## The CENTRE for EDUCATION
### in MATHEMATICS and COMPUTING
**cemc.uwaterloo.ca**

# *Cayley Contest*

## *(Grade 10)*

**Tuesday, February 25, 2020**
*(in North America and South America)*

**Wednesday, February 26, 2020**
*(outside of North America and South America)*

### UNIVERSITY OF WATERLOO

**Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.**

**Instructions**

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper right corner.

5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

*Do not discuss the problems or solutions from this contest online for the next 48 hours.*

*The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.*

**Part A: Each correct answer is worth 5.**

1.  The value of $\dfrac{20 - 20}{20 + 20}$ is

    **(A)** 0      **(B)** 1      **(C)** 10      **(D)** $-2$      **(E)** 2

2.  When $x = 3$ and $y = 4$, the value of $xy - x$ is

    **(A)** 3      **(B)** 4      **(C)** 12      **(D)** 9      **(E)** 15

3.  The points $O(0,0)$, $P(0,3)$, $Q$, and $R(5,0)$ form a rectangle, as shown. The coordinates of $Q$ are

    **(A)** $(5,5)$      **(B)** $(5,3)$      **(C)** $(3,3)$

    **(D)** $(2.5, 1.5)$      **(E)** $(0,5)$
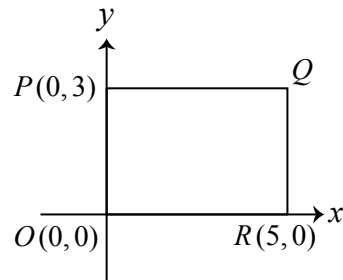
4.  Which of the following numbers is less than $\frac{1}{20}$?

    **(A)** $\frac{1}{15}$      **(B)** $\frac{1}{25}$      **(C)** $0.5$      **(D)** $0.055$      **(E)** $\frac{1}{10}$

5.  In the diagram, point $Q$ lies on $PR$ and point $S$ lies on $QT$. What is the value of $x$?

    **(A)** 10      **(B)** 30      **(C)** 50

    **(D)** 40      **(E)** 20
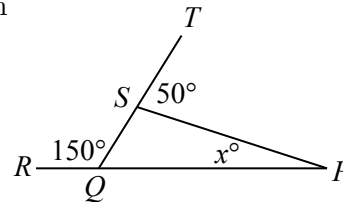
6.  Matilda counted the birds that visited her bird feeder yesterday. She summarized the data in the bar graph shown. The percentage of birds that were goldfinches is

    **(A)** 15%      **(B)** 20%      **(C)** 30%

    **(D)** 45%      **(E)** 60%
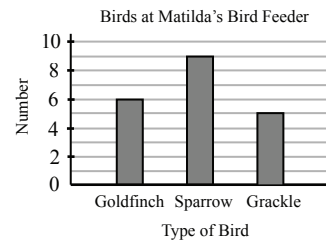
7.  The average of the two positive integers $m$ and $n$ is 5. What is the largest possible value for $n$?

    **(A)** 5      **(B)** 7      **(C)** 9      **(D)** 11      **(E)** 13

8.  Roman wins a contest with a prize of $200. He gives 30% of the prize to Jackie. He then splits 15% *of what remains* equally between Dale and Natalia. How much money does Roman give Dale?

    **(A)** $10.50      **(B)** $15.00      **(C)** $4.50      **(D)** $25.50      **(E)** $59.50

9. Shaded and unshaded squares are arranged in rows so that:

    • the first row consists of one unshaded square,

    • each row begins with an unshaded square,

    • the squares in each row alternate between unshaded and shaded, and

    • each row after the first has two more squares than the previous row.
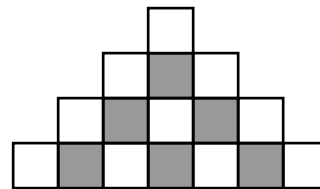
The first 4 rows are shown. The number of shaded squares in the 2020th row is

(A) 2022      (B) 2021      (C) 2020

(D) 2019      (E) 2018

10. In the diagram, pentagon $PQRST$ has $PQ = 13$, $QR = 18$, $ST = 30$, and a perimeter of 82. Also, $\angle QRS = \angle RST = \angle STP = 90°$. The area of the pentagon $PQRST$ is

(A) 306      (B) 297      (C) 288

(D) 279      (E) 270

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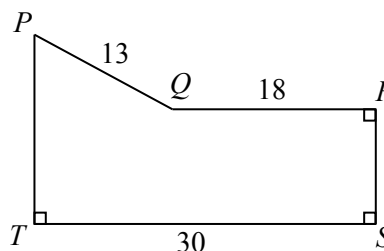
**Part B: Each correct answer is worth 6.**

11. The sum of the first 9 positive integers is 45; in other words,

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

What is the sum of the first 9 positive multiples of 5? In other words, what is the value of $5 + 10 + 15 + \cdots + 40 + 45$?

(A) 225      (B) 250      (C) 180      (D) 150      (E) 450

12. The volume of a rectangular prism is 21. Its length, width and height are all different positive integers. The sum of its length, width and height is

(A) 11      (B) 13      (C) 15      (D) 9      (E) 17

13. If $2^n = 8^{20}$, what is the value of $n$?

(A) 10      (B) 60      (C) 40      (D) 16      (E) 17

14. Juliana chooses three different numbers from the set $\{-6, -4, -2, 0, 1, 3, 5, 7\}$ and multiplies them together to obtain the integer $n$. What is the greatest possible value of $n$?

(A) 168      (B) 0      (C) 15      (D) 105      (E) 210

15. A bag contains only green, yellow and red marbles. The ratio of green marbles to yellow marbles to red marbles in the bag is $3 : 4 : 2$. If 63 of the marbles in the bag are *not* red, the number of red marbles in the bag is

(A) 14      (B) 18      (C) 27      (D) 36      (E) 81

16. In the diagram, the circle has centre $O$ and square $OPQR$ has vertex $Q$ on the circle. If the area of the circle is $72\pi$, the area of the square is

(A) 38      (B) 48      (C) 25

(D) 12      (E) 36
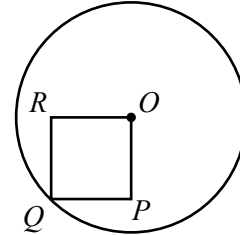
17. Carley made treat bags. Each bag contained exactly 1 chocolate, 1 mint, and 1 caramel. The chocolates came in boxes of 50. The mints came in boxes of 40. The caramels came in boxes of 25. Carley made no incomplete treat bags and there were no unused chocolates, mints or caramels. What is the minimum total number of boxes that Carley could have bought?

(A) 19      (B) 17      (C) 44      (D) 25      (E) 9

18. Nate is driving to see his grandmother. If he drives at a constant speed of 40 km/h, he will arrive 1 hour late. If he drives at a constant speed of 60 km/h, he will arrive 1 hour early. At what constant speed should he drive to arrive just in time?

(A) 56 km/h    (B) 80 km/h    (C) 54 km/h    (D) 48 km/h    (E) 58 km/h

19. A multiple choice test has 10 questions on it. Each question answered correctly is worth 5 points, each unanswered question is worth 1 point, and each question answered incorrectly is worth 0 points. How many of the integers between 30 and 50, inclusive, are *not* possible total scores?

(A) 2      (B) 3      (C) 4      (D) 6      (E) 5

20. For how many pairs $(m, n)$ with $m$ and $n$ integers satisfying $1 \leq m \leq 100$ and $101 \leq n \leq 205$ is $3^m + 7^n$ divisible by 10?

(A) 2600      (B) 2626      (C) 2601      (D) 2650      (E) 2625

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**Part C: Each correct answer is worth 8.**

21. How many points $(x, y)$, with $x$ and $y$ both integers, are on the line with equation $y = 4x + 3$ and inside the region bounded by $x = 25$, $x = 75$, $y = 120$, and $y = 250$?

(A) 44      (B) 36      (C) 40      (D) 32      (E) 48

22. In the diagram, points $S$ and $T$ are on sides $QR$ and $PQ$, respectively, of $\triangle PQR$ so that $PS$ is perpendicular to $QR$ and $RT$ is perpendicular to $PQ$. If $PT = 1$, $TQ = 4$, and $QS = 3$, what is the length of $SR$?

(A) 3      (B) $\frac{11}{3}$      (C) $\frac{15}{4}$

(D) $\frac{7}{2}$      (E) 4
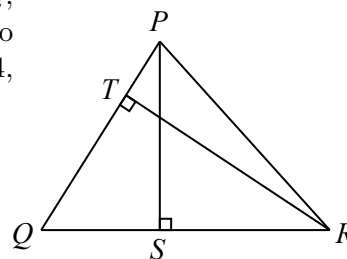
23. Ricardo wants to arrange three 1s, three 2s, two 3s, and one 4 to form nine-digit positive integers with the properties that

- when reading from left to right, there is at least one 1 before the first 2, at least one 2 before the first 3, and at least one 3 before the 4, and

- no digit 2 can be next to another 2.

(For example, the integer $121\,321\,234$ satisfies these properties.) In total, how many such nine-digit positive integers can Ricardo make?

(A) 278      (B) 260      (C) 254      (D) 272      (E) 266

24. A cube with vertices $FGHJKLMN$ has edge length 200. Point $P$ is on $HG$, as shown. The shortest distance from $G$ to a point inside $\triangle PFM$ is 100. Which of the following is closest to the length of $HP$?

(A) 53      (B) 55      (C) 57

(D) 59      (E) 61
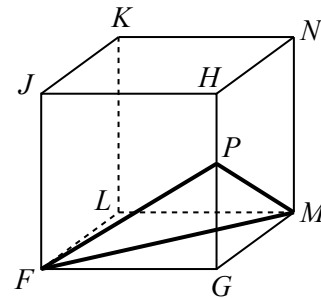


25. How many positive integers $n \leq 20\,000$ have the properties that $2n$ has 64 positive divisors including 1 and $2n$, and $5n$ has 60 positive divisors including 1 and $5n$?

(A) 4      (B) 5      (C) 3      (D) 2      (E) 6

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**For students...**

Thank you for writing the 2020 Cayley Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Galois Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Galois Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

**For teachers...**

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results