The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2019 Gauss Contests

(Grades 7 and 8)

Wednesday, May 15, 2019
(in North America and South America)

Thursday, May 16, 2019
(outside of North America and South America)

Solutions

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Grade 7

1. Erin receives \$3 a day. To receive a total of \$30, it will take Erin $\dfrac{\$30}{\$3} = 10$ days.

   Answer: (E)

2. Beginning at the origin $(0, 0)$, the point $(2, 4)$ is located right 2 units and up 4 units.
   The point $(2, 4)$ is located at D.

   Answer: (D)

3. One of four identical squares is shaded and so the fraction of square $PQRS$ that is shaded is $\frac{1}{4}$ which is equivalent to 25%.

   Answer: (C)

4. Adding, we get $0.9 + 0.09 = 0.99$.

   Answer: (D)

5. The mode is the amount of rainfall that occurs most frequently.
   Reading from the graph, the daily rainfall amounts for Sunday through to Saturday are 6 mm, 15 mm, 3 mm, 6 mm, 3 mm, 3 mm, and 9 mm.
   The mode for the amount of rainfall for the week is 3 mm.

   Answer: (C)

6. When $x = 3$,

$$\begin{aligned} 2x &= 2 \times 3 &= 6 \\ 3x - 1 &= 3 \times 3 - 1 &= 8 \\ x + 5 &= 3 + 5 &= 8 \\ 7 - x &= 7 - 3 &= 4 \\ 6 + 2x &= 6 + 2 \times 3 &= 12 \end{aligned}$$

   Of the given answers, $3x - 1 = 8$ is the only one of the given equations which is true when $x = 3$.

   Answer: (B)

7. When -37 and 11 are added, the result is $-37 + 11 = -26$. The correct answer is (A).
   Alternately, the result can be found by subtracting, $-26 - 11 = -37$.

   Answer: (A)

8. *Solution 1*
   One third of 396 is $396 \div 3 = 132$. Thus, Joshua has read 132 pages of the book.
   To finish the book, Joshua has $396 - 132 = 264$ pages left to read.
   Solution 2
   Joshua has read the first third of the book only, and so he has $1 - \frac{1}{3} = \frac{2}{3}$ of the book left to read.
   Two thirds of 396 is $396 \times \frac{2}{3} = \frac{396 \times 2}{3} = \frac{792}{3} = 264$.
   Joshua has 264 pages left to read.

   Answer: (A)

9. One complete rotation equals 360°.
   Therefore, $k^\circ + 90^\circ = 360^\circ$ and so $k = 360 - 90 = 270$.

   Answer: (D)

10. *Solution 1*

The mean of the numbers $20, 30, 40$ is $\dfrac{20+30+40}{3} = \dfrac{90}{3} = 30$.

Since each of the given answers has three numbers, then for the mean to equal 30, the sum of the three numbers must also equal $30 \times 3 = 90$.

Of the given answers, only (D) has numbers whose sum is 90 $(23 + 30 + 37 = 90)$.

Solution 2

Since 20 is 10 less than 30 and 40 is 10 more than 30, the mean of the numbers $20, 30, 40$ is 30.

In each of the given answers, 30 is the middle number in the list of three numbers.

Thus, for the mean of the three numbers to equal 30, the first and last numbers must be equal "distances" away from 30 (with one number being less than 30 and the other greater than 30).

Looking at answer (D), 23 is 7 less than 30 and 37 is 7 more than 30 and so the mean of these three numbers is 30. (We may check that this isn't the case for each of the other four answers.)

ANSWER: (D)

11. Evaluating, $\sqrt{81} = 9$ and $9 = 3^2$, so $\sqrt{81} = 3^2$.

ANSWER: (B)

12. The width of rectangle $PQRS$ is the horizontal distance between points P and Q (or points S and R), since these points have equal y-coordinates.

This distance is equal to the difference between their x-coordinates or $4 - (-4) = 8$.

Similarly, the height of rectangle $PQRS$ is equal the vertical distance between points S and P (or points R and Q), since these points have equal x-coordinates.

This distance is equal to the difference between their y-coordinates or $2 - (-2) = 4$.

The area of rectangle $PQRS$ is $8 \times 4 = 32$.

ANSWER: (B)

13. The repeating pattern of ABCDEFG has 7 white keys.

Since the first white key is A, the pattern repeats after each number of keys that is a multiple of 7.

Since 28 is a multiple of 7, then the 28^{th} white key is G and so the 29^{th} white key is A, the 30^{th} white key is B, the 31^{st} white key is C, the 32^{nd} white key is D, and the 33^{rd} white key is E.

ANSWER: (E)

14. On the spinner given, the prime numbers that are odd are $3, 5$ and 7.

Since the spinner is divided into 8 equal sections, the probability that the arrow stops in a section containing a prime number that is odd is $\frac{3}{8}$.

ANSWER: (C)

15. Barry's 12 coins include at least one of each of the 5 coins of different values.

The total value of these 5 coins is $\$2.00 + \$1.00 + \$0.25 + \$0.10 + \$0.05 = \$3.40$.

Barry has the smallest total amount of money that he could have if each of his remaining $12 - 5 = 7$ coins has value $\$0.05$ (the smallest possible value of a coin).

Thus, the smallest total amount of money that Barry could have is

$$\$3.40 + 7 \times \$0.05 = \$3.40 + \$0.35 = \$3.75.$$

ANSWER: (A)

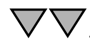
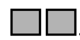
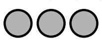
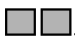
16. *Solution 1*

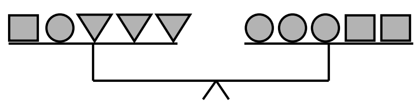
The are 10 palindromes between 100 and 200: $101, 111, 121, 131, 141, 151, 161, 171, 181$, and 191.
The are 10 palindromes between 200 and 300: $202, 212, 222, 232, 242, 252, 262, 272, 282$, and 292.
Similary, there are 10 palindromes between each of 300 and 400, 400 and 500, 500 and 600, 600 and 700, 700 and 800, 800 and 900, 900 and 1000.
That is, there are 10 palindromes between each of the 9 pairs of consecutive multiples of 100 from 100 to 1000.
The number of palindromes between 100 and 1000 is $10 \times 9 = 90$.

Solution 2

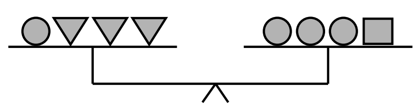
Each palindrome between 100 and 1000 is a 3-digit number of the form aba, where a is a digit from 1 to 9 inclusive and b is a digit from 0 to 9 inclusive, and a and b are not necessarily different digits.
Thus, there are 9 choices for the first digit a and each of those choices determines the third digit which must also be a.
There are 10 choices for the second digit b, and so there are $9 \times 10 = 90$ choices for a and b.
Thus, there are 90 palindromes between 100 and 1000.

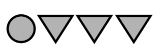
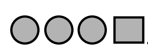
Answer: (B)

17. The first scale shows that  has the same mass as .
The second scale shows that  has the same mass as .
Therefore, the sum of the masses of  and  is equal to the sum of the masses of  and .
That is, the equal-arm scale shown below is balanced.



The left and right sides of this scale each contain a , and the scale remains balanced if one  is removed from each side of the scale (since they are equal in mass).
That is, the equal-arm scale shown below is balanced.



Therefore of the answers given,  has the same mass as .

Answer: (D)

18. The area of the rectangle with length x and width y is $x \times y$.
The area of the triangle with base 16 and height x is $\frac{1}{2} \times 16 \times x$ or $8 \times x$.
The area of the rectangle is equal to the area of the triangle, or $x \times y = 8 \times x$, and so $y = 8$.

Answer: (C)

19. *Solution 1*

Each of the four fractions is equal to the other three fractions, and each fraction has numerator 1, so then each denominator must be equal to the other three denominators.
That is, $a - 2 = b + 2 = c + 1 = d - 3$.
If we let $b = 4$, then $a - 2 = 4 + 2 = c + 1 = d - 3$ or $a - 2 = 6 = c + 1 = d - 3$, and so

$a = 8, c = 5$, and $d = 9$.
Thus, the correct ordering is $b < c < a < d$.

Solution 2
Each of the four fractions is equal to the other three fractions, and each fraction has numerator 1, so then each denominator must be equal to the other three denominators.
That is, $a-2 = b+2 = c+1 = d-3$, and by adding 3 to each, we get $a+1 = b+5 = c+4 = d$.
Since $d = a + 1$, then d is one more than a and so $a < d$.
Since $a + 1 = c + 4$, then a is 3 more than c and so $c < a$.
Since $c + 4 = b + 5$, then c is 1 more than b and so $b < c$.
Thus, the correct ordering is $b < c < a < d$.

ANSWER: (E)

20. Each of 14 and 21 is a divisor of n.
Since $14 = 2 \times 7$, then each of 2 and 7 is also a divisor of n.
Since $21 = 3 \times 7$, then 3 is a divisor of n (as is 7 which we already noted).
So far, the positive divisors of n are: $1, 2, 3, 7, 14$, and 21.
Since 2 and 3 are divisors of n, then their product $2 \times 3 = 6$ is a divisor of n.
Since $2, 3$ and 7 are divisors of n, then their product $2 \times 3 \times 7 = 42$ is a divisor of n.
The positive divisors of n are: $1, 2, 3, 6, 7, 14, 21$, and 42.
We are given that n has exactly 8 positive divisors including 1 and n, and so we have found them all, and thus $n = 42$.
The sum of these 8 positive divisors is $1 + 2 + 3 + 6 + 7 + 14 + 21 + 42 = 96$.

ANSWER: (D)

21. We begin by separating the given information, as follows:

- Kathy owns more cats than Alice
- Kathy owns more dogs than Bruce
- Alice owns more dogs than Kathy
- Bruce owns more cats than Alice

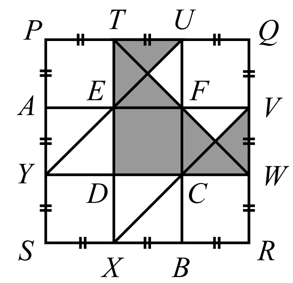
From bullets 2 and 3, we can conclude that Alice owns more dogs than both Kathy and Bruce.
From bullet 4, we can conclude that answer (A) is not true.
From bullets 1 and 4, we can conclude that both Kathy and Bruce own more cats than Alice.
However, we cannot determine if Kathy owns more cats than Bruce, or vice versa.
Therefore, we cannot conclude that (B) or (C) *must* be true.
From bullet 2, we can conclude that (E) is not true.
Thus the statement which *must* be true is (D).

ANSWER: (D)

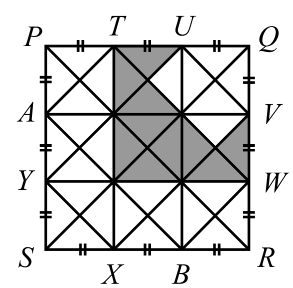
22. The single-digit divisors of 36 are: $1, 2, 3, 4, 6$, and 9.
The groups of 3 of these digits whose product is 36 are: $1, 4, 9$; $1, 6, 6$; $2, 2, 9$; $2, 3, 6$, and $3, 3, 4$.
Next, we count the number of ways to arrange each of these 5 groups of digits.
The digits $1, 4, 9$ can be arranged to form: $149, 194, 419, 491, 914, 941$.
The digits $1, 6, 6$ can be arranged to form: $166, 616, 661$.
The digits $2, 2, 9$ can be arranged to form: $229, 292, 922$.
The digits $2, 3, 6$ can be arranged to form: $236, 263, 326, 362, 623, 632$.
The digits $3, 3, 4$ can be arranged to form: $334, 343, 433$.
The number of 3-digit positive integers having digits whose product is 36 is $6+3+3+6+3 = 21$.

ANSWER: (A)

23. Begin by constructing AV perpendicular to TX and UB perpendicular to YW, as shown.



The four segments TX, UB, AV, and YW divide $PQRS$ into 9 identical squares.
Label the intersections of the perpendicular pairs of these four segments as points C, D, E, and F.
Segment UY is a diagonal of square $PUCY$ and so UY passes through E, the centre of square $PUCY$.
Segment UE is a diagonal of square $TUFE$.
Segment TW is a diagonal of square $TQWD$ and TF is a diagonal of square $TUFE$.
The diagonals in any square divide the square into 4 identical triangles.
For example, the diagonals TF and UE divide the square $TUFE$ into 4 identical triangles, 3 of which are shaded.
Similarly, we can show that diagonals FW and VC divide square $FVWC$ into 4 identical triangles, 3 of which are shaded.
We may construct the missing diagonals in each of the 9 squares, as shown.



These diagonals divide square $PQRS$ into $9 \times 4 = 36$ identical triangles.
Since 10 of these triangles are shaded, then $\frac{10}{36} = \frac{5}{18}$ of square $PQRS$ is shaded.

Answer: (A)

24. *Solution 1*
The ten moves have lengths 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
If the first move is vertical, then the five vertical moves have lengths 1, 3, 5, 7, 9 and the five horizontal moves have lengths 2, 4, 6, 8, 10.
If the first move is horizontal, then the five horizontal moves have lengths 1, 3, 5, 7, 9 and the five vertical moves have lengths 2, 4, 6, 8, 10.
If a horizontal move is to the right, then the length of the move is added to the x-coordinate.
If a horizontal move is to the left, then the length of the move is subtracted from the x-coordinate.
If a vertical move is up, then the length of the move is added to the y-coordinate.
If a vertical move is down, then the length of the move is subtracted from the y-coordinate.
Therefore, once the ten moves have been made, the change in one of the coordinates is a combination of adding and subtracting 1, 3, 5, 7, 9 and the change in the other coordinate is a combination of adding and subtracting 2, 4, 6, 8, 10.
For example, if the dot moves right 1, down 2, right 3, up 4, right 5, down 6, right 7, up 8, left 9, and up 10, then its final x-coordinate is

$$20 + 1 + 3 + 5 + 7 - 9 = 27$$

and its final y-coordinate is

$$19 - 2 + 4 - 6 + 8 + 10 = 33$$

making its final location $(27, 33)$, which is choice (A). This means that choice (A) is not the answer.
We note that, in the direction with the moves of even length, the final change in coordinate

will be even, since whenever we add and subtract even integers, we obtain an even integer.
In the other direction, the final change in coordinate will be odd, since adding or subtracting an odd number of odd integers results in an odd integer. (Since odd plus odd is even and odd minus odd is even, then after two moves of odd length, the change to date is even, and after four moves of odd length, the change to date is still even, which means that the final change after the fifth move of odd length is completed must be odd, since even plus or minus odd is odd.)
In the table below, these observations allow us to determine in which direction to put the moves of odd length and in which direction to put the moves of even length.

Choice	Change in x	Change in y	Horizontal moves	Vertical moves
(A) $(27, 33)$	7	14	$1+3+5+7-9=7$	$-2+4-6+8+10=14$
(B) $(30, 40)$	10	21	$2-4-6+8+10=10$	
(C) $(21, 21)$	1	2	$1-3+5+7-9=1$	$2+4-6-8+10=2$
(D) $(42, 44)$	22	25	$2-4+6+8+10=22$	$1+3+5+7+9=25$
(E) $(37, 37)$	17	18	$-1-3+5+7+9=17$	$2+4-6+8+10=18$

Since each of the locations (A), (C), (D), and (E) is possible, then the location that is not possible must be (B).
We note that in the case of (B), it is the change in the y-coordinate that is not possible to make.
In other words, we cannot obtain a total of 21 by adding and subtracting 1, 3, 5, 7, 9.
Can you see why?

Solution 2
Let a be the horizontal change from the initial to the final position of the point and let b be the vertical change from the initial to the final position.
For example, if $a=-5$ and $b=6$, the point's final position is 5 units to the left and 6 units up from its original position of $(20, 19)$.
Notice that if (x, y) is the final position of the point, then a and b can be calculated as $a+20=x$ or $a=x-20$ and $b+19=y$ or $b=y-19$.
First, assume the initial move is in the horizontal direction.
This means the second move will be in the vertical direction, the third will be horizontal, and so on.
In all, the first, third, fifth, seventh, and ninth moves will be horizontal and the others will be vertical.
Also, the first move is by one unit, the second is by two units, the third is by three units, and so on, so the horizontal moves are by 1, 3, 5, 7, and 9 units.
Each of these moves is either to the left or the right.
If all of the horizontal moves are to the right, then $a=1+3+5+7+9=25$.
If the point moves, left on the first move, right on the third, then left on the fifth, seventh, and ninth moves, a will be $-1+3-5-7-9=-19$.
In this case, the final position is 19 units to the left of the initial position (and has potentially moved up or down, as well).
In each of the five horizontal moves, the point either moves to the left or to the right.
This means there are two choices (left or right) for each of the five horizontal moves, so there are $2\times2\times2\times2\times2=32$ possible configurations similar to the examples above.
Going through these carefully, the table below computes all possible values of a:

$$\begin{array}{rcrcrcr}
1+3+5+7+9 & = & 25 & \qquad & 1+3+5+7-9 & = & 7 \\
-1+3+5+7+9 & = & 23 & & -1+3+5+7-9 & = & 5 \\
1-3+5+7+9 & = & 19 & & 1-3+5+7-9 & = & 1 \\
-1-3+5+7+9 & = & 17 & & -1-3+5+7-9 & = & -1 \\
1+3-5+7+9 & = & 15 & & 1+3-5+7-9 & = & -3 \\
-1+3-5+7+9 & = & 13 & & -1+3-5+7-9 & = & -5 \\
1-3-5+7+9 & = & 9 & & 1-3-5+7-9 & = & -9 \\
-1-3-5+7+9 & = & 7 & & -1-3-5+7-9 & = & -11 \\
1+3+5-7+9 & = & 11 & & 1+3+5-7-9 & = & -8 \\
-1+3+5-7+9 & = & 9 & & -1+3+5-7-9 & = & -9 \\
1-3+5-7+9 & = & 5 & & 1-3+5-7-9 & = & -13 \\
-1-3+5-7+9 & = & 3 & & -1-3+5-7-9 & = & -15 \\
1+3-5-7+9 & = & 1 & & 1+3-5-7-9 & = & -17 \\
-1+3-5-7+9 & = & -1 & & -1+3-5-7-9 & = & -19 \\
1-3-5-7+9 & = & -5 & & 1-3-5-7-9 & = & -23 \\
-1-3-5-7+9 & = & -7 & & -1-3-5-7-9 & = & -25
\end{array}$$

Some numbers appear more than once, but after inspecting the list, we see that when the first move is in the horizontal direction, a can be any odd number between -25 and 25 inclusive except for -21 and 21.

When the first move is horizontal, the second, fourth, sixth, eighth, and tenth moves will be vertical and have lengths 2, 4, 6, 8, and 10 units.
Using the same idea as the previous case, all 32 possible values of b when the first move is horizontal are calculated as follows:

$$\begin{array}{rcrcrcr}
2+4+6+8+10 & = & 30 & \qquad & 2+4+6+8-10 & = & 10 \\
-2+4+6+8+10 & = & 26 & & -2+4+6+8-10 & = & 6 \\
2-4+6+8+10 & = & 22 & & 2-4+6+8-10 & = & 2 \\
-2-4+6+8+10 & = & 18 & & -2-4+6+8-10 & = & -2 \\
2+4-6+8+10 & = & 18 & & 2+4-6+8-10 & = & -2 \\
-2+4-6+8+10 & = & 14 & & -2+4-6+8-10 & = & -6 \\
2-4-6+8+10 & = & 10 & & 2-4-6+8-10 & = & -10 \\
-2-4-6+8+10 & = & 6 & & -2-4-6+8-10 & = & -14 \\
2+4+6-8+10 & = & 14 & & 2+4+6-8-10 & = & -6 \\
-2+4+6-8+10 & = & 10 & & -2+4+6-8-10 & = & -10 \\
2-4+6-8+10 & = & 6 & & 2-4+6-8-10 & = & -14 \\
-2-4+6-8+10 & = & 2 & & -2-4+6-8-10 & = & -18 \\
2+4-6-8+10 & = & 2 & & 2+4-6-8-10 & = & -18 \\
-2+4-6-8+10 & = & -2 & & -2+4-6-8-10 & = & -22 \\
2-4-6-8+10 & = & -6 & & 2-4-6-8-10 & = & -26 \\
-2-4-6-8+10 & = & -10 & & -2-4-6-8-10 & = & -30
\end{array}$$

Again, some numbers appear more than once, but examination of the table shows that when the first move is horizontal, the possible values of b are the even numbers between -30 and 30 inclusive which are not multiples of 4.
We now begin to inspect the possible answers. The point in (A) is $(27, 33)$.
In this case, we have that $a = 27 - 20 = 7$ and $b = 33 - 19 = 14$.
These values for a and b fit the descriptions above, so $(27, 33)$ is a possible final position.

The point $(21, 21)$ in (C) has $a = 1$ and $b = 2$, which means $(21, 21)$ is also a possible final position for the point.
The point $(37, 37)$ in (E) has $a = 17$ and $b = 18$, so this point is also a possible final position.
We have shown that the answer must be either (B) or (D).
Note that in both cases we have that a is even and b is odd, which means that if either of these points is a possible final position, the first move cannot have been horizontal, which means it must have been vertical.
If the first move is vertical, then the third, fifth, seventh, and ninth moves are also vertical, and the other moves are horizontal.
Going through similar analysis as before, we will see that the restrictions on a and b have switched.
That is, a must be an even number between -30 and 30 inclusive which is not a multiple of 4, and b must be an odd number between -25 and 25 inclusive other than -21 and 21.
The point $(42, 44)$ in (D) has $a = 22$ and $b = 25$, which is possible with a vertical first move.
However, the point $(30, 40)$ in (B) has $a = 10$ and $b = 21$.
Since 21 is not a possible value for b, then $(30, 40)$ is not a possible final position.

Answer: (B)

25. The rectangular prism has two faces whose area is $8 \times 8 = 64$, and four faces each of whose area is $8 \times n$.
Therefore, A is $2 \times 64 + 4 \times 8 \times n = 128 + 32n$.
The prism is made up of $8 \times 8 \times n = 64 \times n$ cubes, each of which has dimensions $1 \times 1 \times 1$.
Each $1 \times 1 \times 1$ cube has surface area 6 because each has 6 faces which are all 1×1 squares.
Therefore, $B = 6 \times 64 \times n = 384n$.
Thus, we get

$$\frac{B}{A} = \frac{384n}{128 + 32n}.$$

This expression can be simplified by recognizing that each of $384, 128$ and 32 is divisible by 32.
After dividing the numerator and denominator by 32, we get $\frac{B}{A} = \frac{12n}{4+n}$. We require that $\dfrac{B}{A}$ be equal to some integer, so we will determine which integers $\dfrac{12n}{4+n}$ can be.
First, note that n is positive, so both $12n$ and $4+n$ are positive, which means $\dfrac{12n}{4+n}$ is positive.
This means $\dfrac{12n}{4+n}$ is a positive integer, so we determine which positive integers $\dfrac{12n}{4+n}$ can equal.
If $\dfrac{12n}{4+n} = 1$, then $12n = 4 + n$ which can be rearranged to give $11n = 4$ or $n = \frac{4}{11}$.
Since n must be an integer, we conclude that $\dfrac{12n}{4+n}$ cannot be equal to 1.
What if $\dfrac{12n}{4+n} = 2$? In this case, we need $12n$ to be twice as large as $4 + n$, or $12n = 8 + 2n$.
This can be rearranged to give $10n = 8$ or $n = \frac{8}{10}$.
Again, this value of n is not an integer, so we conclude that $\dfrac{12n}{4+n}$ is not 2.
Following this reasoning, if $\dfrac{12n}{4+n}$ is 3, we find that n must be $\frac{4}{3}$, which is also not an integer, so $\dfrac{12n}{4+n}$ is not equal to 3.
If $\dfrac{12n}{4+n}$ is equal to 4, we have that $12n$ is four times $4 + n$, or $12n = 16 + 4n$.

Rearranging this gives $8n = 16$ which means $n = 2$. Therefore, $\dfrac{12n}{4+n}$ can be 4, and it happens when $n = 2$.

We continue in this way for all possible positive integer values of $\dfrac{12n}{4+n}$ up to and including $\dfrac{12n}{4+n} = 11$.

The results are summarized in the table below.

$\frac{12n}{n+4}$	n	n is an integer
1	$\frac{4}{11}$	×
2	$\frac{8}{10}$	×
3	$\frac{4}{3}$	×
4	2	✓
5	$\frac{20}{7}$	×
6	4	✓
7	$\frac{28}{5}$	×
8	8	✓
9	12	✓
10	20	✓
11	44	✓

According to the table, $\dfrac{12n}{4+n}$ can be any of the integers $4, 6, 8, 9, 10$, and 11 and these occur when n is equal to $2, 4, 8, 12, 20$, and 44, respectively.

We now consider what happens when $\dfrac{12n}{4+n}$ is 12 or greater.

If $\dfrac{12n}{4+n} = 12$, then $12n$ is 12 times as large as $4 + n$, or $12n = 48 + 12n$.

Since $48 + 12n$ is always greater than $12n$, there is no value of n for which $12n = 48 + 12n$.

Similarly, since n is a positive integer, there is no value of n for which $\dfrac{12n}{4+n}$ is 13 or greater.

We conclude that the only possible positive integer values of $\dfrac{12n}{4+n}$ are those in the table, so the only values of n which make $\dfrac{12n}{4+n}$ an integer are $2, 4, 8, 12, 20$, and 44.

The sum of these numbers is $2 + 4 + 8 + 12 + 20 + 44 = 90$.

Answer: (B)

Grade 8

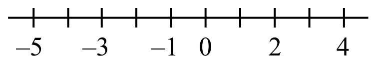
1. As a percentage, one half of a muffin is equivalent to $\frac{1}{2} \times 100\% = 50\%$ of a muffin.

   Answer: (E)

2. The sum of the angles in a triangle is 180°.
   Thus $x + x + x = 180$ or $3x = 180$, and so $x = \frac{180}{3} = 60$.

   Answer: (B)

3. Place zero and the given answers in their correct locations on the number line, as shown.
   The integer closest to 0 is -1.

   Answer: (A)

4. A number gives a remainder of 3 when divided by 5 if it is 3 more than a multiple of 5.
   Since 88 is 3 more than 85, and 85 is a multiple of 5, then 88 gives a remainder of 3 when divided by 5.
   We may check that this is the only answer which gives a remainder of 3 when divided by 5.

   Answer: (D)

5. Recall that a prime number is an integer greater than 1 whose only divisors are 1 and itself.
   The prime numbers between 10 and 20 are: $11, 13, 17$, and 19.
   Thus, there are 4 integers between 10 and 20 which are prime numbers.

   Answer: (E)

6. Reading from the graph, 15 vehicles had an average speed from 80 km/h to 89 km/h, 30 vehicles had an average speed from 90 km/h to 99 km/h, and 5 vehicles had an average speed from 100 km/h to 109 km/h.
   The average speed of each of the remaining vehicles was less than 80 km/h.
   The number of vehicles that had an average speed of at least 80 km/h was $15 + 30 + 5 = 50$.

   Answer: (E)

7. Any positive integer that is divisible by both 3 and 7 is divisible by their product, $3 \times 7 = 21$.
   That is, we are being asked to count the number of positive integers less than 100 that are multiples of 21.
   These integers are: $21, 42, 63$, and 84.
   We note that the next multiple of 21 is $21 \times 5 = 105$, which is greater than 100.
   There are 4 positive integers less than 100 that are divisible by both 3 and 7.

   Answer: (C)

8. The circumference, C, of a circle is given by the formula $C = \pi \times d$, where d is the diameter of the circle.
   If the circumference is 100, then $100 = \pi \times d$ and so $d = \dfrac{100}{\pi}$.

   Answer: (C)

9. The area, A, of a triangle is given by the formula $A = \dfrac{b \times h}{2}$, where b is the length of the base and h is the perpendicular height of the triangle.
   If the area of a triangle is 6, then substituting we get $6 = \dfrac{b \times h}{2}$, and so $b \times h = 12$.
   Consider the base, b, of the triangle to be PQ and so $b = 4$.
   Since $b \times h = 12$ and $b = 4$, then the perpendicular height, h, is equal to 3.
   The point which lies a perpendicular distance of 3 units above PQ is E.

   Answer: (E)

10. Barry's 12 coins include at least one of each of the 5 coins of different values.
The total value of these 5 coins is $\$2.00 + \$1.00 + \$0.25 + \$0.10 + \$0.05 = \$3.40$.
Barry has the smallest total amount of money that he could have if each of his remaining $12 - 5 = 7$ coins has value \$0.05 (the smallest possible value of a coin).
Thus, the smallest total amount of money that Barry could have is

$$\$3.40 + 7 \times \$0.05 = \$3.40 + \$0.35 = \$3.75.$$

Answer: (A)

11. An isosceles triangle has two sides of equal length.
Since we are given that two of the side lengths in an isosceles triangle are 6 and 8, then it is possible that the three side lengths are 6, 6 and 8.
In this case, the perimeter of the triangle is $6 + 6 + 8 = 20$, but this is not one of the given answers.
The only other possibility is that the side lengths are $6, 8$ and 8.
In this case, the perimeter is $6 + 8 + 8 = 22$.

Answer: (C)

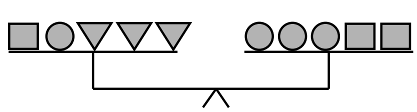
12. The angles measuring 60° and $(y+5)^\circ$ lie along the line segment PQ, and so they add to 180°.
Thus, $60 + y + 5 = 180$ or $y + 65 = 180$, and so $y = 180 - 65 = 115$.
The angles measuring $(4x)^\circ$ and $(y+5)^\circ$ are opposite angles and so $4x = y + 5$.
Since $y = 115$, we get $4x = 120$ and so $x = 30$.
Therefore, $x + y = 30 + 115 = 145$.

Answer: (A)

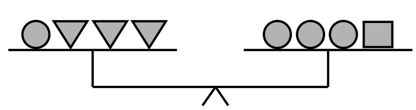
13. For the five numbers $12, 9, 11, 16, x$ to have a unique mode, x must equal one of the existing numbers in the list: $12, 9, 11$, or 16.
If $x = 9$, then the mode is 9, but the mean of the five numbers $9, 9, 11, 12, 16$ is greater than 9.
If $x = 16$, then the mode is 16, but the mean of the five numbers $9, 11, 12, 16, 16$ is less than 16.
Therefore, x must either be equal to 11 or 12.
If $x = 11$, the mean of the numbers is $\frac{9+11+11+12+16}{5} = \frac{59}{5}$ which is not equal to the mode 11.
If $x = 12$, the mean of the numbers is $\frac{9+11+12+12+16}{5} = \frac{60}{5} = 12$ which is equal to the mode 12.
And finally we confirm that if $x = 12$, then the median (the middle number in the ordered list $9, 11, 12, 12, 16$) is also equal to 12.

Answer: (C)

14. The first scale shows that ●●● has the same mass as ▼▼.
The second scale shows that ■●▼ has the same mass as ■■.
Therefore, the sum of the masses of ▼▼ and ■●▼ is equal to the sum of the masses of ●●● and ■■.
That is, the equal-arm scale shown below is balanced.



The left and right sides of this scale each contain a ■, and the scale remains balanced if one ■ is removed from each side of the scale (since they are equal in mass).
That is, the equal-arm scale shown below is balanced.



Therefore of the answers given, ●▼▼▼ has the same mass as ●●●■.

Answer: (D)

15. *Solution 1*

After the arrow is spun twice, the possible outcomes are: red, red; red, blue; red, green; blue, red; blue, blue; blue, green, green, red; green, blue; green, green.
That is, there are 9 possible outcomes.
Of these 9 outcomes, 3 have the same colour appearing twice: red, red; blue, blue; green, green.
The probability that the arrow lands on the same colour twice is $\frac{3}{9} = \frac{1}{3}$.

Solution 2

First, we count the total number of possible outcomes given that the arrow is spun twice.
When the arrow is spun the first time, there are 3 possible outcomes (red, blue, green).
When the arrow is spun the second time, there are again 3 possible outcomes.
Thus when the arrow is spun twice, there are $3 \times 3 = 9$ possible outcomes.
Next, we count the number of outcomes in which the arrow lands on the same colour twice.
When the arrow is spun the first time, there are 3 possible outcomes (red, blue, green).
When the arrow is spun the second time, there is 1 possible outcome (since the colour must match the colour that the arrow landed on after the first spin).
Thus when the arrow is spun twice, there are $3 \times 1 = 3$ possible outcomes in which the arrow lands on the same colour twice.
The probability that the arrow lands on the same colour twice is $\frac{3}{9} = \frac{1}{3}$.

Solution 3

The first spin will land on one of the three colours.
For the arrow to land on the same colour twice, the second spin must land on the colour matching the first spin.
Since exactly 1 of the 3 colours matches the first spin, the probability that the arrow lands on the same colour twice is $\frac{1}{3}$.

Answer: (D)

16. The lightbulb is used for exactly 2 hours every day and it will work for 24 999 hours.
Thus, the lightbulb will work for $24\,999 \div 2 = 12\,499.5$ days, which means that it will stop working on the $12\,500^{th}$ day.
How many weeks are there in 12 500 days?
Since $12\,500 \div 7 \approx 1785.71$, there are between 1785 and 1786 weeks in 12 500 days.
Specifically, $12\,500 - 7 \times 1785 = 12\,500 - 12\,495 = 5$, and so there are 1785 complete weeks and 5 additional days in 12 500 days.
The lightbulb starts being used on a Monday and so the last day of 1785 complete weeks will be a Sunday.
The lightbulb will stop working 5 days past Sunday, which is a Friday.

Answer: (B)

17. Since $w + x = 45$ and $x + y = 51$, then $x + y$ is 6 more than $w + x$ (since 51 is 6 more than 45).
Both $x + y$ and $w + x$ contain x, and so the difference between these two expressions, 6, must be the difference between y and w.
That is, y is 6 more than w (or $y = 6 + w$).
In the final equation given, $y + z = 28$, y is 6 more than w, and so 6 more than w added to z equals 28.
Since $6 + 22 = 28$, then $w + z = 22$.

Answer: (B)

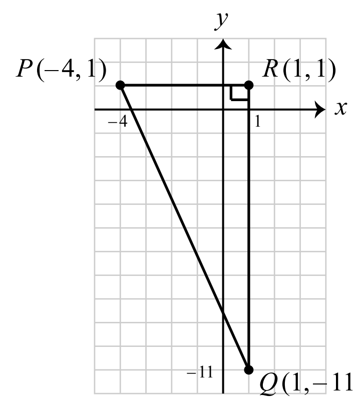
18. We begin by separating the given information, as follows:

- Kathy owns more cats than Alice
- Kathy owns more dogs than Bruce
- Alice owns more dogs than Kathy
- Bruce owns more cats than Alice

From bullets 2 and 3, we can conclude that Alice owns more dogs than both Kathy and Bruce.
From bullet 4, we can conclude that answer (A) is not true.
From bullets 1 and 4, we can conclude that both Kathy and Bruce own more cats than Alice.
However, we cannot determine if Kathy owns more cats than Bruce, or vice versa.
Therefore, we cannot conclude that (B) or (C) *must* be true.
From bullet 2, we can conclude that (E) is not true.
Thus the statement which *must* be true is (D).

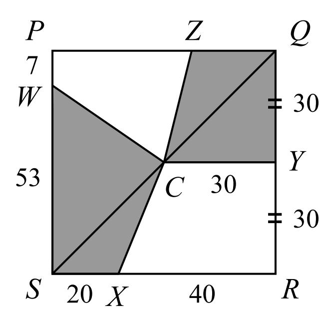
ANSWER: (D)

19. The horizontal line through P intersects the vertical line through Q at $R(1,1)$, as shown.
Joining P, Q, R creates right-angled $\triangle PQR$, with hypotenuse PQ.
The x-coordinate of P is -4 and the x-coordinate of R is 1, and so PR has length $1-(-4)=5$ (since P and R have equal y-coordinates).
The y-coordinate of Q is -11 and the y-coordinate of R is 1, and so QR has length $1-(-11)=12$ (since Q and R have equal x-coordinates).
Using the Pythagorean Theorem, $PQ^2 = PR^2 + QR^2$ or $PQ^2 = 5^2 + 12^2 = 25 + 144 = 169$, and so $PQ = \sqrt{169} = 13$ (since $PQ > 0$).
(Alternatively, we could have drawn the vertical line through P and the horizontal line through Q which meet at $(-4,-11)$.)



ANSWER: (A)

20. *Solution 1*
Square $PQRS$ has side length 60, and so its area is $60^2 = 3600$.
We require the total area of the shaded regions to equal the total area of the non-shaded regions, and so the total area of the shaded regions is to be half the area of the square, or 1800.
Since C is the centre of the square, its distance to each of the sides of the square is $\frac{1}{2}(60) = 30$.
To determine the area of each of the shaded regions, we begin by labelling each of the given lengths (as well as those that we can determine), and constructing diagonal SQ, as shown.



Quadrilateral $WCXS$ is made up of two triangles: $\triangle SCX$ and $\triangle SCW$.
In $\triangle SCX$, the base $SX = 20$ and the perpendicular height from C to SR is 30 (since C is the centre of the square $PQRS$).
Thus, the area of $\triangle SCX$ is $\frac{1}{2}(20)(30) = 300$.
In $\triangle SCW$, the base $SW = 53$ and the perpendicular height from C to SW is 30.
The area of $\triangle SCW$ is $\frac{1}{2}(53)(30) = 795$.

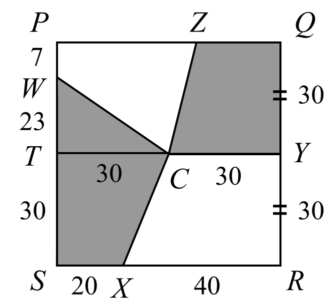
Therefore, the area of quadrilateral $WCXS$ is $300 + 795 = 1095$.

Quadrilateral $ZQYC$ is made up of two triangles: $\triangle CQY$ and $\triangle CQZ$.
The midpoint of QR is Y and C is the centre of the square, so CY is perpendicular to QY.
In $\triangle CQY$, the base $CY = 30$, the perpendicular height $YQ = 30$, and so the area of $\triangle CQY$ is $\frac{1}{2}(30)(30) = 450$.
In $\triangle CQZ$, if the base is ZQ, then the perpendicular height from C to PQ is 30.
The area of $\triangle CQZ$ is $\frac{1}{2}(ZQ)(30) = 15(ZQ)$.
Therefore, the area of quadrilateral $ZQYC$ is $15 \times ZQ + 450$.

Finally, adding the areas of the shaded regions, we get $1095 + 15 \times ZQ + 450 = 1800$ or $15 \times ZQ = 1800 - 1095 - 450 = 255$, and so $ZQ = \frac{255}{15} = 17$.

Solution 2

As in Solution 1, we require the total area of the shaded regions to equal 1800.
We begin by labelling T, the midpoint of side PS.
Since Y is the midpoint of QR, then TY passes through C and is perpendicular to each of PS and QR.
Since C is the centre of the square, its distance to each of the sides of the square is $\frac{1}{2}(60) = 30$.
We label each of the given lengths (as well as those that we can determine), as shown.



Quadrilateral $WCXS$ is made up of $\triangle WCT$ and trapezoid $TCXS$ (TC is parallel SX and thus $TCXS$ is a trapezoid).
In $\triangle WCT$, base $WT = WS - TS = 53 - 30 = 23$ and the perpendicular height CT is 30.
Thus, the area of $\triangle WCT$ is $\frac{1}{2}(23)(30) = 345$.
In trapezoid $TCXS$, the parallel sides have lengths $TC = 30$ and $SX = 20$, and the perpendicular height is $TS = 30$.
The area of trapezoid $TCXS$ is $\frac{1}{2}(30)(30 + 20) = 750$.
Therefore, the area of quadrilateral $WCXS$ is $345 + 750 = 1095$.

Quadrilateral $ZQYC$ is also a trapezoid (since ZQ is parallel to CY).
The area of trapezoid $ZQYC$ is $\frac{1}{2}(QY)(ZQ + 30) = 15(ZQ + 30)$.

Adding the areas of the shaded regions, we get $1095 + 15(ZQ + 30) = 1800$ or $15(ZQ + 30) = 1800 - 1095 = 705$ or $ZQ + 30 = \frac{705}{15} = 47$, and so $ZQ = 17$.

ANSWER: (E)

21. Let the number of teams in Jen's baseball league be n.
Each of these n teams plays 6 games against each of the $n - 1$ other teams in the league.
Since there are 2 teams in each of these games, the total number of games played is $\dfrac{6n(n-1)}{2}$.
The total number of games played is 396, so $\dfrac{6n(n-1)}{2} = 396$ or $3n(n - 1) = 396$ or $n(n - 1) = 132$.
The numbers n and $n - 1$ differ by 1, and so we are looking for two consecutive positive integers whose product is 132.
Since $12 \times 11 = 132$, the number of teams in Jen's league is 12.

ANSWER: (A)

22. Let Rich's 4-digit positive integer be $abcd$, where d is the units (ones) digit, c is the tens digit, b is the hundreds digit, and a is the thousands digit of the original number .
We begin by determining which of the 4 digits is erased.
If Rich erases the digit a, then the 3-digit integer that remains is bcd, and the sum of the two integers is $abcd + bcd$.
The units digit of this sum is determined by adding the units digits of $abcd$ and bcd, or $d+d = 2d$.
In this case, the units digit of the sum is even since $2d$ is even.
The sum of these two integers is 6031 which has an odd units digit, and so the digit that is erased cannot be a.
Using a similar argument, the digit that is erased cannot be b or c, and thus must be d.
Next, we determine the digits a, b, c, d so that

$$\begin{array}{rr} & a\,bcd \\ + & abc \\ \hline & 6\,031 \end{array}$$

The hundreds column of this sum gives that $b + a$ added to the 'carry' from the tens column has units digit 0. That is, this sum is either 0, 10 or 20.
If the sum is 0, then $a = 0$, but looking to the thousands column we see that a cannot be 0.
If the sum is 20, then $a + b = 18$ (since any digit can be at most 9 and the carry from the tens column can be at most 2).
If $a + b = 18$, then $a = 9$, but looking to the thousands column we see that a cannot be 9.
Therefore, $b + a$ added to the carry from the tens column is 10, and so the carry from the hundreds column to the thousands column is 1.
The thousands column then gives $a + 1 = 6$ and so $a = 5$.

$$\begin{array}{rr} & 5\,bcd \\ + & 5bc \\ \hline & 6\,031 \end{array}$$

The hundreds column gives $b + 5$ added to the carry from the tens column is 10.
If the carry from the tens column is 0, then $b + 5 = 10$ and so $b = 5$.
In the tens column, if $b = 5$ then $c + b = c + 5$ is greater than 3 and so must be 13.
This tells us that the carry from the tens colunn cannot be 0.
If the carry from the tens column is 1, then $b + 5 + 1 = 10$ and so $b = 4$.
(Note that sum of the tens column has units digit 3 and so the carry cannot be 2.)

$$\begin{array}{rr} & 5\,4cd \\ + & 54c \\ \hline & 6\,031 \end{array}$$

The units digit of the sum in the tens column is 3, and so $c + 4$ added to the carry from the units column is 13 (it can't be 3 or 23).
If there is no carry from the units column, then $c + 4 = 13$ and so $c = 9$.
But if $c = 9$ then the units column gives that $d + 9$ must be 11 and so there is carry from the units column. Hence c is not 9.
If the carry from the units column is 1, then $c + 4 + 1 = 13$ and so $c = 8$ and $d = 3$.
The final sum is shown here:

$$\begin{array}{rr} & 5\,483 \\ + & 548 \\ \hline & 6\,031 \end{array}$$

The sum of the digits of the original 4-digit number is $a + b + c + d = 5 + 4 + 8 + 3 = 20$.

Answer: (B)

23. We begin by recognizing that each of the given answers has a common numerator of (20!)(19!). Since 20! is equal to the product of the integers from 1 to 20 inclusive, we can consider that 20! is equal to the product of the integers from 1 to 19 inclusive, multiplied by 20.
However, the product of the integers from 1 to 19 inclusive is equal to 19!, and so $20! = 19! \times 20$.
That is, the common numerator (20!)(19!) can be rewritten as $(19! \times 20)(19!)$ or $(19!)^2 \times 20$.
Next, we consider the result after dividing $(19!)^2 \times 20$ by each of the denominators:

$$\frac{(20!)(19!)}{1} = \frac{(19!)^2 \times 20}{1} = (19!)^2 \times 20$$

$$\frac{(20!)(19!)}{2} = \frac{(19!)^2 \times 20}{2} = (19!)^2 \times 10$$

$$\frac{(20!)(19!)}{3} = \frac{(19!)^2 \times 20}{3} = (19!)^2 \times \frac{20}{3}$$

$$\frac{(20!)(19!)}{4} = \frac{(19!)^2 \times 20}{4} = (19!)^2 \times 5$$

$$\frac{(20!)(19!)}{5} = \frac{(19!)^2 \times 20}{5} = (19!)^2 \times 4$$

Since $(19!)^2$ is the square of an integer (19!), it is a perfect square.
The product of a perfect square and some positive integer factor f, is equal a perfect square only if f is a perfect square.
An answer satisfying this condition is (E).
Why is $(19!)^2 \times 4$ a perfect square?
Rewriting, $(19!)^2 \times 4 = (19!)^2 \times 2^2 = (19! \times 2)^2$, which is the square of the integer $19! \times 2$ and thus is a perfect square.
Can you explain why each of the other four answers is not equal to a perfect square?

ANSWER: (E)

24. First, notice that the given list of 10 numbers has a sum of $0+1+2+3+4+5+6+7+8+9 = 45$.
If n is the number of groups and m is the total in each group, then we must have $mn = 45$.
This means 45 is a multiple of the number of groups.
We also require that the number of groups be at least 2, so the number of groups is one of $3, 5, 9, 15, 45$.
If the number of groups is 9, then the total in each group is $\frac{45}{9} = 5$, but this is not possible since one of the groups must contain 9 and therefore cannot have a sum of 5.
Similarly, if the number of groups is 15 or 45, the sum of each group must be 3 or 1 respectively, which is too small.
Therefore, there are either 5 groups or 3 groups.

Let's first assume that there are five groups.
In this case, the total in each group must be $\frac{45}{5} = 9$.
Since 0 does not contribute anything to the sum, we will ignore it for now.
Since the total in each group must be 9, we are forced to have 9 in a group by itself.
The number 8 must be paired with 1 since adding any other positive integer to 8 will give a sum greater than 9.
That is, one of the groups has the numbers 1 and 8. We will denote this group by $\{1, 8\}$.
The number 7 cannot be in a group with any number larger than 2, and 1 is already paired with 8, so another group must be $\{2, 7\}$.
Continuing with this reasoning, we get that $\{3, 6\}$ must be a group, and $\{4, 5\}$ must also be a group.

Therefore, if there are 5 groups, they must be $\{9\}, \{1, 8\}, \{2, 7\}, \{3, 6\}$, and $\{4, 5\}$.
As mentioned before, 0 does not contribute anything to the sum, so it can be placed in any of the 5 groups without changing the sum.
There are 5 ways to do this: $\{0, 9\}, \{1, 8\}, \{2, 7\}, \{3, 6\}, \{4, 5\}$, or $\{9\}, \{0, 1, 8\}, \{2, 7\}, \{3, 6\}, \{4, 5\}$, and so on.
We have shown that there are 5 ways to separate the numbers 0 through 9 into 5 groups so that each group has the same sum.

Let's now assume that there are three groups.
As we did when there were 5 groups, we ignore 0 for now.
Since there are three groups, the sum in each group is $\frac{45}{3} = 15$.
We now find all groups which add up to 15.
There are 17 of them, so we will label them using A through Q:

A	$\{6, 9\}$	G	$\{2, 5, 8\}$	M	$\{1, 3, 4, 7\}$
B	$\{1, 5, 9\}$	H	$\{3, 4, 8\}$	N	$\{4, 5, 6\}$
C	$\{2, 4, 9\}$	I	$\{1, 2, 4, 8\}$	O	$\{1, 3, 5, 6\}$
D	$\{1, 2, 3, 9\}$	J	$\{2, 6, 7\}$	P	$\{2, 3, 4, 6\}$
E	$\{7, 8\}$	K	$\{3, 5, 7\}$	Q	$\{1, 2, 3, 4, 5\}$
F	$\{1, 6, 8\}$	L	$\{1, 2, 5, 7\}$		

Of these 17 groups, only A, B, C, and D contain the number 9.
Therefore, any way of separating the integers into three groups must use exactly one of these four groups.

Assume $A = \{6, 9\}$ is one of the groups. This means the other two groups do not contain 6 or 9.
The groups which lack both 6 and 9 are E, G, H, I, K, L, M, and Q.
If $E = \{7, 8\}$ is one of the groups, we will have used $6, 7, 8$, and 9, so the remaining group must be $Q = \{1, 2, 3, 4, 5\}$.
If $G = \{2, 5, 8\}$ is one of the groups, we will have used $2, 5, 6, 8$, and 9.
This forces the third group to be $M = \{1, 3, 4, 7\}$.
Similarly, if H is one of the groups, so is L, and if I is one of the groups, so is K.
At this point, we can stop checking.
This is because, for example, if we assume M is one of the groups, we are forced to take G as the third group, but we have already identified the case where we separate into A, M, and G.
To summarize, if A is one of the groups, the possible configurations are

$$A, E, Q \qquad A, G, M \qquad A, H, L \qquad A, I, K$$

If we assume B is one of the groups, the other two groups must not contain any of 1, 5, and 9.
The groups satisfying this condition are E, H, J, and P.
By the same reasoning as in the previous paragraph, the configurations including B are

$$B, E, P \qquad B, H, J$$

The groups which have no members in common with C are E, F, K, and O, and the configurations which use C are

$$C, E, O \qquad C, F, K$$

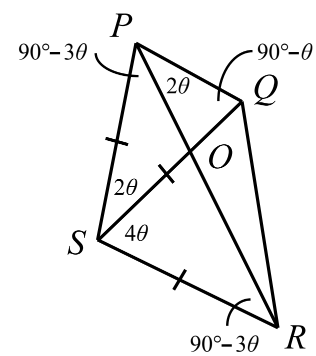
The only groups which have no members in common with D are E and N, so the only configuration using D is

$$D, E, N.$$

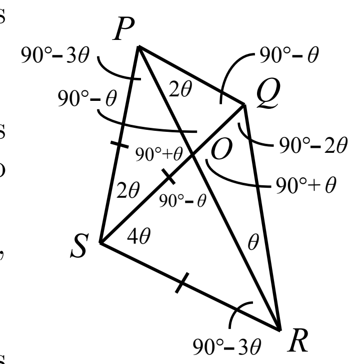
In total, we have found that there are 9 possible ways to separate the numbers from 1 through 9 into three groups each having the same sum.
As before, placing 0 in a group has no effect on the sum of that group. Since there are three groups, there are three possible ways to include 0 for each configuration, so there are $9 \times 3 = 27$ ways to separate the integers from 0 to 9 into three groups where each group has the same sum.
Adding this to the five from earlier, the number of ways to separate the list $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ into at least two groups so that the sum of the numbers in each group is the same is $27 + 5 = 32$.

ANSWER: (E)

25. We begin by labeling $\angle QSP = 2\theta$. Since $\angle QSR = 2\angle QSP$, $\angle QSR = 2 \times 2\theta = 4\theta$.
We will now find the measures of all 12 angles in terms of θ.
Let $x = \angle SPR$. Since $\triangle SPR$ has $SP = SR$, it is isosceles which means $\angle SRP = \angle SPR = x$.
Also, $\angle PSR = 2\theta + 4\theta = 6\theta$, so $180^\circ = 6\theta + x + x$.
Rearranging this equation, we have $2x = 180^\circ - 6\theta$, so $x = 90^\circ - 3\theta$.
In $\triangle SPQ$, we know $\angle SPQ + \angle SQP + \angle PSQ = 180^\circ$, so rearranging and using $\angle PSQ = 2\theta$ gives $\angle SPQ + \angle SQP = 180^\circ - 2\theta$.



Since $SP = SR$, $\triangle SPQ$ is isosceles which means $\angle SPQ = \angle SQP$.
Substituting this into the above equation, we have $2\angle SQP = 180^\circ - 2\theta$ or $\angle SQP = 90^\circ - \theta$.
Note that this means $\angle SPQ = 90^\circ - \theta$ as well, so $\angle SPR + \angle RPQ = 90^\circ - \theta$.
Substituting $\angle SPR = 90^\circ - 3\theta$, we get that $90^\circ - 3\theta + \angle RPQ = 90^\circ - \theta$, so $\angle RPQ = 2\theta$.
Since $\angle POQ + \angle PQO + \angle OPQ = 180^\circ$, substitution gives $\angle POQ + (90^\circ - \theta) + 2\theta = 180^\circ$.
Solving this, we get $\angle POQ = 90^\circ - \theta$.
Angles $\angle POQ$ and $\angle ROS$ are opposite, which means $\angle ROS = \angle POQ = 90^\circ - \theta$. Also, $\angle POQ + \angle QOR = 180^\circ$, so $\angle QOR = 180^\circ - \angle POQ = 180^\circ - (90^\circ - \theta) = 90^\circ + \theta$.
Since $\angle POS$ and $\angle ROQ$ are opposite, they are equal, so $\angle POS = 90^\circ + \theta$.
We now set $\angle SQR = y$.
Since $SQ = SR$, $\triangle SQR$ is isosceles, which means $\angle SRQ = \angle SQR = y$.



Substituting this and $\angle QSR = 4\theta$ into $\angle QSR + \angle SRQ + \angle SQR = 180^\circ$ gives $4\theta + 2y = 180^\circ$, so $2y = 180^\circ - 4\theta$ or $y = 90^\circ - 2\theta$.
Finally, we have $\angle ROQ + \angle OQR + \angle QRO = 180^\circ$, so $\angle QRO = 180^\circ - (90^\circ + \theta) - (90^\circ - 2\theta) = \theta$.

In summary, the 12 angles in terms of θ are

$$\begin{aligned}
\angle QSP = \angle RPQ &= 2\theta \\
\angle QSR &= 4\theta \\
\angle SRP = \angle RPS &= 90^\circ - 3\theta \\
\angle QRP &= \theta \\
\angle RQS &= 90^\circ - 2\theta \\
\angle SQP = \angle POQ = \angle ROS &= 90^\circ - \theta \\
\angle POS = \angle QOR &= 90^\circ + \theta
\end{aligned}$$

We are given that the measure of each angle in degrees is an integer.
In particular, since $\angle QRP = \theta$, we get that θ must be an integer number of degrees.
We also know that PR and QS intersect inside $PQRS$, so it must be that $6\theta = \angle PSR < 180^\circ$.
This means $\theta < 30^\circ$.

Since 4θ is even (and greater than 2), it cannot be a prime number for any integer θ.
The integers 2θ and 3θ are only prime numbers when $\theta = 1^\circ$.
In this case, the twelve angles are

$$2^\circ, 2^\circ, 4^\circ, 87^\circ, 87^\circ, 1^\circ, 88^\circ, 89^\circ, 89^\circ, 89^\circ, 91^\circ, 91^\circ$$

of which five are prime numbers: two copies of 2° and three copies of 89°.
Therefore, $\theta \neq 1^\circ$.

By factoring, we have that $90^\circ - 2\theta = 2(45^\circ - \theta)$ which is even.
Therefore, this number can only be prime when $45^\circ - \theta = 1$ or $\theta = 44^\circ$.
We know that $\theta < 30^\circ$, so $90^\circ - 2\theta$ must be composite.
Similarly, $90^\circ - 3\theta = 3(30^\circ - \theta)$ can only be a prime number when $30^\circ - \theta = 1^\circ$ or $\theta = 29^\circ$.
When $\theta = 29^\circ$, the twelve angles are

$$58^\circ, 58^\circ, 116^\circ, 3^\circ, 3^\circ, 29^\circ, 32^\circ, 61^\circ, 61^\circ, 61^\circ, 119^\circ, 119^\circ$$

and exactly 6 of these are prime: 29°, the two copies of 3°, and the three copies of 61°.

When $\theta < 30^\circ$ is different from 1° and 29°, each of the six angles

$$2\theta, 4\theta, 90^\circ - 3\theta, 90^\circ - 2\theta, 2\theta, 90^\circ - 3\theta$$

is composite.
In order to satisfy the condition that the measures of exactly 6 of the angles in degrees is a prime number, we therefore require that each of θ, $90^\circ - \theta$, and $90^\circ + \theta$ is prime.
The prime numbers less than 30 are $2, 3, 5, 7, 11, 13, 17, 19, 23$, and 29.
We have already investigated what happens when $\theta = 29^\circ$, so the values for the angles $\theta, 90^\circ + \theta$, and $90^\circ - \theta$ in the other cases are calculated in the table:

θ	$90^\circ - \theta$	$90^\circ + \theta$	All 3 angles prime?
2°	88°	92°	×
3°	87°	93°	×
5°	85°	95°	×
7°	83°	97°	✓
11°	79°	101°	✓
13°	77°	103°	×
17°	73°	107°	✓
19°	71°	109°	✓
23°	67°	113°	✓

Of the angles that appear in the second two columns, the numbers which are prime are

$$67^\circ, 71^\circ, 73^\circ, 79^\circ, 83^\circ, 97^\circ, 101^\circ, 103^\circ, 107^\circ, 109^\circ, 113^\circ.$$

Therefore, all three angles are prime when θ is one of $7^\circ, 11^\circ, 17^\circ, 19^\circ, 23^\circ$.
Combining these with $\theta = 29^\circ$ from earlier, this gives a total of 6 quadrilaterals with the given properties.

Answer: (D)