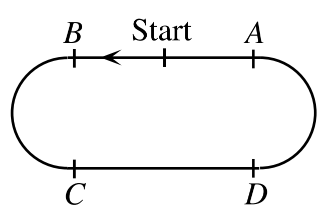
(A) 31 **(B)** 37 **(C)** 35 **(D)** 34 **(E)** 36

12. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. The area of the triangle formed by these two lines and the line $x = -2$ is



(A) 15 **(B)** 27 **(C)** 30 **(D)** 36 **(E)** 45

13. A 400 m track is constructed so that the points A, B, C, and D divide the track into four segments of equal length. The Start is half-way between A and B. Andrew begins at the Start and walks at a steady rate of 1.4 m/s in a counter-clockwise direction. After exactly 30 minutes, to what point will Andrew be closest?

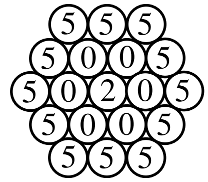


(A) A **(B)** B **(C)** C **(D)** D **(E)** Start

14. If x is a *positive* integer less than 100, how many values of x make $\sqrt{1 + 2 + 3 + 4 + x}$ an integer?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

15. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?



(A) 36 **(B)** 24 **(C)** 12 **(D)** 18 **(E)** 6

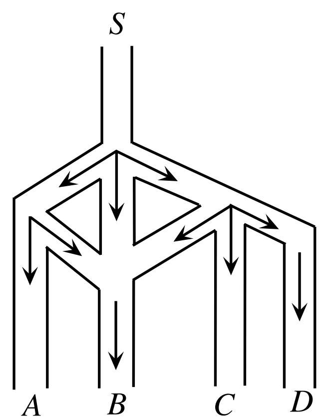
16. The *non-negative difference* between two numbers a and b is $a - b$ or $b - a$, whichever is greater than or equal to 0. For example, the non-negative difference between 24 and 64 is 40. In the sequence 88, 24, 64, 40, 24, $\ldots$, each number after the second is obtained by finding the non-negative difference between the previous 2 numbers. The sum of the first 100 numbers in this sequence is

(A) 496 **(B)** 760 **(C)** 752 **(D)** 776 **(E)** 405

17. 10^{100} is a googol. 1000^{100} equals

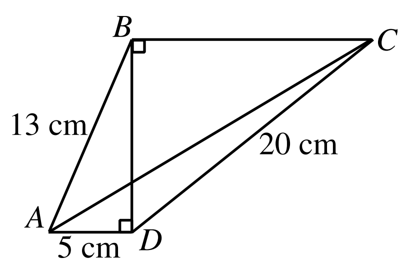
(A) 100 googol **(B)** 3 googol **(C)** googol$^{\text{googol}}$ **(D)** googol^2 **(E)** googol^3

18. Harry the Hamster is put in a maze, and he starts at point S. The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at B?



(A) $\frac{2}{3}$ **(B)** $\frac{13}{18}$ **(C)** $\frac{11}{18}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{4}$

19. In the diagram, $AB = 13$ cm, $DC = 20$ cm, and $AD = 5$ cm. The length of AC, to the nearest tenth of a centimetre, is



(A) 24.2 **(B)** 20.6 **(C)** 25.2 **(D)** 23.4 **(E)** 24.9

20. There are 81 cars in the CMC parking lot, which are all Acuras, Beetles, or Camrys. There are half as many Acuras as Beetles. The number of Camrys is 80% of the number of Acuras and Beetles together. How many of the 81 cars are Beetles?

(A) 36 **(B)** 30 **(C)** 45 **(D)** 51 **(E)** 66

Part C: Each correct answer is worth 8.

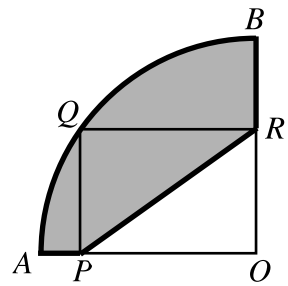
21. In Yacleyland, the unit of money used is called the Yacley. There are only two denominations of paper money: the 17 Yacley bill and the 5 Yacley bill. How many different combinations of these bills total 453 Yacley?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

22. In the diagram, AOB is a quarter circle of radius 10 and $PQRO$ is a rectangle of perimeter 26. The perimeter of the shaded region is

(A) $7 + 5\pi$ **(B)** $13 + 5\pi$ **(C)** $17 + 5\pi$

(D) $7 + 25\pi$ **(E)** $17 + 25\pi$



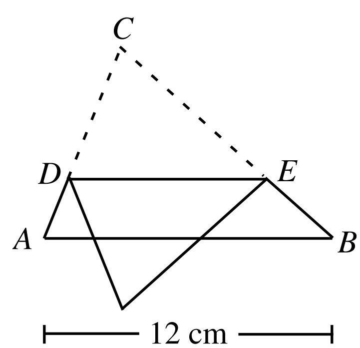
23. At 12:00 noon, Anna and Bill left home and walked in the same direction. Anna walked at 4 km/h and Bill walked at 3 km/h. At 12:15 their dog Dexter, running at 6 km/h, left home to run after them. The dog ran until it caught up to Anna, then it ran back to Bill. (In his excitement, Dexter lost no time in turning around once he reached Anna.) At what time did Bill meet Dexter on Dexter's way back?

(A) 1:00 p.m. **(B)** 1:15 p.m. **(C)** 12:45 p.m. **(D)** 1:05 p.m. **(E)** 12:50 p.m.

24. The base of a triangular piece of paper ABC is 12 cm long. The paper is folded down over the base, with the crease DE parallel to the base of the paper. The area of the triangle that projects below the base is 16% that of the area of the triangle ABC. The length of DE, in cm, is

(A) 9.6 **(B)** 8.4 **(C)** 7.2

(D) 4.8 **(E)** 6.96



25. The positive integers a, b and c satisfy $\dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{c^2}$.

The sum of all possible values of $a \leq 100$ is

(A) 315 **(B)** 615 **(C)** 680 **(D)** 555 **(E)** 620

2005 Cayley Contest (English)

Canadian Mathematics Competition

For students...

Thank you for writing the 2005 Cayley Contest!
In 2004, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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- **Register your students for the Fryer, Galois and Hypatia Contests which will be written on April 20, 2005**
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Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 18, 2004

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Time: 1 hour

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

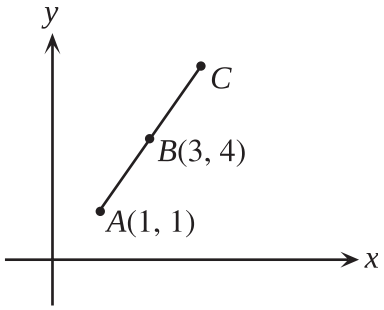
1. The value of $2^2+1^2+0^2+(-1)^2+(-2)^2$ is

(**A**) 5 (**B**) -6 (**C**) 3 (**D**) 6 (**E**) 10

2. 25% of 2004 is equal to

(**A**) 50% of 4008 (**B**) 50% of 1002 (**C**) 100% of 1002
(**D**) 10% of 8016 (**E**) 20% of 3006

3. Point $B(3,4)$ is the midpoint of the line segment joining the points $A(1,1)$ and C. The coordinates of C are



(**A**) $(2,3)$ (**B**) $(2,2)$ (**C**) $(4,6)$
(**D**) $(5,8)$ (**E**) $(5,7)$

4. If $x+1-2+3-4=5-6+7-8$, the value of x is

(**A**) -2 (**B**) -1 (**C**) 0 (**D**) 1 (**E**) 2

5. In the sequence, each figure is made up of small squares of side length 1. What is the outer perimeter of the fifth figure in the sequence?

(**A**) 9 (**B**) 18 (**C**) 20 (**D**) 24 (**E**) 36

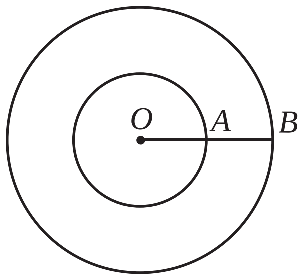
6. If $x+6y=17$, the value of $7x+42y$ is

(**A**) 24 (**B**) 42 (**C**) 49 (**D**) 102 (**E**) 119

7. If $3^2+3^2+3^2=3^a$, the value of a is

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) 8

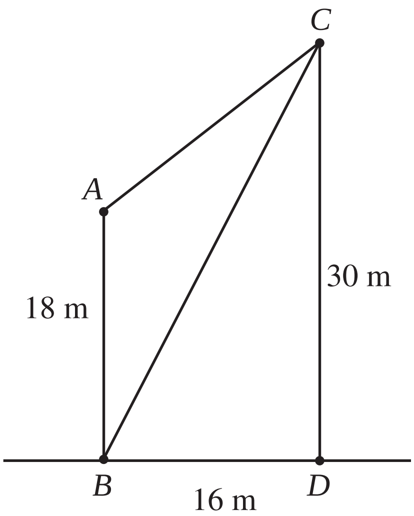
8. In the diagram, O is the centre of each circle. The circumferences of the circles are 24π and 14π. B is a point on the outer circle and OB intersects the inner circle at A. The length of AB is



(**A**) $\sqrt{10}$ (**B**) 5 (**C**) 7
(**D**) 10π (**E**) 3

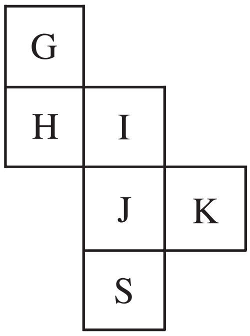
9. Two vertical towers, AB and CD, are located 16 m apart on flat ground, as shown. Tower AB is 18 m tall and tower CD is 30 m tall. Ropes are tied from A to C and from B to C. Assuming the ropes are taut, the total length of rope, in m, is

(**A**) 54 (**B**) 64 (**C**) 44
(**D**) 48 (**E**) 59



10. If the figure shown is folded to make a cube, what letter is opposite G?

(**A**) S (**B**) H (**C**) I
(**D**) J (**E**) K



Part B: Each correct answer is worth 6.

11. In the sequence of five numbers x, ______, 3, ______, 18, each number after the second is obtained by multiplying the two previous numbers. The value of x is

(**A**) $\frac{2}{3}$ (**B**) $\frac{3}{2}$ (**C**) 1 (**D**) -9 (**E**) -1

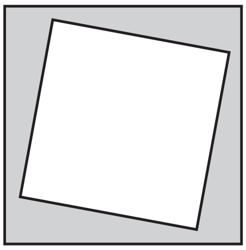
12. In the magic square, the sum of the three numbers in any row, column or diagonal is the same. The sum of the three numbers in any row is

(**A**) 0 (**B**) 1 (**C**) 3
(**D**) 7 (**E**) 9

$2x$	3	2
		-3
0	x	

13. In the diagram, a smaller square lies inside a larger square. The perimeter of the smaller square is 72 cm. The shaded area is 160 cm^2. The perimeter of the larger square, in cm, is

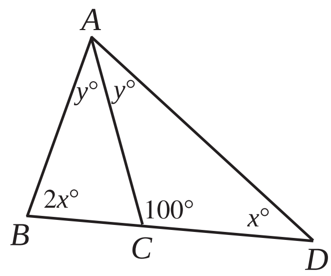
(**A**) 58 (**B**) 88 (**C**) 116
(**D**) 121 (**E**) 112



14. If x and y are positive numbers and the average of 4, 20 and x is equal to the average of y and 16, then the ratio $x:y$ is

(**A**) 3:2 (**B**) 2:3 (**C**) 1:1 (**D**) 2:5 (**E**) 5:2

15. In the diagram, B, C and D lie on a straight line, with $\angle ACD = 100^\circ$, $\angle ADB = x^\circ$, $\angle ABD = 2x^\circ$, and $\angle DAC = \angle BAC = y^\circ$. The value of x is



(**A**) 10 (**B**) 45 (**C**) 30
(**D**) 50 (**E**) 20

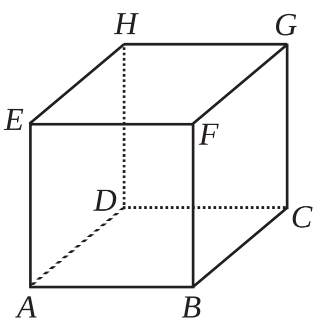
16. In a dice game, a player rolls two dice. His score is the larger of the two numbers on the dice. For example, if he rolls 3 and 5, his score is 5, and if he rolls 4 and 4, his score is 4. What is the probability that his score is 3 or less?

(**A**) $\frac{1}{4}$ (**B**) $\frac{7}{36}$ (**C**) $\frac{5}{36}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{2}{9}$

17. The two whole numbers m and n satisfy $m+n=20$ and $\frac{1}{m}+\frac{1}{n}=\frac{5}{24}$. The product mn is equal to

(**A**) 72 (**B**) 36 (**C**) 48 (**D**) 96 (**E**) 24

18. In the diagram, $ABCDEFGH$ is a cube with an edge length of 12 cm. An ant sits on the cube at vertex A. The ant can only walk along the edges of the cube, and cannot walk along any edge more than once. What is the greatest distance that the ant can walk before it cannot continue?

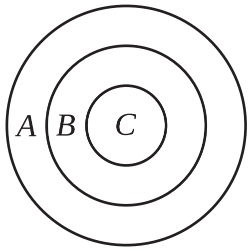


(**A**) 96 cm (**B**) 144 cm (**C**) 84 cm
(**D**) 108 cm (**E**) 132 cm

19. $\frac{1}{2}+\frac{2^1}{2^2}+\frac{2^2}{2^3}+\cdots+\frac{2^{2002}}{2^{2003}}+\frac{2^{2003}}{2^{2004}}$ is equal to

(**A**) 1002 (**B**) 501 (**C**) $\frac{1}{2^{2004}}$ (**D**) 2004 (**E**) $\frac{2004}{2^{2004}}$

20. An archery target has 3 regions, each worth a different value if it is hit. Three archers shoot two arrows each and record scores as follows:



First archer: 1 arrow in C and 1 arrow in A for a score of 15 points
Second archer: 1 arrow in C and 1 arrow in B for a score of 18 points
Third archer: 1 arrow in B and 1 arrow in A for a score of 13 points

If a fourth archer shoots 2 arrows into ring B, her score is

(**A**) 10 (**B**) 14 (**C**) 16 (**D**) 18 (**E**) 20

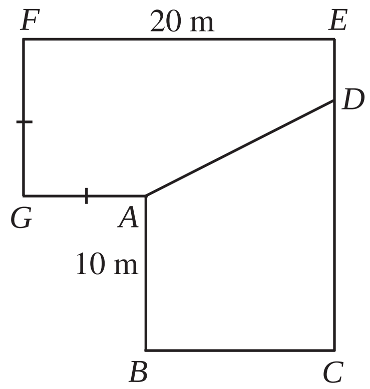
continued ...

Part C: Each correct answer is worth 8.

21. In a pack of construction paper, the numbers of blue and red sheets are originally in the ratio 2:7. Each day, Laura uses 1 blue sheet and 3 red sheets. One day, she uses 3 red sheets and the last blue sheet, leaving her with 15 red sheets. How many sheets of construction paper were in the pack originally?

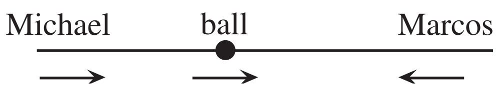
(**A**) 144 (**B**) 252 (**C**) 135 (**D**) 270 (**E**) 105

22. In the diagram, *ABCDEFG* is a room having square corners, with $EF = 20$ m, $AB = 10$ m, and $AG = GF$. The total area of the room is 280 m^2. A wall is built from *A* to *D* creating two rooms of equal area. What is the distance, in metres, from *C* to *D*?



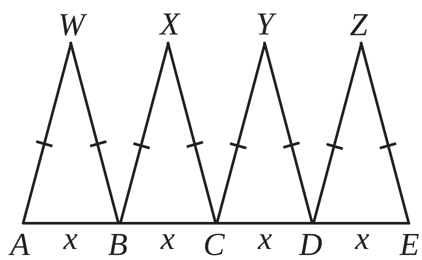
(**A**) 15 (**B**) $\frac{50}{3}$ (**C**) 12
(**D**) 13 (**E**) $\frac{40}{3}$

23. A soccer ball rolls at 4 m/s towards Marcos in a direct line from Michael. The ball is 15 m ahead of Michael who is chasing it at 9 m/s. Marcos is 30 m away from the ball and is running towards it at 8 m/s. The distance between Michael and Marcos when the ball is touched for the first time by one of them is closest to



(**A**) 2.00 m (**B**) 2.25 m (**C**) 2.50 m
(**D**) 2.75 m (**E**) 3.00 m

24. Four identical isosceles triangles *AWB*, *BXC*, *CYD*, and *DZE* are arranged, as shown, with points *A*, *B*, *C*, *D*, and *E* lying on the same straight line. A new triangle is formed with sides the same lengths as *AX*, *AY* and *AZ*. If $AZ = AE$, the largest integer value of *x* such that the area of this new triangle is less than 2004 is



(**A**) 18 (**B**) 19 (**C**) 20
(**D**) 21 (**E**) 22

25. The number of positive integers *x* with $x \leq 60$ such that each of the rational expressions

$$\frac{7x+1}{2}, \frac{7x+2}{3}, \frac{7x+3}{4}, \ldots, \frac{7x+300}{301}$$

is in lowest terms (i.e. in each expression, the numerator and denominator have no common factors) is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

PUBLICATIONS

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Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 19, 2003

C.M.C. Sponsors:

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C.M.C. Contributors:

Manulife Financial

Time: 1 hour

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{3-(-3)}{2-1}$ is

   (**A**) 2 (**B**) 0 (**C**) 3 (**D**) 6 (**E**) -3

2. $17^2 - 15^2$ equals

   (**A**) 8^2 (**B**) 2^2 (**C**) 4^2 (**D**) 7^2 (**E**) 6^2

3. The integer 42 is

   (**A**) an odd number (**B**) a prime number (**C**) a perfect cube
   (**D**) divisible by 7 (**E**) a perfect square

4. If 5% of a number is 8, what is 25% of the same number?

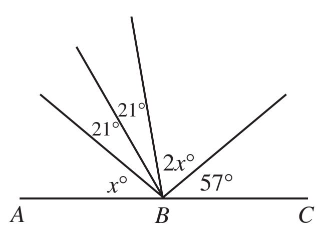
   (**A**) 40 (**B**) 0.1 (**C**) 320 (**D**) 10 (**E**) 200

5. The integer closest to the value of $\frac{3}{2} \times \frac{4}{9} + \frac{7}{2}$ is

   (**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

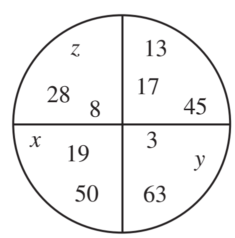
6. In the diagram, *ABC* is a straight line. The value of x is

   (**A**) 27 (**B**) 33 (**C**) 24
   (**D**) 87 (**E**) 81



7. In the diagram, the sum of the numbers in each quarter circle is the same. The value of $x + y + z$ is

   (**A**) 75 (**B**) 64 (**C**) 54
   (**D**) 171 (**E**) 300



8. An equilateral triangle has a side length of 20. If a square has the same perimeter as this triangle, the area of the square is

   (**A**) 25 (**B**) 400 (**C**) 225 (**D**) 60 (**E**) 100

9. If $\dfrac{1}{x+\frac{1}{5}}=\dfrac{5}{3}$, then x equals

(**A**) $\frac{2}{5}$ (**B**) $\frac{4}{5}$ (**C**) $\frac{1}{5}$ (**D**) $-\frac{2}{5}$ (**E**) $-\frac{22}{5}$

10. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that $\frac{5}{8}$ of the players are girls?

(**A**) 6 (**B**) 3 (**C**) 5 (**D**) 8 (**E**) 7

Part B: Each correct answer is worth 6.

11. Let $N=10^3+10^4+10^5+10^6+10^7+10^8+10^9$. The sum of the digits of N is

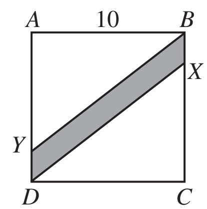
(**A**) 12 (**B**) 1 (**C**) 6 (**D**) 9 (**E**) 7

12. The points $A(a,1)$, $B(9,0)$ and $C(-3,4)$ lie on a straight line. The value of a is

(**A**) 3 (**B**) $\frac{8}{3}$ (**C**) $\frac{7}{2}$ (**D**) 6 (**E**) $\frac{5}{2}$

13. In the diagram, $ABCD$ is a square with a side length of 10. If $AY=CX=8$, the area of the shaded region is

(**A**) 16 (**B**) 20 (**C**) 40
(**D**) 48 (**E**) 24

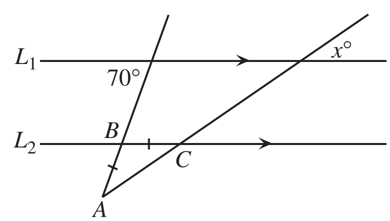


14. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

(**A**) 16 (**B**) 9 (**C**) 36 (**D**) 12 (**E**) 18

15. In the diagram, line L_1 is parallel to line L_2 and $BA=BC$. The value of x is

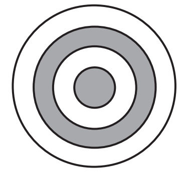
(**A**) 35 (**B**) 30 (**C**) 37.5
(**D**) 45 (**E**) 40



16. The value of $\dfrac{(4^{2003})(3^{2002})}{(6^{2002})(2^{2003})}$ is

(**A**) 1 (**B**) 2 (**C**) 12 (**D**) 4 (**E**) $\frac{1}{2}$

17. In the diagram, the four circles have a common centre, and have radii of 1, 2, 3, and 4. The ratio of the area of the shaded regions to the area of the largest circle is



(**A**) 5 : 8 (**B**) 1 : 4 (**C**) 7 : 16
(**D**) 1 : 2 (**E**) 3 : 8

18. If $496 = 2^m - 2^n$, where m and n are integers, then $m + n$ is equal to

(**A**) 13 (**B**) 9 (**C**) 4 (**D**) 14 (**E**) 5

19. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is

(**A**) 18 (**B**) 19 (**C**) 23 (**D**) 25 (**E**) 22

20. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car's engine at a cost of $400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs $0.80 per litre, this distance, in kilometres, is between

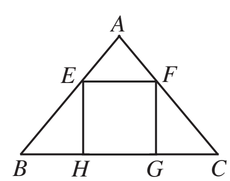
(**A**) 10 000 and 14 000 (**B**) 14 000 and 18 000 (**C**) 18 000 and 22 000
(**D**) 22 000 and 26 000 (**E**) 26 000 and 30 000

Part C: Each correct answer is worth 8.

21. Troye and Daniella are running at constant speeds in opposite directions around a circular track. Troye completes one lap every 56 seconds and meets Daniella every 24 seconds. How many seconds does it take Daniella to complete one lap?

(**A**) 32 (**B**) 36 (**C**) 40 (**D**) 48 (**E**) 42

22. In the diagram, ΔABC is isosceles with $AB = AC$ and $BC = 30$ cm. Square $EFGH$, which has a side length of 12 cm, is inscribed in ΔABC, as shown. The area of ΔAEF, in cm^2, is



(**A**) 27 (**B**) 54 (**C**) 51
(**D**) 48 (**E**) 60

23. A pyramid has a square base which has an area of 1440 cm^2. Each of the pyramid's triangular faces is identical and each has an area of 840 cm^2. The height of the pyramid, in cm, is

(**A**) $30\sqrt{2}$ (**B**) 40 (**C**) $20\sqrt{6}$ (**D**) $20\sqrt{3}$ (**E**) 30

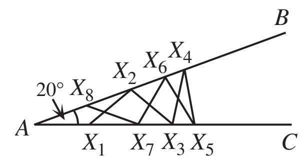
24. In how many ways can a, b, c, and d be chosen from the set $\{0, 1, 2, ..., 9\}$ so that $a < b < c < d$ and $a + b + c + d$ is a multiple of three?

(**A**) 54 (**B**) 64 (**C**) 63 (**D**) 90 (**E**) 72

continued ...

25. $\angle BAC$ is said to be "laceable" if distinct points $X_1, X_2, \ldots, X_{2n}$ can be found so that

- X_{2k-1} is on AC for each value of k,
- X_{2k} is on AB for each value of k, and
- $AX_1 = X_1X_2 = X_2X_3 = \cdots = X_{2n-1}X_{2n} = X_{2n}A$.



For example, the angle 20° is laceable, as shown. The number of laceable acute angles, whose sizes in degrees are integers, is

(**A**) 3 (**B**) 4 (**C**) 5
(**D**) 6 (**E**) 7

❍ ❍ ❍ ❍ ❍

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Cayley Contest (Grade 10)

Wednesday, February 20, 2002

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C.M.C. Supporters: Canadian Institute of Actuaries; Great West Life and London Life; Sybase Inc. (Waterloo); *i*Anywhere Solutions

C.M.C. Contributors: Manulife Financial; Equitable Life of Canada

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

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2. You may use rulers, compasses and paper for rough work.
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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. $5x+2(4+x)$ is equal to

   (**A**) $5x+8$ (**B**) $3x+6$ (**C**) $7x+8$ (**D**) $7x+6$ (**E**) $6x+8$

2. The value of $(2+3)^2-\left(2^2+3^2\right)$ is

   (**A**) 12 (**B**) 0 (**C**) 30 (**D**) 16 (**E**) -3

3. If $x=-3$, the numerical value of $x^2-4(x-5)$ is

   (**A**) 40 (**B**) 38 (**C**) -23 (**D**) 41 (**E**) -26

4. If n is $\frac{5}{6}$ of 240, then $\frac{2}{5}$ of n is

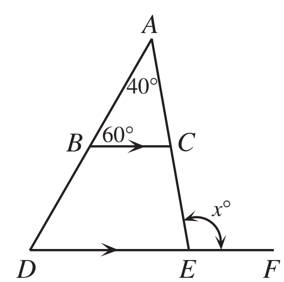
   (**A**) 288 (**B**) 80 (**C**) 96 (**D**) 200 (**E**) 500

5. The numerical value of $2^{-2}\times 2^{-1}\times 2^{0}\times 2^{1}\times 2^{2}$ is

   (**A**) 4 (**B**) 1 (**C**) 0 (**D**) $\frac{1}{4}$ (**E**) $\frac{1}{2}$

6. In the diagram, the value of x is

   (**A**) 130 (**B**) 120 (**C**) 110
   (**D**) 100 (**E**) 80



7. If the point $(-2,4)$ is on a line with slope $\frac{1}{2}$, then the y-intercept of this line is

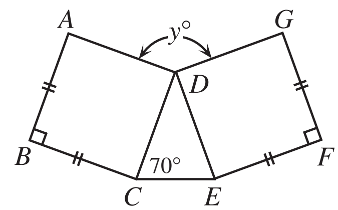
   (**A**) 5 (**B**) -4 (**C**) 3 (**D**) 0 (**E**) 8

8. After having played three basketball games, Megan had scored an average of 18 points per game. After her fourth game, her scoring average had dropped to 17 points per game. How many points did Megan score in her fourth game?

   (**A**) 18 (**B**) 17 (**C**) 16 (**D**) 15 (**E**) 14

9. In the diagram, $ABCD$ and $DEFG$ are squares with equal side lengths, and $\angle DCE = 70°$. The value of y is

(**A**) 120 (**B**) 160 (**C**) 130
(**D**) 110 (**E**) 140



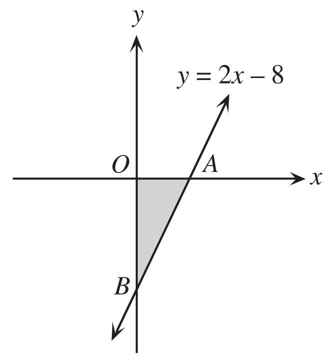
10. Faruq subtracted 5 from a number and then divided by 4. Next, he subtracted 4 from the original number and then divided by 5. He got the same final answer both times. The original number was

(**A**) 4 (**B**) 15 (**C**) 9 (**D**) 20 (**E**) -9

Part B: Each correct answer is worth 6.

11. In the diagram, the line with equation $y = 2x - 8$ crosses the x-axis at A and the y-axis at B. The area of ΔAOB is

(**A**) 8 (**B**) 16 (**C**) 12
(**D**) 32 (**E**) 4

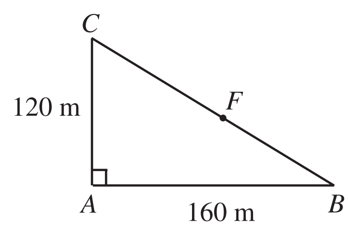


12. A compact disc originally sells for \$10.00. If the price of the compact disc is increased by 40% and this new price is later decreased by 30%, what is the final price?

(**A**) \$9.80 (**B**) \$17.00 (**C**) \$9.00 (**D**) \$19.80 (**E**) \$9.60

13. In the diagram, ABC represents a triangular jogging path. Jack jogs along the path from A to B to F. Jill jogs from A to C to F. Each jogs the same distance. The distance from F to B, in metres, is

(**A**) 40 (**B**) 120 (**C**) 100
(**D**) 80 (**E**) 200

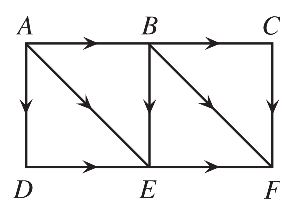


14. If $a(c+d)+b(c+d) = 42$ and $c+d = 3$, what is the value of $a+b+c+d$?

(**A**) 14 (**B**) 56 (**C**) 3 (**D**) 17 (**E**) 39

15. In the grid shown, it is only possible to travel along an edge in the direction indicated by the arrow. The number of different paths from A to F is

(**A**) 9 (**B**) 5 (**C**) 3
(**D**) 6 (**E**) 4



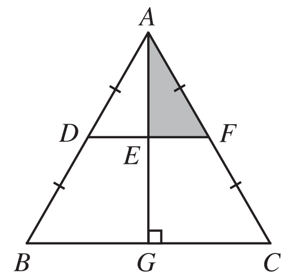
16. If the product of four consecutive positive integers is 358 800, then the sum of these four integers is

(**A**) 102 (**B**) 98 (**C**) 94 (**D**) 90 (**E**) 106

17. A "double-single" number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

(**A**) 81 (**B**) 18 (**C**) 72 (**D**) 64 (**E**) 90

18. In the diagram, triangle ABC is isosceles with $AB = AC$, and AG is perpendicular to BC. Point D is the midpoint of AB, point F is the midpoint of AC, and E is the point of intersection of DF and AG. What fraction of the area of ΔABC does the shaded area represent?



(**A**) $\frac{1}{12}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{4}$

(**D**) $\frac{1}{10}$ (**E**) $\frac{1}{8}$

19. The sum of the digits of the integer equal to $777\,777\,777\,777\,777^2 - 222\,222\,222\,222\,223^2$ is

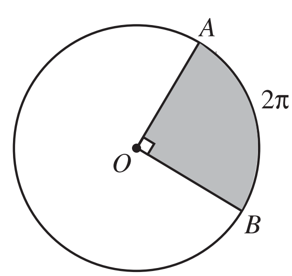
(**A**) 148 (**B**) 84 (**C**) 74 (**D**) 69 (**E**) 79

20. Two cylindrical tanks sit side by side on a level surface. The first tank has a radius of 4 metres, a height of 10 metres, and is full of water. The second tank has a radius of 6 metres, a height of 8 metres, and is empty. Water is pumped from the first tank to the second until the depth of water in both tanks is the same. The depth of water in each tank, in metres, is

(**A**) 4 (**B**) 5 (**C**) $\frac{46}{15}$ (**D**) $\frac{52}{17}$ (**E**) $\frac{40}{13}$

Part C: Each correct answer is worth 8.

21. In the diagram, the circle has centre O. The shaded sector AOB has sector angle 90°, and AB has arc length 2π units. The area of sector AOB is

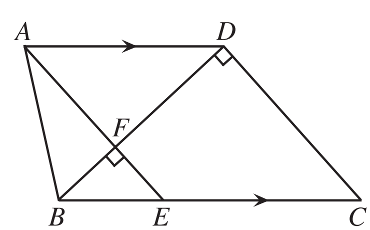


(**A**) 4π (**B**) 16π (**C**) 6π
(**D**) 24π (**E**) 8π

22. In how many ways can 75 be expressed as the sum of two or more consecutive positive integers?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

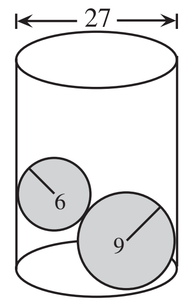
23. In trapezoid $ABCD$, AD is parallel to BC. Also, BD is perpendicular to DC. The point F is chosen on line BD so that AF is perpendicular to BD. AF is extended to meet BC at point E. If $AB = 41$, $AD = 50$ and $BF = 9$, what is the area of quadrilateral $FECD$?



(**A**) 900 (**B**) 1523.5 (**C**) 960
(**D**) 1560 (**E**) 1300

continued ...

24. A cylinder, which has a diameter of 27 and a height of 30, contains two lead spheres with radii 6 and 9, with the larger sphere sitting on the bottom of the cylinder, as shown. Water is poured into the cylinder so that it just covers both spheres. The volume of water required is



(**A**) 3672π (**B**) 3660π (**C**) 3375π
(**D**) 3114π (**E**) 4374π

25. A *lattice point* is a point (x, y) where both x and y are integers For how many different integer values of k will the two lines $kx - 5y + 7 = 0$ and $k^2x - 5y + 1 = 0$ intersect at a lattice point?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 21, 2001

C.M.C. Sponsors:

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Great West Life and London Life

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Equitable Life of Canada

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 20.

Part A: Each correct answer is worth 5.

1. The value of $\frac{5(6)-3(4)}{6+3}$ is

   (**A**) 1 (**B**) 2 (**C**) 6 (**D**) 12 (**E**) 31

2. When $\frac{1}{4}$ of 15 is multiplied by $\frac{1}{3}$ of 10, the answer is

   (**A**) 5 (**B**) $\frac{25}{2}$ (**C**) $\frac{85}{12}$ (**D**) $\frac{99}{8}$ (**E**) $\frac{25}{7}$

3. If $x=\frac{1}{4}$, which of the following has the largest value?

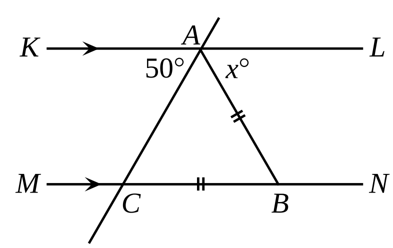
   (**A**) x (**B**) x^2 (**C**) $\frac{1}{2}x$ (**D**) $\frac{1}{x}$ (**E**) $\sqrt{x}$

4. In a school, 30 boys and 20 girls entered the Cayley competition. Certificates were awarded to 10% of the boys and 20% of the girls. Of the students who participated, the percentage that received certificates was

   (**A**) 14 (**B**) 15 (**C**) 16 (**D**) 30 (**E**) 50

5. In the diagram, KL is parallel to MN, $AB=BC$, and $\angle KAC=50°$. The value of x is

   (**A**) 40 (**B**) 65 (**C**) 25
   (**D**) 100 (**E**) 80

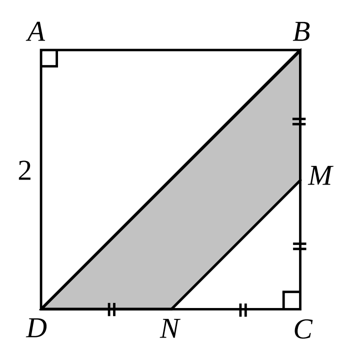


6. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

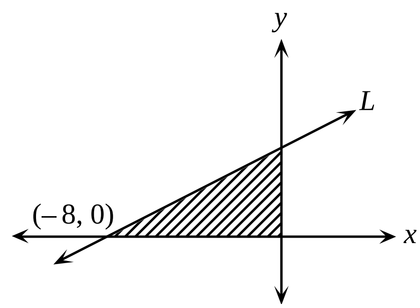
   (**A**) 153 (**B**) 171 (**C**) 180 (**D**) 266 (**E**) 144

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD. The area of the shaded region $BMND$ is

   (**A**) 1 (**B**) $2\sqrt{2}$ (**C**) $\frac{4}{3}$
   (**D**) $\frac{3}{2}$ (**E**) $4-\frac{3}{2}\sqrt{2}$



8. The line L crosses the x-axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line L?



(A) $\frac{1}{2}$ **(B)** 4 **(C)** $-\frac{1}{2}$

(D) 2 **(E)** -2

9. If $\left[\left(10^3\right)\left(10^x\right)\right]^2 = 10^{18}$, the value of x is

(A) $\sqrt{2}$ **(B)** 12 **(C)** 6 **(D)** 1 **(E)** 3

10. The sum of five consecutive integers is 75. The sum of the largest and smallest of these five integers is

(A) 15 **(B)** 25 **(C)** 26 **(D)** 30 **(E)** 32

Part B: Each correct answer is worth 6.

11. When a positive integer N is divided by 60, the remainder is 49. When N is divided by 15, the remainder is

(A) 0 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 8

12. The 6 members of an executive committee want to call a meeting. Each of them phones 6 different people, who in turn each calls 6 other people. If no one is called more than once, how many people will know about the meeting?

(A) 18 **(B)** 36 **(C)** 216 **(D)** 252 **(E)** 258

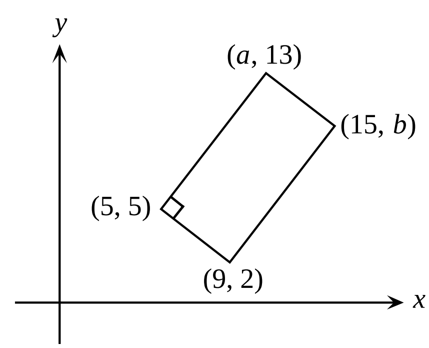
13. The sequences 3, 20, 37, 54, 71, … and 16, 27, 38, 49, 60, 71, … each have 71 as a common term. The next term that these sequences have in common is

(A) 115 **(B)** 187 **(C)** 258 **(D)** 445 **(E)** 1006

14. In the rectangle shown, the value of $a - b$ is

(A) -3 **(B)** -1 **(C)** 0

(D) 3 **(E)** 1



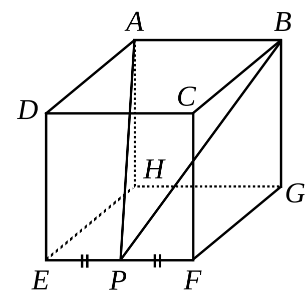
15. A small island has $\frac{2}{5}$ of its surface covered by forest and $\frac{1}{4}$ of the remainder of its surface by sand dunes. The island also has 90 hectares covered by farm land. If the island is made up of only forest, sand dunes and farm land, what is the total area of the island, to the nearest hectare?

(A) 163 **(B)** 120 **(C)** 200 **(D)** 138 **(E)** 257

16. How many integer values of x satisfy $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

17. $ABCDEFGH$ is a cube having a side length of 2. P is the midpoint of EF, as shown. The area of ΔAPB is



(**A**) $\sqrt{8}$ (**B**) 3 (**C**) $\sqrt{32}$
(**D**) $\sqrt{2}$ (**E**) 6

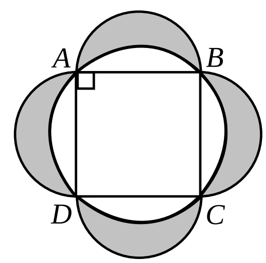
18. How many five-digit positive integers, divisible by 9, can be written using only the digits 3 and 6?

(**A**) 5 (**B**) 2 (**C**) 12 (**D**) 10 (**E**) 8

19. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is

(**A**) 34 (**B**) 35 (**C**) 36 (**D**) 37 (**E**) 38

20. Square $ABCD$ with side length 2 is inscribed in a circle, as shown. Using each side of the square as a diameter, semi-circular arcs are drawn. The area of the shaded region outside the circle and inside the semi-circles is



(**A**) π (**B**) 4 (**C**) $2\pi - 2$
(**D**) $\pi + 1$ (**E**) $2\pi - 4$

Part C: Each correct answer is worth 8.

21. Point P is on the line $y = 5x + 3$. The coordinates of point Q are $(3, -2)$. If M is the midpoint of PQ, then M must lie on the line

(**A**) $y = \frac{5}{2}x - \frac{7}{2}$ (**B**) $y = 5x + 1$ (**C**) $y = -\frac{1}{5}x - \frac{7}{5}$ (**D**) $y = \frac{5}{2}x + \frac{1}{2}$ (**E**) $y = 5x - 7$

22. What is the shortest distance between two circles, the first having centre $A(5, 3)$ and radius 12, and the other with centre $B(2, -1)$ and radius 6?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

continued ...

23. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

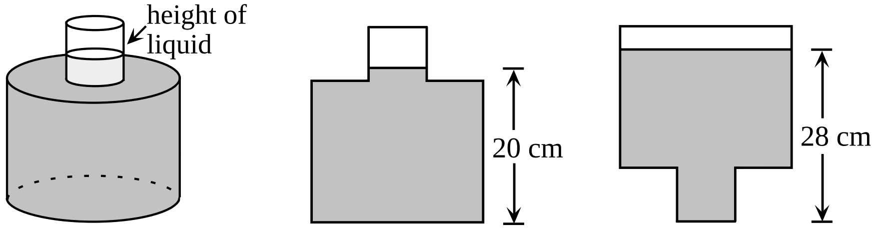


Figure A Figure B Figure C

(**A**) 29 (**B**) 30 (**C**) 31 (**D**) 32 (**E**) 48

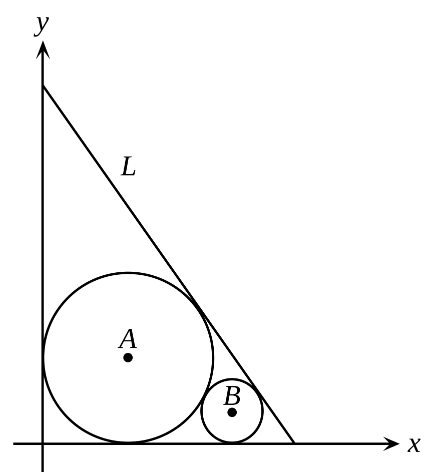
24. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

(**A**) 28 (**B**) 32 (**C**) 36 (**D**) 40 (**E**) 44

25. The circle with centre A has radius 3 and is tangent to both the positive x-axis and positive y-axis, as shown. Also, the circle with centre B has radius 1 and is tangent to both the positive x-axis and the circle with centre A. The line L is tangent to both circles. The y-intercept of line L is

(**A**) $3+6\sqrt{3}$ (**B**) $10+3\sqrt{2}$ (**C**) $8\sqrt{3}$

(**D**) $10+2\sqrt{3}$ (**E**) $9+3\sqrt{3}$



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Canadian Mathematics Competition

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Cayley Contest (Grade 10)

Wednesday, February 23, 2000

C.M.C. Sponsors:

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

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9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

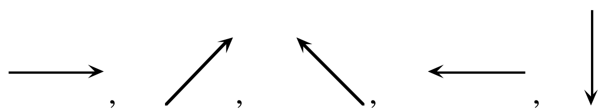
Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

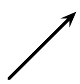
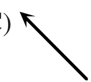
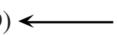
Part A: Each correct answer is worth 5.

1. The value of $2(5-2)-5^2$ is

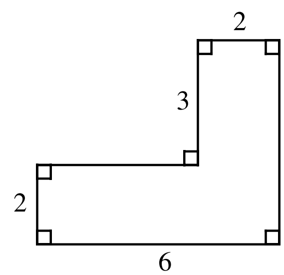
(**A**) -19 (**B**) -4 (**C**) 1 (**D**) -11 (**E**) -17

2. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?



(**A**) (**B**)  (**C**)  (**D**)  (**E**) 

3. In the given diagram, the numbers shown are the lengths of the sides. What is the perimeter of the figure?

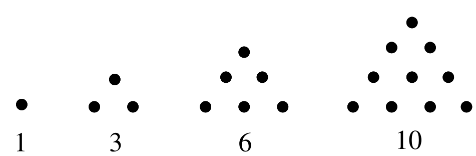


(**A**) 13 (**B**) 18 (**C**) 22
(**D**) 21 (**E**) 19

4. A farmer has 7 cows, 8 sheep and 6 goats. How many more goats should be bought so that half of her animals will be goats?

(**A**) 18 (**B**) 15 (**C**) 21 (**D**) 9 (**E**) 6

5. The first four triangular numbers 1, 3, 6 and 10 are illustrated in the diagram. What is the tenth triangular number?

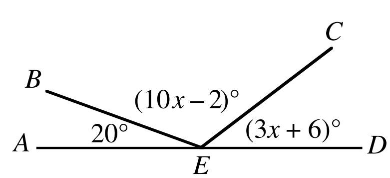


(**A**) 55 (**B**) 45 (**C**) 66
(**D**) 78 (**E**) 50

6. The sum of the digits of an even ten digit integer is 89. The last digit is

(**A**) 0 (**B**) 2 (**C**) 4 (**D**) 6 (**E**) 8

7. If AD is a straight line segment and E is a point on AD, determine the measure of $\angle CED$.



(**A**) 20° (**B**) 12° (**C**) 42°
(**D**) 30° (**E**) 45°

8. On a 240 kilometre trip, Corey's father drove $\frac{1}{2}$ of the distance. His mother drove $\frac{3}{8}$ of the total distance and Corey drove the remaining distance. How many kilometres did Corey drive?

(**A**) 80 (**B**) 40 (**C**) 210 (**D**) 30 (**E**) 55

9. Evaluate $(-50)+(-48)+(-46)+\ldots+54+56$.

(**A**) 156 (**B**) 10 (**C**) 56 (**D**) 110 (**E**) 162

10. The ages of three contestants in the Cayley Contest are 15 years, 9 months; 16 years, 1 month; and 15 years, 8 months. Their average (mean) age is

(**A**) 15 years, 8 months (**B**) 15 years, 9 months (**C**) 15 years, 10 months
(**D**) 15 years, 11 months (**E**) 16 years

Part B: Each correct answer is worth 6.

11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?

(**A**) \$10.00 (**B**) \$13.50 (**C**) \$14.00 (**D**) \$14.50 (**E**) \$15.00

12. Natural numbers are equally spaced around a circle in order from 1 to n. If the number 5 is directly opposite the number 14, then n is

(**A**) 14 (**B**) 15 (**C**) 16 (**D**) 18 (**E**) 20

13. The average of 19 consecutive integers is 99. The largest of these integers is

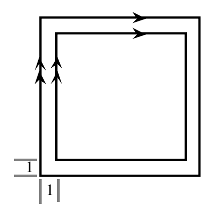
(**A**) 118 (**B**) 108 (**C**) 109 (**D**) 117 (**E**) 107

14. A positive integer is to be placed in each box. The product of any four adjacent integers is always 120. What is the value of x?

		2			4			x			3		

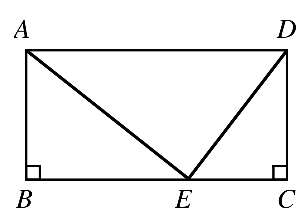
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

15. Eight squares with the same centre have parallel sides and are one unit apart. The two largest squares are shown. If the largest square has a perimeter of 96, what is the perimeter of the smallest square?



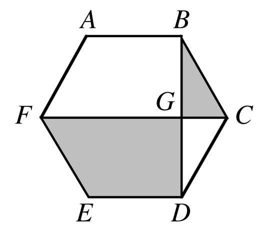
(**A**) 40 (**B**) 68 (**C**) 32
(**D**) 64 (**E**) 89

16. In the diagram, $ABCD$ is a rectangle with $AD=13$, $DE=5$ and $EA=12$. The area of $ABCD$ is



(**A**) 39 (**B**) 60 (**C**) 52
(**D**) 30 (**E**) 25

17. In the regular hexagon $ABCDEF$, two of the diagonals, FC and BD, intersect at G. The ratio of the area of quadrilateral $FEDG$ to ΔBCG is

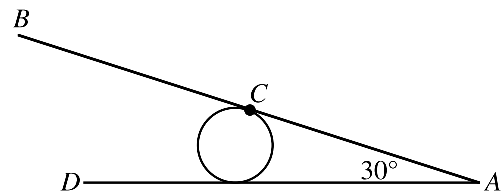


(**A**) $3\sqrt{3}:1$ (**B**) 4:1 (**C**) 6:1
(**D**) $2\sqrt{3}:1$ (**E**) 5:1

18. If a, b and c are distinct positive integers such that $abc = 16$, then the largest possible value of $a^b - b^c + c^a$ is

(**A**) 253 (**B**) 63 (**C**) 249 (**D**) 263 (**E**) 259

19. A metal rod with ends A and B is welded at its middle, C, to a cylindrical drum of diameter 12. The rod touches the ground at A making a $30°$ angle. The drum starts to roll along AD in the direction of D. How far along AD must the drum roll for B to touch the ground?



(**A**) π (**B**) 2π (**C**) 3π
(**D**) 4π (**E**) 5π

20. Twenty pairs of integers are formed using each of the integers 1, 2, 3, ..., 40 once. The positive difference between the integers in each pair is 1 or 3. (For example, 5 can be paired with 2, 4, 6 or 8.) If the resulting differences are added together, the greatest possible sum is

(**A**) 50 (**B**) 54 (**C**) 56 (**D**) 58 (**E**) 60

Part C: Each correct answer is worth 8.

21. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

(**A**) 9:2 (**B**) 9:4 (**C**) 6:1 (**D**) 3:1 (**E**) 5:2

22. An ant walks inside a 18 cm by 150 cm rectangle. The ant's path follows straight lines which always make angles of $45°$ to the sides of the rectangle. The ant starts from a point X on one of the shorter sides. The first time the ant reaches the opposite side, it arrives at the midpoint. What is the distance, in centimetres, from X to the nearest corner of the rectangle?

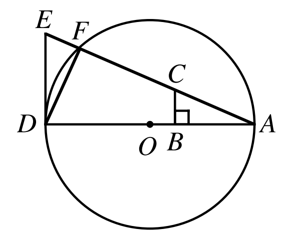
(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

22. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either 'a' or 'b'. What is the value of $a + b$?

(**A**) 3 (**B**) 7 (**C**) 4 (**D**) 10 (**E**) 17

continued ...

24. In the diagram shown, $\angle ABC = 90°$, $CB \parallel ED$, $AB = DF$, $AD = 24$, $AE = 25$ and O is the centre of the circle. Determine the perimeter of $CBDF$.



(**A**) 39 (**B**) 40 (**C**) 42
(**D**) 43 (**E**) 44

25. For the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$, the sum of the real y values that satisfy the equations is

(**A**) 20 (**B**) 2 (**C**) $\frac{3}{2}$ (**D**) $\frac{55}{2}$ (**E**) $\frac{5}{2}$

Canadian
Mathematics
Competition

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Manulife
Financial

Equitable Life
of Canada

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

1. The value of $3^2 + 7^2 - 5^2$ is

(**A**) 75 (**B**) 83 (**C**) 33 (**D**) 25 (**E**) 10

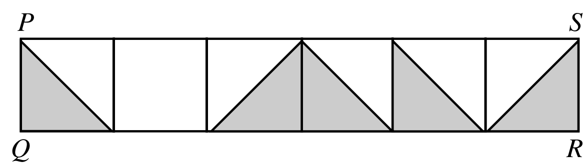
2. If 8 is added to the square of 5 the result is divisible by

(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 23 (**E**) 11

3. Today is Wednesday. What day of the week will it be 100 days from now?

(**A**) Monday (**B**) Tuesday (**C**) Thursday (**D**) Friday (**E**) Saturday

4. The rectangle $PQRS$ is divided into six equal squares and shaded as shown. What fraction of $PQRS$ is shaded?



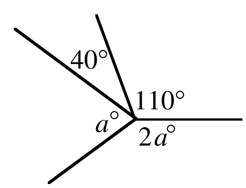
(**A**) $\frac{1}{2}$ (**B**) $\frac{7}{12}$ (**C**) $\frac{5}{11}$ (**D**) $\frac{6}{11}$ (**E**) $\frac{5}{12}$

5. If $x = 4$ and $y = 3x$ and $z = 2y$, then the value of $y + z$ is

(**A**) 12 (**B**) 20 (**C**) 40 (**D**) 24 (**E**) 36

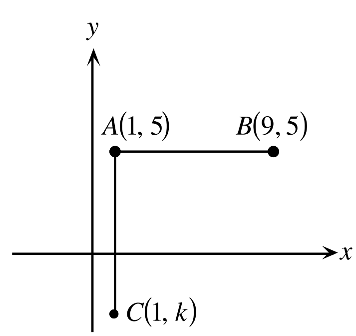
6. In the diagram, the value of a is

(**A**) 50 (**B**) 65 (**C**) 70
(**D**) 105 (**E**) 110



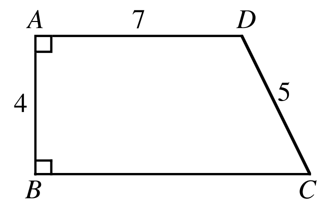
7. In the diagram, AB and AC have equal lengths. What is the value of k?

(**A**) -3 (**B**) -4 (**C**) -5
(**D**) -7 (**E**) -8



8. In the diagram, $AD < BC$. What is the perimeter of $ABCD$?

(**A**) 23 (**B**) 26 (**C**) 27
(**D**) 28 (**E**) 30

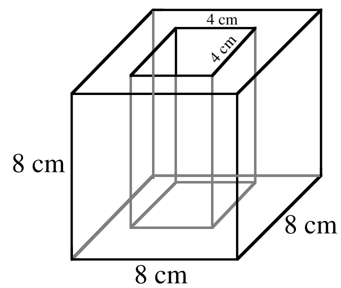


9. Three CD's are bought at an average cost of \$15 each. If a fourth CD is purchased, the average cost becomes \$16. What is the cost of the fourth CD?

(**A**) \$16 (**B**) \$17 (**C**) \$18 (**D**) \$19 (**E**) \$20

10. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in cm^3?

(**A**) 64 (**B**) 128 (**C**) 256
(**D**) 384 (**E**) 448



Part B: Each question is worth 6 credits.

11. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

(**A**) 71 (**B**) 72 (**C**) 255 (**D**) 316 (**E**) 436

12. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

(**A**) 33 (**B**) 40 (**C**) 47 (**D**) 49 (**E**) 53

13. The units digit in the product $(5^2+1)(5^3+1)(5^{23}+1)$ is

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 5 (**E**) 6

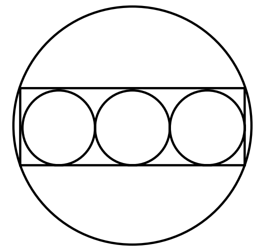
14. In an election for class president, 61 votes are cast by students who are voting to choose one of four candidates. Each student must vote for only one candidate. The candidate with the highest number of votes is the winner. The smallest number of votes the winner can receive is

(**A**) 15 (**B**) 16 (**C**) 21 (**D**) 30 (**E**) 31

15. A chocolate drink is 6% pure chocolate, by volume. If 10 litres of pure milk are added to 50 litres of this drink, the percent of chocolate in the new drink is

(**A**) 5 (**B**) 16 (**C**) 10 (**D**) 3 (**E**) 26

16. Three circles, each with a radius of 10 cm, are drawn tangent to each other so that their centres are all in a straight line. These circles are inscribed in a rectangle which is inscribed in another circle. The area of the largest circle is

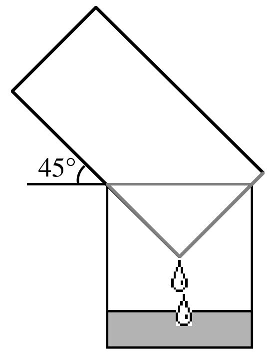


(**A**) 1000π (**B**) 1700π (**C**) 900π
(**D**) 1600π (**E**) 1300π

17. Let N be the smallest positive integer whose digits have a product of 2000. The sum of the digits of N is

(**A**) 21 (**B**) 23 (**C**) 25 (**D**) 27 (**E**) 29

18. A cylindrical pail containing water drains into a cylindrical tub 40 cm across and 50 cm deep, while resting at an angle of $45°$ to the horizontal, as shown. How deep is the water in the tub when its level reaches the pail?

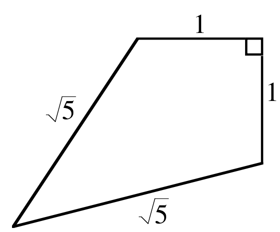


(**A**) 10 cm (**B**) 20 cm (**C**) 30 cm
(**D**) 35 cm (**E**) 40 cm

19. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (e.g. 10 is Beprisque but 12 is not). The number of *two-digit* Beprisque numbers (including 10) is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

20. The area of the given quadrilateral is



(**A**) $\frac{3}{2}$ (**B**) $\sqrt{5}$ (**C**) $\frac{1+\sqrt{10}}{2}$
(**D**) 2 (**E**) 3

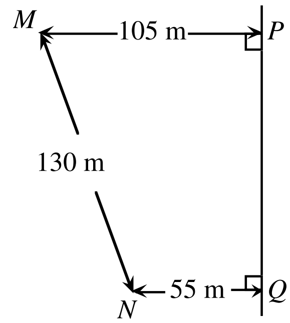
Part C: Each question is worth 8 credits.

21. A number is formed using the digits 1, 2, ..., 9. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?

(**A**) 72 (**B**) 73 (**C**) 144 (**D**) 145 (**E**) 91

continued ...

22. A main gas line runs through P and Q. From some point T on PQ, a supply line runs to a house at point M. A second supply line from T runs to a house at point N. What is the minimum total length of pipe required for the two supply lines?



(**A**) 200 (**B**) 202 (**C**) 198
(**D**) 210 (**E**) 214

23. How many integers can be expressed as a sum of three distinct numbers chosen from the set $\{4, 7, 10, 13, ..., 46\}$?

(**A**) 45 (**B**) 37 (**C**) 36 (**D**) 43 (**E**) 42

24. The sum of all values of x that satisfy the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is

(**A**) -4 (**B**) 3 (**C**) 1 (**D**) 5 (**E**) 6

25. If $a = 3^p$, $b = 3^q$, $c = 3^r$, and $d = 3^s$ and if p, q, r, and s are positive integers, determine the smallest value of $p + q + r + s$ such that $a^2 + b^3 + c^5 = d^7$.

(**A**) 17 (**B**) 31 (**C**) 106 (**D**) 247 (**E**) 353

35th Anniversary 1963 – 1998

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 18, 1998

Time: 1 hour

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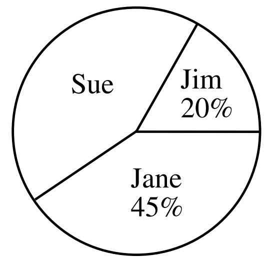
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

1. The value of $(0.3)^2 + 0.1$ is
(**A**) 0.7 (**B**) 1 (**C**) 0.1 (**D**) 0.19 (**E**) 0.109

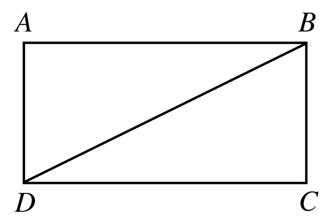
2. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?
(**A**) 550 (**B**) 350 (**C**) 330
(**D**) 450 (**E**) 935



3. The expression $\dfrac{a^9 \times a^{15}}{a^3}$ is equal to
(**A**) a^{45} (**B**) a^8 (**C**) a^{18} (**D**) a^{14} (**E**) a^{21}

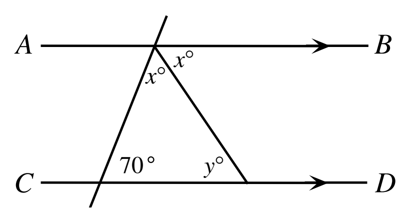
4. The product of two positive integers p and q is 100. What is the largest possible value of $p+q$?
(**A**) 52 (**B**) 101 (**C**) 20 (**D**) 29 (**E**) 25

5. In the diagram, $ABCD$ is a rectangle with $DC = 12$. If the area of triangle BDC is 30, what is the perimeter of rectangle $ABCD$?
(**A**) 34 (**B**) 44 (**C**) 30
(**D**) 29 (**E**) 60



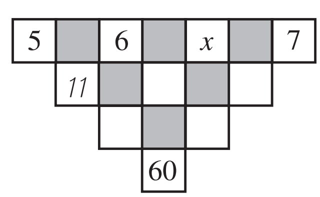
6. If $x = 2$ is a solution of the equation $qx - 3 = 11$, the value of q is
(**A**) 4 (**B**) 7 (**C**) 14 (**D**) -7 (**E**) -4

7. In the diagram, AB is parallel to CD. What is the value of y?
(**A**) 75 (**B**) 40 (**C**) 35
(**D**) 55 (**E**) 50



8. The vertices of a triangle have coordinates $(1,1)$, $(7,1)$ and $(5,3)$. What is the area of this triangle?
(**A**) 12 (**B**) 8 (**C**) 6 (**D**) 7 (**E**) 9

9. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of x must be
(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10

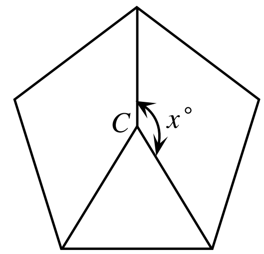


10. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) The number of such integers is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Part B: Each question is worth 6 credits.

11. If $x+y+z=25$, $x+y=19$ and $y+z=18$, then y equals
(**A**) 13 (**B**) 17 (**C**) 12 (**D**) 6 (**E**) -6

12. A regular pentagon with centre C is shown. The value of x is
(**A**) 144 (**B**) 150 (**C**) 120
(**D**) 108 (**E**) 72

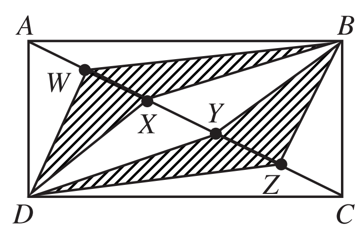


13. If the surface area of a cube is 54, what is its volume?
(**A**) 36 (**B**) 9 (**C**) $\frac{81\sqrt{3}}{8}$ (**D**) 27 (**E**) $162\sqrt{6}$

14. The number of solutions (x, y) of the equation $3x+y=100$, where x and y are positive integers, is
(**A**) 33 (**B**) 35 (**C**) 100 (**D**) 101 (**E**) 97

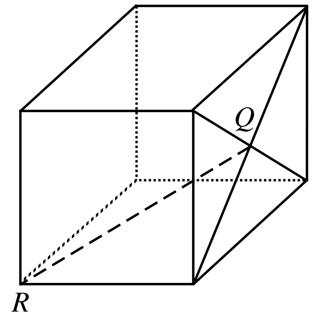
15. If $\sqrt{y-5}=5$ and $2^x=8$, then $x+y$ equals
(**A**) 13 (**B**) 28 (**C**) 33 (**D**) 35 (**E**) 38

16. Rectangle $ABCD$ has length 9 and width 5. Diagonal AC is divided into 5 equal parts at W, X, Y, and Z. Determine the area of the shaded region.
(**A**) 36 (**B**) $\frac{36}{5}$ (**C**) 18
(**D**) $\frac{4\sqrt{106}}{5}$ (**E**) $\frac{2\sqrt{106}}{5}$



17. If $N=\left(7^{p+4}\right)\left(5^q\right)\left(2^3\right)$ is a perfect cube, where p and q are positive integers, the smallest possible value of $p+q$ is
(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 6 (**E**) 12

18. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is
(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$



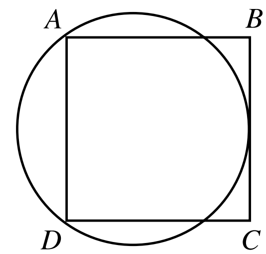
19. Mr. Anderson has more than 25 students in his class. He has more than 2 but fewer than 10 boys and more than 14 but fewer than 23 girls in his class. How many different class *sizes* would satisfy these conditions?

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 3 (**E**) 4

20. Each side of square $ABCD$ is 8. A circle is drawn through A and D so that it is tangent to BC. What is the radius of this circle?

(**A**) 4 (**B**) 5 (**C**) 6

(**D**) $4\sqrt{2}$ (**E**) 5.25



Part C: Each question is worth 8 credits.

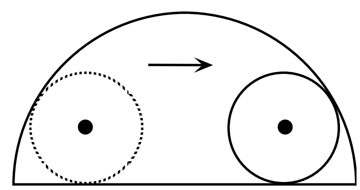
21. When Betty substitutes $x = 1$ into the expression $ax^3 - 2x + c$ its value is -5. When she substitutes $x = 4$ the expression has value 52. One value of x that makes the expression equal to zero is

(**A**) 2 (**B**) $\frac{5}{2}$ (**C**) 3 (**D**) $\frac{7}{2}$ (**E**) 4

22. A wheel of radius 8 rolls along the diameter of a semicircle of radius 25 until it bumps into this semicircle. What is the length of the portion of the diameter that cannot be touched by the wheel?

(**A**) 8 (**B**) 12 (**C**) 15

(**D**) 17 (**E**) 20



23. There are four unequal, positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

(**A**) 13 (**B**) 17 (**C**) 22 (**D**) 33 (**E**) 36

24. Three rugs have a combined area of $200\ \text{m}^2$. By overlapping the rugs to cover a floor area of $140\ \text{m}^2$, the area which is covered by exactly two layers of rug is $24\ \text{m}^2$. What area of floor is covered by three layers of rug?

(**A**) $12\ \text{m}^2$ (**B**) $18\ \text{m}^2$ (**C**) $24\ \text{m}^2$ (**D**) $36\ \text{m}^2$ (**E**) $42\ \text{m}^2$

25. One way to pack a 100 by 100 square with 10 000 circles, each of diameter 1, is to put them in 100 rows with 100 circles in each row. If the circles are repacked so that the centres of any three tangent circles form an equilateral triangle, what is the maximum number of additional circles that can be packed?

(**A**) 647 (**B**) 1442 (**C**) 1343 (**D**) 1443 (**E**) 1344

Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 19, 1997

Part A: Each question is worth 5 credits.

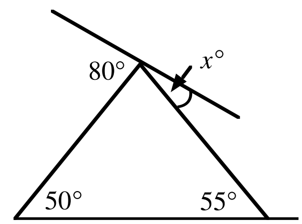
1. The value of $2\frac{1}{10}+3\frac{11}{100}$ is
(**A**) 5.11 (**B**) 5.111 (**C**) 5.12 (**D**) 5.21 (**E**) 5.3

2. The value of $(1)^{10}+(-1)^{8}+(-1)^{7}+(1)^{5}$ is
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 16 (**E**) 4

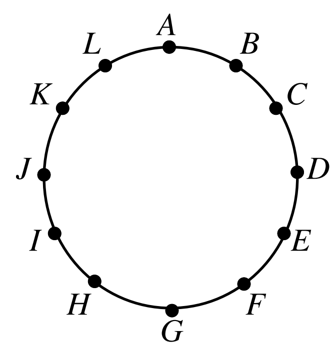
3. An integer is multiplied by 2 and the result is then multiplied by 5. The final result could be
(**A**) 64 (**B**) 32 (**C**) 12 (**D**) 25 (**E**) 30

4. The greatest number of Mondays that can occur in 45 consecutive days is
(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

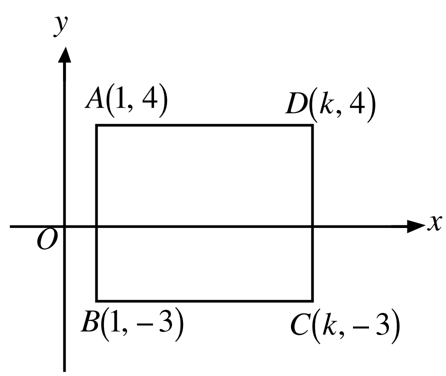
5. The value of x is
(**A**) 25 (**B**) 30 (**C**) 50
(**D**) 55 (**E**) 20



6. Twelve balloons are arranged in a circle as shown. Counting clockwise, every third balloon is popped, with C the first one popped. This process continues around the circle until two unpopped balloons remain. The last two remaining balloons are
(**A**) B, H (**B**) B, G (**C**) A, E
(**D**) E, J (**E**) F, K



7. In the diagram, rectangle $ABCD$ has area 70 and k is positive. The value of k is
(**A**) 8 (**B**) 9 (**C**) 10
(**D**) 11 (**E**) 12



8. If p, q, r, s , and t are numbers such that $r < s$, $t > q$, $q > p$, and $t < r$, which of these numbers is greatest?
(**A**) t (**B**) s (**C**) r (**D**) q (**E**) p

9. The sum of seven consecutive integers is 77. The smallest of these integers is
(**A**) 5 (**B**) 7 (**C**) 8 (**D**) 11 (**E**) 14

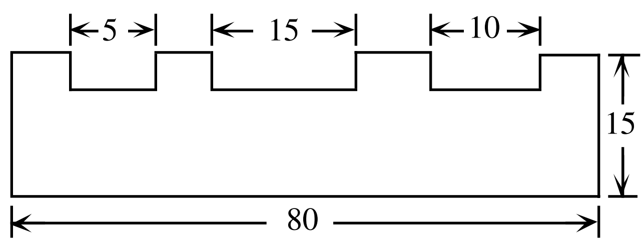
10. Each of the numbers 1, 2, 3, and 4 is assigned, in some order, to p, q, r, and s. The largest possible value of $p^q + r^s$ is
(**A**) 12 (**B**) 19 (**C**) 66 (**D**) 82 (**E**) 83

Part B: Each question is worth 6 credits.

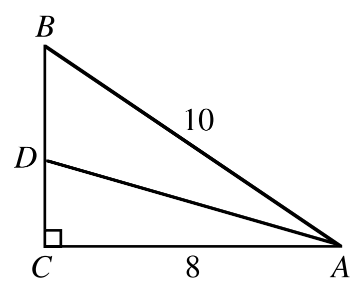
11. In the chart, the products of the numbers represented by the letters in each of the rows and columns are given. For example, $xy = 6$ and $xz = 12$. If x, y, z, and w are integers, what is the value of xw?
(**A**) 150 (**B**) 300 (**C**) 31
(**D**) 75 (**E**) 30

x	y	6
z	w	50
12	25	

12. Three small rectangles, of the same depth, are cut from a rectangular sheet of metal. The area of the remaining piece is 990. What is the depth of each cut?
(**A**) 8 (**B**) 7 (**C**) 6
(**D**) 5 (**E**) 4



13. Triangle ABC is right-angled with $AB = 10$ and $AC = 8$. If $BC = 3DC$, then AD equals
(**A**) 9 (**B**) $\sqrt{65}$ (**C**) $\sqrt{80}$
(**D**) $\sqrt{73}$ (**E**) $\sqrt{68}$



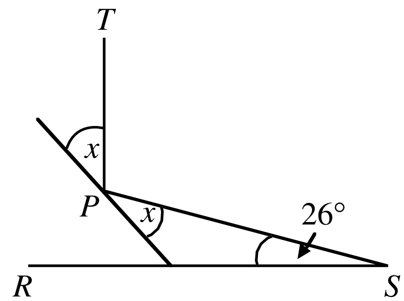
14. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four digit numbers. If these twenty-four numbers are then listed from smallest to largest, in what position is 3142?
(**A**) 13th (**B**) 14th (**C**) 15th (**D**) 16th (**E**) 17th

15. The product of 20^{50} and 50^{20} is written as an integer in expanded form. The number of zeros at the end of the resulting integer is

(**A**) 70 (**B**) 71 (**C**) 90 (**D**) 140 (**E**) 210

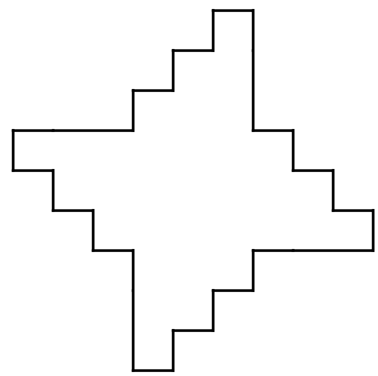
16. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. Then x is

(**A**) 32° (**B**) 37° (**C**) 45°
(**D**) 26° (**E**) 38°



17. In the diagram adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The area of the shape is 528. What is the perimeter?

(**A**) 132 (**B**) 264 (**C**) 92
(**D**) 72 (**E**) 144



18. If $\frac{30}{7} = x + \cfrac{1}{y + \cfrac{1}{z}}$, where $x, y,$ and z are positive integers, then what is the value of $x + y + z$?

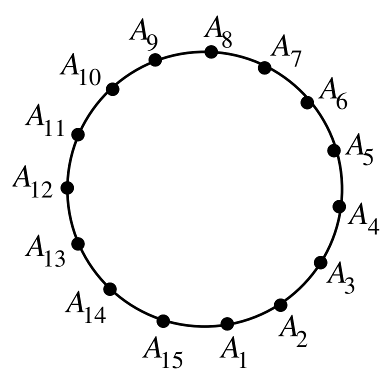
(**A**) 13 (**B**) 9 (**C**) 11 (**D**) 37 (**E**) 30

19. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then xyz equals

(**A**) 7 (**B**) 7^2 (**C**) 7^3 (**D**) 7^8 (**E**) 7^9

20. On a circle, fifteen points $A_1, A_2, A_3, \ldots, A_{15}$ are equally spaced. What is the size of angle $A_1A_3A_7$?

(**A**) 96° (**B**) 100° (**C**) 104°
(**D**) 108° (**E**) 120°

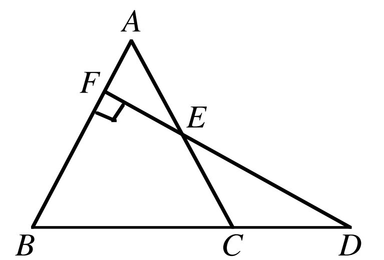


Part C: Each question is worth 8 credits.

21. If $\dfrac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=11$, where a, b, and c are positive integers, the number of different ordered triples (a, b, c) such that $a+2b+c \leq 40$ is

(**A**) 33 (**B**) 37 (**C**) 40 (**D**) 42 (**E**) 45

22. In the diagram, ΔABC is equilateral, $BC = 2CD$, $AF = 6$, and DEF is perpendicular to AB. What is the area of quadrilateral $FBCE$?



(**A**) $144\sqrt{3}$ (**B**) $138\sqrt{3}$ (**C**) $126\sqrt{3}$

(**D**) $108\sqrt{3}$ (**E**) $66\sqrt{3}$

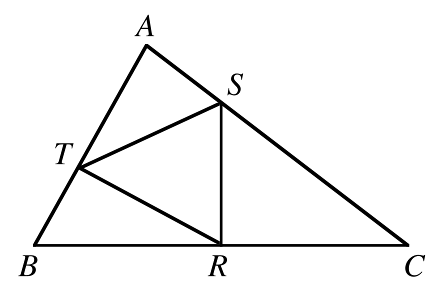
23. Given the set $\{1, 2, 3, 5, 8, 13, 21, 34, 55\}$, how many integers between 3 and 89 cannot be written as the sum of exactly two elements of the set?

(**A**) 51 (**B**) 57 (**C**) 55 (**D**) 34 (**E**) 43

24. In a convex polygon, exactly five of the interior angles are obtuse. The largest possible number of sides for this polygon is

(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

25. In triangle ABC, $BR = RC$, $CS = 3SA$, and $\dfrac{AT}{TB}=\dfrac{p}{q}$. If the area of ΔRST is twice the area of ΔTBR, then $\dfrac{p}{q}$ is equal to



(**A**) $\frac{2}{1}$ (**B**) $\frac{8}{3}$ (**C**) $\frac{5}{2}$

(**D**) $\frac{7}{4}$ (**E**) $\frac{7}{3}$