The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

2022 Canadian Team Mathematics Contest

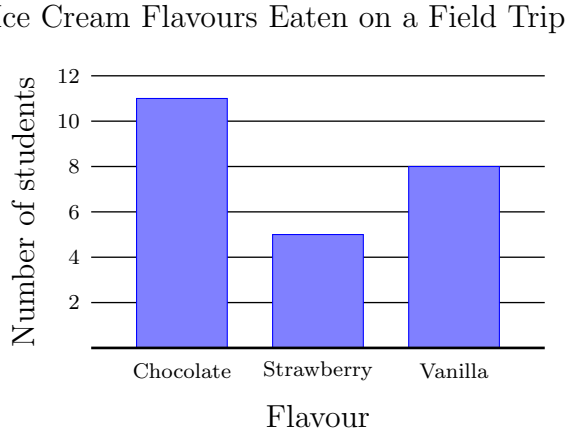
Individual Problems (45 minutes)

IMPORTANT NOTES:

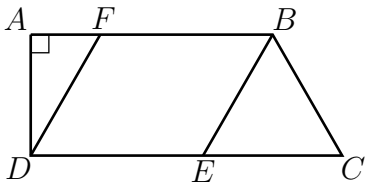
- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

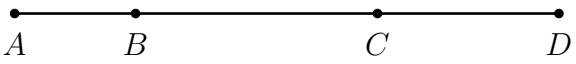
1. The bar graph below shows how many students chose each flavour of ice cream on a recent field trip. What fraction of the students chose chocolate ice cream?



2. In trapezoid $ABCD$, AB is parallel to DC and $\angle DAF = 90°$. Point E is on DC so that $EB = BC = CE$. Point F is on AB so that DF is parallel to EB. In degrees, what is the measure of $\angle FDA$?



3. Line segment AD is divided into three segments by points B and C, so that $AB : BC = 1 : 2$ and $BC : CD = 6 : 5$. The length of AD is 56 units. What is the length of AB?



4. The series below includes the consecutive even integers from 2 to 2022 inclusive, where the signs of the terms alternate between positive and negative:

$$S = 2 - 4 + 6 - 8 + 10 - \cdots - 2016 + 2018 - 2020 + 2022$$

What is the value of S?

5. What is the largest integer n with the properties that $200 < n < 250$ and that $12n$ is a perfect square?

6. Each of A, B, C, and D is a positive two-digit integer. These integers satisfy each of the equations

$$\begin{aligned} B &= 3C \\ D &= 2B - C \\ A &= B + D \end{aligned}$$

What is the largest possible value of $A + B + C + D$?

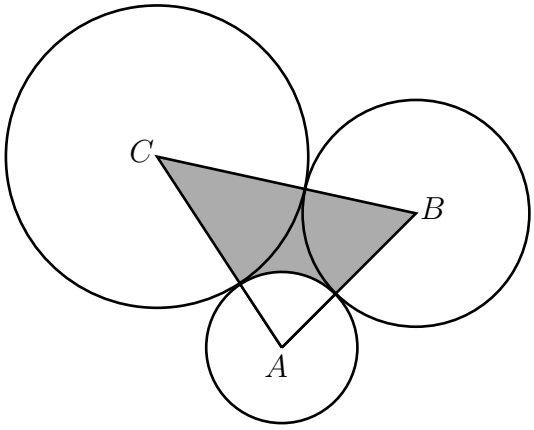
7. What is the sum of the digits of the integer equal to $3 \times 10^{500} - 2022 \times 10^{497} - 2022$?

8. The integers a and b have the property that the expression

$$\frac{2n^3 + 3n^2 + an + b}{n^2 + 1}$$

is an integer for every integer n. What is the value of the expression above when $n = 4$?

9. Three circles with centres A, B and C have radii $\sqrt{3} - 1$, $3 - \sqrt{3}$ and $1 + \sqrt{3}$ respectively. Each circle is externally tangent to the other two as shown. The area of the shaded region is of the form $a\sqrt{3} + b\pi + c\pi\sqrt{3}$ for some rational numbers a, b and c. What is the value of $a + b + c$?



10. Starting with a four-digit integer that is not a multiple of 1000, an integer with fewer digits can be obtained by removing the leading digit and ignoring leading zeros. For example, removing the leading digit from 1023 gives the integer 23, and removing the leading digit from 2165 gives 165. How many integers from 1001 to 4999, inclusive, other than multiples of 1000, have the property that the integer obtained by removing the leading digit is a factor of the original integer?