The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Wednesday, April 12, 2017
(in North America and South America)

Thursday, April 13, 2017
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by 
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by 
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions supplied by your supervising teacher must be inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked  , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked  , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. A store sells packages of red pens and packages of blue pens. Red pens are sold only in packages of 6 pens. Blue pens are sold only in packages of 9 pens.


(a) Igor bought 5 packages of red pens and 3 packages of blue pens. How many pens did he buy altogether?


(b) Robin bought 369 pens. She bought 21 packages of red pens. How many packages of blue pens did she buy?


(c) Explain why it is not possible for Susan to buy exactly 31 pens.

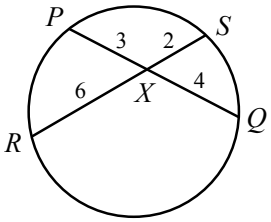
2. By finding a common denominator, we see that $\frac{1}{3}$ is greater than $\frac{1}{7}$ because $\frac{7}{21} > \frac{3}{21}$. Similarly, we see that $\frac{1}{3}$ is less than $\frac{1}{2}$ because $\frac{2}{6} < \frac{3}{6}$.


(a) Determine the integer n so that $\frac{n}{40}$ is greater than $\frac{1}{5}$ and less than $\frac{1}{4}$.


(b) Determine all possible integers m so that $\frac{m}{8}$ is greater than $\frac{1}{3}$ and $\frac{m+1}{8}$ is less than $\frac{2}{3}$.


(c) Fiona calculates her *win ratio* by dividing the number of games that she has won by the total number of games that she has played. At the start of a weekend, Fiona has played 30 games, has w wins, and her win ratio is greater than 0.5. During the weekend, she plays five games and wins three of these games. At the end of the weekend, Fiona's win ratio is less than 0.7. Determine all possible values of w.

3. When two chords intersect each other inside a circle, the products of the lengths of their segments are equal. That is, when chords PQ and RS intersect at X, $(PX)(QX) = (RX)(SX)$.



(a) In Figure A below, chords DE and FG intersect at X so that $EX = 8$, $FX = 6$, and $GX = 4$. What is the length of DX?

(b) In Figure B, chords JK and LM intersect at X so that $JX = 8y$, $KX = 10$, $LX = 16$, and $MX = y + 9$. Determine the value of y.

(c) In Figure C, chord ST intersects chords PQ and PR at U and V, respectively, so that $PU = m$, $QU = 5$, $RV = 8$, $SU = 3$, $UV = PV = n$, and $TV = 6$. Determine the values of m and n.

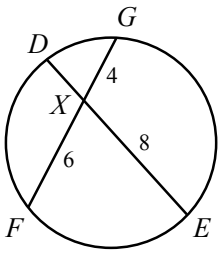


Figure A

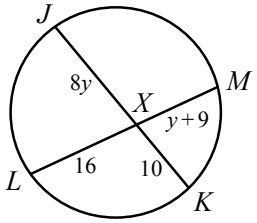


Figure B

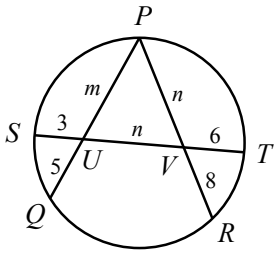


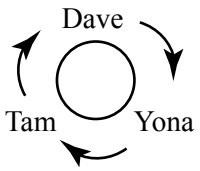
Figure C

4. Three students sit around a table. Each student has some number of candies. They share their candies using the following procedure:

- Step 1: Each student with an odd number of candies discards one candy. Students with an even number of candies do nothing.
- Step 2: Each student passes half of the candies that they had after Step 1 clockwise to the person beside them.
- Step 1 and Step 2 are repeated until each of the three students has an equal number of candies. The procedure then ends.

On Monday, Dave, Yona and Tam start with 3, 7 and 10 candies, respectively. After Step 1 and Step 2, the number of candies that each student has is given in the following table:

	Dave	Yona	Tam
Start	3	7	10
After Step 1	2	6	10
After Step 2	6	4	8



(a) When the procedure in the example above is completed, how many candies does each student have when the procedure ends?

(b) On Tuesday, Dave starts with 16 candies. Each of Yona and Tam starts with zero candies. How many candies does each student have when the procedure ends?

(c) On Wednesday, Dave starts with $2n$ candies. Each of Yona and Tam starts with $2n+3$ candies. Determine, with justification, the number of candies in terms of n that each student has when the procedure ends.

(d) On Thursday, Dave starts with 2^{2017} candies. Each of Yona and Tam starts with zero candies. Determine, with justification, the number of candies that each student has when the procedure ends.

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

For students...

Thank you for writing the 2017 Fryer Contest! Each year, more than 220 000 students from more than 60 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2017.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2017/2018 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
- Look at our free online courseware for senior high school students
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results